A Measurement of the Gross-Llewellyn Smith Sum Rule from the CCFR xF_3 Structure Function

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We report a measurement of the Gross-Llewellyn Smith Sum Rule: $\int \frac{dx}{x} xF_3(x, Q^2 = 3\ \mathrm{GeV}^2) = 2.50 \pm .018(\ \mathrm{stat}) \pm .078(\ \mathrm{syst})$.

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The Gross-Llewellyn Smith (GLS) Sum Rule[1] predicts that the integral of xF_3, weighted by $1/x$, equals the number of valence quarks inside a nucleon — three in the naive quark parton model. With next to leading order QCD corrections, the GLS sum rule can be written as

$$S_{\mathrm{GLS}} \equiv \int_0^1 \frac{dx}{x} xF_3(x, Q^2) = 3\left[1 - \frac{12}{(33 - 2N_f)ln(Q^2/\Lambda^2)} + \mathcal{O}(Q^{-2})\right], \quad (1)$$

where N_f is the number of quark flavors (=4) and Λ is the mass parameter of QCD. Higher twist effects, of the order $\mathcal{O}(Q^{-2})$, are expected to be small ($< 1\%$ of S_{GLS} at $x \approx 0.01$).[2] Until now, the most precise measurement of the GLS Sum Rule has come from the Narrow Band Beam (NBB) neutrino data of the CCFR collaboration[3]. The factor of 18 increase in the $\overline{\nu}$-induced charged current (CC) sample of the new data, compared to our earlier experiment, provides a much more precise determination of xF_3, and an improved measurement of S_{GLS}. In an accompanying letter, we have reported a high statistics determination of $F_2(x, Q^2)$ and $xF_3(x, Q^2)$.[4]

Due to the $1/x$ weighting in Eq.1, the small x region ($x < 0.1$) is particularly important. Accurate measurements of the following ensure small systematic errors: (a) the muon angle (θ_μ);[5] and (b) the relative $\overline{\nu}/\nu$ flux. Since xF_3 is obtained from the difference of ν and $\overline{\nu}$ cross-sections, small relative normalization errors can become magnified by the weighting in the integral. The absolute normalization uses an average of ν-N cross-section measurements.[4] Here we describe procedures for obtaining the *relative* flux: ratios of neutrino flux from energy to energy, and between $\overline{\nu}_\mu$ and ν_μ. Two methods have been used in extracting the relative flux $[\Phi(E)]$: the fixed ν-cut method and y-intercept method.[6] The two techniques yielded consistent measures of $\Phi(E)$.

The fixed ν-cut method uses the most general form for the differential cross section for the V-A neutrino nucleon interaction (Eq.1 of Ref.[4]) which requires that the number of events with $\nu < \nu_0$ in a E_ν bin, $\mathcal{N}(\nu < \nu_0)$, is proportional to the relative flux $\Phi(E_\nu)$ at that bin, up to corrections of order of $\mathcal{O}(\nu_0/E_\nu)$:

$$\mathcal{N}(\nu < \nu_0) = C\Phi(E_\nu)\nu_0\left[\mathcal{A} + (\frac{\nu_0}{E_\nu})\mathcal{B} + (\frac{\nu_0}{E_\nu})^2\mathcal{C} + \mathcal{O}(\frac{\nu_0}{E_\nu})^3\right]. \quad (2)$$

The parameter, ν_0, was chosen to be 20 GeV to simultaneously optimize statistical precision while keeping corrections small. There are 426,000 ν- and 146,000 $\overline{\nu}$-induced events in the fixed ν-cut flux analysis.

The y-intercept method comes from a simple helicity argument: the differential cross sections, $d\sigma/dy$, for ν- and $\overline{\nu}$-induced events should be equal for forward scattering, *i.e.*, as y→0.

$$\left[\frac{1}{E}\frac{d\sigma^\nu}{dy}\right]_{y=0} = \left[\frac{1}{E}\frac{d\sigma^{\overline{\nu}}}{dy}\right]_{y=0} = \text{Constant}. \quad (3)$$

Thus, in a plot of number of events versus y, the y-intercept obtained from a fit to the entire y-region is proportional to the relative flux. The fixed ν-cut and y-intercept methods of $\Phi(E)$ determination typically agreed to about 1.5% with no measureable systematic difference. A smoothing procedure was applied to minimize the effects of point-to-point flux variations.[7]

