TEST OF GOLDSTONE BOSON EQUIVALENCE THEOREM

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OSU Preprint 279

April, 1993

ABSTRACT

We study the convergence (with energy) of the exact longitudinal gauge boson scattering amplitude to that given by the equivalance theorem. For low Higgs boson masses, this convergence is rather slow, and can have significant effect at the SSC energies. We find that in addition to $O(m_w / \sqrt{s})$ terms, the two amplitudes differ by terms of $O(m_H / \sqrt{s})$. We incorporate the presence of such terms in the general proof of the equivalence theorem.

The Goldstone boson equivalence theorem [1-5] in the electro-weak gauge theory is a very popular method for approximating difficult calculations. It states that if $A(W_L^{\pm}, Z_L, H)$ is an amplitude for scattering of the longitudinal gauge bosons and the physical Higgs particle in the electroweak theory, and if $A(\phi^{\pm}, \phi_2, h)$ is the analogues amplitude for the scalar sector of the theory in the R_ξ gauge, then, for $s >> m_w^2$, m_z^2,

$$A(W_L^{\pm}, Z_L, H) = A(\phi^{\pm}, \phi_2, h) + O(m_w / \sqrt{s}) \tag{1}$$

Here, $W_L^{\pm}, Z_L$ represent the external physical particles, whereas $\phi^{\pm}, \phi_2$ represent the corresponding Goldstone bosons. s is the square of the center of mass energy. The theorem was explicitly checked for several processes at the tree level in the limit of large λ so that the gauge contributions could be ignored [3]. (Here, $\lambda = m_H^2 / 2v^2$, m_H is the Higgs boson mass and v is the Higgs vacuum expectation value.) The theorem was proved in the 't Hooft Feynman gauge [1,3] and was generalized [4] to R_ξ gauge to all orders in the scalar self coupling λ , using power counting method for the external lines. The validity of the theorem to the one loop level has also been investigated [6]. The theorem has been used extensively for many processes, as the amplitudes involving the Goldstone bosons, substituted for the external gauge bosons, are much easier to calculate.

In this work, we ask two questions:

i) How quickly, with $\sqrt{s}$, the exact amplitude (A_{EX}) converges to that given by the equivalence theorem (A_{EQ})? How large the Higgs mass need to be?

ii) Is the statement of the equivalence theorem complete?

To address the first question, we have calculated the exact amplitude for the process

$$Z_L Z_L \to W_L^+ W_L^- \tag{2}$$

and also that given by the equivalence theorem. Our results are as follows:

a) The exact amplitude (A_{EX}) converges rather slowly with $m_w / \sqrt{s}$ to A_{EQ}. This is particularly true for low Higgs masses, $m_H < 200 GeV$. Since W^+W^- luminosity in hadron collider falls like $\frac{1}{s}$, this effect is significant.

b) The difference, A_{EX} -A_{EQ} also depends on $m_H / \sqrt{s}$, in addition to $m_w / \sqrt{s}$. This has big effect when $m_H \sim \sqrt{s}$.

To address the second question, we have studied what happens to the equivalence theorem when m_H is of the order of $\sqrt{s}$. We have extended the proof given in ref. 4 to include the effect of Higgs boson (H) or heavy fermion (f) exchanges inside the blob of the S-matrix. With this generalization, the more complete statement of the Equivalence theorem is

$$A(W_L^{\pm}, Z_L, H, f) = A(\phi^{\pm}, \phi_2, h, f) + O\left(\frac{m_w}{\sqrt{s}}, \frac{m_H}{\sqrt{s}}, \frac{m_f}{\sqrt{s}}\right) \tag{3}$$

In the following, we now give our calculations and discuss the results. The Feynman diagrams for the process (2) for the calculation of the exact amplitude and Equivalence theorem amplitude is shown in Fig. 1. Note that the only external longitudinal gauge bosons are replaced by the corresponding Goldstone bosons.

Defining $e \equiv 4m_w^2 / s$, $x \equiv 4m_H^2 / s$, $a \equiv m_Z^2 / m_w^2$ and $\cos\theta_{cm} \equiv n$, total exact amplitude [7] is obtained to be

$$A_{EX} = g^2 N_{EX} / D \tag{4}$$

where N_{EX} = -24 +48 e + 4 a e + 4 a^2 e - 24 a e^2 + 2 a^2 e^2 - 2 a^3 e^2 + a^2 e^3 - 8 n^2 - 40 e n^2 + 28 a e n^2 - 8 a^2 e n^2 - 32 e^2 n^2 + 12 a e^2 n^2 + 12 a^2 e^2 n^2 - 4 a^2 e^3 n^2 + 4 x + <u>4 e x</u> <u>-6 a e x</u> - 12 e^2 x + 2 a e^2 x + 2 a^2 e^3 x + 6 a e^3 x - 2 a^2 e^3 x - 4 n^2 x + <u>8 e n^2 x</u> + <u>6 a n^2 x</u> + 8 e^2 n^2 x - 16 a e^2 n^2 x + 8 e^3 n^2 x and

