KFTI 91-21

Kharkov Institute

of Physics and Technology

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THREE-DIMENSIONAL N=4 EXTENDED [illegible] MECHANICS

Preprint

Moscow-Atominform-1991

UDC 539.12

BEREZOVOJ V.P., PASHNEV A.I. Three-dimensional N =4 extended supersymmetrical quantum mechanics.- Preprint KFTI 91-2I . - Kharkov,: KFTI 1991. - 11 p.

A description of 3-dimensional N = 4 extended Supersymmetrical Quantum Mechanics is proposed, based on the superfield construction of the action. The main feature of the approach is the unification of 3-dimensional bosonic coordinate vector and fermionic spinor of O(3) in one irreducible representation of N =4 supersymmetry algebra.

13 refs.

УДК 539.I2

БЕРЕЗОВОЙ В.П., ПАШНЕВ А.И. Трёхмерная N =4 расширенная суперсимметричная квантовая механика.- Препринт ХФТИ 9I-2I . - Харьков : ХФТИ I99I.- 11 с.

Предложено описание трёхмерной N =4 расширенной квантовой механики, основанное на суперполевом представлении действия. Основной особенностью подхода является объединение 3-мерного бозонного вектора координат и фермионного спинора O(3) в одном неприводимом представлении N =4 алгебры суперсимметрии.

Список лит. - I3 назв.

1. Historically Supersymmetric Quantum Mechanics (SQM) appeared as a very simple and simultaneously very efficient laboratory for the investigation of supersymmetry and its consequences in particle physics[1]. In turn, the presence of some number of supercharges commuting with the Hamiltonian determines the structure of the SQM Hamiltonian in terms of usual coordinates and momenta and additional anticommuting variables, which can be represented by matrices. Due to this matrix structure, the diagonalized Hamiltonian of SQM consists of some number of usual Quantum Mechanics Hamiltonians related to each other by some relations. In the case of $N = 2$ SQM these relations represent the well known Darboux transformations[2,3]. If the number of supercharges grows, such extended supersymmetry leads to additional interesting consequences and in the case of $N = 4$ (or $N = 2$ if we consider complex supercharges) SQM is very closely related to the Inverse Scattering Problem[4].

There is another rather important reason due to which the $N = 4$ SQM is of particular interest in the study of supersymmetry in real world. In spite of the fact that supersymmetry is very attractive from the point of view of theoretical investigations in particle physics it is difficult to extract supersymmetry implications in the frames of field theoretical approach. On the other hand, the supersymmetry algebra

$$\{Q_\alpha, Q_{\dot\beta}\} = (\sigma_\mu)_{\alpha\dot\beta} P^\mu$$

in the center of mass system becomes the algebra of $N = 4$ SQM due to equality $P^i = 0 (i = 1,2,3)$. The time component P^0 is the Hamiltonian H of the system and there are 4 real supercharges Q_m. So, the knowledge of H and Q_m allows us to investigate the spectrum of the system and at the same time the questions of supersymmetry breaking by traditional methods of ordinary Quantum Mechanics.

All dynamical variables in models of SQM, as well as in models of spinning particles, usually are considered as a components of scalar superfields Φ_μ with external vector index μ. The zero bosonic component $X_\mu$of such superfield plays the role of space (space-time) coordinate. Corresponding fermionic components Ψ^a_μ with additional index a describe the spin degrees of freedom and there are some difficulties to extend this scheme on higher N. If N increases, the number of Ψ^a_μ is proportional to N and we must have very high internal symmetry group under which index a is transformed[5]. Besides, in this approach spinor is described with the help of vector variables which are analogs of the Dirac γ-matrices.

In this paper we show how one can overcome these difficulties in the case of $N = 4$ by the use of nontrivial representation of $N = 4$ extended supersymmetry (see also [6] for $N = 2$ SQM). In the second part of the paper we describe such representation and construct for it the more general action. The only dynamical variables in this action are three bosonic and four fermionic variables transforming as a vector and complex spinor of rotation group $O(3)$.

The quantization of the model is performed in the third part of the paper and possible physical applications are discussed in conclusions.

