# Averaging Lifetimes for B Hadron Species

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**Abstract**

The measurement of the lifetimes of the individual B species are of great interest. Many of these measurements are well below the 10 % level of precision. However, in order to reach the precision necessary to test the current theoretical predictions, the results from different experiments need to be averaged. Therefore, the relevant systematic uncertainties of each measurement need to be well defined in order to understand the correlations between the results from different experiments.

In this paper we discuss the dominant sources of systematic errors which lead to correlations between the different measurements. We point out problems connected with the conventional approach of combining lifetime data and discuss methods which overcome these problems.

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# 1 Introduction

The measurements of individual B hadron lifetimes are currently among the most interesting physics results since, after the success of the spectator model [1] in explaining the order of magnitude of the average lifetime of B hadrons, they should allow a test of corrections to this model provided by the Heavy Quark Effective Theory [2].

The average B hadron lifetime is now known to a precision of $\approx 1.2\%$ [3, 4]. The precision on individual B hadron lifetimes continues to improve with the increasing size of the available data samples and the improved understanding of the systematic uncertainties. Nevertheless the precision required on individual B hadron lifetimes ($< 5\%$) to test the theoretical predictions [5] can only be reached by combining the results of different experiments.

The task of averaging these measurements plays an important role since the resulting averages can differ from each other by an amount comparable to the required precision, depending on the way the statistical error is treated and on the assumptions made concerning the correlated systematic uncertainties. Moreover, the task of averaging these results is complicated by the fact that different experiments use different assumptions concerning these systematics. To facilitate averaging it is therefore important to specify how the results depend on the value assumed for the relevant input parameters.

In this paper the dominant sources of systematic errors which lead to correlations between measurements are discussed: the estimation of the background contamination, the evaluation of the B hadron momentum and the decay length reconstruction.

In the last section the treatment of the statistical and systematic errors is discussed and two different methods of combining lifetime measurements are presented and compared.

# 2 Background estimation

An important source of correlated uncertainties between measurements arises from the imprecise knowledge of the amount of background events in the data sample and their proper-time distribution. The background contamination can be due either to physics processes leading to the same final state used to tag the signal process or to accidental combinations of tracks which simulate the decay of interest.

An example of the first kind of background ("physics" background) is the process $B^0 \to D_sDX$ (with $D \to \ell X$) in the measurement of the $B_s$ lifetime based on $D_s\ell$ correlation [6, 7, 8]. When "physics" background is present the experimental uncertainties on the branching ratios of the background processes and on the lifetime of the background particles lead to a systematic error which is correlated between different experiments. Another example occurs in the measurement of the average B hadron lifetime [9, 10, 11, 12, 13], where the main source of background is due to charmed particles which have lifetimes of the same order of magnitude as B hadrons [14]. The average B lifetime is computed assuming measured charmed hadron branching ratios and lifetimes; systematic uncertainties are evaluated by varying these quantities within the experimental errors. This procedure introduces a correlation among the results from different measurements.

When the background is combinatorial the amount and/or the lifetime of the background "particles" is normally extracted from the data using the sidebands of mass plots or wrong sign combinations. In this case the related systematic uncertainty is due to the limited statistics of the data sample used and is not correlated between experiments. An example is represented by the measurements of $B^0$ and of $B^+$ lifetimes based on the selection of $D^{(*)}\ell$ samples [15, 16, 17, 18, 19]. The combinatorial background is suppressed using identification and kinematic requirements but is often the source of the largest systematic error. The lifetime distribution of the background is estimated from the data using the events in the sidebands of $D^{(*)}$ mass distributions and the $D^{(*)}\ell$ combinations that have the wrong charge correlation.

There is also an example of combinatorial background extracted from the data which needs a correction which is common to all experiments. In the measurements of the $b$ baryon lifetime using $\Lambda\ell$ correlation [20, 21, 22], the amount of accidental background is obtained from the number of wrong sign combinations ($\Lambda\ell^+$). However a correction has to be applied to this number to take into account the production asymmetry of accidental $\Lambda\ell$ pairs. This correction is evaluated using simulation and is correlated between experiments.