D = [-2 + a e - 2 Sqrt(1 - e) Sqrt(1 - a e) n]

[-2 + a e + 2 Sqrt(1 - e) Sqrt(1 - a e) n] (-4 + e x)

The total Equivalence Theorem amplitude is obtained to be

$$A_{EQ} = g^2 N_{EQ} / D \tag{5}$$

N_{EQ} = -24 + 4 e + 16 a e - 2 a e^2 - 2 a^2 e^2 - 8 n^2 + 8 e n^2 + 8 a e n^2 - 8 a e^2 n^2 + 4 x + 6 e x - 4 a e x - e^2 x - 4 a e^2 x + a^2 e^2 x + ½ a e^3 x + ½ a^2 e^3 x -4 n^2 x + 6 e n^2 x +4 a e n^2 x - 2 e^2 n^2 x -6 a e^2 n^2 x + 4 a e^3 n^2 x.

To see how fast the exact amplitudes converges to the equivalent amplitude, we plot both amplitudes against $\sqrt{s}$ for m_H = 60, 100 and 150 *GeV* in Fig. 2. The solid (dotted) curves represent the A_{EX} (A_{EQ}) amplitude. We see that for low Higgs masses the convergence is very slow. Over 1 *TeV* of energy in the W^+W^- center of mass is required for the equivalence amplitude to be a good approximation. For a given mass, at low center of mass energies, the two amplitudes differ by more than 50%, thus the order $m_w / \sqrt{s}$ terms are very important. This is specially significant in hadron supercolliders (such as Large Hadron collider or Superconducting Supercollider) because W^+W^- luminosity falls like 1/s. The Fig. 2 is for $\cos\theta_{CM} = 0$; however, the above conclusions are also true for other values of the angle. In Fig. 3, we have plotted the partial wave, a_0 against $\sqrt{s}$ for m_H = 60, 100 and 150 *GeV*. Again, we see, the convergence of the two amplitudes is rather slow, showing the importance of the $O(m_w / \sqrt{s})$ terms. The plots of a_0 against $\sqrt{s}$ is also shown for m_H = 400 *GeV* and 1,000 *GeV* in Fig. 4 and 5 respectively. Again, the exact a_0 differ significantly from that given by the equivalence theorem.

We now consider the effects of the $m_H / \sqrt{s}$ terms. Those terms in A_{EX} and A_{EQ} have been underlined in Eq. (4) and (5). We note that the coefficients of the

$ex \equiv 4m_H^2 / s$ terms are different in A_{EX} and A_{EQ}. Thus, the two amplitudes differ not only by the m_w^2 / s terms, but also by the m_H^2 / s terms. In the limit of $e \equiv 4m_w^2 / s = 0$, the difference.

$$A_{EX} - A_{EQ} = 2g^2(1+a)\frac{m_H^2}{s - m_H^2} \tag{6}$$

Including the width of the Higgs boson ala Chanowitz and Gaillard Ref. (4), we replace $\frac{1}{s-m_H^2}$ $by \frac{(1 - i\Gamma_H / m_H)}{s - m_H^2 + i\Gamma_H m_H}$. (the factor in the numerator is to satisfy the low energy theorem as discussed in Ref. (4)). Then, we obtain.

$$\left|A_{EX} - A_{EQ}\right| = 2g^2(1+a)\sqrt{\frac{1 + \Gamma_H^2 / m_H^2}{(s / m_H^2 - 1)^2 + \Gamma_H^2 / m_H^2}} \tag{7}$$

The dependence of the difference on m_H^2 / s is shown in Fig. 6 for $m_H = 150$, 400 and 1000 GeV. We see that the two amplitudes differ significantly for a wide range of values of m_H^2 / s. We also note that the A_{EX} and A_{EQ} also differ by m_H^2 / s terms for the process $W^+W^- \to HH$ [8] and $W^+W^+ \to W^+W^+$ [9]. Thus we conclude that a more complete statement of the equivalence theorem is as given in Eq. (3) [10]. The presence of such terms could be incorporated in the general proof of the equivalence theorem àla Chanowitz and Gaillard. Following Ref. (4), we consider

$$\Delta(r,s,m) = \sum_{M_j=o}^{4} \sum_{\mu_j=o}^{3} \left(\prod_{i=1}^{r} m_{a_i}^{-1} D_{a_i}^{M_i}(p_i) \right)$$

$$\left(\prod_{j=1}^{s} \varepsilon_{(L)b_j}^{\mu_j}(q_j) \right) S_{M_1 \cdots M_r, \mu_1 \cdots \mu_s, A_1 \ldots A_m}^{a_1 \ldots a_r, b_1 \ldots b_s}(p,q,r) = 0 \quad , \quad r \geq 1, s \geq 0 \tag{8}$$

This is a Ward identity for the S-matrix element of r vectors and/or Goldstone bosons with momenta $p_1, \cdots, p_r$, s longitudinally polarized vector bosons with momenta $q_1, \cdots, q_s$, and in other particles (physical Higgs particles, fermions,etc) with all the external particles on the mass shell. $D_M^a(p) \equiv (ip_\mu, m^a)$ corresponds to the 5-component vector field $\tilde{V}_M^a = (V_\mu^a, \phi^a)$ where ϕ^a is the Goldstone boson eaten by the vector field V_μ^a.