Structure functions were extracted from the CC data in the kinematic domain $E_{had} > 10$ GeV, $Q^2 > 1$ GeV^2 and $E_\nu > 30$ GeV. In this sample, there were 1,050,000 ν- and 180,000 $\overline{\nu}$-induced events. Accepted events were separated into twelve x bins and sixteen Q^2 bins from 1 to 600 GeV^2. Integrating the ν-N differential cross-section (Eq.1 of Ref.[4]) times the flux over each x and Q^2 bin gives two equations for the number of neutrino and antineutrino events in the bin in terms of the structure functions at the bin centers, x_0 and Q_0^2.

$$\Delta N^{\nu,\overline{\nu}} = \left(\int a\Phi(E)^{\nu,\overline{\nu}} dE\right)(\mathrm{F}_2(x_0, Q_0^2)) \pm \left(\int b\Phi(E)^{\nu,\overline{\nu}} dE\right)(\mathrm{xF}_3(x_0, Q_0^2)) \tag{4}$$

where a and b are known functions of x, y, E and $R(x, Q^2)$;[4] and $\Phi(E)$ is the flux. The observed numbers of events, N^ν & $N^{\overline{\nu}}$, were corrected with an iterative Monte Carlo procedure for acceptance and resolution smearing.

To solve these equations for F_2 and xF_3 certain known corrections have to be applied. We assumed a parameterization of $R(x, Q^2)$ determined from the SLAC measurements,[8] and applied corrections for the 6.85% excess of neutrons over protons in iron. We used the magnitude and the x-dependence of the strange sea determined from our opposite-sign dimuon analysis.[9] The threshold dependence of charm quark production was corrected with the slow rescaling model,[10] where the relevant charm quark mass parameter, $m_c = 1.34 \pm 0.31$GeV, was determined from our data.[9] Radiative corrections followed the calculation by De Rújula *et al.*;[11] and the cross-sections were corrected for the massive W-boson propagator. The charm-threshold, strange sea, and radiative corrections were largely independent of Q^2. For F_2, they ranged from $\pm 10\%$ at $x = .015$, to $\pm 3\%$ at $x = 0.125$, to $^{+0\%}_{-5\%}$ at $x = 0.65$ over our Q^2 range. For xF_3 they ranged from $^{+4\%}_{-0\%}$ at $x = .015$, to $^{+1.5\%}_{-0\%}$ at $x = 0.125$, to $^{+0\%}_{-6\%}$ at $x = 0.65$. Resolution smearing was corrected using a Monte Carlo calculation which incorporated the measured resolution functions from dedicated test run data.[5] We have excluded the highest x-bin, $0.7 \leq x \leq 1.0$, due to its susceptibility to Fermi motion (which was not included in the smearing correction).

To measure $S_{\rm GLS}$, the values of xF_3 were interpolated or extrapolated to $Q_0^2 = 3$ GeV^2, which is the mean Q^2 of the data in the lowest x-bin which contributes most heavily to the integral. Figure 1 shows the data and the Q^2-dependent fits used to extract $xF_3(x, Q^2 = 3)$ in three x-bins. The resulting xF_3 is then fit to a function of the form: $f(x) = Ax^b(1-x)^c$ $(b > 0)$. The best fit values are $A = 5.976 \pm 0.148$, $b = 0.766 \pm 0.010$, and $c = 3.101 \pm 0.036$. The integral of the fit weighted by $1/x$ gives $S_{\rm GLS}$. Figure 2 shows the measured $xF_3(x)$ at $Q^2 = 3$ GeV^2, as a function of x, the fits and their integrals. The measurement of the sum rule yields:[12]

$S_{\rm GLS} = \int_0^1 \frac{xF_3}{x} dx = 2.50 \pm 0.018(\text{stat.})$

Fitting different functional forms to our data,[7] gives answers within $\pm 1.5\%$ of the above. We estimate ± 0.040 to be the systematic error on $S_{\rm GLS}$ due to fitting. The dominant systematic error of the measurement comes from the uncertainty in determining the absolute level of the flux, which is 2.2%. The other two systematic errors are 1.5% from uncertainties in relative $\bar{\nu}$ to ν flux measurement and 1% from uncertainties in E_μ calibration.[7] The systematic errors are detailed in Table 1. Our value for $S_{\rm GLS}$ is:

$$S_{\rm GLS} = \int_0^1 \frac{\rm xF_3}{x} dx = 2.50 \pm 0.018(\text{ stat.}) \pm 0.078(\text{ syst.}) \qquad (5)$$

The theoretical prediction of $S_{\rm GLS}$, for the measured $\Lambda = 210 \pm 50$ MeV from the evolution of the non-singlet structure function,[7] is 2.66 ± 0.04 (Eq.1). The prediction assumes negligible contributions from higher twist effects, target mass corrections,[13] and higher order QCD corrections. [8] The world status of $S_{\rm GLS}$ measurements is shown in Fig.3.

