The Behavior of Cross Sections at Very High Energies*

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In the fall of 1967, T.T. Wu and I started to calculate the high-energy Delbrück scattering amplitude. The calculations were long and tedious, and occupied almost all of our time for a whole year. When it was completed, nobody paid any attention. The only comment I remember is that this calculation had already been done by Bethe and Rohrlich,[1] as was recorded in the classic textbook of Jauch and Rohlrich[2]. It appeared that we had toiled for a year for nothing. I must admit that this is a hard way to earn a living.

What motivated us to do this torturous calculation to begin with? Consider the scattering of a photon of a few GEV (which was high energy then) from a proton, the kind of events seen in the CEA those days. The lowest-order process is of the second order: the proton absorbs a photon and then emits another one (or the other way around), just like what an electron does in Compton scattering. This amplitude is of the dimension of the inverse of mass and at high energies is approximately energy independent. It is then easy to show that, in the forward direction, this amplitude in the high-energy limit is of the order of

$$\frac{e^2}{M}, \tag{1}$$

where M is the mass of the proton, the only mass scale (aside from the energy) in the process. Fourth-order processes are usually ignored, the amplitude being smaller than the second-order amplitude by a factor of e^2. But then something curious happens in the sixth-order: the amplitude is much bigger than even the second-order amplitude. This is because, in the high energy limit, the sixth-order amplitude is linearly proportional to the photon laboratory energy ω. One of the sixth-order diagrams is illustrated in Figure 1. One may argue on the basis of dimension that this amplitude in the forward direction is of the order of

$$e^6 \frac{\omega}{m^2}, \tag{2}$$

where m is the mass of the electron. The ratio of (2) with (1) is

$$e^4 \frac{\omega M}{m^2}. \tag{3}$$

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At $\omega = 1GeV$, $\frac{\omega M}{m^2}$ is approximately 4x10^6. Therefore, although it has to pay a dear price for the factor e^4, the sixth-order amplitude is still over one hundred times larger than the second order amplitude. This is one rare instance where a higher-order amplitude dominates over the lowest-order amplitude.

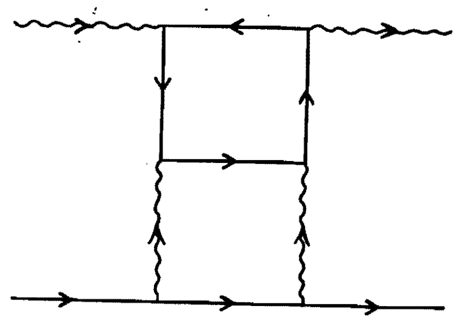

Figure 1

If the proton is replaced by a heavy nucleus of charge Z, the process of photon scattering through pair creation and recombination is known as Delbrück scattering. When Z is large (for lead, $Z\alpha \approx 0.6$, when α is the fine structure constant) it no longer suffices to calculate the sixth-order amplitude alone. Indeed, all multi-photon exchange processes are appreciable, and it would be desirable to calculate and sum all of them. By a stroke of luck, we found that we could do precisely that. The sum is just a slight modification of the lowest-order (sixth-order) term. Our answer[3], different from that of Bethe and Rohrlich, was later verified experimentally by Jarlskog, et. al[4].

It is funny, but almost everyone who looked at our result thought it complicated, while it appeared wonderously simple to us. The reason for this discrepancy in aesthetic appreciation is that we had gone through the process of evaluating the traces in the amplitude, introducing Feynman parameters, carrying out the momentum integration, and seeing the expressions turning into a complicated mess. But as we made the high-energy approximation and added things up, most terms just dropped by the wayside and only a few terms survived. We believed that there must be good reasons for this to happen. So, soon after we completed the grinding, we began to look for them.