$$\varepsilon_{\mu(L)}^a(p) = \frac{p_\mu}{m_a} + \upsilon_\mu^a(p) \tag{9}$$

where $\upsilon_\mu^a(p) = 0(m_w / E)$ for $E >> m_w$. We now define

$$V(\ell,n) = \left(\prod_{j=1}^{n} \varepsilon_{(L)b_j}^{\mu_j}(p) \right) \left[S(a,4,p)_\ell (b,\mu,q)_n \right] \tag{10}$$

where $(a,4,p)_\ell$ represent ℓ Goldstone bosons, and $(b,\mu,q)_n$ represent n longitudinal gauge bosons. For simplicity, we have dropped the fermions in the external legs. Chanowitz and Gaillard argued that $\varepsilon_{(L)b_j}^{\mu_j}$ should be replaced by $\upsilon_{(L)b_j}^{\mu_j}$, since terms proportional to p^μ have to be cancelled to satisfy appropriate high enery behavior. Thus, they conclude that

$$V(\ell,n)=\prod_{j=1}^{n}\upsilon_{(L)b_j}^{\mu_j}\left[S(a,4,p)_\ell(b,\mu,q)_n\right]=0\left[(m_w/E)^n\right] \tag{11}$$

However, if the interaction blob of the S-matrix involves exchange particles like Higgs or heavy fermions, then, the powers of energy coming from the product $\prod_j=(p_{\mu_j}/m_j)$ terms can also get cancelled by the inverse powers of energy coming from the expansion of the propagator. This will give rise to $O(m_H/E)$ terms in addition to $O(m_w/E)$ terms. As a result, the equation (11) becomes.

$$V(\ell,n)=\sum_{c=o}^{n}0\left[(m_w/\sqrt{s})^{n-c}\right]0\left[(m_H/\sqrt{s})^c\right] \tag{12}$$

Now, defining

$$X(\ell,r,s)=\left(\prod_{i=1}^{r}m_{b_i}^{-1}q^{\mu_i}\right)\left(\prod_{j=1}^{s}\varepsilon_{(L)c_i}^{\upsilon_j}(\kappa_j)\right)\left[S(a,4,p)_\ell(b,\mu,q)_r(c,\upsilon,\kappa)_s\right] \tag{13}$$

with notations same as in Ref. (4), and following the steps as in Ref. (4), we obtain,

$$\overline{V}(n,o)=V(n,o)=(i)^n X(o,o,n)+0(m_w/\sqrt{s})+0(m_H/\sqrt{s}) \tag{14}$$

From Eqs. (12), (13) and (14), it follows that

$$S\left[(a,4,p)_n\right]=(i)^n\left(\prod_{J=1}^{n}\varepsilon_{(L)a_j}^{\mu_j}\right)S\left[(a,\mu,p)_j\right]+O(m_w/\sqrt{s})+O(m_H/\sqrt{s}) \tag{15}$$

ACKNOWLEDGEMENT

We thank M. Berger, D. A. Dicus, X. Li and S.S.C. Willenbrock for many helpful discussions. The authors also benefitted from the comments and criticism from the participants of the Higgs Workshop Group of the SSC Physics Symposium held in Madison, Wisconsin. Finally, we thank D. Alspach of the OSU Mathematics Department, MLRC staff and A. Mian for helping us with the computational facilities. This work was supported in part by the U.S. Department of Energy, DE-FG05-85ER 40215.

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7. The exact amplitude for this process was also calculated by M. C. Bento and C. H. Lellynsmith, Nucl. Phys. B289, 36 (1987). Our result agrees with them.

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10. We note however that for this process (and also for WW $\rightarrow$ WW), the following statement is true: $A_{EX} = A_{EQ}\left[1 + 0(m_w / \sqrt{s})\right]$.

FIGURE CAPTIONS

Fig. 1: Feynman diagrams for the process $Z_L Z_L \rightarrow W_L^+ W_L^-$.

Fig 2: Variation of A_{EX} and A_{EQ} against $\sqrt{s}$ for $m_H = 60$, 100 and 150 GeV.

Fig. 3, 4 & 5: Variation of the zeroth partial wave amplitudes, $a_{\circ}$ (exact), $a_{\circ}$ (equivalent) for the Higgs masses shown.

Fig. 6: The dependence of the difference, $|A_{EX} - A_{EQ}|$, on m_H^2 / s for $m_H = 150$,

400 and 1,000 GeV in the limit $m_w^2 / s = 0$.