# 3 B momentum estimation

Most of the exclusive B lifetime measurements are based on the reconstruction of B decay lengths. In some analyses these decay lengths are converted into proper times event by event [6], while in other analyses a statistical approach is used [15]. In both cases the relativistic boost of the B hadron needs to be estimated.

In most of the analyses the B particles are only partially reconstructed and their energies are estimated from the energies of the detected decay products [22]. The estimator often includes scale factors or corrections obtained from Monte Carlo simulations [21]. No matter what estimator is used, however, systematic errors must be evaluated for the following effects:

- Uncertainties in the $b$ quark fragmentation function. The mean energy fraction of B and charmed hadrons in $Z$ decays has been measured and used in numerous heavy flavour analyses [23]. This uncertainty affects also the measurements based on the impact parameter distribution of B decay particles [9, 10, 12, 13, 20, 22]. Care should be taken that the fragmentation function might be different for different B species (especially $\Lambda_b$ and $B_s$). Due to the lack of experimental results no lifetime measurement has so far taken this effect into account.

- Uncertainties in branching ratios of B and charmed hadrons. The branching ratio uncertainties of importance in the B energy estimate tend to be those describing the production of additional particles which escape detection in partially reconstructed modes. For instance in the measurement of the $B_s$ lifetime using $D_s\ell$ correlations [6, 7, 8], the presence of missing particles such as the photon affects the boost estimate (i.e. $B_s \to D_s^*\ell\nu$ with $D_s^* \to D_s\gamma$).

- Uncertainties in B hadron masses. The effect of the uncertainties on the B hadron masses is important mainly for $b$ baryons since the $\Lambda_b$ mass has the largest ($\pm 50$ MeV/c$^2$) uncertainty among the observed B hadron states and since some of the selected events may come from other $b$ baryons, e. g. $\Xi_b$, which are expected to have masses about 0.2-0.3 GeV/c$^2$ greater.

- Uncertainties in $b$ baryon polarization. The $b$ quark polarization in $Z \to b\bar{b}$ decays is expected to survive (at least partially) the hadronization phase. The momentum spectrum and the impact parameter distribution of the leptons from $b$ baryon decays depend on the amount of polarization of the decaying particle. All current LEP measurements of the lifetimes of $b$ baryons are based on semileptonic decays and the $b$ baryon momentum is estimated from the observed decay products. Therefore a systematic uncertainty in the estimated momentum arises from imprecise knowledge of the $b$ baryon polarization.

- Uncertainties in modelling neutral hadronic energy and in detector momentum and energy resolution. The uncertainties in charged momentum resolution are almost certainly independent between experiments. However, uncertainties due to neutral energy modelling (in, e.g., GEANT) and uncertainties due to decay topologies with overlapping particles which cannot be measured separately may be correlated. No measurement has so far taken this effect into account. It would be helpful if variations in the models used in evaluating the uncertainty in detector response to hadronic showers could be standardized.

Not all of the items are relevant to each analysis; however, all are sources of correlation between different measurements.

# 4 Correlated uncertainties within an experiment

There may be several measurements of the same quantity made by the same experiment using different techniques in order to obtain their best possible result. In this case systematic uncertainties normally treated as being uncorrelated with measurements from other experiments will be correlated between the measurements of the same experiment. Sources of uncertainties of this kind are due to primary and secondary vertex reconstruction procedures, detector resolution, tracking errors, B flight direction reconstruction and detector alignment uncertainties and will be discussed in the following subsections. To make the task of averaging easier and more reliable, experiments should quote which systematics are correlated and the size of these correlations.

An additional experimental correlated uncertainty is represented by the statistical correlation between measurements and is discussed in the last subsection.

## 4.1 Primary vertex reconstruction

Some information on the primary vertex is already given by the known size of the beam overlap region. However the position of the interaction region may change during a fill (because of orbit corrections), which makes it necessary to monitor it. The precision with which this can be done depends of course on the performance of the tracking detectors.

