GEM TN-93-343

Calculation of Bellow Parallel Offset

Sanyi Zheng - SSCL

April 14, 1993

Abstract:

This note gives two approaches to design the bellows which are employed in GEM beam pipe system. One approach is to calculate the allowable parallel offset of bellows based on their parameters. The other is to design the suitable parameters of bellows based on the expected parallel offset.

Vacuum Group	
Technical Note # GEM TN - 93 - 343	Date: 04-14-93
Title: CALCULATION OF BELLOW PARALLEL OFFSET	Pages: 9
Requested By: Gerry Chapman	Checked By: Jiaxiang Zhou
Originator: Sanyi Zheng	Approved By: Gerry Chapman

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This note gives two approaches to design the bellows which are employed in GEM bean pipe system. One approach is to calculate the allowable parallel offset of bellows based on their parameters. The other is to design the suitable parameters of bellows based on the expected parallel offset.

References:

1) Standards of the Expansion Jiont Manufactures Association, Inc., 5th edition, 1980

Distributions:

CACULATION OF BELLOW PARALLEL OFFSET

Sanyi Zheng
Physics Research Division
EFD Vacuum Group
Dec. 92

1. The Parameter of the Bellow

d = 80 mm
d is the outside diameter of cylindrical tengent of the bellow

w = 10 mm
w is the convolution depth

$d_p = d + w = 80 + 10 = 90$ mm
d_p is the mean diameter of bellows

N = 15
N is the total number of convolution in one bellows

C = 70 mm
C is the free length of bellows

L = 70 + 70 + 70 = 210 mm
L is the distance between outmost end of the convolution in a universal expansion

-The Density of Materials:

1) Stainless Steel: $\rho_{s.s.} = 7.86\ g/cm^3$
2) Beryllium: $\rho_b = 1.85\ g/cm^3$

2. The Loading on the Bellow in the Beam Pipe (Approxiamately)

The length of beam pipe section between the left and of bellows and nest support point:

$\ell_L = 2" = 50.8mm$, Ø83 OD_L, Ø80 ID_L
material: Beryllium

The length of beam pipe section between the right end of bellows and next support point:

$\ell_R = 5000$ mm, Ø250 mm OD_R, Ø246 mm ID_R
material: Stainless Steel

The weight of beam pipe section on the left side of bellows W_L:

$W_L = V_L\ \rho_B$
where V_L is the volume of beam pipe

$$W_L = \left(\pi R_O^2 - \pi R_I^2\right) \cdot \ell_L \cdot \rho_B$$

$$= \frac{\pi}{4} \ell_L \cdot \rho_B \left(OD_L^2 - ID_L^2\right) = \frac{\pi}{4} \cdot 50.8 \cdot 1.85 \cdot 10^{-3} \cdot \left(83^2 - 80^2\right)$$

$$W_L = 36 \text{ g}$$

The weight of beam pipes section on the right sides of bellows W_R:

$$W_R = \left(\pi R_0^2 - \pi R_I^2\right) \ell_R \cdot \rho_{S.S.}$$

$$= \frac{\pi}{4} \ell_R \, \rho_{S.S.} \left(OD_R^2 - ID_R^2\right) = \frac{\pi}{4} \cdot 5000 \cdot 7.86 \cdot 10^{-3} \left(250^2 - 246^2\right)$$

$$W_R = 61207 \text{ g}$$

The moment acting on the bellow, M:

$$M \approx \frac{1}{2} \ell_R \cdot W_R = \frac{5000}{2} \cdot 61207$$

$$M = 1.55 \text{x} 10^8 \text{ mm} \cdot \text{g}$$

3. The Parallel Offset of Bellows

Note: Please refer to Reference 1 for the nomeclature and the equation number in this section

Refering to Fig. 2, it show the free-body design of bellows

$$e_x = \frac{x}{2N} \qquad (c-2)$$

$$e_y = \frac{K d_P \, y}{2N\left(L - C \pm x/2\right)} \qquad (c-4a \, x \, b)$$

$$F = f_W \cdot e_x \qquad (c-8)$$

$$V = \frac{f_W \cdot d_P \cdot e_y}{2\,L} \qquad (c-11)$$

$$M = \frac{f_W \cdot d_P \cdot e_y}{4} \qquad (c-9)$$

also, for static equilibrium

$$M = F \cdot y + V \cdot (L \pm x) \qquad (4)$$

$$f_{iu} = 1.7\frac{d_p \cdot E_b \cdot t_p^3\, n}{W^3\, C_f} \qquad (c-31)$$

$$t_p = t\left(\frac{d}{d_p}\right)^{\frac{1}{2}} = \left(\frac{1}{15}\right)\cdot 25.4\cdot\left(\frac{80}{90}\right)^{\frac{1}{2}}$$

$$t_p = 1.6\ \text{mm}$$

t is the thickness of the bellows wall (mm)

$$q/(2W) = \frac{70/15}{2\cdot 10} = 0.25$$

q is the bellow pitch (mm)