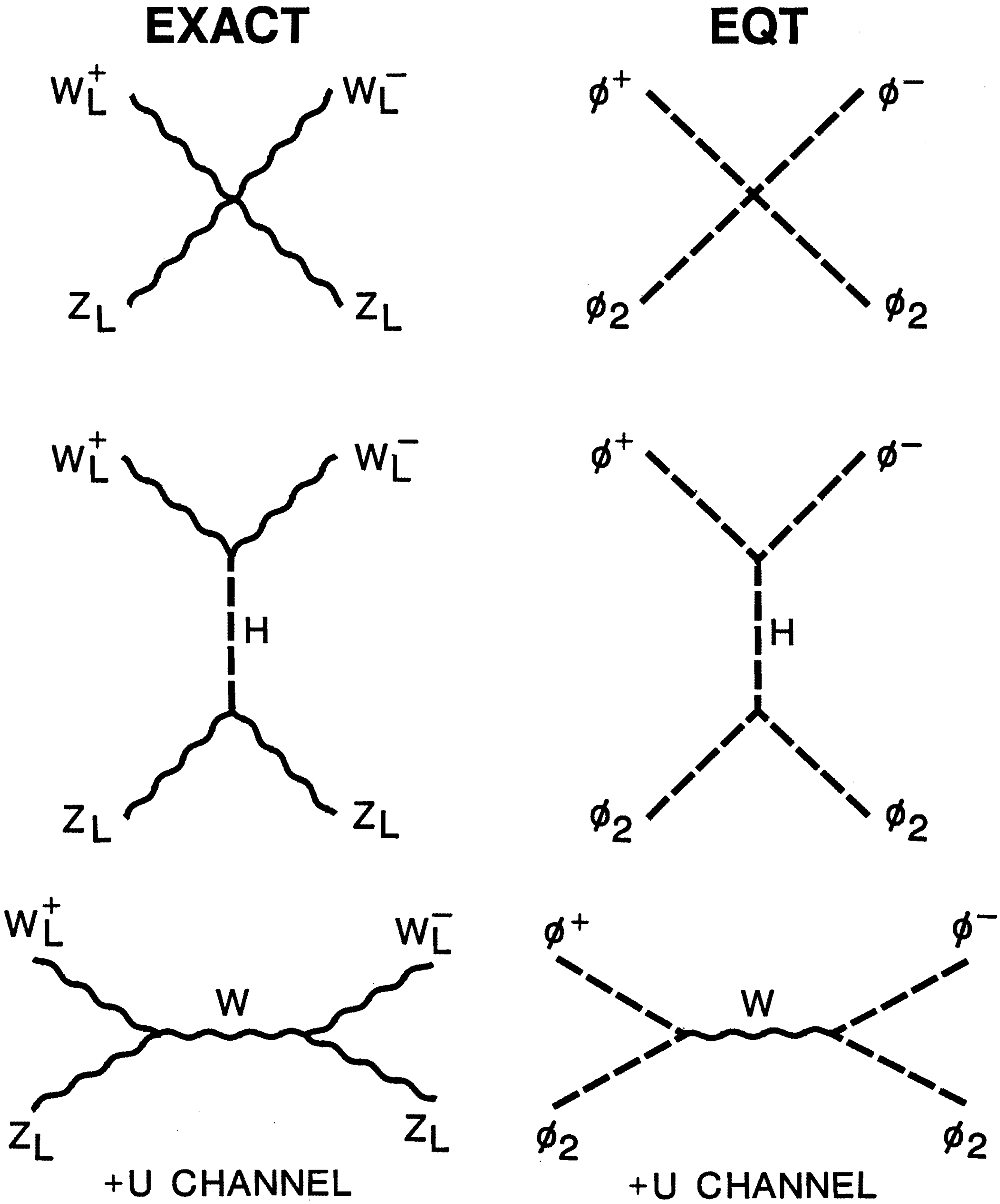
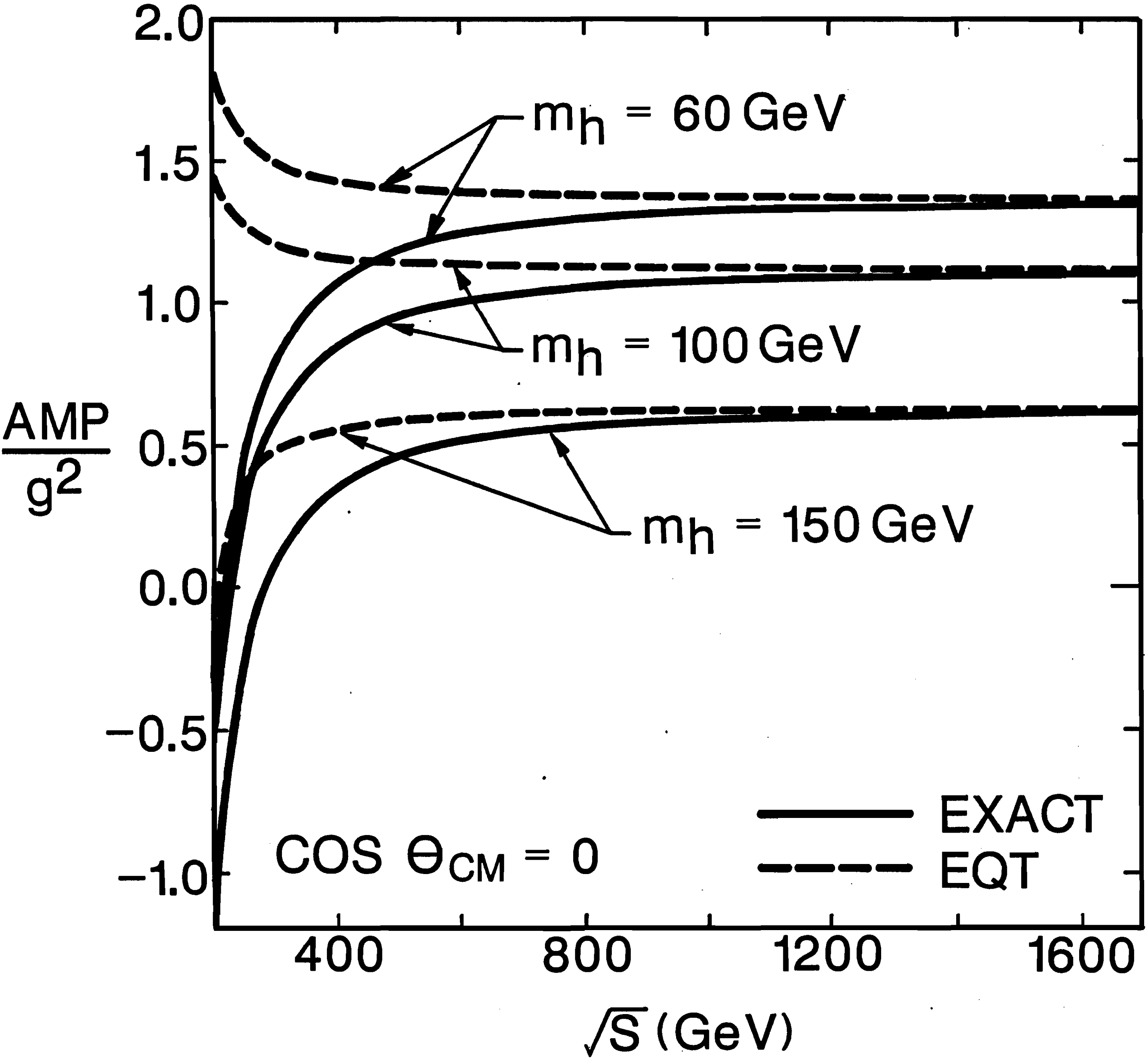