Because of the rather complex algorithms used to reconstruct the primary vertex the errors can be regarded as uncorrelated amongst the LEP experiments. However they should be completely correlated for different measurements done at the same experiment.

## 4.2 Secondary vertex reconstruction and tracking resolution

The secondary vertex reconstruction error depends on the resolution of the tracking device. Furthermore there are contributions due to multiple scattering, pattern recognition errors, and alignment. In complex topological vertex searches (e.g. looking inclusively for displaced vertices), systematic errors of the algorithm used have to be added.

A good measure of the tracking performance is the impact parameter resolution, which can be obtained using *uds* events or the tails with apparently negative lifetime of the impact parameter distribution (which then also includes errors of the primary vertex reconstruction). Sometimes corrections or scale factors derived from Monte Carlo simulations are applied to the measured "resolution function" [10, 22].

For secondary vertex reconstruction the resolution is often obtained using simulated events. Corrections which take into account any deficiencies of the

Monte Carlo simulations are applied.

The resolution can be treated as uncorrelated for different experiments and as fully correlated for measurements at the same detector, except if the resolution is dominated by errors due to the reconstruction algorithm specific to a certain analysis.

### 4.3 Flight direction

The reconstruction of the flight direction is important for the sign of the impact parameter and for all projective vertex measurements using $r - \phi$ information only. In addition some methods for calculating the total decay length (most likely decay length) and certain topological vertex algorithms depend on the knowledge of the B flight direction. In these cases, as the B hadron is normally only partially reconstructed, the B flight direction must be approximated. Most of the analyses use the jet axis, while some use the direction of reconstructed secondaries as the estimate of the B direction. This implies systematic uncertainties which are deduced from Monte Carlo simulations. Jet axis reconstruction depends also on the algorithm and what information is used (charged tracks with/without calorimeter information). Systematic effects can arise from uncertainties in $b$ fragmentation and B decay.

It is very difficult to estimate how much these errors are correlated between different experiments. Correlations are probably small, as the algorithms used are often different, and are so far neglected. The correlation could be estimated if papers specify how the use of a jet algorithm (when jets are used to estimated the B direction) or of a decay model (when the B direction is estimated from secondaries) affects the result.

### 4.4 Statistical correlation

When several measurements of the same quantity are made by the same experiment with different techniques, the statistical overlap of the selected data samples should be taken into account by the averaging procedure.

To allow a reliable combination of measurements, experiments should quote the amount of statistical correlation giving the correlation coefficient. If an experiment provides an average of several statistically correlated results, a breakdown of the common systematic errors should also be given.

## 5 The averaging procedure

Various methods have previously been used to average lifetime measurements from different experiments [14]. When the individual estimates from different experiments have uncorrelated errors, the standard approach is simply to weight the measurements according to their error, thus for a measurement $\tau_i \pm \sigma_i$ the weight is taken as $W_i = 1/\sigma_i^2$. Lifetime measurements however have an underlying exponential distribution, so $\sigma_i \propto \tau_i$. Therefore if a measurement fluctuates low then its weight in the average will increase, leading to a bias towards low values. An alternative method, to avoid this bias, is to calculate the weight using the *relative* error $r_i = \sigma_i/\tau_i$ [24]. With this procedure the combined lifetime is obtained as the following weighted average :

$$\bar{\tau} = \frac{\Sigma_i W_i \cdot \tau_i}{\Sigma_i W_i}$$

with

$$W_i = \frac{1}{r_i^2}.$$

The use of the relative error instead of the absolute error can produce different results for the combined lifetime. This fact can be illustrated by comparing the world averages quoted for the $B_s$ at the Winter Conferences 1994:

$$\begin{aligned} \tau(B_s) &= (1.38 \pm 0.17)\,\text{ps} \quad \text{(la Thuile [25])}\ , \qquad (1) \\ \tau(B_s) &= (1.66 \pm 0.22)\,\text{ps} \quad \text{(Moriond [26])}\ . \end{aligned}$$

Both averages were performed using essentially the same data; in the first case the absolute error was used in the weight, whilst in the second case the relative error was used.