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[12] Our present value of $S_{\rm GLS}$ (2.50) is lower than the earlier preliminary presentations (2.66) [see S.R.Mishra, Talk at Lepton-Photon, 1991, Geneva]. Two small changes in the *assumptions* of the analysis lowered $S_{\rm GLS}$. These changes, we believe, are more accurate than those employed earlier. See Ref.[7].

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D.B.MacFarlane *et al.*, *Z.Phys.* **C26**, 1(1984); d)WA25: D.Allasia *et al.*, *Phys.Lett.* **B135**, 231(1984), *ibid*, *Z.Phys.* **C28**, 321(1985).

Table 1: **Error on the Gross-Llewellyn Smith sum rule:** The statistical and systematic errors on S_{GLS} are presented.

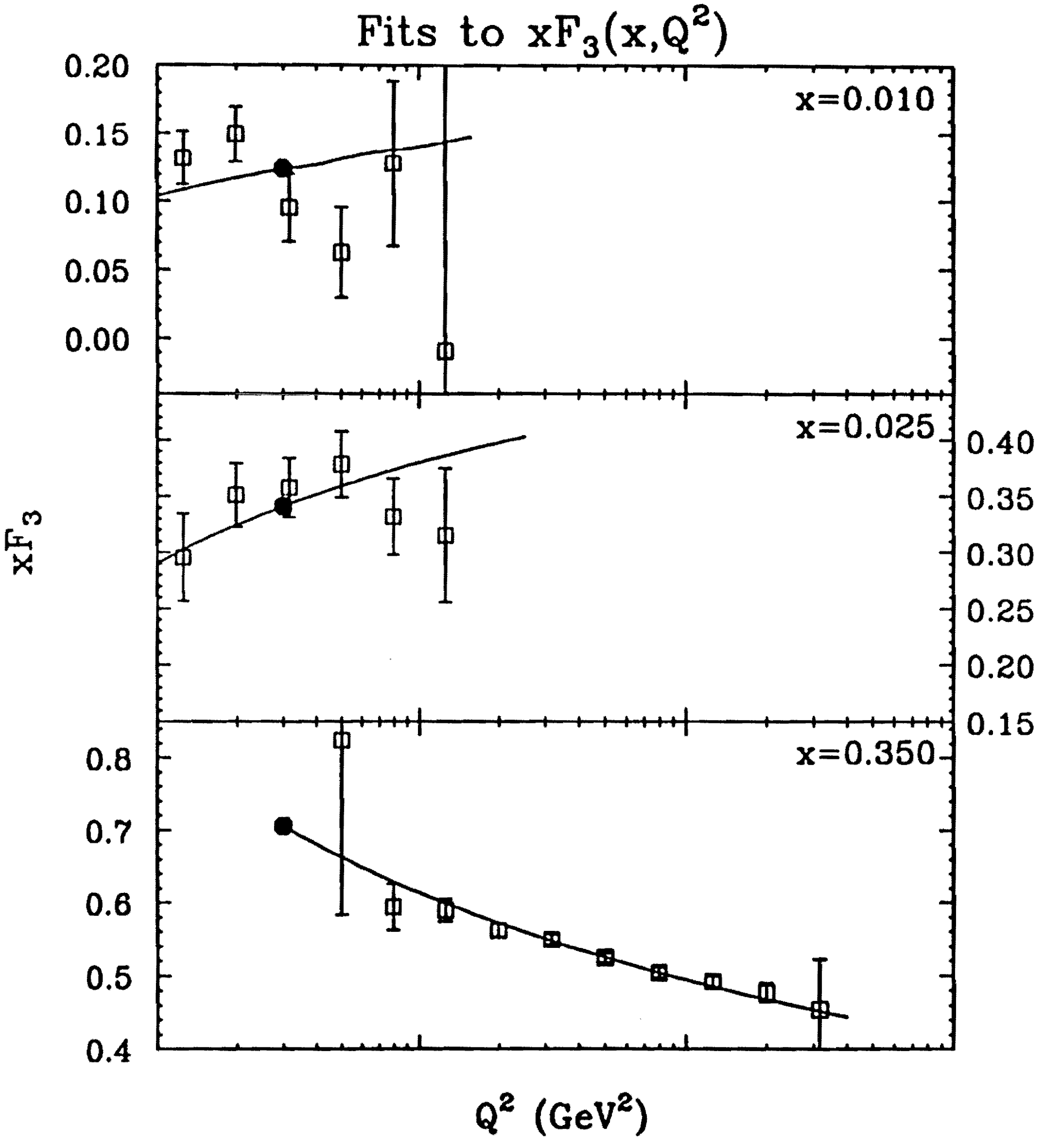
Error	Variation	ΔS_{GLS}
Statistical		$\pm$ .018
Systematic		
Fit	different fits	$\pm$ .040
$\sigma^{\nu N}$ Level	$\pm 2.1\%$	$\mp$ .056
$\frac{\sigma^{\bar{\nu} N}}{\sigma^{\nu N}}$ Level	$\pm 1.0\%$	$\mp$ .034
Energy Scale	$\pm 1.0\%$	$\pm$ .001
Rel. Calibr.	$\pm 0.6\%$	$\mp$ .010
Flux Shape	smoothing on/off	$\pm$ .006
Total		$\pm$ .078