2. The algebra of N-extended SQM has the form

$$\{Q_\alpha, Q_\beta\} = \delta_{\alpha\beta} H,$$
$$[H, Q_\alpha] = 0, (\alpha, \beta = 1, 2, ..., N) \tag{1}$$

and its automorphysm group is $SO(N)$. For $N = 4$ we can introduce complex supercharges Q_a and $\overline{Q}^a \equiv (Q_a)^\star, (a = 1, 2)$ with the following commutation relations

$$\{Q_a, \overline{Q}^b\} = \delta_a^b H,$$
$$[H, Q_a] = [H, \overline{Q}^a] = 0. \tag{2}$$

The automorphysm group is now $SO(4) = SU(2) \times SU(2)$ and Q_a transforms as a spinor of one of the $SU(2)$ groups.

In the superspace with one bosonic coordinate τ and two complex fermionic coordinates θ^a the supersymmetry transformations have the following form *

$$\delta\tau = \frac{i}{2}(\epsilon^a \overline{\theta}_a + \overline{\epsilon}_a \theta^a),$$
$$\delta\theta^a = \epsilon^a,$$
$$\delta\overline{\theta}_a = \overline{\epsilon}_a, \tag{3}$$

where ϵ^a are infinitesimal anticommuting parameters.

The covariant derivatives

$$D_a = \frac{\partial}{\partial\theta^a} - \frac{i}{2}\overline{\theta}_a \frac{\partial}{\partial\tau},$$

* Our convensions for spinors are as follows: $\overline{\theta}_a \equiv (\theta^a)^\star, \theta_a \equiv \theta^b \varepsilon_{ba}, \theta^a = \varepsilon^{ab}\theta_b, \overline{\theta}^a \equiv \varepsilon^{ab}\overline{\theta}_b, \overline{\theta}_a = \overline{\theta}^b \varepsilon_{ba}, \overline{\theta}^a = -(\theta_a)^\star, (\theta\theta) \equiv \theta^a\theta_a = -2\theta^1\theta^2, (\overline{\theta}\overline{\theta}) \equiv \overline{\theta}_a\overline{\theta}^a = (\theta\theta)^\star$.

$$\overline{D}^a = \frac{\partial}{\partial\overline{\theta}_a} - \frac{i}{2}\theta^a\frac{\partial}{\partial\tau}, \tag{4}$$

always play the important role in superspace constructions. In particular various irreducible representations can be extracted from the general superfields with the help of D_a and $\overline{D}^a$. The examples are the chiral ($\overline{D}^a\Phi = 0$) and antichiral ($D_a\Phi = 0$) superfields each having two bosonic and four fermionic dynamical degrees of freedom. In view describing one-dimensional SQM, we have to reduce bosonic degrees of freedom to one and consider another representations. Two types of them are singled-out from real superfields by constraints [7]

$$\{\varepsilon^{ac}D_c\overline{D}^b + \varepsilon^{bc}D_c\overline{D}^a\}\Phi = 0, \tag{5}$$

or[8,9]

$$D^aD_a\Phi = \overline{D}_a\overline{D}^a\Phi = [D_a, \overline{D}^a]\Phi = 0. \tag{6}$$

This last superfield has the following form

$$\Phi = X + \theta^a\overline{\Psi}_a - \overline{\theta}_a\Psi^a + \theta^a(\sigma_i)_a^b\overline{\theta}_bB_i + \frac{i}{4}(\theta\theta)\overline{\theta}_a\dot{\overline{\Psi}}^a - \frac{i}{4}(\overline{\theta}\overline{\theta})\theta^a\dot{\Psi}_a + \frac{1}{16}(\theta\theta)(\overline{\theta}\overline{\theta})\ddot{X} \tag{7}$$

and can be used to describe one-dimensional $N = 4$ SQM [7-9] as well as $N = 4$ spinning particle if superfield Φ has additional vectorlike index μ: $\Phi \rightarrow \Phi_\mu$ [10]. If, instead, we consider three dimensional vector superfield Φ_i with external index i we can describe three dimensional $N = 4$ SQM but encounter the difficulties enumerated in introduction.

