N. S. Serikbaev¹, Kur. Myrzakul¹, S. K. Saiymbetova¹,
A. D. Koshkinbaev¹, R. Myrzakulov²

SELF-DUAL YANG–MILLS EQUATION
AND DEFORMATION OF SURFACES

¹Institute of Physics and Technology, 480082, Almaty, Kazakhstan
²E-mail: myrzakul@thsun1.jinr.ru, cnlpmyra@satsun.sci.kz
1 Introduction

Several nonlinear phenomena in physics, modelled by the nonlinear differential
equations, can describe also the evolution of surfaces in time. The interaction
between differential geometry of surfaces and nonlinear differential equations has
been studied since the 19th century. This relationship is based on the fact that
most of the local properties of surfaces are expressed in terms of nonlinear dif­
ferential equations. Since the famous sine-Gordon and Liouville equations, the
interrelation between nonlinear differential equations of the classical differential
geometry of surfaces and modern soliton equations has been studied from vari­
ous points of view in numerous papers. In particular, the relationship between
deformations of surfaces and integrable systems in 2+1 dimensions was studied
by several authors [1-14, 27-28, 30].

The Self-Dual Yang - Mills equation (SDYME) is a famous example of nonlin­
erar differential equations in four dimensions integrable by the inverse scattering
method [16]-[17]. Ward conjectured that all integrable (1+1)-dimensional nonlin­
erar differential equations may be obtained from SDYME by reduction [18] (see
the book [19] and references therein). More recently, many soliton equations in
2+1 dimensions have been found as reductions of the SDYME [20]-[23].

In this paper we study the deformation of surfaces in the context of its con­
nection with integrable systems in 2+1 and 3+1 dimensions. We show that many
integrable (2+1)-dimensional nonlinear differential equations can be obtained
from the deformed or (2+1)-dimensional Gauss - Mainardi - Codazzi equation
(GMCE) describing the deformation (motion) of the surface, as exact particular
cases. At the same time, integrable isotropic spin systems in 2+1 dimensions are
exact reductions of the (2+1)-dimensional or, in other words, deformed Gauss
- Weingarten equation (GWE). Also we show that the deformed GMCE is the
exact reduction of two famous multidimensional integrable system, namely, the
Yang - Mills - Higgs - Bogomolny equation and the SDYME.
2 Fundamental facts on the theory of surfaces

Let us consider a smooth surface in $\mathbb{R}^3$ with local coordinates $x$ and $t$, where $r(x, t)$ is a position vector. The first and second fundamental forms of this surface are given by

$$I = dr^2 = E dx^2 + 2Fdxdt + Gdt^2, \quad (1a)$$
$$II = dr \cdot n = L dx^2 + 2Mdxdt + Ndt^2, \quad (1b)$$

where by definition

$$E = r_x^2, \quad F = r_x \cdot r_t, \quad G = r_t^2,$$
$$L = r_{xx} \cdot n, \quad M = r_{xt} \cdot n, \quad N = r_{tt} \cdot n.$$

The unit normal vector $n$ to the surface is given by

$$n = \frac{r_x \times r_t}{|r_x \times r_t|}.$$

There exists the third fundamental form

$$III = d\mathbf{n} \cdot d\mathbf{n} = edx^2 + 2fdxdt + gd^2, \quad (2)$$

This form, in contrast to $II$, does not depend on the choice of $n$ and contains no new information, since it is expressible in terms of $I$ and $II$ as

$$III = 2II \cdot n - K \cdot I, \quad (3)$$

where $K$, $H$ are the gaussian and mean curvatures, respectively. As is well known in surface theory, the GWE for surface can be written as

$$r_{xx} = \Gamma^1_{11} r_x + \Gamma^1_{12} r_t + L n, \quad (4a)$$
$$r_{xt} = \Gamma^1_{12} r_x + \Gamma^1_{22} r_t + M n, \quad (4b)$$
$$r_{tt} = \Gamma^1_{22} r_x + \Gamma^1_{22} r_t + N n, \quad (4c)$$
$$n_x = \Gamma^2_{11} r_x + \Gamma^2_{12} r_t, \quad (4d)$$
$$n_t = \Gamma^2_{22} r_x + \Gamma^2_{22} r_t, \quad (4e)$$