For $q/(2W) = 0.25$, $t_p = 1.6$ from Fig C19 of Reference 1

$$q/\left(2.2\sqrt{d_p t_p}\right) = \frac{70/15}{2.2\sqrt{90\cdot 16}} = 0.177$$

$C_f = 1.4$

Modules of Elasticity: 2.11 x 10^6 Kg/cm^2 for stainless steel

Therefore, from eq. $(c-31)$

$$f_{iu} = 1.7\cdot\frac{90\cdot 2.11\cdot 10^6\cdot 1.6\cdot 1\cdot 0.01}{10^3\cdot 1.4} = 5556\ \text{kg/mm}$$

within elastic range

$$f_i = f_w = f_{iu}$$

it results,

$$f_i = 5556\ \text{kg/mm}$$

From eq. $(c-9)$

$$e_y = \frac{4M}{f_w\cdot d_p} = \frac{4\cdot 1.55\cdot 10^8}{(5556\cdot 1000)\cdot 90}$$

$$e_y = 1.240\ \text{mm}$$

From eq. $(c-11)$

$$V = \frac{f_W \cdot d_P \cdot e_y}{2L} = \frac{5556 \cdot 90 \cdot 1.24}{2 \cdot 210}$$

$V = 1476$ kg

By rewriting the equations

$$e_x = \frac{x}{2N} \qquad (c-2)$$

$$e_y = \frac{K \cdot d_P \cdot y}{2N\left(L - C \pm \frac{x}{2}\right)} \qquad (c\text{-}4a \times b)$$

$$F = f_W \cdot e_X \qquad (c-8)$$

$$M = F \cdot y + V \cdot (L + x) \qquad (1)$$

Substitute eq. $(c-2)$ into eq. $(c-8)$

$$F = f_W \cdot \frac{x}{2N} \qquad (2)$$

From eq. $(c - 4a \ \& b)$ it results

$$\underline{+}x = 2\left\{\frac{Kd_P y}{2N\, e_y} - L + C\right\} \qquad (3)$$

Case 1

For the positive displacement on x direction (+x) and substitute eq (3) into eg (2), it results

$$F = \frac{f_W}{N}\left\{\frac{Kd_P\, y}{2Ne_y}\right\} - L + C \qquad (4)$$

Substitute of eq. (4) into eq. (1)

$$M = \frac{f_W}{N}\left\{\frac{Kd_P\, y}{2Ne_y} - L + C\right\}y + V\left\{L + 2\left(\frac{Kd_P\, y}{2Ne_y} - L + C\right)\right\}$$

regrouping above equation, it results

$$\left(\frac{f_W \cdot K \cdot d_P}{2 \cdot N^2 \cdot e_y}\right)y^2 + \left\{(C-L)\frac{f_W}{N} + \frac{V \cdot K \cdot d_P}{N \cdot e_y}\right\}y + (2VC - VL - M) = 0$$

(15)

For	$M = 1.55x10^5$	Kg.mm
	$f_w = 5556$	Kg/mm
	$K = 1.385$	
	$d_p = 90$	mm
	$N = 15$	
	$e_y = 1.24$	mm
	$C = 70$	mm
	$L = 210$	mm
	$V = 1476$	Kg

The equation (5) will result

$$1241.41\ y^2 - 42005y - 258320 = 0$$

$$y = \begin{cases} 39.16 \\ -5.32 \end{cases} \text{mm}$$

$y = 39.16$ mm will be taken as the parallel offset value in the case 1, positive displacment in x direction

Case 2

For the negative displacement on x direction $(-x)$ the eq (3) will become

$$x = 2\left(-\frac{Kd_P \cdot y}{2N \cdot e_y}\right) + L - C \qquad (3')$$

Substitute eq. $(3')$ into (2)

$$F = \frac{f_W}{N}\left(L - \frac{K \cdot d_P \cdot y}{2N \cdot e_y} - C\right)$$