First of all, it is meaningful to distinguish longitudinal momenta from transverse momenta. For example, we found that the photons exchanged between the nucleus and the electron-positron pair carry transverse momenta only, while the electron and the positron in the pair carry longitudinal momenta. An important point is that the longitudinal momenta of these two particles are equal to $\beta\omega$ and $(1-\beta)\omega$, respectively, with

$$0 < \beta < 1. \tag{4}$$

This means that both particles share a positive fraction of the longitudinal momentum of the incoming photon – in support of Yang's theory of limiting fragmentation.[5] Later on, this was discovered independently by Feynman and has been used extensively in the parton model, although in a somewhat different context.

It is also easy to recognize that the multi-photon exchange amplitude can be cast into an exponentiation form. The Delbrück scattering amplitude is equal to the product of two eikonal expressions, one for each particle in the pair, integrated over the momentum distribution of the e^+e^- pair. This distribution is represented by a structure factor that we call "the impact factor", which is simply the overlapping integral between the incoming and the outgoing wavefunctions of the external particle, very much like a corresponding expression in Glauber's earlier model of deuteron scattering. This feature has since been incorporated in the parton model, with electrons and positrons replaced by quarks, antiquarks, and gluons.

It is the eighth-order diagrams which signaled a real departure from conventional thinking. Soon after we finished Delbrück scattering, we turned to other elastic scattering reactions ($ee \rightarrow ee$, $\gamma\gamma \rightarrow \gamma\gamma, etc.$) in QED. It turned out that, algebraically, $ee \rightarrow ee$ is the simplest one to do and we concentrated on it. The interesting eighth-order diagram for $e-e$ scattering is the one illustrated in Figure 2.

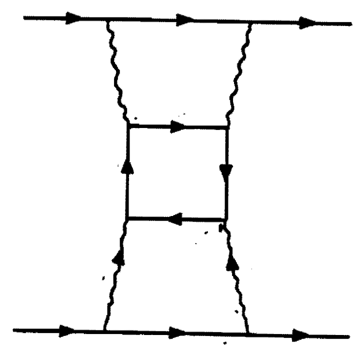

Figure 2

As we can see, in these diagrams, an e^+e^- pair is created and then annihilated, just as in γP scattering illustrated in Figure 1. However, there is an important distinction.

While the e^+e^- pair in γP scattering is clearly associated with one of the external particles, i.e., the photon, this is not the case with the e^+e^- pair in $e-e$ scattering. The e^+e^- pair in $e-e$ scattering is created jointly by both incoming particles, and does not favor either one. Therefore, in the center of mass system, the momentum of the pair takes any positive as well as negative value between $-\omega$ and ω. Thus $-1 < \beta < 1$. Calculations show that the distribution in longitudinal momenta is uniform. The important point is that the distribution curve does not vanish in the central region of $\beta \approx 0$. Indeed, the range of longitudinal momentum in which a pair may be created is from $-\omega$ to ω. As ω becomes larger, this range becomes larger. As a result, the integrated cross section grows like

$$lns. \tag{5}$$

If one further calculates the lowest-order diagrams for the creation of n-pairs, one may find that the integrated cross section for n-pairs creation increases like

$$(lns)^n.$$

Summing over n, one may obtain a cross section of the order of

$$s^a \tag{6a}$$

and hence an amplitude of the order of

$$s^{1+a}, \tag{6b}$$

where [6,7]

$$a = \frac{11}{32}\alpha^2\pi, \tag{7}$$

(logarithmic factor of s not exhibited).

While the amplitude in (6b) is merely the sum of lowest-order terms of pair-creation, it signals that the scattering process becomes strongly absorptive as energy increases. This is a feature not contained in potential scattering models. To explore further the significance of (6), let us express the scattering amplitude in the impact distance respresentation

$$M \simeq \frac{is}{2m^2}\int d^2b\ e^{-i\vec{\Delta}\cdot\vec{b}}[1 - S(\vec{b}, s)], \tag{8}$$

where $\vec{\Delta}$ is the momentum transfer, and $\vec{b}$ is the impact distance. Then $[1 - S(\vec{b}, s)]$ gives the amount of scattering at impact distance $\vec{b}$ and energy s. In the region where $\vec{b}$ is finite and s is large, almost all diagrams contribute to the amplitude $(1 - S)$. But when $\vec{b}$ is very large, it is possible to prove that the lowest-order diagrams of pair creation dominate. Thus we may prove that,for $|\vec{b}| \to \infty$,