where

$$\Gamma^1_{11} = \frac{GE_x - 2FF_x + FE_t}{2g}, \quad \Gamma^1_{12} = \frac{2EF_x - EE_x - FE_t}{2g},$$
$$\Gamma^1_{22} = \frac{GE_t - FG_x}{2g}, \quad \Gamma^2_{12} = \frac{EG_x - FE_t}{2g},$$
$$\Gamma^2_{22} = \frac{2GF_t - GG_x - FG_t}{2g}, \quad \Gamma^2_{22} = \frac{EG_t - 2FF_t + FG_t}{2g}, \quad (5)$$

2
Here
\[ g = EG - F^2. \]

Now we introduce the orthogonal basis as
\[ e_1 = \frac{r_x}{\sqrt{E}}, \]
\[ e_2 = n, \]
\[ e_3 = e_1 \wedge e_2. \]

Hence
\[ r_x = \frac{F}{\sqrt{E}} e_1 - \sqrt{E} e_3. \]

Then the GWE takes the form
\[ \begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix} = A \begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix}, \]
\[ \begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix} = C \begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix}, \]
where
\[ A = \begin{pmatrix} 0 & \kappa & -\sigma \\ -\kappa & 0 & \tau \\ \sigma & -\tau & 0 \end{pmatrix}, \]
\[ C = \begin{pmatrix} 0 & \omega_3 & -\omega_2 \\ -\omega_3 & 0 & \omega_1 \\ \omega_2 & -\omega_1 & 0 \end{pmatrix}. \]

and
\[ \kappa_3 = \kappa = \frac{l_3}{\sqrt{E}}, \]
\[ \tau_3 = \tau = \sqrt{\frac{g}{E} P_{31}^2}, \]
\[ \kappa_3 = \sigma = \frac{\sqrt{g}}{E} P_{31}^2, \]
and
\[ \omega_3 = -\frac{\sqrt{g}}{E} P_{31}^2. \]
Here \( \kappa_n, \kappa_g, \tau_g \) are called the normal curvature, geodesic curvature and geodesic torsion, respectively. In the case
\[
\sigma = 0
\]
the first equation of this GWE coincides in fact with the Frenet equation for the curves. So, all that we are doing in the next sections is true for the motion (deformation) of curves when \( \sigma = 0 \).

The compatibility condition for the GWE (6) gives the GMCE as
\[
A_x - C_x + [A, C] = 0,
\]
or in elements
\[
\kappa_2 = \omega_{2x} + \tau \omega_2 - \sigma \omega_1, \tag{9a}
\]
\[
\tau_g = \omega_{1x} + \sigma \omega_3 - \kappa \omega_2, \tag{9b}
\]
\[
\sigma_2 = \omega_{22} + \kappa \omega_1 - \tau \omega_2. \tag{9c}
\]
We can reformulate the linear system (6) in 2 x 2 matrix form as
\[
\begin{align*}
\psi_x &= U \psi, \\
\psi_t &= V \psi,
\end{align*}
\]
where
\[
U = \frac{1}{2i} \begin{pmatrix} \tau & \kappa + i \sigma \\ \kappa - i \sigma & -\tau \end{pmatrix},
\]
\[
V = \frac{1}{2i} \begin{pmatrix} \omega_1 & \omega_3 + i \omega_2 \\ \omega_3 - i \omega_2 & -\omega_1 \end{pmatrix}.
\]

3 Deformation of surfaces

Now we would like to consider the deformation of the surface with respect to \( y \).
We postulate that such deformation or motion of the surface is governed by the system
\[
\begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}_x = A \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}, \tag{10a}
\]

where $I_1$ and $I_2$ are real functions. The system (10) will be called the deformed or (2+1)-dimensional GWE. We remark that first and third equations of the system (10) are the equations (6) and $A$, $C$ coincide with formulas (7). The compatibility conditions of the deformed GWE (10) gives the deformed or (2+1)-dimensional GMCE of the form

\[
\begin{align*}
A_t - C_x + [A, C] &= 0, \quad (12a) \\
B_t - C_y + [B, C] &= 0, \quad (12b) \\
C_t - A_y + [A, B] &= 0. \quad (12c)
\end{align*}
\]