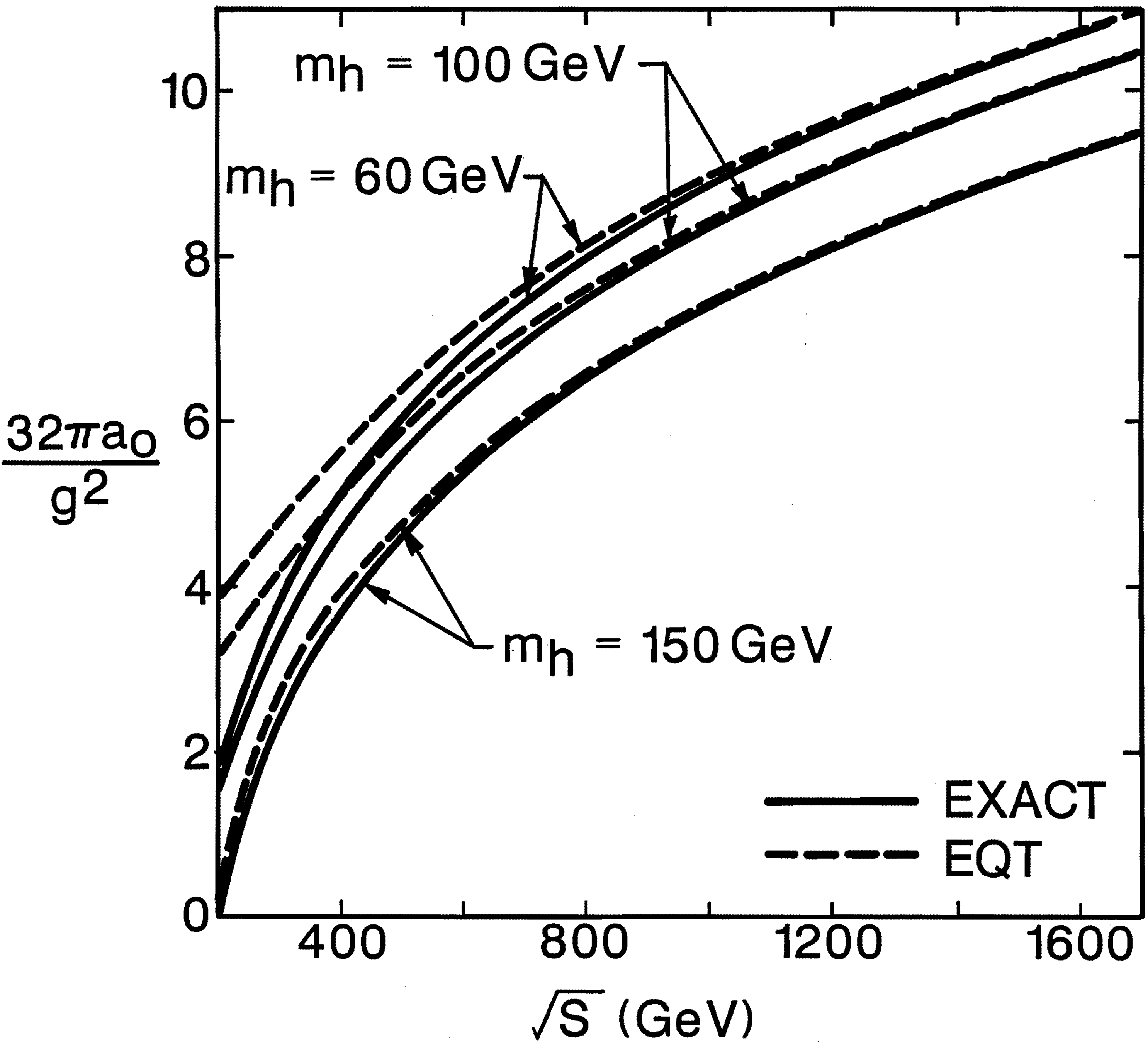