$$1 - S(\vec{b}, s) \propto e^{-\mu b} s^a \tag{9}$$

where μ is a constant. From (9), we see that the absorption at a large impact distance increases like s^a as $s \to \infty$. This means that, even at a large impact distance, scattering can become appreciable as energy becomes sufficiently high. Theoretically speaking, a proton can become as big as a house, although it is very far from it in reality. This suggests that a particle behaves like a growing black disk. The radius of the black disk is obtained by setting the right side of (9) to unity. Thus[8]

$$R \simeq \frac{a}{\mu} lns \tag{10}$$

As a result, the total cross section increases like

$$\sigma_{total} \approx 2\pi(\frac{a}{\mu})^2(lns)^2, \tag{11}$$

which is the Froissart bound.

In 1970, when T.T. Wu and I first presented our theory of expanding hadrons,[8,9] it was met with almost universal skepticism. In retrospect this is perhaps not surprising, as all major theories of high-energy scattering in those days placed the constancy of total cross sections as one of the key foundation blocks of the theory, other than as a physical behavior to be determined. Thus people tended to dismiss any suggestion of non-constant behavior, without examining the theoretical evidences with complete objectivity. Three years later, the announcement of the ISR results [14] that the $p-p$ total cross section does rise in the energy range up to $\sqrt{s} = 54$ GeV took the world of high-energy physics by surprise. While these important works drastically changed the experimental picture, many theorists were still reluctant to embrace our concept of rising total cross section. Perhaps the cross section would only rise a little bit, say a few milli-barns, and then level off? Perhaps it would exhibit an oscillatory damped behavior? Speculations of all kinds were abound those days. Today, this has all changed. On the experimental side, beautiful works performed in UA4.2, E710, E760, and CDF show various cross sections continue to rise with energy, all the way up to $\sqrt{s} = 1.8$ TeV, with the $p-p$ cross section almost doubling that at the ISR energies as presented in a number of reports here. The elastic cross sections are also seen to rise dramatically, although the ratio between the elastic cross section and the total cross section has only increased slightly. The ratio ρ passes from negative values to positive values, as predicted, although it is by no means clear yet that it is dropping to zero. The value of b, the inverse of which measures the width of the the diffractive peak, continues to increase, also as predicted. In addition, Dr. Halzen informed me that much of the rise of the total cross section can be accounted for by the creation of relatively low energy jets in the C.M. system. This is consistent with the theoretical picture I illustrated in Figure 2, where the $e^+ - e^-$ pair should be replaced by quarks,antiquarks, and gluons, which appear physically as jets in the laboratory. Such creation processes are not diffractive, and hence the jets are of wide angle, as is observed.

In the face of these realities, theorists have entirely given up fitting the experimental data with theories of $\alpha(0) = 1$. Now everybody uses $\alpha(0) > 1$, as in (6b), in one form or another: some people fit the data with $ln^2 s$, others with lns; still others with the exponentiated form of (6b) with various factors of lns associated with it, or simply with the form of (6b), or with a few terms of the eikonal form. All of these works fit the data with success. And with the way these variations are being named, I am just glad that I did not bring my wife to this conference. If she had sat through these talks, I would have trouble explaining to her why she had to be left alone to fend for herself, in the first three years of our marriage.