Figure Captions

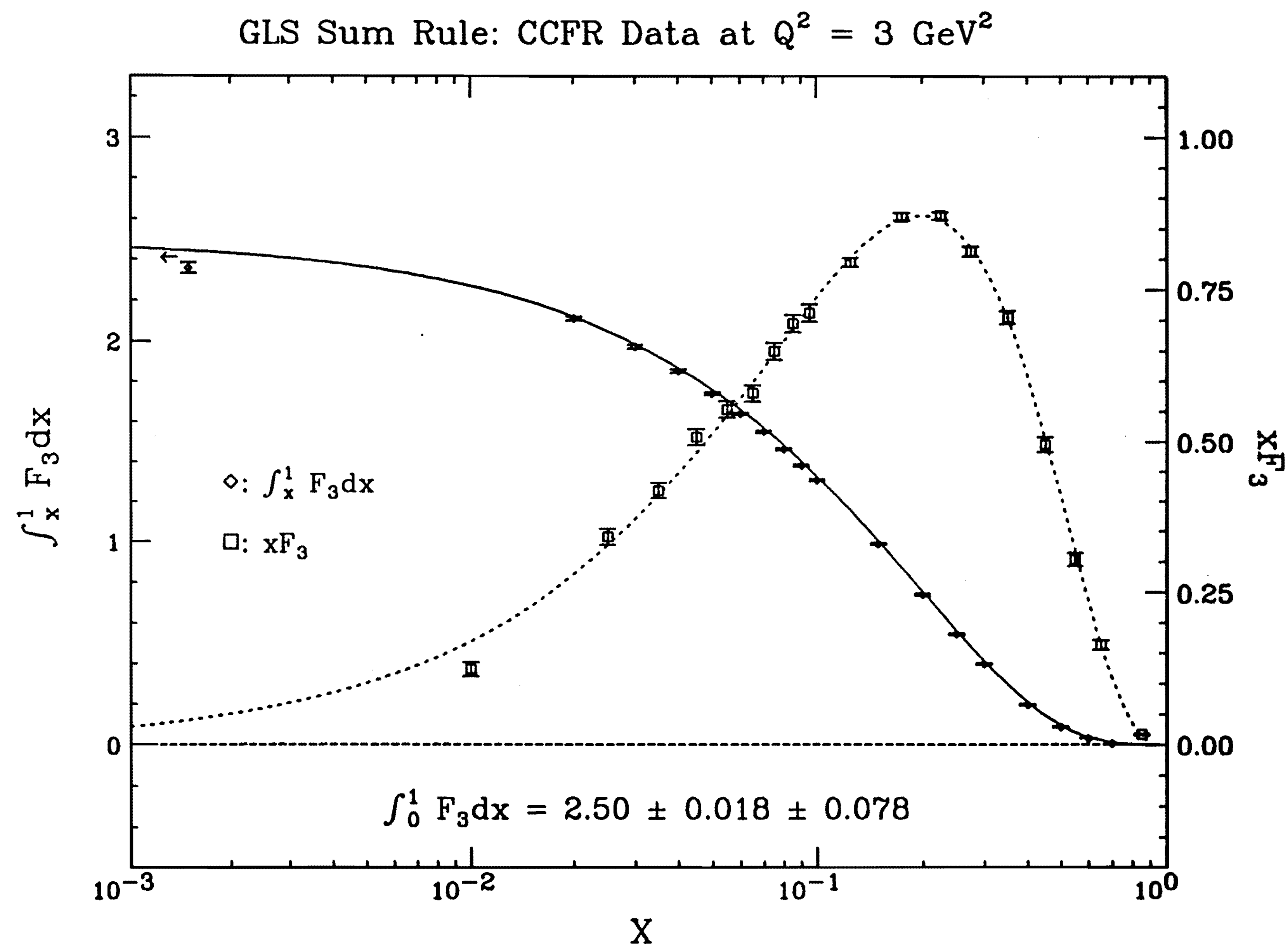
Figure 1: Fits to Q^2-dependence of xF_3 in 3 x-bins (the 2 lowest x-bins and a middle x-bin). xF_3 at $Q_0^2 = 3$ GeV^2 (squares) is obtained by interpolation, as in the 2 lowest x-bin and shown by a dark symbol, or by extrapolation as in the middle x-bin (dark symbol).

