Calculation of Bellow Parallel Offset

Sanyi Zheng - SSCL

April 14, 1993

Abstract:

This note gives two approaches to design the bellows which are employed in GEM beam pipe system. One approach is to calculate the allowable parallel offset of bellows based on their parameters. The other is to design the suitable parameters of bellows based on the expected parallel offset.
Vacuum Group

Technical Note # GEM TN - 93 - 343  Date: 04-14-93

Title: CALCULATION OF BELLOW PARALLEL OFFSET  Pages: 9

Requested By: Gerry Chapman  Checked By: Jiaxiang Zhou

Originator: Sanyi Zheng  Approved By: Gerry Chapman

Abstracts:
This note gives two approaches to design the bellows which are employed in GEM beam pipe system. One approach is to calculate the allowable parallel offset of bellows based on their parameters. The other is to design the suitable parameters of bellows based on the expected parallel offset.

References:

Distributions:
1. The Parameter of the Bellow

\[ d = 80 \text{ mm} \]
\[ d \text{ is the outside diameter of cylindrical tangent of the bellow} \]

\[ w = 10 \text{ mm} \]
\[ w \text{ is the convolution depth} \]

\[ d_p = d + w = 80 + 10 = 90 \text{ mm} \]
\[ d_p \text{ is the mean diameter of bellows} \]

\[ N = 15 \]
\[ N \text{ is the total number of convolution in one bellows} \]

\[ C = 70 \text{ mm} \]
\[ C \text{ is the free length of bellows} \]

\[ L = 70 + 70 + 70 = 210 \text{ mm} \]
\[ L \text{ is the distance between outmost end of the convolution in a universal expansion} \]

-The Density of Materials:

1) Stainless Steel: \( \rho_{st} = 7.86 \text{ g/cm}^3 \)
2) Beryllium: \( \rho_b = 1.85 \text{ g/cm}^3 \)

2. The Loading on the Bellow in the Beam Pipe (Approximately)

The length of beam pipe section between the left end of bellows and next support point:

\[ \ell_L = 2" = 50.8 \text{ mm}, \varnothing83 \text{ OD}_L, \varnothing80 \text{ ID}_L \]
\[ \text{material: Beryllium} \]

The length of beam pipe section between the right end of bellows and next support point:

\[ \ell_R = 5000 \text{ mm}, \varnothing250 \text{ mm OD}_R, \varnothing246 \text{ mm ID}_R \]
\[ \text{material: Stainless Steel} \]

The weight of beam pipe section on the left side of bellows \( W_L \):

\[ W_L = V_L \rho_b \]
\[ \text{where } V_L \text{ is the volume of beam pipe} \]
The weight of beam pipes section on the right sides of bellows \( W_R \):

\[
W_R = \left( 2 \pi R_0^2 - \pi R_i^2 \right) \ell_R \cdot \rho_{s.s.}
\]

\[
= \frac{\pi}{4} \ell_R \cdot \rho_{s.s.} \left( OD_R^2 - ID_R^2 \right) = \frac{\pi}{4} \cdot 5000 \cdot 7.86 \cdot 10^{-3} \left( 205^2 - 246^2 \right)
\]

\[W_R = 61207 \text{ g}\]

The moment acting on the bellow, \( M \):

\[
M = \frac{1}{2} \ell_R \cdot W_R = \frac{5000}{2} \cdot 61207
\]

\[M = 1.55 \times 10^8 \text{ mm} \cdot \text{g}\]

3. The Parallel Offset of Bellows

Note: Please refer to Reference 1 for the nomenclature and the equation number in this section.

Refering to Fig. 2, it show the free-body design of bellows

\[
e_x = \frac{x}{2N} \quad (c-2)
\]

\[
e_y = \frac{Kd_p y}{2N \left( L - C \pm \frac{x}{2} \right)} \quad (c-4a \times b)
\]

\[F = f_w \cdot e_x \quad (c-8)
\]

\[V = \frac{f_w \cdot d_p \cdot e_y}{2 \cdot L} \quad (c-11)
\]

\[M = \frac{f_w \cdot d_p \cdot e_y}{4} \quad (c-9)
\]

also, for static equilibrium

\[M = F \cdot y + V \cdot (L \pm x) \quad (4)
\]
\[ f_w = 1.7 \frac{d_p \cdot E_b \cdot t^3 \cdot n}{W^3 \cdot C_t} \quad (c-31) \]

\[ t_p = t \left( \frac{d}{d_p} \right)^{\frac{1}{4}} = \left( \frac{1}{15} \right) \cdot 25.4 \cdot \left( \frac{80}{90} \right)^{\frac{1}{4}} \]

\[ t_p = 1.6 \text{ mm} \]

\( t \) is the thickness of the bellows wall (mm)

\[ q/(2W) = \frac{70/15}{2 \cdot 10} = 0.25 \]

\( q \) is the bellow pitch (mm)

For \( q/(2W) = 0.25, \ t_p = 1.6 \) from Fig C19 of Reference 1

\[ q/(2.2 \sqrt{d_p \cdot t_p}) = \frac{70/15}{2.2 \sqrt{90 \cdot 16}} = 0.177 \]