This issue has been studied [27] using a simple Monte Carlo simulation. A sample of $N$ events is generated according to an exponential distribution (with $\tau = 1$), smeared by a Gaussian resolution function (with r.m.s width $w$). The mean $\tau_i$ and variance $\sigma_i^2$ of the events is then calculated, simulating a single lifetime measurement. This is then repeated for many samples, and their weighted mean calculated (see Figure 1). Weighting with the absolute error, as shown in Figure 1 (a), a bias to low values is seen, as expected. For perfect resolution ($w = 0$) the bias is about 10 % when the sample size is 20 events, decreasing for higher sample sizes; the effect of finite resolution is to reduce the bias. If instead the samples are weighted according to their relative error, as shown in Figure 1 (b), then for perfect resolution there is no bias. However, as the resolution is degraded a bias appears towards *higher* values. Experiments measuring lifetimes using microvertex detectors have a typical resolution of $w \lesssim \tau/10$. In this case the bias is a few percent or less. Nevertheless it seems worthwhile to try to avoid it.

In an ideal world each experiment would provide the log-likelihood function they calculated for their events, and these would be summed and then fitted for the combined lifetime. In practice this would be difficult to organize, and there is the additional question of how to include systematic errors. Instead, one could attempt to reconstruct the likelihood function of each experiment from the quoted asymmetric errors [27]. For an experiment with perfect resolution,

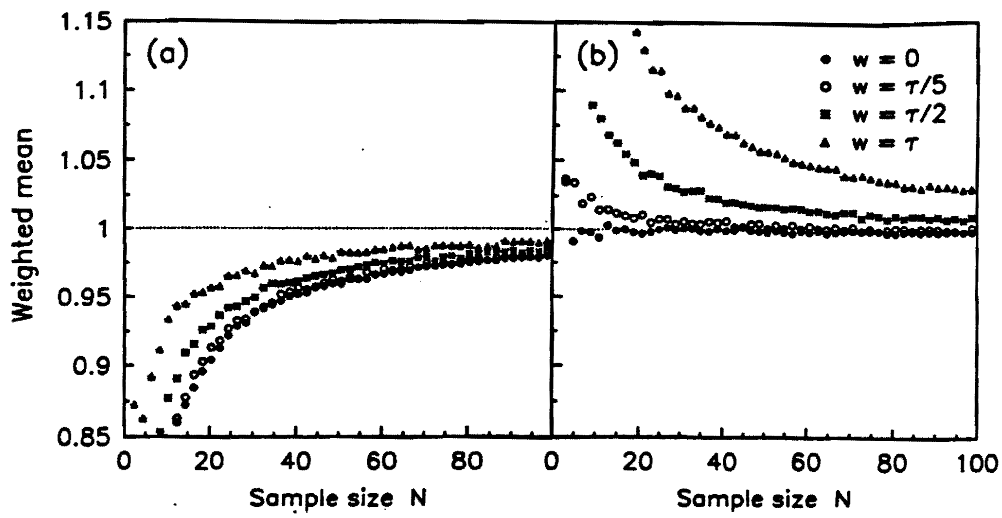


Figure 1: Weighted mean of many samples, each of $N$ events: (a) weighting with the absolute error $\sigma_i$, (b) weighting with the relative error $\sigma_i/\tau_i$. w is the resolution.

with an underlying exponential distribution, the form of the likelihood function is maximally asymmetric and can be calculated:

$$\ln \mathcal{L}_{\mathrm{E}}(\tau) = -N\left(\frac{\tau_i}{\tau} + \ln \tau\right) . \tag{2}$$

In the limit of poor resolution the likelihood function is symmetric:

$$\ln \mathcal{L}_{\mathrm{G}}(\tau) = -\frac{1}{2}\left(\frac{\tau_i - \tau}{\sigma}\right)^2 . \tag{3}$$