As we see, equation (12a) is in fact the GMCE (8). This fact explains why we call (12) the deformed GMCE. The linear problem (Lax representation) associated with the system (12) can be written as

\[
\begin{align*}
\Phi_x &= \lambda^2 \Phi_x + (F^* - \lambda^2 F^+) \Phi, \quad (13a) \\
\Phi_t &= -i\lambda \Phi_x + (C + i\lambda F^+) \Phi, \quad (13b)
\end{align*}
\]

where

\[
F^\pm = A \pm iB,
\]

\[
z = \frac{1}{2} (x + iy),
\]

\[
z = \frac{1}{2} (x - iy).
\]

So we can confirm that the deformed GMCE (12) is a candidate to be integrable in the sense that for it there exists the Lax representation with the spectral
parameter (13). Higher hierarchy of the deformed GMCE (12) can be obtained as the compatibility condition of the linear system

$$\Psi_x = \lambda^2 \Psi_z + (F^* - \lambda^2 F^+)\Psi, \quad (14a)$$
$$\Psi_t = -i\lambda^2 \Psi_z + \sum_{j=0}^{n} \lambda^j F_j \Psi. \quad (14b)$$

4 Deformation of surfaces induced by (2+1)-dimensional integrable systems

In this section we would like to attract your attention to some aspects of the relation between the deformation of surfaces and integrable systems in 2+1 dimensions.

4.1 Integrable systems in 2+1 dimensions and the deformed GMCE

Our first observation is that some important integrable systems in 2+1 dimensions are particular reductions of equations (12). In fact, the well-known (2+1)-dimensional integrable systems such as the Kadomtsev - Petviashvili equation, the Davey - Stewartson equation, and so on, can be obtained from the deformed GMCE (12) as some reductions. We support this statement by presenting some examples.

**Example 1. The Davey - Stewartson II equation.** Let the matrices $A, B, C$ in the equations (12) have the following form

$$A = \sqrt{2i\lambda} \sigma_3 + \frac{1}{\sqrt{2}} \sigma^1 \sigma^+ + \frac{1}{\sqrt{2}} \sigma^{-}, \quad (15a)$$
$$B = -i\lambda \sigma_3 + \frac{1}{\sqrt{2}} \sigma^1 \sigma^+ + \frac{1}{\sqrt{2}} \sigma^{-}, \quad (15b)$$
$$C = -\frac{i}{2} (|\psi|^2 + \phi \psi + 3\lambda^2) \sigma_3 - 3\lambda \phi \sigma^+ - 3\lambda \phi \sigma^{-}, \quad (15c)$$
where we used the isomorphism \( so(3) \cong su(2) \) to write the matrices \( A, B, C \) in \( 2 \times 2 \) form,
\[
\sigma^k = \sigma_1 \pm i \sigma_2.
\]
Substituting (15) into the system (12) after some algebra we get the Davey - Stewartson II equation [24]:
\[
\begin{align*}
\dot{\eta} + \frac{1}{2}(\eta_{xx} + \eta_{yy}) - (|\eta|^2 + \phi)\eta &= 0, \quad (16a) \\
\phi_{xx} + \phi_{yy} + 2(|\eta|^2)_x &= 0. \quad (16b)
\end{align*}
\]