Then Substitute eq. (4') into eq. (1)

$$\left(\frac{f_W \cdot K \cdot d_P}{2N^2 e_y}\right) y^2 + \left\{(C-L)\frac{f_W}{N} - \frac{V \cdot K \cdot d_P}{N \cdot e_y}\right\} y + (VL + M - 2VC) = 0 \qquad (5')$$

The equation $(5')$ will result

$$1241.14\, y^2 - 61707y + 258320 = 0$$

$$y = \begin{cases} 49.3 \\ 0.42 \end{cases} \text{mm}$$

$y = 49.3$ mm will be taken as the parallel offset value in Case2, negative displacement in x direction

Note: the factor K is the factor establishing relationship between equivalent axial displacement per convolution due to lateral deflection and radio $L/2c$ (see Fig. C-1 in Reference 1)

$$K = \frac{3L^2 - 3CL}{3L^2 - 6CL + 4C^2} = \frac{3 \cdot 210^2 - 3 \cdot 70 \cdot 210}{3 \cdot 210^2 - 6 \cdot 70 \cdot 210 + 4 \cdot 70^2}$$

$$K = 1.385$$

4. Discussion and Conclusion

In case of positive in x. The parallel offset is 39.16 mm

In case of negative displacement in x the parallel offset is 49.3 mm

We take 39.16 mm as the parallel offset value to calculate the allowable parallel offset y_{allow}.

If the safety factor is 1.5

$$y_{allow} = \frac{y}{\text{safety factor}}$$

$$y_{allow} = \frac{39}{1.5} = 26 \text{ mm}$$

Therefore, the allowable parallel offset of bellows under the certain loading which is estimated in this report is 26 mm.

5. Another Approach of Design Calculation

If the bellow movements is expected as follow:

Axial: 0.0; Lateral: 1.188 in (30 mm)
Angular: 0.0

and bellow ply n = 3, convelution N = 5

Bellows Data

I.D. = 3.23 in (82 mm);
O.D. = 4.0 in (102 mm);
Con. HT. W = 0.385 in (10 mm);
Pitch q = 0.551 in/con (14 mm/con)
Temperature 68°F (20°C)
Pressure 15.0 psig (760 Torr)

Material: SA 240 – T304
Modules: $E_b = 29.4x10^6$ psi
Thickness t = 0.01 in (0.254 mm)

Constants

$d_p = d + w = 3.675$ in; (93.3 mm)
$t_p = t\ (d/dp)\ 0.5 = 0.0095$ in.; (0.24 mm)
$q/(2.2\ (d_p t_p)^{0.5}) = 1.34$ in (34 mm)

$q/2w = 0.71$
$C_p = 0.55$ (Fig. C - 18)
$C_f = 1.38$ (Fig. C - 19)
$C_d = 2.40$ (Fig. C - 20)

Pressure Stresses

$$S_1' = PdE_c / 2[t_cE_c + ntE_b] \qquad \text{(C - 23b)}$$
$$= 15.0(3.31)(2.9) / 2[0.01(2.9) + 3(0.01)(2.9)]$$

$$S_1' = 620 \text{ psi } \left(0.46\text{kg} / \text{mm}^2\right)$$

$$S_2 = \left(Pd_p / 2nt\right)(0.571 + 2w / q)^{-1} \qquad \text{(C - 24)}$$
$$= 15.0(3.675) / [2(3)(0.0095)][0.571 + 2(0.385 / 0.551)]^{-1}$$

$$S_2 = 493 \text{ psi } \left(0.35\text{kg} / \text{mm}^2\right)$$

$$S_3 = P_w / \left(2nt_p\right) \qquad \text{(C - 25)}$$
$$= 15.0(0.385) / [2(3)(0.0095)]$$

$$S_3 = 101 \text{ psi } \left(0.07\text{kg} / \text{mm}^2\right)$$

$$S_4 = \left(P\,C_p / 2n\right)\left(W / t_p\right)^2 \qquad (C-26)$$
$$= (15)(0.55)/(2(3))(0.385/0.0095)^2$$

$$S_4 = 2274 \text{ psi } \left(1.60\text{kg}/\text{mm}^2\right)$$

Deflection Stresses

$$S_5 = E_b t_p^2 e / \left(2w^3 C_f\right) \qquad (C-27)$$
$$= 28.30\left(10^6\right)(0.0095)^2(0.11)/\left[2(0.385)^3(1.38)\right]$$

