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The Superconducting Super Collider

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Abstract

Radon activity may be a source of radiation exposure in the SSC main ring tunnel. Typical radon activity concentrations for rock and soil on site are calculated. The effects of mitigation by tunnel sealing, lining and ventilating are discussed.

I. Introduction

Radon is a naturally occurring noble gas (Z=86) all of whose isotopes are radioactive. Because of its chemical inertness it does not bond to the surface of material, in marked contrast to its heavy metal daughters. From a health physics point of view, the main hazard is the alpha radiation dose to the lungs. This dose is mainly due to direct radiation from inhaled dust particles on which the radon daughter nuclides' ions have become attached. Nero and Nazaroff[1] state: "Exposures of the lung to the decay products of $^{222}\text{Rn}$ and $^{220}\text{Rn}$ contribute approximately half of the average total effective dose equivalent of 2 mSv (0.2 rem) per year suffered by the general population from naturally occurring radionuclides. Most of the radon-daughter exposure occurs indoors, where substantial variability in time-averaged exposures to the daughters is observed, ranging from
rates more than a factor of 10 lower to a factor of 50 or more higher than aver
even among the general population. This variability arises partly from diffe
equilibrium factors, but is caused mainly by the difference in indoor radon
centration from one building to another. Furthermore, although the indoor re
concentration is definitely affected by the ventilation rate, the major cause of
variability in radon concentration from one building to another appears to
differences in the rate at which radon enters the indoor atmosphere from its
ious sources." The same should be true for tunnels. The radon concen-
in an unlined rock tunnel will govern the dose received by workers replacin
repairing equipment in the tunnel. Dagenais[2] quotes Department of En
Order 5480.1A, Chapter 11, p. 3, as listing a concentration guide for radon
in air under controlled conditions of 100 pCi/l (units: 1 Ci = 1 curie = 3.7×
Bq, where 1 Bq = 1 becquerel = 1 disintegration/second = 1 s⁻¹. 1 liter =
= 10⁻³ m³. Therefore, 100 pCi/l = 3.7 Bq/l = 3.7×10⁻³ Bq/m³, or 1 Bq m⁻
0.027027 ... pCi/l).

This report is based on an earlier note that did not take the specific prope:
of site materials into account [3]. It is the purpose of this report to estimate
likely radon concentrations in tunnels, with and without substantial ventila
In the SSC, the tunnel is not occupied under operating conditions; it is li:
that ventilation would be required only prior to and during occupation of
tunnel by work crews. Ventilation applied to the entire tunnel can be expen
Peterson and Théilacker[4], indicate that power expenditures of the orde
megawatts (MW) can be achieved in cooling and dehumidifying tunnel air
a continuous basis. Using outside air directly for ventilation purposes prod;
problems of condensation of moisture in the tunnel, with attendant problems with corrosion and increased likelihood of electrical breakdown. Therefore it is of interest to examine the case in which the radon concentration is allowed to build up to equilibrium levels in the absence of ventilation, and then examine the time constants associated with subsequent ventilation. The release rates will also be a concern, since they will govern dose to the general public at the site boundaries.

The porosity $\varepsilon$ of rock, soil or building materials is the ratio of void space volume due to pores to the total bulk volume of the material. This pore space is connected to a certain degree, so that radon in the pore space can diffuse through the material. The release of radon from a porous material is thus due to Ra alpha decay driving the recoiling Rn atom into a pore space, from whence it diffuses (or is forced by a pressure gradient) into the surrounding environment. The porosity of rocks at tunnel depth will be mostly secondary (due to fractures).

In what follows, the diffusion equation for radon is reviewed following the treatment by Nazaroff, Moed and Sextro[5] (NMS) and solved for the case of cylindrical geometry, which is taken to approximate the shape of a tunnel. Ventilation is then discussed in terms of the model of Dagenais. A brief discussion of the venting problem concludes the report.
II. Radon Diffusion

Notation (units):

C<sub>Rn</sub> = concentration (m<sup>-3</sup>) = number of radon atoms/volume

\( \lambda_{Rn} \) = decay constant of Rn = \( 2.0982 \times 10^{-6} \) s<sup>-1</sup> for \(^{222}\)Rn; for \(^{220}\)Rn, decay constant is \( 1.247 \times 10^{-2} \) s<sup>-1</sup>.

\( I_{Rn} = C_{Rn} \lambda_{Rn} \) = activity concentration of Rn (Bq m<sup>-3</sup>) in gas or pore sp.

\( f = \) emanation fraction, emanating power or emanation coefficient. \( f \) (0.005 - 0.40) for rock; a value of 0.025 will be used here as representative site rocks. For soils, \( f = (0.05 - 0.7) \) with a representative value of 0.1 for dry soil, or around 0.2 for representative soil moisture, with values of 0.4 -common as saturation is approached. The emanation coefficient is the fraction of the radon generated in the grains that enters the pore volume.

\( \rho_g = \) density of solid grains in rock or soil; \( \rho_g = (2.2 - 3.0) \times 10^3 \) kg m<sup>-3</sup> with a typical value for continental rock minerals of \( \rho_g = 2.65 \times 10^3 \) kg m<sup>-3</sup>. Densities of chalk, marl and clay are around \( 2.4 \times 10^3 \) kg m<sup>-3</sup> for site materials.

\( \varepsilon = \) porosity = (effective pore space volume)/(total volume). A typical value for silt with a sizeable clay fraction is 0.5. In reservoir rocks this may vary from 0.20 near surface to less than 0.05 at depth. The value of \( \varepsilon = 0.20 \) is probably a slight overestimate for the porosity of the site rocks.