**Example 2. The Kadomtsev - Petviashvili equation.** Now we consider the case when
\[
\tau = \sigma = \omega_1 = \omega_2 = \gamma_1 = \gamma_2 = 0.
\]
Then the matrices \( A, B, C \) take the form
\[
A = \begin{pmatrix} 0 & \kappa \\ -\kappa & 0 \end{pmatrix}, \quad (17a)
\]
\[
B = \begin{pmatrix} 0 & \gamma_1 \\ -\gamma_1 & 0 \end{pmatrix}, \quad (17b)
\]
\[
C = \begin{pmatrix} 0 & \omega_3 \\ -\omega_3 & 0 \end{pmatrix}. \quad (17c)
\]
Substituting these expressions into (12) we get
\[
\begin{align*}
\kappa_{yy} &= \gamma_1, \quad (18a) \\
\gamma_{yy} &= \omega_3, \quad (18b) \\
\kappa_{y} &= \omega_{11}. \quad (18c)
\end{align*}
\]
Now we assume that the function \( \omega_3 \) has the form
\[
\omega_3 = -\kappa_{yy} - 3\kappa^2 - \frac{3}{2}\kappa_{xy}^2 \gamma_{xy}. \quad (19)
\]
Hence and from (18c) we obtain
\[
\begin{align*}
\kappa_{y} + 6\kappa \kappa_y + \kappa_{xy} + 3\kappa^2 \gamma_{yy} &= 0, \quad (20a) \\
\gamma_{yy} &= \kappa_y. \quad (20b)
\end{align*}
\]
or
\[
(\kappa_{y} + 6\kappa \kappa_y + \kappa_{xy})_x + 3\kappa^2 \kappa_{yy} = 0. \quad (21)
\]
It is the famous Kadomtsev - Petviashvili equation.
Example 3. The Lame equation. Now we consider the case when the matrices $A, B, C$ take the form (see, e. g. the Ref. [26])

$$A = \begin{pmatrix} 0 & -\beta_{11} & -\beta_{13} \\ \beta_{21} & 0 & 0 \\ \beta_{31} & 0 & 0 \end{pmatrix}, \quad (22a)$$

$$B = \begin{pmatrix} 0 & \beta_{12} & 0 \\ -\beta_{12} & 0 & -\beta_{22} \\ 0 & \beta_{22} & 0 \end{pmatrix}, \quad (22b)$$

$$C = \begin{pmatrix} 0 & 0 & \beta_{33} \\ 0 & 0 & \beta_{33} \\ -\beta_{13} & -\beta_{23} & 0 \end{pmatrix}. \quad (22c)$$

In this case, the $(2+1)$-dimensional GMCE (12) takes the form

$$\frac{\partial \tilde{\alpha}_3}{\partial u^1} = \beta_{4k} \tilde{\alpha}_3, \quad (23a)$$

$$\frac{\partial \tilde{\alpha}_3}{\partial u^1} + \frac{\partial \tilde{\alpha}_3}{\partial u^2} + \sum_{n \neq 3} \beta_{ni} \tilde{\alpha}_n = 0, \quad (23b)$$

where $u^1 = x, \quad u^2 = y, \quad u^3 = t$.

It is the Lame equation, integrability of which was proved by V. E. Zakharov in [2].

So we have presented three examples of integrable equations in $2+1$ dimensions which are the particular reductions of the deformed GMCE (12).

4.2 Integrable spin systems in 2+1 dimensions and the deformed GWE

Now let us consider the relation between integrable spin systems in $2+1$ dimensions and the deformed GWE (10). Our second observation is that many integrable isotropic spin systems in $2+1$ dimensions are particular reductions of equations (10). We support this statement by citing two examples.

Example 4. The M-I equation. First, we consider the M-I equation which reads as

$$S_t = (S \wedge S_y + uS)_x, \quad (24a)$$

$$u_x = -S \cdot (S_x \wedge S_y) \quad (24b)$$

8
and which is integrable (see, e. g. Refs. [9, 13]). In this case we take the identification
\[ e_1 = S, \]  
where \( S \) is the solution of the M-I equation (24) and
\[ e_1^2 = \kappa^2 + \sigma^2 = S_x. \]

Then the M-I equation (24) becomes
\[
e_{11} = (e_1 \wedge e_1 + u e_1)_x, \tag{26a}
\]
\[ u_x = -e_1 \cdot (e_1 \wedge e_1). \tag{26b}\]

Now let us assume
\[
\tau = f_x, \\
\gamma_1 = f_x + u, \\
\omega_1 = f_x + \partial_x^{-1}(\sigma \omega_2 - \kappa \omega_3), \\
\omega_2 = -\gamma_3 x - \gamma_2 x + u \sigma, \\
\omega_3 = \gamma_2 x - \gamma_3 x + u \kappa, \tag{27}
\]
where \( f(x, y, t, \lambda) \) is a real function. Taking into account the formulas (27) and after eliminating the vectors \( e_2 \) and \( e_3 \), the system (10) takes the form (26). This means that the M-I equation (24) is the particular exact reduction of the (2+1)-dimensional GWE (10) with the choice (27).