The approximation is made that the likelihood function for a given experiment is a linear combination of these two forms:

$$\ln \mathcal{L} = a \ln \mathcal{L}_{\mathrm{E}} + b \ln \mathcal{L}_{\mathrm{G}} , \tag{4}$$

and the coefficients $a$ and $b$ are determined from the quoted errors, using (for a value $\tau\,{}^{+\sigma_1}_{-\sigma_2}$) $\ln \mathcal{L}(\tau + \sigma_1) = \ln \mathcal{L}(\tau - \sigma_2) = \ln \mathcal{L}(\tau) - \frac{1}{2}$. The functions $-\ln \mathcal{L}$ are then summed for all of the experiments, and a fit is made for the minimum of their sum, which gives the average. This procedure takes into account asymmetric errors on the individual estimates from different experiments and allows for asymmetric errors on the combined result.

To treat the case in which the measurements are affected by correlated systematic uncertainties additional parameters are added to the fit. These parameters allow a common movement of the mean, with a Gaussian constraint applied according to the correlated errors. The above technique has been implemented in the averaging program COMBY [28] and has been shown to reproduce correctly the generated lifetime on Monte Carlo simulation data samples. The currently available version of the COMBY program can handle up to eight different sets of correlated uncertainties between the various results.

An alternative technique of averaging which handles any number of different sets of correlated uncertainties, relies on the use of the error matrix [24]. The combined lifetime is obtained by minimizing with respect to $\bar{\tau}$ :

$$\chi^2 = \Sigma_{ij}(\bar{\tau} - \tau_i) \cdot E_{ij}^{-1} \cdot (\bar{\tau} - \tau_j),$$

where $E^{-1}$ is the inverse of the error matrix, $\bar{\tau}$ is our best estimate and $\tau_i$ are the separate estimates derived from the data. This yields:

$$\bar{\tau} = \frac{\Sigma_{ij}\tau_i \cdot (E^{-1})_{ij}}{\Sigma_{ij}(E^{-1})_{ij}}$$

with error :

$$\bar{\sigma} = [\Sigma_{ij}(E^{-1})_{ij}]^{-1/2} \cdot \bar{\tau}.$$

The error matrix is constructed assuming that the error on the measured lifetime is fractional. The diagonal terms take into account the total errors, the off-diagonal terms contain the correlated part of the total errors. Systematic errors of the same category quoted by different measurements are conservatively taken to be fully correlated.

This technique is implemented in the averaging program COMBINE [29] and correctly treats correlations among the different measurements, however the uncertainties become symmetrized and the method does not allow for asymmetric errors on the final result. Also, this second technique does not properly handle the bias introduced due to detector resolution (as mentioned above, see Figure 1).

In the COMBINE program the way of accounting for systematic uncertainties is different to the COMBY approach: COMBINE treats systematic errors as relative errors while COMBY treats them as absolute. Some of the systematic uncertainties are certainly absolute like resolution, while other might be relative, depending on the size of the data sample used to evaluate them. Therefore the two programs are complementary to each other, and their difference reflects our uncertainty on the correct averaging procedure. For a situation close to the actual measurements which need to be combined ($N > 30$), studies of Monte Carlo simulations have shown that the results from two averaging methods described differ from the true input value of lifetime by about 1%. This difference should be attributed to a systematic uncertainty from the averaging procedure, but is typically negligible compared to the overall error.

# 6 Conclusions

In this note the relevant sources of correlation among B lifetime measurements are presented.

To make the task of averaging easier and more reliable the determination of all systematic errors should be explained in the description of the analyses. Experiments should give a breakdown of the systematic errors ensuring that all input parameters used and their range of variation are specified or documented. The covariance matrix should be given if constrained fits to input parameters are used. Experiments should state if measurements are statistically correlated and quote the size of this correlation.

In this note the importance of taking into account in the averaging procedure the underlying exponential behaviour of the lifetime measurements has been shown. This has been done either by using asymmetric errors or treating the statistical error as a relative error. The two methods proposed agree at the percent level, and this difference may be regarded as a systematic error of the averaging procedure.

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