\( A_{Ra} = \) radium activity concentration in bulk material (Bq kg<sup>-1</sup>). A reasonable average value for rocks on the site is \( A(^{226}Ra) = 22 \) Bq kg<sup>-1</sup>. See Table

\( D_0 = \) diffusion coefficient of radon in open air. \( D_0 = 1.2 \times 10^{-5} \) m<sup>2</sup> s<sup>-1</sup>. 


$D_e = \text{effective or interstitial diffusion coefficient in the pores of the material.}$

$D_e = (0.7 - 5) \times 10^{-6} \, \text{m}^2 \, \text{s}^{-1}$ for dry soil, with a typical value of $2 \times 10^{-6} \, \text{m}^2 \, \text{s}^{-1}$. For mud with high moisture content, $D_e$ is of the order of $10^{-10} \, \text{m}^2 \, \text{s}^{-1}$, essentially impervious to radon. Diffusion coefficients in crystalline material are several orders of magnitude smaller. The value $D_e = 4.7 \times 10^{-8} \, \text{m}^2 \, \text{s}^{-1}$ adopted here is essentially due to diffusion along fractures.

$D = \text{bulk diffusion coefficient} = \epsilon D_e \, (\text{m}^2 \, \text{s}^{-1})$

$J_{Ra} = \text{magnitude of the activity flux density} \, (\text{Bq} \, \text{m}^2 \, \text{s}^{-1})$

\begin{equation}
\vec{J}_{Ra} = I_{Ra} \vec{v}_{air} - D \nabla I_{Ra},
\end{equation}

where the first term on the right-hand side (RHS) is due to the motion of the air with velocity $\vec{v}_{air}$ and the second term is the diffusive flux density for the bulk material, or the geometric diffusive radon activity flux density:

\begin{equation}
\vec{J}_{Ra}^{dG} = - D \nabla I_{Ra}.
\end{equation}

The diffusion through the pores is given by the effective diffusion coefficient $D_e$, so

\begin{equation}
\vec{J}_{Ra}^d = - D_e \nabla I_{Ra};
\end{equation}

since $D = \epsilon D_e$

\begin{equation}
\vec{J}_{Ra}^{dG} = \epsilon \vec{J}_{Ra}^d
\end{equation}

$G = \text{volumetric generation rate of radon in pore space} \, (\text{Bq} \, \text{m}^{-3} \, \text{s}^{-1})$. 

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\[ G = f \rho_g A_{Ra} \lambda_{Ra} \frac{1}{\epsilon} \]

For \( f = 0.025, \rho_g = 2.4 \times 10^3 \text{ kg m}^{-3}, \epsilon = 0.2 \) and \( A_{Ra} = 22 \text{ Bq kg}^{-1}, \)
\[ G = 0.0111 \text{ Bq m}^{-3} \text{ s}^{-1}. \]

\[ \ell = (D_e/\lambda_{Ra})^{1/2} \text{ diffusion length for radon in material (m)}. \]

For example, if \( D_e = 2.1 \times 10^{-6} \text{ m}^2 \text{ s}^{-1}, \ell = 1.0 \text{ m}. \) For \( D_e = 4.7 \times 10^{-6} \text{ m}^2 \text{ s}^{-1}, \) the effective diffusion length in rocks is 0.15 m.

The local change with time in numbers of radon atoms is given by the rate of generation due to decay of the radium parents, the decay rate into daughter radon and the rate of diffusion of radon from elsewhere. Corresponding terms with apply for the numbers per unit volume, the radon concentrations. Thus, we could multiply through by \( \lambda_{Ra} \) to write the equation in terms of the activity concentration \( I_{Ra} \). The increase in activity concentration due to diffusion is

\[ \nabla \cdot \mathbf{J} = \int \int \int (\nabla \cdot \mathbf{J}) dV, \]

where the infinitesimal area vector over the closed surface spanning the volume is directed outward. The increase in the activity per volume is then

\[ -\nabla \cdot \mathbf{J}^d = D_e \nabla^2 I_{Ra}, \]

where \( D_e \) is taken as constant.

The total diffusion equation for the activity concentration in the pores is

\[ \partial_t I_{Ra} = D_e \nabla^2 I_{Ra} - \lambda_{Ra} I_{Ra} + G, \]
where the terms on the RHS correspond to the diffusion term, loss rate due to radon decay and generation rate due to radium decay and radon recoil into the pore space.

As an example, consider the case of “all outdoors” modeled as an infinite half space with typical soil values for the parameters. If ventilation of uncovered soil reduces the radon concentration at the surface to zero, then the steady-state equation for the radon activity concentration in the soil half space ($z$ positive down) is

$$\frac{G}{\lambda_{Rn}} = \left(\frac{D_e}{\lambda_{Rn}}\right) \frac{d^2}{dz^2} (I_{Rn}) - I_{Rn}. \tag{7}$$

The solution to the homogeneous equation

$$\frac{d^2}{dz^2} (I_{Rn}) - I_{Rn} = 0 \tag{8}$$

is $I_{Rn} = I_1 e^{-r}$ where $r = z/\ell$ and $\ell = (D_e/\lambda_{Rn})^{1/2}$ is the diffusion length. The particular (constant) solution is $I_{Rn,p} = G/\lambda_{Rn} = I_\infty$. In the example for $G$ given above, $I_\infty = 5280 \text{ Bq m}^{-3}$.