**Example 5.** The Ishimori equation. Now we assume that the functions \( \gamma_i, \omega_i \) are given by the formulas
\[
\tau = f_x + \frac{1}{2} u_y, \\
\gamma_1 = f_x + \frac{1}{2} u_x, \tag{28}
\]
\[
\omega_1 = f_x + \partial_x^{-1}\left(\frac{1}{2} u_y + \sigma \omega_2 - \kappa \omega_3\right),
\]
where as in the previous case, \( f(x, y, t, \lambda) \) is a real function. Using these expressions and doing as for the M-I equation, i. e., eliminating the vectors \( e_2, e_3 \) from the system (10), we obtain the following equation for the unit vector \( e_1 \)
\[
e_{11} = e_1 \wedge (e_1 x + \alpha^2 e_{1y}) + u e_1 + u_x e_1, \tag{29a}
\]
\[
\alpha^2 u_{yy} - u_{xx} = 2\alpha \cdot e_1 \cdot (e_1 \wedge e_1). \tag{29b}\]

After the identification (25), the equation (29) takes the form
\[
S_t = S \wedge (S_x + \alpha^2 S_y) + u_S + u_x S_x, \tag{30a}\]
that is, the Ishimori equation [25]. So we have shown that the M-I and Ishimori equations are the particular reductions of the deformed GWE (10). Similarly, we can show that the other isotropic spin systems in 2+1 dimensions are the exact reductions of the system (10) at least for existing known integrable isotropic (2+1)-dimensional spin systems.

4.3 Integrable spin systems in 2+1 dimensions as exact reductions of the M-O equation

Now let us consider the (2+1)-dimensional isotropic M-O equation (about our notations, see, e.g., [13-15], [31-32])

\[ e_{ij} = \omega_1 e_{i1} + \omega_2 e_{i2}, \quad (31a) \]

\[ \tau_y - \gamma_{1e} = e_1 \cdot (e_{12} \wedge e_{13}), \quad (31b) \]

which sometimes we write in terms of \( S \) as

\[ S_x = \theta_1 S_x + \theta_2 S_y, \quad (32a) \]

\[ \tau_y - \gamma_{1e} = S \cdot (S_x \wedge S_y), \quad (32b) \]

where \( \theta_j \) are some real functions. We note that many integrable isotropic spin systems in 2+1 dimensions are particular reductions of the (2+1)-dimensional isotropic M-O equation (31), (32). For example, the M-I equation (24) is the particular case of (32) as

\[ \theta_1 = \frac{\omega_1 \gamma_2 - \omega_2 \gamma_3}{\kappa \gamma_2 - \sigma \gamma_3}, \quad (33a) \]

\[ \theta_2 = \frac{\omega_2 \kappa - \omega_3 \sigma}{\kappa \gamma_2 - \sigma \gamma_3}. \quad (33b) \]
5 The (2+1)-dimensional GMCE as exact reduction of the Yang - Mills - Higgs - Bogomolny equation

One of the most interesting and important integrable equations in 2+1 dimensions is the following Yang - Mills - Higgs - Bogomolny equation [19]

\[ \Phi_y + [\Phi, B] + C_x - A_t + [C, A] = 0, \quad (34a) \]
\[ \Phi_t + [\Phi, C] + A_y - B_x + [A, B] = 0, \quad (34b) \]
\[ \Phi_x + [\Phi, A] + B_t - C_y + [B, C] = 0. \quad (34c) \]

The third important observation is that the deformed GMCE (12) is the particular case of the equation (34). In fact, if in the Yang - Mills - Higgs - Bogomolny equation we put

\[ \Phi = 0, \]

then it becomes the deformed GMCE (12). So we can suggest that the deformed GMCE is a candidate to be integrable as the exact reduction of the integrable equation (34).