Finally, where do we go from here? Scientists strive for better and deeper understanding of physical phenomena, and it is the scientific spirit to remain unsatisfied with the status quo. High-energy physicists are of course no exception. However, allow an old-timer to offer this perspective. In 1973, soon after the announcement of the ISR results, a conference was held at Batavia on small-angle high-energy scattering. What had just been observed in ISR was so puzzling to some physicists that one of the speakers kept emphasizing that we did not have an Einstein, and that the job was too large for mere humans like us. Today, you, workers in high-energy physics, have made the general picture very clear: a high-energy particle becomes increasingly absorptive as energy increases. This increase is a result of particle creation, particularly in the central region. Therefore, you have established that the black disk picture is no more in doubt. This is great progress. However, understanding the general picture of high-energy hadron scattering does not necessarily translate into detailed and quantitative predictions. We are not yet in the asymptopia, and may never be, and fits are just that, fits. So we must not have illusions. I just talked to Dr. A. Martin, and learned that he is making good progress in solving the three-body problem described by the Schrödinger equation. If a three-body problem is just getting solved, how can we expect to solve an infinite-body problem quantitatively, which a quantum field theory is? If we have a small parameter, as in the case of QED, we may make perturbative calculations, as in Kinoshita's elaborate work on $g-2$. But we do not have such a circumstance in hadron physics. Therefore, I am impressed with the valiant efforts made by theorists like Lipatov and White, and hope that one day some of them will be up to the challenge. I would like to mention one more thing: it is my feeling that the pendulum may have swung a little too far the other way; you may now have a bit too much faith in the eikonalization formula we invented. This formula has always been to me a model, extracted from calculating a certain set of diagrams in QED incorporating some of the essential features of high-energy scattering. I must emphasize that, in QCD, <u>no</u> set of diagrams gives the eikonalization of a Regge pole (or cut) term. Calculations of leading terms show that, in non-Abelian gauge theories, this formula should be replaced by an operator eikonal formula [13], while the physical picture remains to be the same. This formula is more difficult to use for numerical calculations. The good news is that it yields a great deal more physics. With a few parameters, we may calculate not only the elastic scattering amplitudes, but also inelastic scattering amplitudes, e.g. the production of jets in the central region. Again, I must emphasize that the operator eikonal formula is no more than a model, as some form of leading term approximations have been used. But it will be interesting to see if a sophisticated model with deeper content on the basis of QCD brings better results.

References

1. H.A. Bethe and F. Rohrlich, Phys. Rev. 86, 10 (1952).

2. J. M. Jauch and F. Rohrlich, Theory of Photons and Electrons, 2nd expanded edition, (Springer-Verlag 1976).

3. H. Cheng and T.T. Wu, Phys . Rev. Letters 22, 666 (1969), see also Phys. Rev. 182, 1873 (1969).

4. G. Jarlskog, L. Jonsson, S. Prunster, H.D. Schulz, H.J. Willutzki, and G.G. Winter, Phys, Rev. *D*8, 3813, 1973.

5. See, for example, C.N. Yang in Proceedings of the Third Topical Conference on High Energy Collisions of Hadrons, Stony Brook, New York (1969).

6. This was first reported by H. Cheng in Proceedings of the Third Topical Conference on High Energy Collisions of Hadrons, Stony Brook, New York (1969).

7. G.V. Frolov, V.N. Gribov, and L.N. Lipatov, Phys Letters 31*B*, 34 (1970).

8. H. Cheng and T.T. Wu, Phys. Rev. Letters 24, 1456 (1970).

9. H. Cheng and T.T. Wu, Phys. Letters 36*B*, 357, (1971).

10. B. McCoy and T.T. Wu, Phys. Rev. *D*12, 3257 (1975), 13, 1076 (1976); L. Tyburski, ibid 13, 1107 (1976); L. N. Lipatov, Yad Fig. 23, 642 (1976) [Sov. J. Nuclear Physics 23, 338 (1976)], L.L. Frankfurt and V.E. Sherman, Yad. Fig 23, 1099 (1976).

11. V.S. Fadin, E.A. Kuraev, and L.N. Lipatove, Phys. Lett. 60*B*, 50 (1975).

12. H. Cheng and C.Y. Lo, Phys Rev. *D*15, 2959 (1977).

13. H. Cheng, J. Dickinson, K. Olaussen, Phys. Rev. *D*23, 534 (1981); H. Cheng, J. Dickinson, P. Yeung, and K. Olaussen, Phys. Rev. *D*23, 1411 (1981).

14. S. R. Amendolia, et. al., Phys. Lett. 44*B*, 119 (1973); U. Amaldi et. al., Phys. Lett. 44*B*, 112 (1973).