The solution that goes to zero at the surface is then

$$I_{Rn}(z) = I_\infty (1 - e^{-z/\ell}) \tag{9}$$

The bulk flux density at $z = 0$ in this case, following NMS, is:

$$J_{Rn}^{dG} = -\varepsilon D_e \frac{d}{dz} \left[I_{Rn}(z)\right]_{z=0} = -\frac{\varepsilon D_e}{\ell} I_\infty$$

$$J_{Rn}^{dG} = -\varepsilon \lambda_{Rn} \ell I_\infty = -(D_e \lambda_{Rn})^{1/2} \rho_f f A_{Ra} (1 - e) \tag{10}$$
where the minus sign means the flux is out of the surface (toward negative
NMS proposed typical values for the case of $^{222}\text{Rn}$ emanating from soil of $D_c$
$2 \times 10^{-6} \text{ m}^2 \text{s}^{-1}$, $\rho_g = 2.65 \times 10^3 \text{ kg} \text{ m}^{-3}$, $f = 0.2$, $A_{Ra} = 30 \text{ Bq kg}^{-1}$
$\varepsilon = 0.5$, which yields $J_{Ra}^{dG} = 0.016 \text{ Bq m}^{-2} \text{s}^{-1}$. The corresponding value
$I_\infty = 1.55 \times 10^4 \text{ Bq m}^{-3} = 419 \text{ pCi/l}$ at depth. The mean worldwide flux[4]
roughly $0.015 \text{ Bq m}^{-2} \text{s}^{-1}$. This value of $D_c$ corresponds to a diffusion length
soil of about a meter, which is also the $e^{-1}$ depth for this case.

The corresponding value for rock with $\ell = 0.15 \text{ m}$, $f = 0.025$, $\rho_g = 2.4 \text{ m}^{-3}$, $A_{Ra} = 22 \text{ Bq kg}^{-1}$ and $\varepsilon = 0.2$ is $J_{Ra}^{dG} = 3.33 \times 10^{-4} \text{ Bq m}^{-2} \text{s}^{-1}$.

The quantities above all depend on the moisture content. An analysis
the effect of moisture on radon generation and diffusion in pores is given in
Appendix. The net result is that the effects of moisture can be taken into accot
by using effective (primed) quantities to represent moist conditions and that
analysis goes through as before.

The isotopes $^{222}\text{Rn}$, $^{220}\text{Rn}$, $^{219}\text{Rn}$ and $^{224}\text{Rn}$ are decay products from
naturally occurring radioactive chains originating with $^{238}\text{U}$, $^{232}\text{Th}$, $^{235}\text{U}$, a
$^{232}\text{Th}$, respectively. The molar flux of $^{220}\text{Rn}$ is about 60 times less than the val
for $^{222}\text{Rn}$. Because of its longer half life (smaller decay constant) the diffusi
length of $^{222}\text{Rn}$ is about 80 times that of $^{220}\text{Rn}$. $^{222}\text{Rn}$ is therefore the isoc
that forms most of the radon concentration in enclosed spaces. The equatic
used thus far could be used separately for each radon isotope.

Because of the difficulties of specifying the dose due to the mixture of rad
isotopes[6] the notion of a “working level” (WL) was introduced; 1 WL = $\infty$
centration of radon daughter products that emit $1.3 \times 10^5$ MeV of $\alpha$ radiation in a liter of air. According to Dagenais [2], this concentration is about 100 pCi/ℓ of the mixture of naturally occurring radon isotopes.

We now focus our attention on a tunnel of radius $a$ in an infinite porous medium. In cylindrical coordinates,

$$\nabla^2 = \partial^2_R + \frac{1}{R} \partial_R + \frac{1}{R^2} \partial^2_\theta + \partial^2_z$$

where the abbreviated notation for partial derivatives (with respect to the subscript) has been employed. For cylindrical symmetry, $I_{\text{Ra}}$ depends only on $R$; the diffusion equation becomes

$$D_e \left( \frac{d^2}{dR^2} + \frac{1}{R} \frac{d}{dR} \right) I_{\text{Ra}} - \lambda_{\text{Ra}} I_{\text{Ra}} = -G. \quad (11)$$

Let $r = R/\ell$, so that $\ell \, d/dR = d/dr = D_r$ in terms of the diffusion length $\ell = (D_e/\lambda_{\text{Ra}})^{1/2}$. Let $F = I_{\text{Ra}}$ for the sake of simplicity; the diffusion equation is then

$$\left( D_r^2 + \frac{1}{r} D_r - 1 \right) F = -\frac{G}{\lambda_{\text{Ra}}} = \text{constant}. \quad (12)$$

The equation

$$z^2 D_z^2 w + z D_z w - (z^2 + \nu^2) w = 0 \quad (13)$$

has solutions $I_{\pm \nu}(z)$ and $K_\nu(z)$, the exponential-like Bessel functions [7]. If we rewrite (13) as

$$\left( D_z^2 + \frac{1}{z} - 1 + \frac{\nu^2}{z^2} \right) w = 0 \quad (14)$$
and note that $\nu = 0$, we see that the solution to the homogeneous equation $F$ is a linear combination of $I_0$ and $K_0$:

$$F = A I_0 + B K_0.$$  

$I_0$ and $K_0$ have the following asymptotic behavior:

$$I_\nu(z) \xrightarrow{z \to 0} \frac{(\frac{1}{2}z)^\nu}{\Gamma(\nu + 1)} \text{, so } I_0(z = 0) = 1$$

$$K_0(z) \xrightarrow{z \to \infty} 0$$

$$I_0(z) \xrightarrow{z \to \infty} \infty$$

$$K_0(z) \xrightarrow{z \to 0} - \ln(z)$$

so $K_0$ is irregular at the origin and regular at infinity, and the reverse is true for $I_0$.

Inside the tunnel, the volume is entirely pore space, so $\epsilon_1 = 1$ and the diffusion length is $\ell_1 = (D_0/\lambda_{Ra})^{-1/2} = 2.39$ m. Also, $A_{Ra} = 0$ so $G = 0$ and

$$\left(D_r^2 + \frac{1}{r}D_r - 1\right)F_i = 0 \text{ for } 0 \leq r \leq \frac{a}{\ell_1},$$

with interior solution $F_i = C_1 I_0(r)$.