To construct another representation of the algebra (2) let us consider the transformation laws for components X, Ψ^a, B_i :

$$\delta X = \epsilon^a\overline{\Psi}_a - \overline{\epsilon}_a\Psi^a,$$

$$\delta\Psi^a = \frac{i}{2}\epsilon^a\dot{X} + \epsilon^b(\sigma_i)_b^aB_i, \tag{8}$$

$$\delta B_i = -\frac{i}{2}\epsilon^a(\sigma)_a^b\dot{\overline{\Psi}}_b + \frac{i}{2}\dot{\Psi}^a(\sigma_i)_a^b\overline{\epsilon}_b \equiv -\frac{i}{2}\epsilon\sigma_i\dot{\overline{\Psi}} + \frac{i}{2}\dot{\Psi}\sigma_i\overline{\epsilon}.$$

In terms of new variables $X_a^b, \chi^a, \overline{\chi}_a, F$ connected with the old ones by relations

$$\dot{X}_a^b = -B_a^b \equiv -B_i(\sigma)_a^b, F = \dot{X}, \chi^a = \Psi^a \tag{9}$$

the transformation law (8) has the form :

$$\delta X_a^b = -\frac{i}{2}(\overline{\chi}_a\epsilon^b - \epsilon_a\overline{\chi}^b + \chi_a\overline{\epsilon}^b - \overline{\epsilon}_a\chi^b),$$

$$\delta\chi^a = -\epsilon^b \dot{X}_b^a + \frac{i}{2}\epsilon^a F, \qquad (10)$$

$$\delta F = \epsilon^a\dot{\overline{\chi}}_a - \overline{\epsilon}_a\dot{\chi}^a.$$

The physical meaning of bosonic variables in (8) and (10) is different. There is only one dynamical and three auxiliary bosonic variables in the transformation law (8) and conversely only one auxiliary and three dynamical bosonic variables in (10). All these variables are components of the superfield

$$\Phi_a^b = X_a^b + \frac{i}{2}(\theta_a\overline{\chi}^b - \overline{\chi}_a\theta^b) + \frac{i}{2}(\overline{\theta}_a\chi^b - \chi_a\overline{\theta}^b) + \frac{1}{4}(\theta_a\overline{\theta}^b - \overline{\theta}_a\theta^b)F-$$

$$-\frac{i}{2}\dot{X}_d^c(\theta_a\overline{\theta}_c\epsilon^{bd} - \overline{\theta}_a\theta^d\delta_c^b + \theta^d\overline{\theta}_c\delta_a^b) + \frac{1}{8}(\overline{\theta}\overline{\theta})(\theta_a\dot{\chi}^b - \dot{\chi}_a\theta^b)+$$

$$+\frac{1}{8}(\theta\theta)(\overline{\theta}_a\dot{\overline{\chi}}^b - \dot{\overline{\chi}}_a\theta^b) + \frac{1}{16}\theta\theta\overline{\theta}\overline{\theta}\ddot{X}_a^b, \qquad (11)$$

or, equivalently, due to tracelessness of Φ_a^b,

$$\Phi_i \equiv \frac{1}{2}\Phi_a^b(\sigma_i)_b^a = X_i + \frac{i}{2}\theta\sigma_i\overline{\chi} - \frac{i}{2}\chi\sigma_i\overline{\theta} + \frac{1}{4}\theta\sigma_i\overline{\theta}F + \frac{1}{2}\varepsilon_{ikl}\dot{X}_k(\theta\sigma_l\overline{\theta})$$

$$+\frac{1}{8}(\theta\theta)\overline{\theta}\sigma_i\dot{\overline{\chi}} + \frac{1}{8}(\overline{\theta}\overline{\theta})\theta\sigma_i\dot{\chi} + \frac{1}{16}\theta\theta\overline{\theta}\overline{\theta}\ddot{X}_i. \qquad (12)$$

The superfield Φ (7) is the prepotential to superfield Φ_a^b . Indeed, the combination $D_a\overline{D}^b\Phi + \frac{i}{2}\delta_a^b\dot{\Phi}$ has exactly the form of expression (11) (see also [13]). Thus, we have the superfield Φ_i which has the vector index i and describe only one irredu cible representation of the supersymmetry algebra (2). It differs from scalar superfield with the external index Φ_i in the form (7), which describes d copies of irreducible representations, where d is the dimension of space (or space-time). In our case index i is not external index and Φ_i transforns as a vector of one of $SU(2)$ subgroup of the automorphysms group $SO(4) = SU(2) \times SU(2)$. X_i and χ^a components transform as a vector and spinor of this $SU(2)$ which is isomorphic to the group $SO(3)$ and can be considered as a rotation group of three-dimensional Eucleadian space. The component F of superfield Φ_i is inert under the rotations, but it changes the sign under the parity transformation $\Phi_i \to -\Phi_i$. It means that F is a pseudoscalar of $O(3)$.