6 The (2+1)-dimensional GCME as exact reduction of the SDYME

Now we study the relationship between the deformed GMCE (12) and the SDYME. The SDYME reads as [20]

\[ F_{\mu\nu} = F_{\mu\nu}^*, \quad (35) \]

where * is the Hodge star operator and the Yang - Mills field defined as

\[ F_{\mu\nu} = \frac{\partial A_{\nu}}{\partial x_\mu} - \frac{\partial A_{\mu}}{\partial x_\nu} - [A_{\mu}, A_{\nu}]. \]

Let

\[ x_\alpha = z + it, \]
\[ x_\beta = z - it, \]
\[ x_\gamma = x + iy, \]
\[ x_\delta = x - iy. \]
be the null-coordinates in the Euclidean space for which the metric has the form
\[
ds^2 = dx_\alpha dx_\beta + dx_\beta dx_\alpha.
\]
Now the SDYME takes the form \([16-17,19]\)
\[
F_{\alpha\beta} = 0, \quad (36a)
F_{\alpha\beta} = 0, \quad (36b)
F_{\alpha\beta} + F_{\beta\alpha} = 0, \quad (36c)
\]
where
\[
A_\alpha = A_x + iA_t,
A_\beta = A_x - iA_t,
A_\gamma = A_x + iA_y,
A_\delta = A_x - iA_y.
\]
The associated linear system is \([19]\)
\[
(\partial_\alpha + \lambda \partial_\beta)\Psi = (A_\alpha + \lambda A_\beta)\Psi, \quad (37a)
(\partial_\beta - \lambda \partial_\beta)\Psi = (A_\beta - \lambda A_\beta)\Psi, \quad (37b)
\]
where \(\lambda\) is the spectral parameter and
\[
\frac{\partial}{\partial x_\alpha} = \frac{\partial}{\partial x} - i\frac{\partial}{\partial t},
\frac{\partial}{\partial x_\beta} = \frac{\partial}{\partial x} + i\frac{\partial}{\partial t},
\frac{\partial}{\partial x_\gamma} = \frac{\partial}{\partial x},
\frac{\partial}{\partial x_\delta} = \frac{\partial}{\partial y}.
\]
Our fourth observation: the deformed GMCE (12) is the particular reduction
of the SDYME (36). In fact, we consider the following reduction of the SDYME
\[
A_\alpha = -iC, \quad (38a)
A_\beta = iC, \quad (38b)
A_\beta = A - iB, \quad (38c)
A_\delta = A + iB, \quad (38d)
\]
and assume that \(A, B, C \in so(3)\) and independent of \(z\). In this case, from
the SDYME (36) we obtain the \((2+1)\)-dimensional GMCE (12) in the Euclidean
coordinates.
7 Conclusion

In this paper, we have considered some deformations or, in other terminology, motions of surfaces. We have shown that the corresponding deformed GMCE is integrable in the sense that the associated linear problem (Lax representation) exists with the spectral parameter. We demonstrated that several important integrable systems in 2+1 dimensions, such as the Davey - Stewartson II, Kadomtsev - Petviashvili and Lame equations are some exact reductions of the deformed GMCE. Although, all known integrable (2+1)-dimensional isotropic spin systems can be obtained from the deformed GMCE, we have showed that such spin systems can be obtained also from the deformed GWE as exact reductions. Finally, we proved that the deformed GMCE is the particular case of two famous integrable systems, namely, the Yang - Mills - Higgs - Bogomolny equation and SDYME. It goes in favour of integrability of the deformed GMCE.

8 Acknowledgements

One of the authors (R. M.) is grateful to M. J. Ablowitz for useful conversation on the reduction of SDYME. Also he would like to thank B. G. Konopelchenko, L. Martina and G. Landolfi for helpful discussions and G. Soliani for careful reading of the manuscript. R. M. is grateful to the Department of Physics, University of Lecce for the kind hospitality. Special thanks are due to the referees for critical remarks and comments which helped us remove some mistakes from the first version of the paper and improve it.