Outside the tunnel in the rock, $\epsilon_2 = \epsilon$ and the outside diffusion length $\ell_2 = (D_\epsilon/\lambda_{Ra})^{-1/2}$. The source term does not vanish, so

$$\left(D_r^2 + \frac{1}{r}D_r - 1\right)F_0 = -\frac{G}{\lambda_{Ra}} \text{ for } r \geq a/\ell_2,$$
with homogeneous solution $F_{0H} = C_2 K_0(\tau)$ and particular solution $F_{0p} = G/\lambda_{Ra} = \text{constant. For } K_0(\tau) > 0, C_2 < 0 \text{ and the outside solution is }$

$$\quad F_0(\tau) = \frac{G}{\lambda_{Ra}} - |C_2|K_0(\tau). \quad (18)$$

With the requirement that the geometric radon activity concentration flux

$$\quad \mathcal{J}_{Ra}^{dG} = -D \nabla F = -D \left( \hat{R} \partial_R + \frac{\hat{\phi}}{R} \partial_\phi + \hat{z} \partial_z \right) F$$

$$\quad = -\hat{R}(D/\ell) [\partial_r F(\tau)] \quad , \quad (19)$$

be continuous at the wall, where $\tau = a/\ell$, and that $D = \epsilon D_e$ then

$$\quad (D_0/\ell_1)C_1 \partial_r [I_0(\tau)]_{r_1 = a/\ell_1} = -\epsilon(D_a/\ell_2)|C_2|\partial_r [K_0(\tau)]_{r_1 = a/\ell_2} \quad . \quad (20)$$

Note that

$$\quad D_r [I_0(\tau)] = I_1(\tau) \quad \text{and} \quad D_r [K_0(\tau)] = -K_1(\tau) \quad (21)$$

from [7].

It is the activity concentration in the pore space that is locally continuous across the boundary. As an example, consider diffusion through a sprinkler head, an impermeable membrane with holes in it.

Matching the activity concentrations and their fluxes at the tunnel wall gives two linear equations in two unknowns that can be used to find the coefficients $C_1$
and \( C_2 \). From matching the net flux Eq. (20) across the boundary where \( R = a \),

\[
\frac{C_1}{|C_2|} = \frac{K_1(a/\ell_2)}{I_1(a/\ell_1)} \cdot \frac{\varepsilon D_e/\ell_2}{D_0/\ell_1} = \frac{K_1(a/\ell_2)}{I_1(a/\ell_1)} \cdot \left( \frac{D_e}{D_0} \right)^{1/2}.
\]

From matching the activity concentrations at \( R = a \),

\[
C_1 I_0(a/\ell_1) = \left[ \frac{G}{\lambda_{Ra}} - |C_2|K_0(a/\ell_2) \right]
\]

or

\[
I_0(a/\ell_1) + \frac{I_1(a/\ell_1)}{K_1(a/\ell_2)} \cdot \frac{(D_0/D_e)^{1/2}}{\varepsilon} \cdot \frac{K_0(a/\ell_2)}{\varepsilon} = \frac{1}{C_1} \left( \frac{G}{\lambda_{Ra}} \right)
\]

so

\[
\frac{C_1}{(G/\lambda_{Ra})} = \frac{\varepsilon}{\varepsilon I_0(a/\ell_1) + K_0(a/\ell_2) \left[ \frac{I_1(a/\ell_1)}{K_1(a/\ell_2)} \cdot (D_0/D_e)^{1/2} \right]}.
\]

The concentration of radon at the center of the tunnel \( I_{Ra}(r = 0) \) which is given in terms of \( I_\infty = G/\lambda_{Ra} \) in Tables 2 and 3, for tunnel radii \( a = 1.5, 1.8, 2.1, 2.4 \) m around 5–8 ft. For a diffusion length of about a meter \( D_e = \ell^2 \lambda_{Ra} = 2.1 \times 10^{-6} \) m\(^2\) s\(^{-1}\), a value typical of soil or extremely porous rock. At the other extreme, NN[1] quote a value of \( \ell = 0.15 \) m for red brick corresponding to a value of \( D_e = 4.7 \times 10^{-8} \) m\(^2\) s\(^{-1}\). Some concretes have similar values[1]. Table 2 contains values of \( C_1/I_\infty \) for \( \ell = 0.15 \) m. The value of \( \varepsilon = 0 \) (typical of chalk) is used in these calculations.

The quantity that depends on the details of the radionuclide distribution in the material is the emanation coefficient \( f \). Diffusion coefficients for \( ^{40} \)Ar for scoria–basalt and some other rocks are found to be in the range \( 10^{-31} - 10^{-69} \) m\(^2\) s\(^{-1}\).
the corresponding radon diffusion lengths are in the range $10^{-13} - 10^{-32}$ m. Therefore, radon diffusion through crystalline material can be neglected. Because the precursors of radium cause radiation damage in the crystal (halos) and because radionuclides are commonly found in accessory minerals (e.g., zircon) or at grain boundaries, radon can readily escape from the mineral grains into the pore space, and diffuse from there. The large values of $f$ for soil (around 0.3–0.4) relative to massive crystalline rock (around 0.02 or less) are due to this.

The effective diffusion coefficients $D_e$ for crystalline rock are very small, even less than the $D_e = 10^{-10}$ m$^2$ s$^{-1}$ characteristic of saturated soil or rock. Most of the diffusion in granite and massive carbonates occurs along fractures in the rock. This process is much more difficult to characterize than the more uniform diffusion in well sorted terrigenous clastics and soils, for which $D_e$ has values similar to that quoted earlier. Also, the process of pressurized flow is much more important in the case of media with a linear fracture density of more than a few centimeters, although it is not expected to vary much in rocks on site at tunnel depth.