The general action in terms of superfield Φ_i is a full superspase integral

$$S_0 = \frac{1}{2} \int d\tau d^4\theta V(\Phi_i)$$

to which one can add parity-violating Fayet-Illiopulos term

$$S_1 = \frac{\alpha}{2} \int d\tau F.$$

After integration over θ the Lagrangian of the theory is completely determined by one arbitrary function of coordinates $W \equiv \partial_i \partial_i V(X_j)$ and by one arbitrary constant α which also characterizes the parity violation :

$$L = W(X)\{\frac{1}{2}\dot{X}_i^2 - \frac{i}{4}\dot{\chi}^d \overline{\chi}_d + \frac{i}{4}\chi^d \dot{\overline{\chi}}_d + \frac{1}{8}F^2\} - \frac{1}{8}F(\chi\sigma_i\overline{\chi})\partial_i W +$$

$$+\frac{1}{4}\varepsilon_{ikl}\dot{X}_k(\chi\sigma_l\overline{\chi})\partial_i W - \frac{1}{32}(\chi\chi)(\overline{\chi}\overline{\chi})\partial_i^2 W + \frac{\alpha}{2}F. \quad (13)$$

Evaluating F due to its equation of motion and taking the redefinition $\Psi^a = \frac{1}{2}\sqrt{W(X)}\chi^a$ we obtain the final expression for the classical Lagrangian in terms of bosonic vector X_i and fermionic spinor Ψ^a

$$L = \frac{1}{2}W(X)\dot{X}_i^2 - i\dot{\Psi}^d\overline{\Psi}_d + i\Psi^d\dot{\overline{\Psi}}_d + \varepsilon_{ikl}\dot{X}_i(\Psi\sigma_k\overline{\Psi})\partial_l \ln W +$$

$$+\alpha(\Psi\sigma_k\overline{\Psi})\frac{\partial_k W}{W^2} - \frac{\alpha^2}{2W} - \frac{1}{2}(\Psi\Psi)(\overline{\Psi}\overline{\Psi})(\frac{\partial_i^2 W}{W^2} - \frac{(\partial_i W)^2}{2W^3}). \quad (14)$$

From the physical point of view this Lagrangian describes the motion of the particle in gravitational background with conformally flat metric $g_{ik} = \delta_{ik}W(X)$. The corresponding classical Hamiltonian has the form

$$H = \frac{1}{2W(X)}P_i^2 - \varepsilon_{ikl}P_i(\Psi\sigma_k\overline{\Psi})\frac{\partial_l W}{W^2} + \frac{\alpha^2}{2W} -$$

$$-\alpha(\Psi\sigma_k\overline{\Psi})\frac{\partial_k W}{W^2} + \frac{1}{2}(\Psi\Psi)(\overline{\Psi}\overline{\Psi})(\frac{\partial^2 W}{W^2} - \frac{3}{2}\frac{\partial_i W^2}{W^3}) \quad (15)$$

with

$$P_i = W(X)\dot{X}_i + \varepsilon_{ikl}(\Psi\sigma_k\overline{\Psi})\frac{\partial_l W}{W}. \quad (16)$$

Applying the Noether procedure to the $N = 4$ invariant Lagrangian (14) we find the conserving supercharges Q_a and $\overline{Q}^b$:

$$Q_a = \frac{1}{\sqrt{W}}\{(\sigma_I)_a^c\overline{\Psi}_c P_i + i\alpha\overline{\Psi}_a + \frac{i}{2}(\sigma_i)_a^c\Psi_c(\overline{\Psi}\overline{\Psi})\frac{\partial_i W}{W}\},$$

$$\overline{Q}^b = \frac{1}{\sqrt{W}}\{\Psi^c(\sigma_k)_c^b P_k - i\alpha\Psi^b + \frac{i}{2}(\sigma_k)_c^b\overline{\Psi}^c(\Psi\Psi)\frac{\partial^k W}{W}\}. \quad (17)$$

3. To quantize the model consider momenta conjugated to the X_i (16) and to the Ψ^a :

$$\pi_a \equiv \frac{\delta L}{\delta\dot{\Psi}^a} = -i\overline{\Psi}_a,$$

$$\overline{\pi}^a \equiv \frac{\delta L}{\delta\dot{\overline{\Psi}}_a} = -i\Psi^a. \quad (18)$$