$$S_5 = 1768 \text{ psi } \left(1.24\text{kg}/\text{mm}^2\right)$$

$$S_6 = 5E_b t_p e / \left[3w^2 C_d\right] \qquad (C-28)$$
$$= 5\left(28.3\left(10^6\right)\right)(0.0095)(0.11)/\left[3(0.385)^2(2.40)\right]$$

$$S_6 = 137928 \text{ psi } \left(96.97\text{kg}/\text{mm}^2\right)$$

Fatigue Life Time

$$N_c = \left[(C\,T_f)/(S_t - B)\right]^a \qquad (C-29)$$

Where $a = 3.4$, $S_t = 0.7\left(S_3 + S_4\right) + \left(S_5 + S_6\right)$
$C = 1.86\left(10^6\right)$, $B = 54000$
$T_f = 1$ at room temperature

Therefore,
$$N_c = \left[1186\left(10^6\right)/(141361 - 54000)\right]^{3.4}$$

$$N_c = 32798 \text{ cycles}$$

Spring Rate

$$F_{iu} = 1.7 d_p E_b t_p^3 n / \left[w^3 C_f\right] \qquad (C-31)$$
$$= 1.7(3.675)\left(29.4\left(10^6\right)\right)(0.0095)^3(3)/\left[(0.385)^3(1.382)\right]$$

$$F_{iu} = 5926$$

Axial Spring Rate $\quad S_x = F_{iu}/2N$
$= 5926/2(5)$

$$S_x = 592 \text{ lb/in } (9.13\text{kg}/\text{mm})$$

Leteral Spring Rate $\left(S_y\right) = d_p{}^2K(S/R)/2L(C_L)$

$$= (3.675)^2(1.39)(592)/(2(8.255)(5.50))$$

$$S_y = 122 \text{ lb/in } (2.18\text{kg/mm})$$

Angular Spring Rate $S_A = 0.004\ S_L R\left(d_p\right)^2$

$$= 0.004(592)(3.675)^2$$

$$S_A = 32 \text{ in} - \text{lb/deg.}(368.7\text{mm} - \text{kg/deg.})$$

During the installation of beam pipe, if the loading on the right hand side of bellow is 6kg in y direction.

$$\text{Lateral Deformation} = \frac{\text{loading}}{S_y} = \frac{6\text{kg}}{2.18\text{kg/mm}}$$