To summarize this section, it is clear that radon activity can range from negligible values in unfractured crystalline rock to significant exposure levels in dry porous material. The effect of concrete slab shielding [1] shows that a 10 cm thick slab can cut the transmitted radon flux to 2–4% of the unshielded case. Even with a 1 cm gap for every meter of slab, the flux penetration rate is still calculated to be only 25% of maximum [1]. The effect of lining and/or sealing the tunnel can be quite significant in reducing the radon activity, subject to certain conditions on performance. Franklin, Bates, Holub and their coworkers have
done considerable work on the measurement and mitigation of radon in mine. Their conclusions may be summarized as follows: two-coat sealing of tun with e.g., latex, can reduce influx on the order of 50–75%, however, any cr. or gaps may episodically release more concentrated radon from pore reserv. Sealed-off underpressured bulkheads are used as radon barriers by the mi. industry. Shotcrete (gunite) as usually used is so porous as to be a neglig hindrance to radon diffusion.

Marked effects due to the superposed velocity flow field of the air are portant in radon levels in houses and mines[1, 5, 8]. Pressures of only a few (N m⁻²) can produce large differences in radon concentrations within struct by bringing in radon from reservoirs in the vicinity. Overpressuring struct: has a marked effect in reducing radon inflow rate. In regions of intense frac ing, this type of radon source may be dominant and extensive grouting may useful in reducing the inflow rate. In cases where the problem is acute eno: local bulkheading and pumps must be installed to control this effect. In the S fracturing significant enough to be of structural concern or allowing signific water inflow would cause the affected region to be fully lined with reinfor concrete. Sections in the site marl and clay units are slated to be so lined in case. This would also serve as an effective radon barrier (see above).
III. Fluxes and Concentrations

The diffusive flux density of radon activity of bulk material was found to be

$$\mathbf{J} = -\epsilon D_s \nabla I_{\text{Ra}},$$  \hspace{1cm} (24)

with peak values at depth of

$$J = \epsilon \ell \lambda_{\text{Ra}} I_\infty = \epsilon (D_s \lambda_{\text{Ra}})^{1/2} I_\infty$$  \hspace{1cm} (25)

where

$$I_\infty = G/\lambda_{\text{Ra}} = f \rho_g \left( \frac{1 - \epsilon}{\epsilon} \right) A_{\text{Ra}}$$  \hspace{1cm} (26)

is the limiting radon activity per volume at depth. In the previous section we found typical values for soil of $J = 0.016 \text{ Bq m}^{-2} \text{ s}^{-1}$ and $I_\infty = 1.55 \times 10^4 \text{ Bq m}^{-3} = 419 \text{ pCi/\ell}$. For rock with $\epsilon = 0.20$, $f = 0.025$, $\rho_g = 2.4 \times 10^3 \text{ kg m}^{-3}$, $A_{\text{Ra}} = 22 \text{ Bq kg}^{-1}$, and $\ell = 0.15 \text{ m}$, then $I_\infty = 5.28 \times 10^3 \text{ Bq m}^{-3} = 143 \text{ pCi/\ell}$ and $J = 3.33 \times 10^{-4} \text{ Bq m}^{-2} \text{ s}^{-1}$.

Consider a simple model of cylindrical tunnel of radius $R$ and length $z$, with a constant concentration of radon at equilibrium between that supplied by the flux of $I_{\text{Ra}}$ through the wall and the rate of loss of $I_{\text{Ra}}$ via decay ($I_{\text{Ra}} \lambda_{\text{Ra}}$). Assuming that the flux into the cylindrical cavity is given by $\mathbf{J}_{\text{wall}} = -\hat{R}J$ (Fig. 1) and the outward area infinitesimal $d\mathbf{a} = Rz d\phi$, then the activity introduced into the tunnel per time will be $-\int \int \mathbf{J}_{\text{wall}} \cdot d\mathbf{a} = 2\pi z RJ$. The rate of change of
activity per volume with time is then

\[ \frac{2 \pi z R}{\pi R^2 z} = \frac{2J}{R^2}. \]

The equilibrium activity concentration of radon is then \( I_{Ra} = 2J/(R\lambda_{Ra}) \). An example, for a tunnel of radius 1.524 m (5 ft) and a flux of 0.01524 Bq m\(^{-2}\) (about the world average for soil) then \( I_{Ra} = 0.952 \times 10^4 \text{ Bq m}^{-3} = 257 \text{ pCi} \), about two and a half times the established WL. On the other, if a value \( J = 3.33 \times 10^{-4} \text{ Bq m}^2 \text{ s}^{-1} \) found for representative site rocks at tunnel de: is used, then \( I_{Ra} = 208 \text{ Bq m}^{-3} = 5.62 \text{ pCi/ℓ}^{-1} \), about an eighteenth of a V. Given this result and the fact that marl and clay sections of the tunnel will lined leads to the conclusion that radon buildup will not be a problem at the in Texas.

For a variety of reasons, including the possibility of radon buildup, it wo­ be desirable to ventilate at least a section of the tunnel during its occupar by people working on equipment stored within the tunnel. Using the model activity flux above, with an activity concentration independent of position wit: the tunnel, the expression for \( I_{Ra} \) is

\[ D_t I_{Ra} = S - \lambda_{Ra} I_{Ra} - \lambda_v I_{Ra}, \]

where the activity source term \( S = 2J/R \) and \( \lambda_v \) is the effective “decay” coe: cient for the removal of radon by ventilation.