Due to definitions (18) we have the following second class constraints

$$\lambda_a = \pi_a + i\overline{\Psi}_a,$$

$$\overline{\lambda}^a = \overline{\pi}^a + i\Psi^a, \quad (19)$$

leading to the Dirac brackets for canonical variables

$$\{X_i, P_j\} = \delta_{ij},$$

$$\{\Psi^a, \overline{\Psi}_b\} = -\frac{i}{2}\delta_b^a. \quad (20)$$

In terms of these Dirac brackets the algebra of conserving charges is

$$\{Q_a, \overline{Q}^b\} = \delta_a^b H, \quad (21)$$

$$\{H, Q_a\} = \{H, \overline{Q}^b\} = 0.$$

After quantization Dirac brackets (20) turn into canonical commutation relations

$$[X_i, P_j] = i\delta_{ij},$$

$$\{\Psi^a, \overline{\Psi}_b\} = \frac{1}{2}\delta^a_b. \tag{22}$$

To obtain the quantum expressions for Hamiltonian and supercharges we must solve the operator ordering ambiguity. Such ambiguities always take place when the operator expression contains the product of noncommuting operators (Ψ^a and $\overline{\Psi}_b$, X_i and P_j in our case). It was shown in [11] that the requirement of general coordinate invariance leads to solution of this ambiguity up to the term proportional to the scalar curvature of the space. The realization of supersymmetry algebra commutation relations on quantum level is the additional requirement [6] and, as was shown in [12] it makes it possible to solve the operator ordering ambiguity. Technically it means that the expressions for quantum operators Q_a and $\overline{Q}^a$ are of the form (17) with additional terms, proportional to the difference between operators with various ordering of noncommuting operators. These operators Q_a and $\overline{Q}^a$ must be conjugated one to another with respect to the natural scalar product

$$< \phi_1, \phi_2 >= \int d^3X \sqrt{g} \phi_1{}^*(X) \phi_2(X), \tag{23}$$

where $\sqrt{g} = W^{\frac{3}{2}}(X)$. The commutation relations (2) fix then all additional terms and define the quantum Hamiltonian simultaneously.

Such procedure leads in our case to the following expressions for Q_a, $\overline{Q}^a$ and H :

$$\hat{Q}_a = ((\sigma_i)^b_a D_i + i\alpha\delta^b_a)\frac{\overline{\Psi}_b}{\sqrt{W}},$$

$$\hat{\overline{Q}}^a = \frac{\Psi^b}{\sqrt{W}}((\sigma_i)^a_b D^*{}_i - i\alpha\delta^a_b), \tag{24}$$

$$\hat{H} = -\frac{1}{2W(X)}\partial_i^2 - \frac{\partial_i W(X)}{4W^2(X)}\partial_i + \frac{1}{2}\frac{\alpha^2}{W(X)} - \frac{3\partial_i^2 W(X)}{8W^2(X)} + \frac{15}{32}\frac{(\partial_i W(X))^2}{W^3(X)}$$

$$+i\varepsilon_{ikl}\Psi\sigma_i\overline{\Psi}\frac{\partial_k W(X)}{W^2(X)}\partial_l + (\overline{\Psi}_a\Psi^a - \frac{1}{2})^2\left(\frac{\partial_i^2 W(X)}{W^2(X)} - \frac{3}{2}\frac{\partial i W(X)^2}{W^3(X)}\right) -$$

$$-\alpha\Psi\sigma_i\overline{\Psi}\frac{\partial_i W(X)}{W^2(X)},$$

where

$$D_i = P_i - i(\overline{\Psi}_a\Psi^a - \frac{1}{2})\partial_i \ln W(X), \tag{25}$$

and

$$P_i = -i\partial_i - \frac{3i}{4}\partial_i \ln W(X)$$

is the hermitian momenta.

