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Abstract

Stochastic quatization is used to derive exact equations connecting multilocal field correlators. An explicit example of φ^3 theory is presented and equations for gluodynamics are discussed.

1. One of the main problems in field theory is to define and calculate the nonperturbative contents of the theory, e.g. nonperturbative contributions to vacuum field correlators. To this end one needs exact equations for vacuum correlators containing both perturbative (P) and nonperturbative (NP) contributions, and some principle of separation. The general path integral for the Green's function (vacuum correlator) of course contains both contributions, but the functional integral over field variables calls for computer simulations. The latter have provided a lot of numerical information on NP effects but the analytic structure of field theory like QCD and its NP contents is still unclear.

One can try to exploit the Dyson-Schwinger equations (DSE) [1], which formally sum up all Feynman amplitudes and presumably also contain NP contributions. However for gauge theory with confinement (like gluodynamics or QCD) DSE are not applicable because they are not gauge-invariant – or in physical terms – because a propagator for a single quark or gluon has no sense without a string connecting it to another color object. There are equations for gauge invariant objects – Wilson loops – the Makeenko-Migdal loop equations [2]. Some progress is being done recently in this direction [3], but the immediate use of them is difficult and therefore one feels justified in looking for alternatives.

Instead of DSE and Bethe-Salpeter equations (BSE) [4] one can exploit for QCD the Feynman-Schwinger representation (FSR) [5] which was applied to quark and gluon systems [6]. All dynamics in FSR is expressed in terms of vacuum field correlators and the latter should be found independently from some fundamental equations. Therefore the main problem is to find these equations from the first principles, i.e. starting from the Lagrangian.

In this letter we approach the problem from another side and use the stochastic quantization method [7] plus cluster (cumulant) expansion [8] to derive exact equations for vacuum correlators, containing both P and NP contributions. The equations obtained are path integrals, but only in quantum mechanical sense (over trajectories of field quanta) and several ways of treating those integrals can be used [9].

The plan of the letter is as follows. In section 2 we formally derive the equations for the φ^3 theory. In section 3 we discuss the P expansion and graphical representation of the equations. In section 4 we consider the case of gluodynamics and propose a method to find explicit set of equations for vacuum correlators. A short conclusion follows in section 5.

2. We use the Euclidean space-time and consider for simplicity the φ^3 Lagrangian

$$L = \frac{m^2\varphi^2}{2} + \frac{1}{2}(\partial_\mu\varphi)^2 - \frac{g}{3}\varphi^3 \tag{1}$$

The Langevin equation of the stochastic quantization method [7] is

$$\frac{\partial}{\partial\tau}\varphi(x,\tau) = -\frac{\delta S}{\delta\varphi} + \eta(x,\tau) = -(m^2-\partial^2)\varphi + g\varphi^2 + \eta(x,\tau) \tag{2}$$

where τ is the Langevin time, $-\infty < \tau < \infty$, and the stochastic source $\eta(x,\tau)$ satisfies

$$< \eta(x,\tau)\eta(x',\tau') >= 2\delta(x-x')\delta(\tau-\tau') \tag{3}$$

while all nonzero multiple averages reduce to quadratic ones (the Gaussian ensemble)

Solution of (2) can be expressed in terms of the retarted Green's function $G(x,y,\tau)$

$$\varphi(x,\tau) = \int_{-\infty}^{\infty} G(x,y;\tau-\tau')\eta(y,\tau')d\tau' dy \tag{4}$$

where G satisfies:

$$\frac{\partial G}{\partial \tau} + (m^2 - \partial^2 - g\varphi(x))G(x,y,\tau) = \delta(x-y)\delta(\tau) \tag{5}$$

In absence of constant classical solutions we can choose a retarded solution for G:

$$G(x,y,\tau) = \Theta(\tau) < x \mid e^{(m^2-\partial^2+g\varphi)\tau} \mid y >=$$
$$= \Theta(\tau)\int (Dz)_{xy} exp[-m^2\tau - \int_0^\tau \frac{\dot{z}^2}{4}d\lambda + g\int_0^\tau \varphi(z(\lambda),\lambda)d\lambda] \tag{6}$$

where the path integral over trajectories $z(\lambda)$ is denoted by

$$(Dz)_{xy} = \lim_{N\to\infty}\prod_{n=1}^{N}\frac{d^4z(n)}{(2\pi\varepsilon)^2}, \quad N\varepsilon = \tau;$$

and boundary conditions $z(\lambda = 0) = y; \quad z(\lambda = \tau) = x$ are implied. One can check straightforwardly that G written in the form (6) satisfies (5). Substituting (6) into (4) we obtain a nonlinear equation for $\varphi(x,\tau)$:

$$\varphi(x,\tau) =$$
$$\int_0^\tau d\tau' dy (Dz)_{xy} exp[-m^2(\tau-\tau') - \int_{\tau'}^\tau \frac{\dot{z}^2(\lambda)}{4}d\lambda + g\int_{\tau'}^\tau \varphi(z(\lambda),\lambda)d\lambda]\eta(y,\tau') \tag{7}$$