As an example, if half the radon in the tunnel were removed by ventilation a time \( T_{v1/2} = 1 \text{ h} = 3600 \text{ s} \), then \( \lambda_v = (\ln 2)/T_{v1/2} = 1.925 \times 10^{-4} \text{ s}^{-1} \), which i
factor of a hundred larger than \( \lambda_{\text{Ra}} \). Therefore ventilation would dominate decay as a removal mechanism for the radon itself. Equations similar to (28) could be written for each of the daughter products in order to compute the working levels, but because of their much shorter half lives, ventilation is much less effective in reducing the activity concentrations once they are produced.

The solution to the homogeneous equation corresponding to (28)

\[
D_t I_{\text{Ra}} + (\lambda_{\text{Ra}} + \lambda_{\ast}) I_{\text{Ra}} = 0
\]  

(29)

is \( I_{\text{Ra}} = I_0 e^{-(\lambda_{\text{Ra}} + \lambda_{\ast})t} \) and a particular solution to (28) is \( I_1 = S/(\lambda_{\text{Ra}} + \lambda_{\ast}) \) so that as \( t \to \infty, I_{\text{Ra}} \to I_1 \). At \( t = 0, I_{\text{Ra}} = I_{\text{max}} = I_1 + I_0 \), the value of the activity concentration before ventilation starts. Thus, \( I_0 = I_{\text{max}} - I_1 \). At equilibrium, \( D_t I_{\text{Ra}} = 0 \) when \( I_{\text{Ra}} = I_1 \). The time to reach any particular activity concentration can be found by inverting the solution

\[
\exp \left[ (-t)(\lambda_{\text{Ra}} + \lambda_{\ast}) \right] = \frac{I_{\text{Ra}} - I_1}{I_0} = \frac{I_{\text{Ra}} - I_1}{I_{\text{max}} - I_1} \]  

(30)

or

\[
t = \frac{1}{(\lambda_{\text{Ra}} + \lambda_{\ast})} \ln \left( \frac{I_{\text{max}} - I_1}{I_{\text{Ra}} - I_1} \right). \]  

(31)

For example, to dilute a given concentration by a factor of two requires (with mixing) a complete "room change" of air, so that with thorough mixing the time for inflow of an amount of air equal to the affected tunnel volume is \( T_{\text{v,1/2}} = \ln 2/\lambda_{\ast} \). For ventilation rates of a room change per hour or less and \( I_1 << I_{\text{max}} \), this would be the approximate time given by (31). The airspeed is given by
the volume to area ratio divided by $T_{v1/2}$: $v = V/(AT_{v1/2})$. Because of the effect of wall friction, actual air speeds will be down to about 2/3 to 1/2 the value. For the tunnel with $z = 4.0$ km and $T_{v1/2} = 3600$ s, this estimate gives $v = z/2T_{v1/2} = 0.55$ m/s, a desirable plume speed.

For $J = 0.01524$ Bq m$^{-2}$s$^{-1}$, $R = 1.524$ m, $S = 2J/R = 0.02$ Bq m$^{-3}$s$^{-1}$; $\lambda_{v} + \lambda_{Rn} = 1.946 \times 10^{-4}$ s$^{-1}$, $I_{1} = S/(\lambda_{Rn} + \lambda_{v}) = 103$ Bq m$^{-3}$ and $I_{max} = 95$ Bq m$^{-3}$ (257 pCi/$\ell$). The time to reduce this activity concentration to $I_{Rn} = 3700$ Bq m$^{-3}$ (100 pCi/$\ell$) is then

$$t = 5139 \text{ s} \ln(9417/3597) = 4946 \text{ s} = 1.37 \text{ h}.$$ 

Therefore, even if the tunnel had a significant radon activity concentration, ventilation for a room change and half would be sufficient to meet exposure standards.

In order to examine the fenceline concentrations of the expelled radon, consider the (over) simplified model where the tunnel air is expelled through a stack to form a plume which is the frustum of a cone, with a radius $R$ near the stack, the same as the tunnel radius (1.524 m) and the radius at the boundary of the service sector (fenceline) the same as the height of the stack, here taken as 15.24 (Fig. 2).

The volume of a unit thickness at each end of the cone will be proportional to the areas. Even disregarding the decay of the radon flux as the plume passes through the canonical outline, the concentration of particles moving at constant (wind) speed will be decreased by the area ratio of the top to the bottom of the cone, here $10^{-2}$. The activity concentration $I_{Rn}$ in the air expelled by the tunnel.
stack will be less than half the corresponding value in the tunnel at any time \( t \). Reducing this by a further factor of 100 converts a level of 2 WL in the tunnel to \( 10^{-2} \) WL or 1 pCi/l outside the fenceline. For comparison, the average concentrations in outdoor air are in the range (0.1–0.5) pCi/l and show wide daily variability [8]. Using the 0.0562 WL found for the best estimate indicates that the fenceline value of 0.028 pCi/l is completely negligible, and indistinguishable from the noise of the background. More sophisticated (Gaussian) plume models would show considerably greater reduction. Even with the close boundaries of the sector service areas, the released radon activity (an episodic event) seems a negligible hazard.
Acknowledgements

The author thanks Tony Nero of the Indoor Environment Program at L1 for help, encouragement and references. Helpful conversations with Al Tann and Jim Otten of the USGS and Robert Holub and John Franklin of the U. Bureau of Mines are also acknowledged. Harold Wollenberg of the LBL Ear Sciences Division supplied considerable information on the natural occurrence of radioactivity in various source rocks. Tim Toohig of the SSC supplied motivation for this calculation, and helped criticize the results. Al Tanner supplied references, a computer program to calculate levels, and helpful criticisms of the earlier note.
References