As a concequence of commutation relations (22) the operators Ψ^a and $\overline{\Psi}_a$ have realization in terms of 4×4 matrices. However, more convenient from the physical point of view is the Fock space representation with $\overline{\Psi}_a$ as a creation and Ψ^a as annihilation operators. If $\mid 0 >$ is a vacuum of the Fock space such that $\Psi^a \mid 0 >= 0$, then the whole space contains four independent vectors

$$\mid 0 >, \overline{\Psi}_a \mid 0 >, \overline{\Psi}_b \overline{\Psi}^b \mid 0 > . \qquad (26)$$

The structure of the Hamiltonian (24) is such that it does not commute with the angular momentum operator $\hat{M}_i = \varepsilon_{ikl} X_k P_l$. Instead, the conserving quantity is

$$\hat{J}_i = \hat{M}_i + \Psi^a (\sigma_i)_a^b \overline{\Psi}_b \equiv \hat{M}_i + \hat{S}_i \qquad (27)$$

with additional term $\hat{S}_i$ describing spin $\frac{1}{2}$. It means that two of physical states (26) $\mid 0 >$ and $\overline{\Psi}_b \overline{\Psi}^b \mid 0 >$ have integer spin and another two states $\overline{\Psi}_a \mid 0 >$ have half integer spin. The Hamiltonian is diagonal on the states with integer spin and has 2×2 matrix structure on the half integer spin states. The operators $\hat{Q}_a$ and $\hat{\overline{Q}}^b$, when applied to the state with definite spin, change the value of spin by half, and connect, thus, the solutions of Schrôdinger equation $\hat{H}\phi = E\phi$ with integer and half integer spin. More detailed analisis of such connection will be given elsewhere.

4. In general the equations (24) describe the model in curved space with conformally-flat metric, depending on the function $W(X)$. In particular, the metric can be flat, leading in some coordinate system to the standard kinetic term in the Hamiltonian. On the other hand we can consider this model as a description of two-particle gravitating system with X_i as relative coordinate. The gravitation interaction must then fix the function $W(X)$ leaving undetermined only the parameter α. Apparently the nonzero α leads to the parity breaking and to the spontaneous breaking of supersymmetry.

After the completion of this investigation we get the paper [13] in which the $N = 4$ superfield approach to the classical Lagrangian (14) with $\alpha = 0$ was constructed.

Acknowledgements: The authors are grateful to D.P.Sorokin for useful discussions. A.P. thanks V.O. Rivelles for interest to the work and useful comments. He is also grateful to Instituto de Fisica, Universidade de Sao Paulo, the organisers of the VI J.A.Swieca Summer School and especially Prof. V.O. Rivelles for their warm hospitality during his stay in Brazil.

References

[1] E.Witten, Nucl.Phys. B188 (1981) 513
[2] C.V.Sukumar, J.Phys.A: Math. and Gen. 18 (1985) 2917
[3] V.P.Berezovoj, A.I.Pashnev, Theor.Math.Phys. 70 (1987) 102
[4] V.P.Berezovoj, A.I.Pashnev, Ibid, 74 (1988) 264
[5] J.C.D'Olivo, L.F.Urrutia, F.Zertuche, Phys.Rev.D 32 (1985) 2174
[6] V.P.Akulov, A.I.Pashnev, Theor.Math.Phys., 65 (1985) 1027
[7] V.P.Berezovoj, A.I.Pashnev, Preprint KFTI 91, Kharkov, (1991)
[8] E.A.Ivanov, S.O.Krivonos, V.M.Leviant, Preprint JINR E2-89-30, Dubna, 1989
[9] E.A.Ivanov, S.O.Krivonos, A.I.Pashnev, Class.Quantum Grav. 8 (1991) 19
[10] A.I.Pashnev, D.P.Sorokin, Preprint KFTI 90-31, Kharkov 1990, Phys.Lett. B (in press)
[11] B.S.De Witt, Phys.Rev. 85 (1952) 653
[12] V.De Alfaro, S.Fubini, S.Furlan, M.Roncadelli, Nucl.Phys. B296 (1988) 402
[13] E.A.Ivanov, A.V.Smilga, Preprint ITEP 95-90, Moscow, 1990

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ТРЕХМЕРНАЯ N=4 РАСШИРЕННАЯ СУПЕРСИММЕТРИЧНАЯ КВАНТОВАЯ МЕХАНИКА

Ответственный за выпуск Л.М.Ракивненко
Научный редактор А.И.Пашнев

Редактор Т.И.Бережная

Подписано в печать 05.02.91. Формат 60x84/16. Бум.писч. № 1.
Офсетн.печ. Усл.п.л. 0,9. Уч.-изд.л. 0,6. Тираж 280. Заказ № 205.
Цена 1 р. 20 к. Индекс 3624.

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