EXACT
EQT
W_L^+
W_L^-
Z_L
Z_L
ϕ^+
ϕ^-
ϕ_2
ϕ_2
H
H
W
W
+U CHANNEL
+U CHANNEL

Figure 1



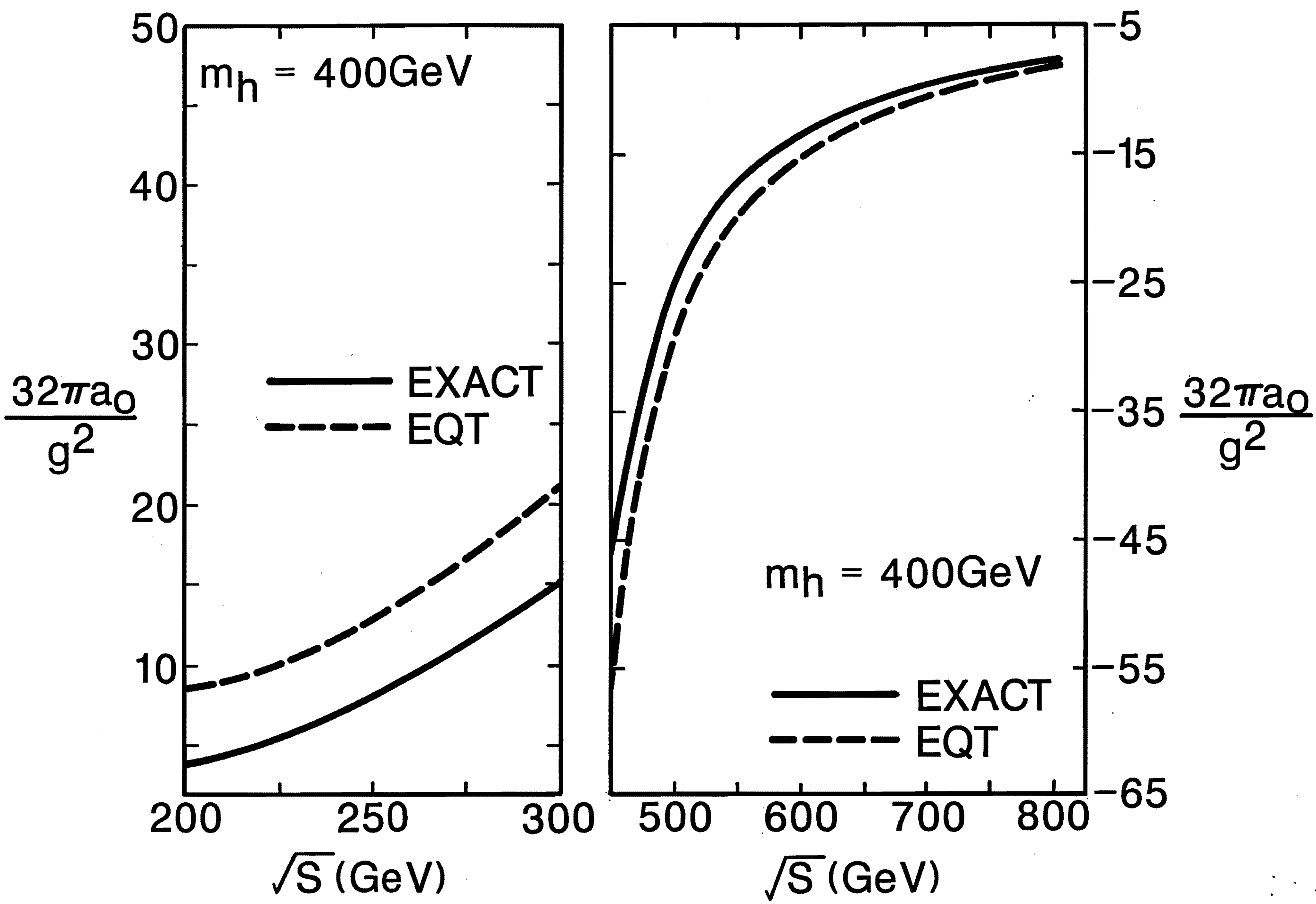
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
AMP / g2
mh = 60 GeV
mh = 100 GeV
mh = 150 GeV
COS ΘCM = 0
EXACT
EQT
400
800
1200
1600
√S (GeV)