References


Received on September 9, 2003.
## SUBJECT CATEGORIES
OF THE JINR PUBLICATIONS

<table>
<thead>
<tr>
<th>Index</th>
<th>Subject</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>High energy experimental physics</td>
</tr>
<tr>
<td>2.</td>
<td>High energy theoretical physics</td>
</tr>
<tr>
<td>3.</td>
<td>Low energy experimental physics</td>
</tr>
<tr>
<td>4.</td>
<td>Low energy theoretical physics</td>
</tr>
<tr>
<td>5.</td>
<td>Mathematics</td>
</tr>
<tr>
<td>6.</td>
<td>Nuclear spectroscopy and radiochemistry</td>
</tr>
<tr>
<td>7.</td>
<td>Heavy ion physics</td>
</tr>
<tr>
<td>8.</td>
<td>Cryogenics</td>
</tr>
<tr>
<td>9.</td>
<td>Accelerators</td>
</tr>
<tr>
<td>10.</td>
<td>Automatization of data processing</td>
</tr>
<tr>
<td>11.</td>
<td>Computing mathematics and technique</td>
</tr>
<tr>
<td>12.</td>
<td>Chemistry</td>
</tr>
<tr>
<td>13.</td>
<td>Experimental techniques and methods</td>
</tr>
<tr>
<td>14.</td>
<td>Solid state physics. Liquids</td>
</tr>
<tr>
<td>15.</td>
<td>Experimental physics of nuclear reactions at low energies</td>
</tr>
<tr>
<td>16.</td>
<td>Health physics. Shieldings</td>
</tr>
<tr>
<td>17.</td>
<td>Theory of condensed matter</td>
</tr>
<tr>
<td>18.</td>
<td>Applied researches</td>
</tr>
<tr>
<td>19.</td>
<td>Biophysics</td>
</tr>
</tbody>
</table>
Серикбаев Н. С. и др.
Самодуальное уравнение Янга–Миллса и деформация поверхностей

Показано, что многие (2 + 1)-мерные интегрируемые системы и спиновые системы могут быть получены из (2 + 1)-мерного уравнения Гаусса–Майнарди–Кодацци (УГМК) и уравнения Гаусса–Вейнгартена соответственно. Также показано, что (2 + 1)-мерное УГМК, описывающее деформацию (движение) поверхностей, является точной редукцией уравнения Янга–Миллса–Хитса–Богомольного и самодуального уравнения Янга–Миллса. На основе этого наблюдения делается предположение, что это УГМК является кандидатом быть интегрируемым. Приведена соответствующая линейная задача (представление Лакса) со спектральным параметром.

Работа выполнена в Лаборатории теоретической физики им. Н. Н. Боголюбова ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна, 2003

Перевод авторов

Serikbaev N. S. et al.
Self-Dual Yang–Mills Equation and Deformation of Surfaces

We show that many integrable systems and integrable spin systems in 2 + 1 dimensions can be obtained from the (2 + 1)-dimensional Gauss–Mainardi–Codazzi and Gauss–Weingarten equations, respectively. We also show that the (2 + 1)-dimensional Gauss–Mainardi–Codazzi equation which describes the deformation (motion) of surfaces is the exact reduction of the Yang–Mills–Higgs–Bogomolny and self-dual Yang–Mills equations. On the basis of this observation, we suggest that the (2 + 1)-dimensional Gauss–Mainardi–Codazzi equation is a candidate to be integrable, and the associated linear problem (Lax representation) with the spectral parameter is presented.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna, 2003
Редактор С. Ю. Романов
Макет Е. В. Сабаевой

Формат 60 × 90/16. Бумага офсетная. Печать офсетная.
Усл. печ. л. 1,0. Уч.-изд. л. 1,26. Тираж 415 экз. Заказ № 54104.

Издательский отдел Объединенного института ядерных исследований
141980, г. Дубна, Московская обл., ул. Жолто-Кюри, 6.
E-mail: publish@pds.jinr.ru
www.jinr.ru/publish/