\( C_a = 1.4 \)

Modules of Elasticity: \( 2.11 \times 10^6 \) Kg/cm² for stainless steel

Therefore, from eq. (c-31)

\[ f_w = 1.7 \cdot \frac{90 \cdot 2.11 \cdot 10^6 \cdot 1.6 \cdot 1.01}{10^3 \cdot 1.4} = 5556 \text{ kg/mm} \]

within elastic range

\( f_i = f_w = f_w \)

it results,

\( f_i = 5556 \text{ kg/mm} \)

From eq. (c-9)

\[ e_y = \frac{4M}{f_w \cdot d_p} = \frac{4 \cdot 1.55 \cdot 10^8}{(5556 \cdot 1000) \cdot 90} \]

\[ e_y = 1.240 \text{ mm} \]

From eq. (c-11)
\[ V = \frac{f_w \cdot d_p \cdot e_x}{2L} = \frac{5556 \cdot 90 \cdot 1.24}{2 \cdot 210} \]

\[ V = 1476 \text{ kg} \]

By rewriting the equations

\[ e_x = \frac{x}{2N} \quad (c - 2) \]

\[ e_y = \frac{K \cdot d_p \cdot y}{2N \left( L \cdot (L + x) \right)} \quad (c - 4a \times b) \]

\[ F = f_w \cdot e_x \quad (c - 8) \]

\[ M = F \cdot y + V \cdot (L + x) \quad (1) \]

Substitute eq. (c - 2) into eq. (c - 8)

\[ F = f_w \cdot \frac{x}{2N} \quad (2) \]

From eq. (c - 4a & b) it results

\[ \pm x = 2 \left( \frac{Kd_p y}{2N e_y} - L + C \right) \quad (3) \]

**Case 1**

For the positive displacement on x direction (+x) and substitute eq (3) into eg (2), it results

\[ F = \frac{f_w}{N} \left( \frac{Kd_p y}{2Ne_y} \right) - L + C \quad (4) \]

Substitute of eq. (4) into eq. (1)

\[ M = \frac{f_w}{N} \left( \frac{Kd_p y}{2Ne_y} - L + C \right) y + V \left\{ L + 2 \left( \frac{Kd_p y}{2Ne_y} - L + C \right) \right\} \]

regrouping above equation, it results
\[
\left( \frac{f_w \cdot K \cdot d_p}{2 \cdot N^2 \cdot e_y} \right) y^2 + \left( (C - L) \frac{f_w}{N} + \frac{V \cdot K \cdot d_p}{N \cdot e_y} \right) y + (2VC - VL - M) = 0
\]

For \( M = 1.55 \times 10^5 \) Kg.mm
\( f_w = 5556 \) Kg/mm
\( K = 1.385 \)
\( d_p = 90 \) mm
\( N = 15 \)
\( e_y = 1.24 \) mm
\( C = 70 \) mm
\( L = 210 \) mm
\( V = 1476 \) Kg

The equation (5) will result

\[ 1241.41 y^2 - 42005y - 258320 = 0 \]

\[ y = \left\{ \begin{array}{l}
39.16 \\
-5.32
\end{array} \right\} \text{ mm} \]

\( y = 39.16 \text{ mm} \) will be taken as the parallel offset value in the case 1, positive displacement in \( x \) direction

**Case 2**

For the negative displacement on \( x \) direction (\(-x\)) the eq (3) will become

\[ x = 2 \left( \frac{Kd_p \cdot y}{2N \cdot e_y} \right) + L - C \]

(3')

Substitute eq. (3') into (2)

\[ F = \frac{f_w}{N} \left( L - \frac{K \cdot d_p \cdot y}{2N \cdot e_y} - C \right) \]

Then Substitute eq. (4') into eq. (1)
\[
\left( \frac{f_w \cdot K \cdot d_p}{2N^2 e_\gamma} \right) y^2 + \left( \frac{(C - L) f_w}{N} - \frac{V \cdot K \cdot d_p}{N \cdot e_\gamma} \right) y + (VL + M - 2VC) = 0
\]

The equation (5') will result

\[
1241.14 y^2 - 61707y + 258320 = 0
\]

\[
y = \begin{cases} 
49.3 \\
0.42 
\end{cases} \text{ mm}
\]

\[
y = 49.3 \text{ mm} \text{ will be taken as the parallel offset value in Case2, negative displacement in x direction}
\]

Note: the factor K is the factor establishing relationship between equivalent axial displacement per convolution due to lateral deflection and ratio \( L/2c \) (see Fig. C-1 in Reference 1)

\[
K = \frac{3L^2 - 3CL}{3L^2 - 6CL + 4C^2} = \frac{3 \cdot 210^2 - 3 \cdot 70 \cdot 210}{3 \cdot 210^2 - 6 \cdot 70 \cdot 210 + 4 \cdot 70^2}
\]

\[
K = 1.385
\]

4. Discussion and Conclusion

In case of positive in x. The parallel offset is 39.16 mm

In case of negative displacement in x the parallel offset is 49.3 mm

We take 39.16 mm as the parallel offset value to calculate the allowable parallel offset \( y_{allow} \).