### Table 1

Porosities and Radium Source Concentrations

<table>
<thead>
<tr>
<th>Material:</th>
<th>Austin Chalk</th>
<th>Ozan Fm (Taylor Marl)</th>
<th>Eagle Ford Clays</th>
</tr>
</thead>
<tbody>
<tr>
<td>Porosities:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>0.16</td>
<td>0.21</td>
<td>0.20</td>
</tr>
<tr>
<td>$\epsilon_u$</td>
<td>0.12</td>
<td>0.16</td>
<td>0.15</td>
</tr>
<tr>
<td>$\epsilon_a$</td>
<td>0.04</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>$\epsilon_a + 0.25\epsilon_u$</td>
<td>0.07</td>
<td>0.09</td>
<td>0.0875</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Ra-226 (Bq/kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>average</td>
</tr>
<tr>
<td>range</td>
</tr>
<tr>
<td>no. samples</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>average density (Mg/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>dry</td>
</tr>
<tr>
<td>grain</td>
</tr>
</tbody>
</table>
Radon Concentrations in Center of Tunnel

Table 2

Parameters: \( l_1 = 2.4 \) m, \( l_2 = 1.0 \) m, \( D_e = 2.1 \times 10^{-8} \) m\(^2\) s\(^{-1}\).

<table>
<thead>
<tr>
<th>( a )</th>
<th>( a/l_1 )</th>
<th>( a/l_2 )</th>
<th>( C_1/I_\infty (10^{-2}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>0.625</td>
<td>1.5</td>
<td>3.2814148</td>
</tr>
<tr>
<td>1.8</td>
<td>0.75</td>
<td>1.8</td>
<td>2.7026333</td>
</tr>
<tr>
<td>2.1</td>
<td>0.875</td>
<td>2.1</td>
<td>2.2679848</td>
</tr>
<tr>
<td>2.4</td>
<td>1.000</td>
<td>2.4</td>
<td>1.9314190</td>
</tr>
</tbody>
</table>
Table 3

Parameters: \( \ell_1 = 2.4 \text{ m}, \ell_2' = 0.14 \text{ m}, D_e' = 4.2 \times 10^{-8} \text{ m}^2 \text{s}^{-1} \).

<table>
<thead>
<tr>
<th>( a )</th>
<th>( a/\ell_2' )</th>
<th>( C_1/I_\infty(10^{-4}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>10</td>
<td>1.0483964</td>
</tr>
<tr>
<td>1.8</td>
<td>12</td>
<td>0.8490810</td>
</tr>
<tr>
<td>2.1</td>
<td>14</td>
<td>0.7061545</td>
</tr>
<tr>
<td>2.4</td>
<td>16</td>
<td>0.5981627</td>
</tr>
</tbody>
</table>
Figure 1:

Figure 2:
APPENDIX 1

Effects of Moisture and Uncertainties in the Diffusion and Emanation Coefficients

The values of the effective diffusion coefficient $D_e = 4.7 \times 10^{-8} \text{ m}^2 \text{ s}^{-1}$ and the emanation coefficient $f = 0.025$ were not measured for site materials. These values were inferred as reasonable based upon the values tabulated for NMS for as similar materials as data is available. Both these parameters are very sensitive to the presence and type of rock fractures present in real site materials. For unfractured crystalline materials both these numbers would be two or three orders of magnitude smaller, and for powdered materials they would be at least an order of magnitude larger. Finally, both these parameters are affected by the presence of moisture; $D_e$ might be decreased and $f$ increased by as much as an order of magnitude as the rock approaches saturation. Because of these interrelations, it is convenient to discuss some of the finer points separately in this appendix.

It is difficult to measure count rates or flow rates that are small enough to represent in situ conditions for site materials. My estimate of the probable range of these quantities is a factor of four. Because these quantities are related to each other and to the moisture content, their definitions may be slightly ambiguous (see below).

The total pore space may be apportioned into the fractions occupied by air and water, respectively, as
\[ \epsilon = \epsilon_a + \epsilon_w \]  

for \( \epsilon_a \) the fraction of the material volume occupied by air in the pores and \( \epsilon_w \) corresponding fraction occupied by water. Table 1 shows representative values of these quantities for site materials.

The diffusion equation becomes

\[
\frac{1}{\epsilon} \partial_t (I_a \epsilon_a + I_w \epsilon_w) = D'_e \nabla^2 I_a - \frac{1}{\epsilon} \lambda_{Rn} (I_a \epsilon_a + I_w \epsilon_w) + G'
\]

with activity concentrations \( I_a \) and \( I_w \) in air and water, respectively. The quantities are related by \( I_w = K I_a \) (Henry's law) where \( K \) is the solubility coefficient for radon in water; for \( T = 20^\circ C \), approximately the site groundwater temperature, \( K \approx 0.25 \). Using the proportionality of \( I_w \) and \( I_a \), the diffusion equation becomes

\[
D'_e \nabla^2 I_a - \frac{1}{\epsilon} \lambda_{Rn} (\epsilon_a + K \epsilon_w) I_a + G' = 0
\]

under equilibrium conditions \( (\partial_t I_a, I_w = 0) \). A little creative rearrangement yie

\[
\left[ \frac{\epsilon D'_e}{(\epsilon_a + K \epsilon_w) \lambda_{Rn}} \right] \nabla^2 I_a - I_a + \frac{\epsilon G'}{\lambda_{Rn} (\epsilon_a + K \epsilon_w)} = 0
\]

with

\[
\xi^2 = \frac{\epsilon D'_e}{(\epsilon_a + K \epsilon_w) \lambda_{Rn}} = \frac{D'_e}{\lambda'_{Rn}},
\]

where

\[
\lambda'_{Rn} = \frac{\lambda_{Rn} (\epsilon_a + K \epsilon_w)}{\epsilon}
\]
giving

\[(\ell')^2 \nabla^2 I_a - I_a + \frac{G'}{\lambda'} = 0 \quad (A.7)\]

as the dimensionless form of the diffusion equation, of the same form as Eq. (6) with primed quantities replacing the unprimed quantities. Thus, if all the ratios are viewed as equivalent, the analysis in the main text still applies provided that \(\ell\) is identified as \(\ell'\), \(D_e\) as \(D'_e\), and \(I_\infty = G/\lambda\) as \(I'_\infty = G'/\lambda'\).