Figure 2



m_h = 100 GeV
m_h = 60 GeV
m_h = 150 GeV
10
8
6
4
2
0
$\frac{32\pi a_0}{g^2}$
400
800
1200
1600
$\sqrt{S}$ (GeV)
EXACT
EQT

Figure 3



50
40
30
20
10
200
250
300
$\frac{32\pi a_0}{g^2}$
m_h = 400GeV
EXACT
EQT
$\sqrt{S}$ (GeV)
-5
-15
-25
-35
-45
-55
-65
500
600
700
800
$\frac{32\pi a_0}{g^2}$
m_h = 400GeV
EXACT
EQT
$\sqrt{S}$ (GeV)

Figure 4

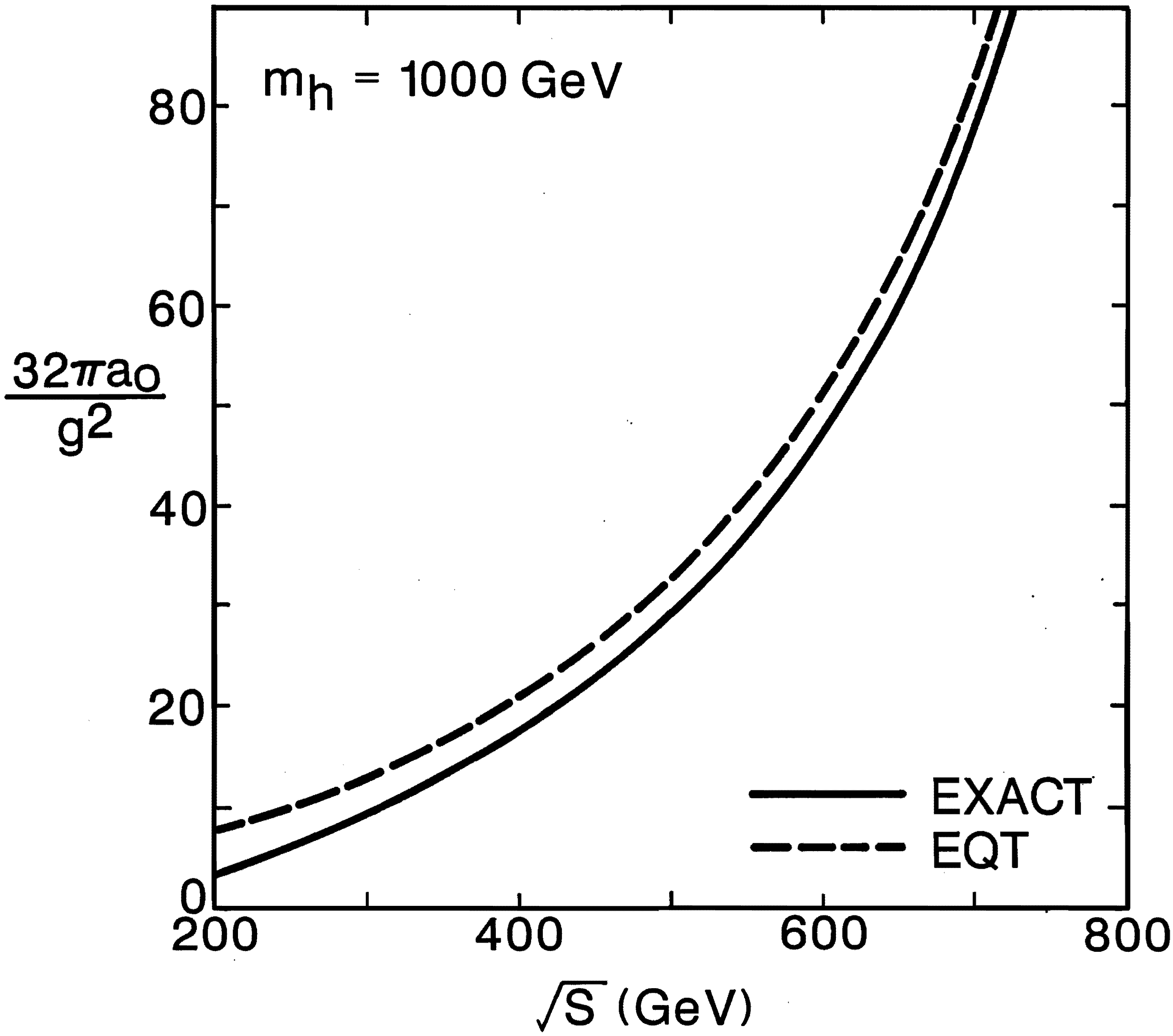
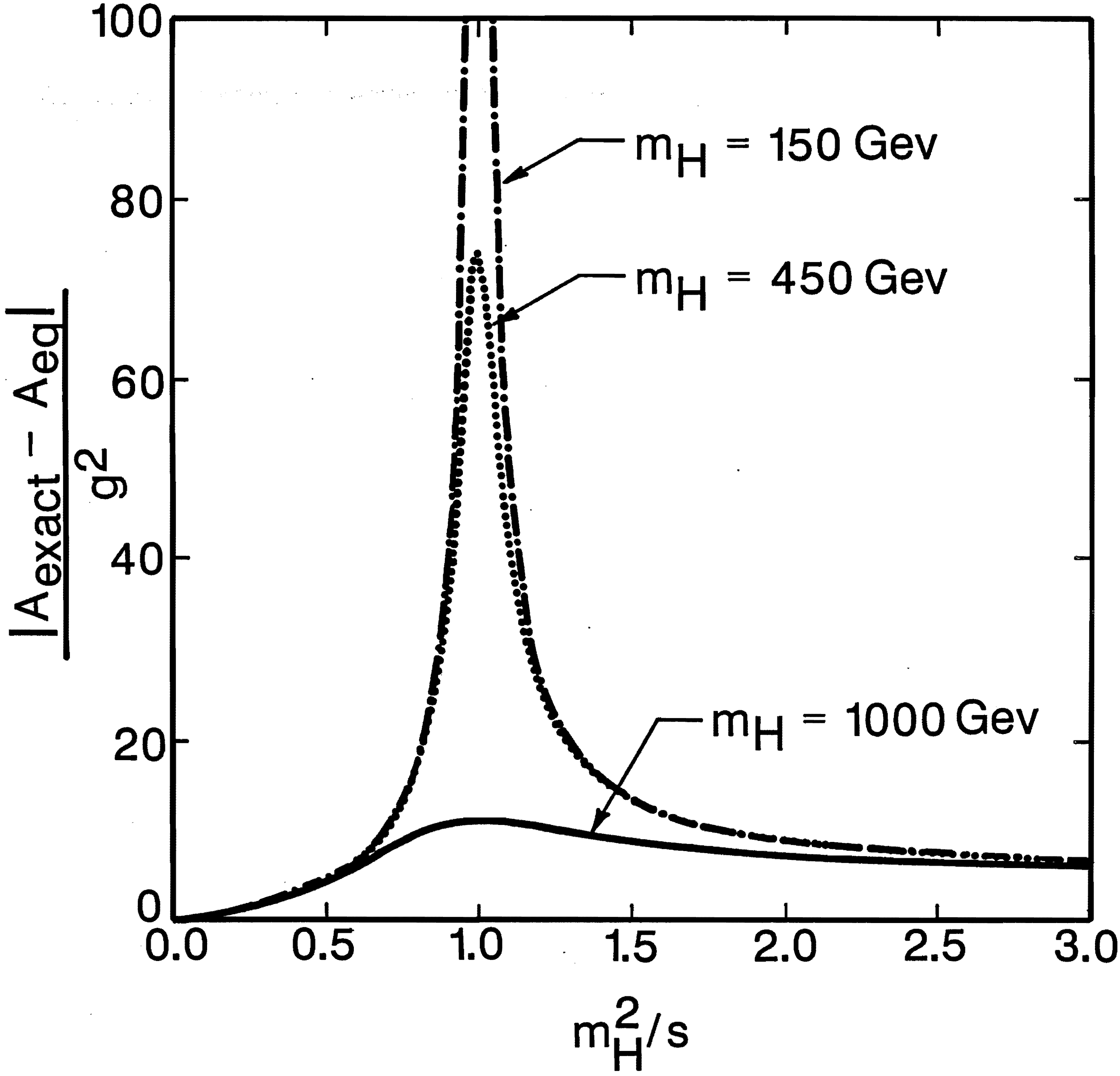

m_h = 1000 GeV
80
60
40
20
0
$\frac{32\pi a_0}{g^2}$
200
400
600
800
$\sqrt{S}$ (GeV)
EXACT
EQT

Figure 5



100
80
60
40
20
0
$\frac{|A_{exact} - A_{eq}|}{g^2}$
m_H = 150 Gev
m_H = 450 Gev
m_H = 1000 Gev
0.0
0.5
1.0
1.5
2.0
2.5
3.0
m_H^2/s

Figure 6

Please attach this page to the paper "A test of Goldstone bosan equivalence theorem" by B. Dutta and S. Nandi, OSU preprint #279.

(i) In page 3, before Eq. (4), replace $x=4m_H^2/s$ by $x=m_H^2/m_W^2$.

(ii) In page 3, after Eq. (4), in the expression for N_{EX}, replace $6\ an^2x$ by $6\ aen^2x$.

(iii) In page 4, in the expression for N_{EQ}, replace $4\ ae^3n^2x$ by $2\ ae^3n^2x$.

(iv) In page 5, Eq. (6), replace 2 by (1/2).

(v) In Fig. 6, for the vertical axis, replace g^2 by $4g^2$.