Figure 2: The GLS sum rule: The squares are $xF_3(x, Q^2 = 3)$ and the dashed line is the fit to $xF_3(x, Q^2 = 3)$ by $Ax^b(1-x)^c$. The solid line is the integral of the fit, $\int_x^1 \frac{dx}{x} xF_3$. The diamonds are an approximation to the integral computed by a weighted sum $[S(x_j)]$ of $xF_3^i = xF_3(x_i, Q^2 = 3)$, *i.e.*, $S(x_j) = \sum_j^n \Delta x_i xF_3^i$.

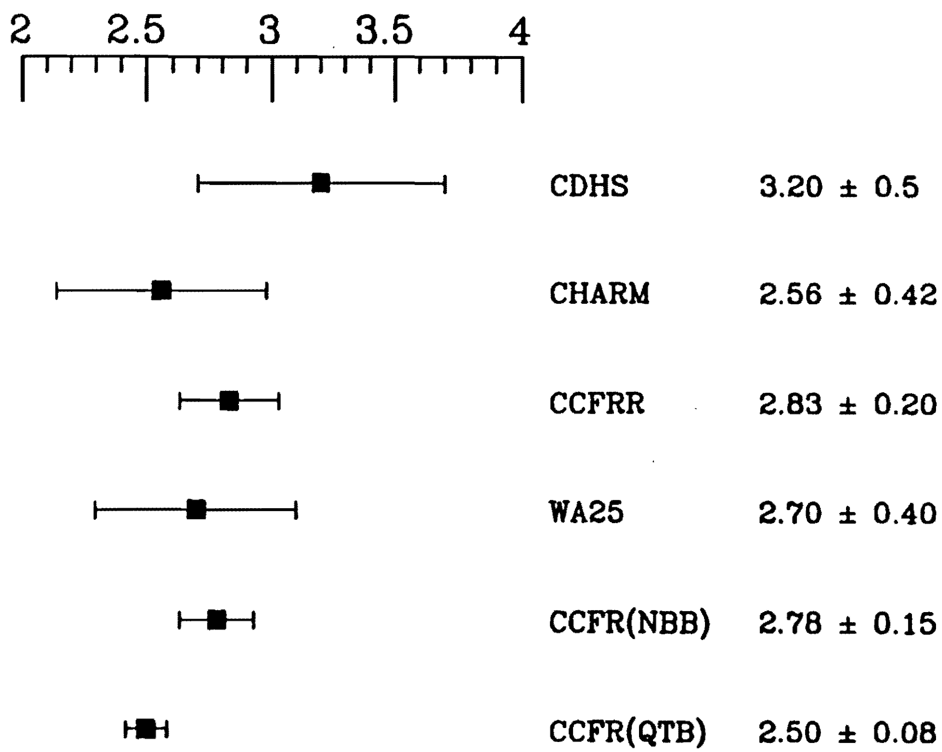
Figure 3: GLS sum rule as measured by previous experiments and these data. The references for other measurements are: CDHS[15a], CHARM[15b], CCFRR[15c], WA25[15d], and CCFR-NBB[3].



Fits to $xF_3(x,Q^2)$
x=0.010
x=0.025
x=0.350
0.20
0.15
0.10
0.05
0.00
0.40
0.35
0.30
0.25
0.20
0.15
0.8
0.7
0.6
0.5
0.4
xF_3
Q^2 (GeV^2)



GLS Sum Rule: CCFR Data at $Q^2 = 3$ GeV^2
$\int_x^1 F_3 dx$
xF_3
◇: $\int_x^1 F_3 dx$
□: xF_3
$\int_0^1 F_3 dx = 2.50 \pm 0.018 \pm 0.078$
X
3
2
1
0
1.00
0.75
0.50
0.25
0.00
10^{-3}
10^{-2}
10^{-1}
10^{0}



The Status of S_{GLS} Measurement