Here \(G'\) is considered to be the rate at which radon generated in the volume fraction occupied by the rock material is transferred to the air in the pores. Recall that \(G = f\lambda_{Ra} \rho_g A_{Ra}(1 - \epsilon)/\epsilon\) for the dry case; the fraction of those decaying transferred to the pore space \(f\), decay rate \(\lambda_{Ra}\) and rate of volume generation of radon \(\rho_g A_{Ra}\) times the ratio of the volume of solid mass to the volume of air. Thus,

\[G' = f' \rho_g A_{Ra} \lambda_{Ra} \left(\frac{1 - \epsilon}{\epsilon_a}\right) \quad (A.8)\]

where \(f'\) is the transfer coefficient from the solid grains to the air in the pore space. More radon recoils are trapped in the system as compared to the case when moisture is not present, due to stopping some of the recoiling radon in the pore space water rather than it ending up in an adjacent grain. Increasing the moisture content from zero to about 75% of saturation increases the emanation coefficient by a factor of four or five.

The generation rate for the dry case is

\[G = f \rho_g A_{Ra} \lambda_{Ra} \left(\frac{1 - \epsilon}{\epsilon}\right)\]
when the pore space is entirely occupied by air. Dividing this into (A.8) multiplying through by $\lambda'/\lambda'$ yields

$$I'_\infty = \frac{G'}{\lambda'} = \left( \frac{G}{\lambda_{\text{Ra}}} \right) \left( \frac{f'}{f} \right) \left( \frac{\epsilon}{\epsilon_a} \right) \left( \frac{\epsilon}{\epsilon_a + K \epsilon_w} \right)$$

as the particular solution to (A.7) for $I_a$ evaluated in the rock material at $h$ distance from the boundary. As an example, suppose $\epsilon_a = 0.05$, $\epsilon_w = 0$ $K = 0.25$, and $f'/f = 4$; then

$$I'_\infty = I_\infty (4) (4) \left( \frac{0.2}{0.0875} \right) = 36.6 I_\infty .$$

The activity concentration in the pore space AIR has been increased by a sizable factor. The $\epsilon/\epsilon_a$ increase in activity concentration simply corresponds to the decrease in the air in the pores. The $f'/f$ increase is due to the increase in the amount of radon trapped in the pore space. The factor $\epsilon/(\epsilon_a + K \epsilon_w)$ corresponds to the assumption that the radon is transferred directly from water to air and then diffuses through the air across the boundary. As the rock approaches saturation, this is no longer the case: increasing amounts of radon are injected directly across the boundary from the water. Formally, $\epsilon/\epsilon_a$ also becomes singular at saturation. A suitable redefinition of the pore space boundary interface could take care of both problems, but it is probably as well not to use this relation for saturation conditions. In any case, the site materials at tunnel depth are not saturated.

Since $K < 1$, the factor $\epsilon/(\epsilon_a + K \epsilon_w)$ increases monotonically from unity $\epsilon_w = 0$ (dry conditions) to $1/K$ at $\epsilon_w = \epsilon$ (saturation).
The corresponding flux

\[ J_{Rn}^{dG} = \varepsilon_a D'_e (\nabla I_a)_{\text{Boundary}} \quad (A.10) \]

is the physical quantity of interest. For the example of the half-space (p6) with

\[ I_{Rn}(z) = I_\infty \left(1 - e - \frac{z}{\ell'} \right) \quad (A.11) \]

\[ J_{Rn}^{dG} = -\varepsilon_a D'_e \frac{d}{dz} [I_{Rn}(z)]_{z=0} = -\frac{\varepsilon_a D'_e}{\ell'} I_\infty \]

\[ = -\varepsilon_a \left[ \frac{(\varepsilon_a + K \varepsilon_\omega)}{\varepsilon} \frac{\lambda_{Rn} D'_e / e}{D'_e} \right]^{1/2} f' \rho_d A_{Rn} \frac{(1 - e)\varepsilon}{\varepsilon_a (\varepsilon_a + K \varepsilon_\omega)} \]

\[ J_{Rn}^{dG} = -(D'_e \lambda_{Rn})^{1/2} \rho_d A_{Rn} f'(1 - e) \left(\frac{\varepsilon}{\varepsilon_a + K \varepsilon_\omega}\right)^{1/2} \quad (A.12) \]

which differs from (10) by replacement of diffusion and emanation coefficients appropriate to the case with water present and by the last factor \( g = [\varepsilon / \varepsilon_a + K \varepsilon_\omega]^{1/2} \), which could be combined with \( D'_e \). Then \( A.12 \) would have the same form as \( 10 \) with \( f \) replaced by \( f' \) and \( D_e \) replaced by \( g D_e \). A similar result obtains in the case of the tunnel where \( \ell_2 = \ell' \). This demonstrates the assertion made above that including the effects of rock moisture was equivalent to using effective values of \( D_e \) and \( f \).

In summary, it is unlikely that the parameters chosen for the site materials produce fluxes or tunnel activity concentrations in error by more than the factor of twenty difference from a WL. I recommend that limits be placed upon the
emanation coefficient \( f \) by measurement of site materials in powdered form. Harbottle at Brookhaven National Laboratory is the expert in this field, and has already volunteered to run this if we ship him a kg or so of typical specimens of chalk, marl and clay.) I also recommend that tracer gas studies be used to evaluate the fracture diffusion coefficient, if preliminary measurements of concentrations during the construction phase noticeably exceed the estimations made here.