If the safety factor is 1.5

\[
y_{allow} = \frac{y}{\text{safety factor}}
\]

\[
y_{allow} = \frac{39}{1.5} = 26 \text{ mm}
\]

Therefore, the allowable parallel offset of bellows under the certain loading which is estimated in this report is 26 mm.
5. Another Approach of Design Calculation

If the bellows movements is expected as follow:

Axial: 0.0; Lateral: 1.188 in (30 mm)
Angular: 0.0

and bellows ply n = 3, convolution N = 5

Bellows Data

I.D. = 3.23 in (82 mm); Material: SA 240 – T304
O.D. = 4.0 in (102 mm); Modules: E_s = 29.4x10^6 psi
Con. HT. W = 0.385 in (10 mm); Thickness t = 0.01 in (0.254 mm)
Pitch q = 0.551 in/con (14 mm/con)

Temperature 68°F (20°C)
Pressure 15.0 psig (760 Torr)

Constants

d_p = d + w = 3.675 in; (93.3 mm)
t_p = t (d/dp) 0.5 = 0.0095 in.; (0.24 mm)
q/(2.2 (d/p) 0.5) = 1.34 in (34 mm)

q/2w = 0.71
C_p = 0.55
C_f = 1.38
C_d = 2.40

Pressure Stresses

S_1' = \frac{PdE_c}{2[t_c E_c + ntE_v]}
= 15.0(3.31)(2.9) / 2[0.01(2.9) + 3(0.01)(2.9)]
= S_1' = 620 psi (0.46kg/mm^2)

S_2 = \left(\frac{Pd_p}{2nt}\right)(0.571 + 2w/q)^{-1}
= 15.0(3.675) / [2(3)(0.0095)][0.571 + 2(0.385 / 0.551)]^{-1}
= S_2 = 493 psi (0.35kg/mm^2)

S_3 = \frac{P_w}{(2nt_p)}
= 15.0(0.385) / [2(3)(0.0095)]
= S_3 = 101 psi (0.07kg/mm^2)
\[ S_4 = \left( \frac{PC_p}{2n} \right) \left( \frac{W}{t_p} \right)^2 \]
\[ = (15)(0.55) / (2(3))(0.385 / 0.0095)^3 \]
\[ S_4 = 2274 \text{ psi (1.60kg/mm}^2) \]

**Deflection Stresses**

\[ S_5 = E \cdot t_e / \left( 2w^3 C_r \right) \]
\[ = 28.30(10^6)(0.0095)^3(0.11) / [2(0.385)^3(1.38)] \]
\[ S_5 = 1768 \text{ psi (1.24kg/mm}^2) \]

\[ S_6 = 5E \cdot t_e / [3w^3 C_e] \]
\[ = 5(28.3(10^6))(0.0095)(0.11) / [3(0.385)^3(2.40)] \]
\[ S_6 = 137928 \text{ psi (96.97kg/mm}^2) \]

**Fatigue Life Time**

\[ N_c = \left[ \left( \frac{C T_r}{S_i - B} \right) \right]^a \]

Where \( a = 3.4, S_i = 0.7(S_3 + S_4) + (S_5 + S_6) \)
\[ C = 1.86(10^6), B = 54000 \]
\[ T_r = 1 \text{ at room temperature} \]

Therefore,
\[ N_c = [1186(10^6) / (141361 - 54000)]^{3.4} \]
\[ N_c = 32798 \text{ cycles} \]

**Spring Rate**

\[ F_w = 1.7d_pE \cdot t_e \cdot \frac{3 n}{w^3 C_r} \]
\[ = 0.17(3.675)(29.4(10^6))(0.0095)^3(3) / [(0.385)^3(1.382)] \]
\[ F_w = 5926 \]

Axial Spring Rate \[ S_x = F_w / 2N \]
\[ = 5926 / 2(5) \]
\[ S_x = 592 \text{ lb/in (9.13kg/mm)} \]
Lateral Spring Rate \( S_y = \frac{d_p^2 K (S/R)}{2L (C_L)} \)
\[ = \frac{(3.675)^2 (1.39)(592)}{(2(8.255)(5.50))} \]
\[ S_y = 122 \text{ lb / in (2.18 kg / mm)} \]

Angular Spring Rate \( S_\alpha = 0.004 S_L R (d_p)^2 \)
\[ = 0.004(592)(3.675)^2 \]
\[ S_\alpha = 32 \text{ in - lb / deg. (368.7 mm - kg / deg.)} \]

During the installation of beam pipe, if the loading on the right hand side of bellow is 6kg in \( y \) direction.

\[
\text{Lateral Deformation} = \frac{\text{loading}}{S_y} = \frac{6\text{kg}}{2.18\text{kg / mm}}
\]

Thus that gives us 2.75mm of lateral deformation.

If loading is 3kg, the lateral deformation will be 1.38mm.

6. List of figure

Figure 1 Beam Pipe Bellows

Figure 2 The loading diagram of bellows and definition of parallel offset of bellows.

7. Reference

Figure 1  Beam pipe bellows
Figure 2  The loading diagram of bellows and the definition of parallel offset of bellows