Thus that gives us 2.75mm of lateral deformation

If loading is 3kg, the lateral deformation will be 1.38mm

6. List of figure

7. Reference

1. Standards of the Expansion Joint Manufactures Association, Inc., 5th edition, 1980

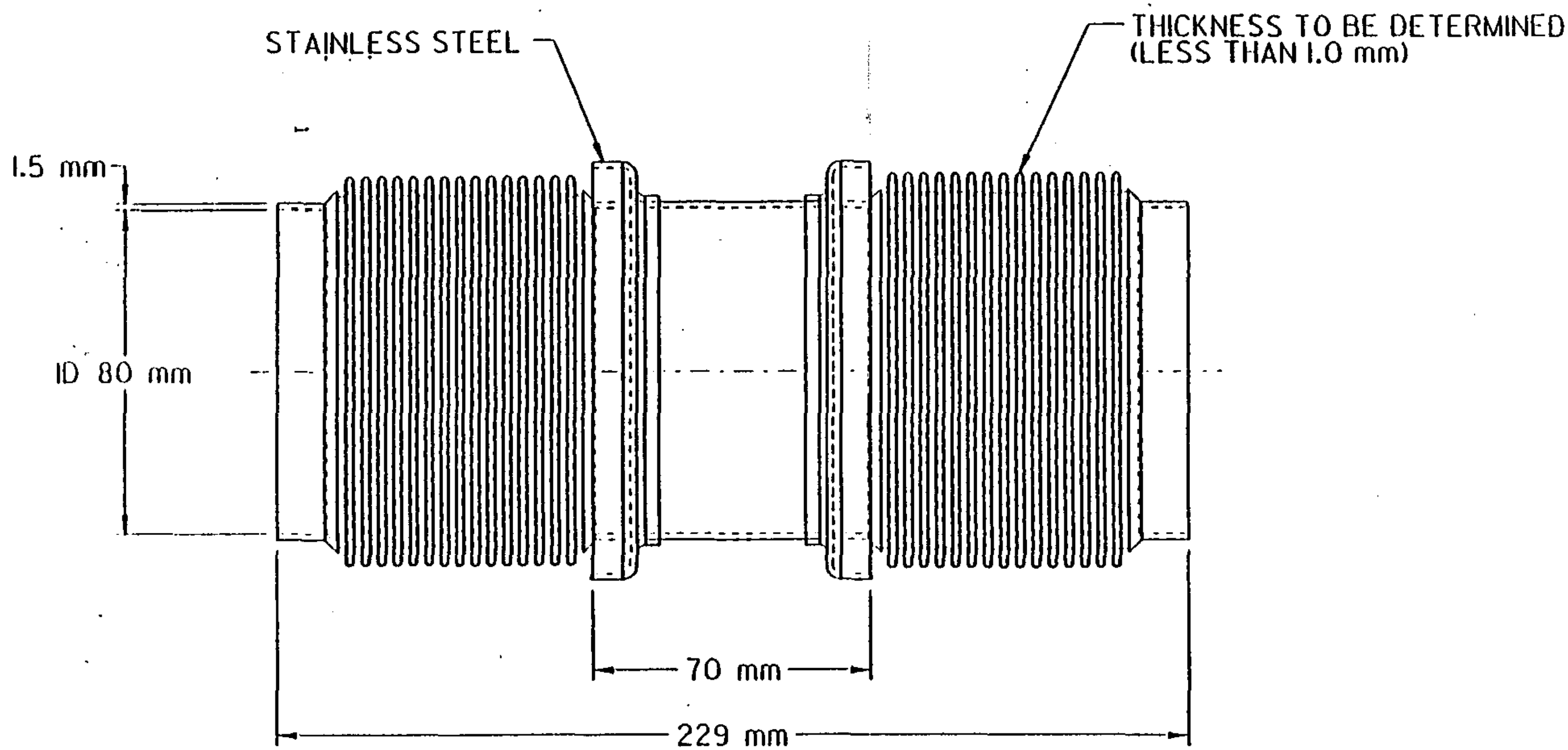

Figure 1 Beam pipe bellows

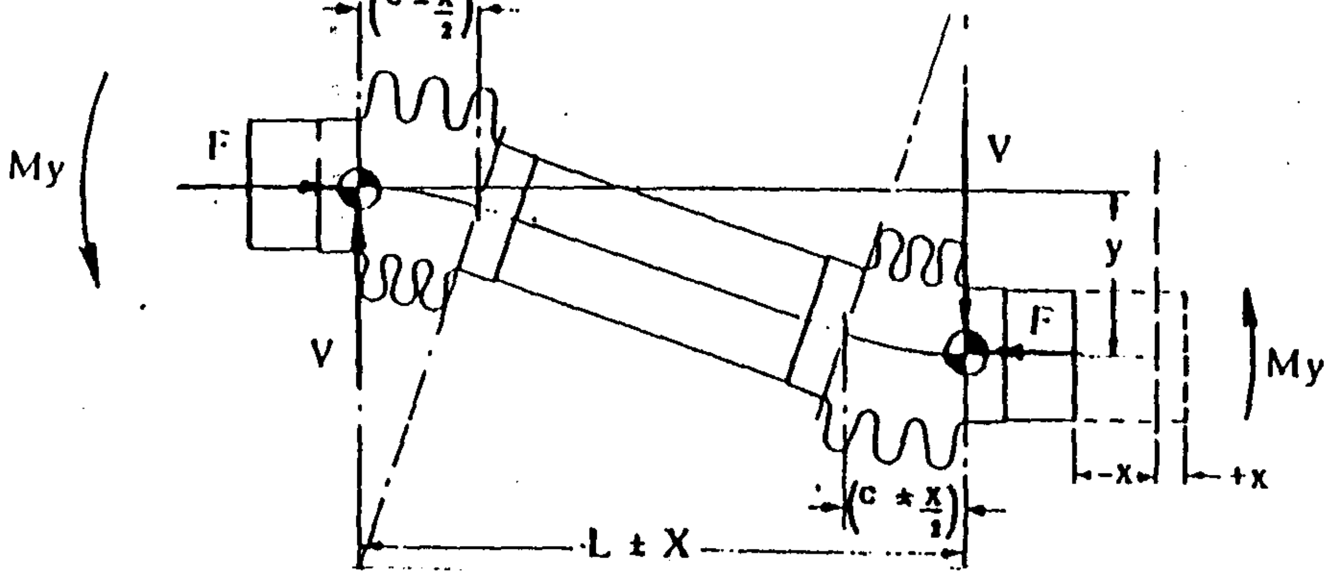

Figure 2 The loading diagram of bellows and the definition of parallel offset of bellows