Eq.(7) defines a stochastic process $\varphi(x,\tau)$ through the Gaussian stochastic source η. The physical quantities are obtained from $\varphi(x,\tau)$ by averaging over η and taking the limit $\tau \to \infty$ [7]. E.g. the quadratic field correlator (Schwinger function) is

$$< \varphi(x)\varphi(x') >_{vac} = \lim_{\tau\sim\bar{\tau}\to\infty} < \varphi(x,\tau)\varphi(x',\bar{\tau}) > \tag{8}$$

where angular brackets on the l.h.s imply vacuum expectation value in the usual sense of field theory (e.g. for Heisenberg operators or in the path integral formalism) while the same brackets on the r.h.s. mean stochastic average over a Gaussian esemble of $\eta(x,\tau)$.

Now using eq.(7) for $\varphi(x,\tau)$ and multiplying by $\varphi(y,\bar{\tau})$ etc. we can obtain a set of equations for correlators. The first one results from averaging both sides of (7)

$$< \varphi(x,\tau) >= \int_0^\tau d\tau' dy (Dz)_{xy} K(\tau,\tau') < \eta(y,\tau')F(\tau,\tau') > \tag{9}$$

where

$$K(\tau,\tau') \equiv \Theta(\tau-\tau')exp[-m^2(\tau-\tau') - \int_{\tau'}^{\tau} \frac{\dot{z}^2(\lambda)}{4} d\lambda], \qquad (10)$$
$$F(\tau,\tau') \equiv exp(+g \int_{\tau'}^{\tau} \varphi(z(\lambda),\lambda)d\lambda)$$

On the r.h.s. of (9) there enters a new quantity $< \eta\varphi(z_1)...\varphi(z_k) >$, which can be also deduced from (7), e.g.

$$< \varphi(x,\tau)\eta(u,t) >= \int_0^{\tau} d\tau' dy (Dz)_{xy} K(\tau,\tau') < \eta(y,\tau')\eta(u,t)F(\tau,\tau') > \qquad (11)$$

In a similar way we obtain a correlator

$$< \varphi(x,\tau)\varphi(x',\bar{\tau}) >= \int_0^{\tau} d\tau' \int_0^{\bar{\tau}} d\bar{\tau}' dy dy' (Dz)_{xy} (Dz')_{x'y'} K(\tau,\tau') K(\bar{\tau},\bar{\tau}') \times$$
$$\times < \eta(y,\tau')\eta(y',\bar{\tau}')F(\tau,\tau')F(\bar{\tau},\bar{\tau}') > \qquad (12)$$

The generalization to higher–order correlators is straightforward.

To obtain a system of equations for $< \varphi >, < \varphi\eta >, < \varphi\varphi >, ...$ we have to express the averages on the r.h.s. of (9,11,12). This can be done using the generating functional

$$\Phi(J) =< F(\tau,\tau')F(\bar{\tau},\bar{\tau}')exp \int d^4u dt J(u,t)\eta(y,t) > \qquad (13)$$

Applying to it the cluster expansion [8] we have

$$\Phi(J) = exp \sum_{n=1}^{\infty} \frac{1}{n!} \ll (g\int \varphi(\lambda)d\lambda + g\int \varphi(\bar{\lambda})d\bar{\lambda} + \int J\eta du dt)^n \gg \qquad (14)$$

The double brackets denote as usual the cumulants, defined in the standard way [8,10]

$$\ll 12 \gg =< 12 > - < 1 >< 2 >$$
$$\ll 123 \gg =< 123 > - < 12 >< 3 > - < 13 >< 2 > - \qquad (15)$$
$$< 1 >< 23 > +2 < 1 >< 2 >< 3 >$$

Differentiating (14) in $J(u,t)$ we express a generic average of the form

$$< \eta(1)...\eta(k)F(\tau_1,\tau_1')F(\tau_2,\tau_2')... >$$

in terms of "elementary" averages $<\varphi>, <\varphi\eta>, <\varphi\varphi>$ etc. We list necessary formulas in the Appendix for the convenience of the reader.

Using these formulas we obtain equations for the lowest correlators $<\varphi>, <\varphi\eta>, <\varphi\varphi>$ Eqs.(9),(11),(12) where on the r.h.s. one makes use of expansions (A12) of Appendix for the quantities $<\eta F>, <\eta\eta F>$ and $<\eta\eta FF>$. In principle one obtains an (infinite) exact system of coupled equations for all correlators, containing both perturbative and nonperturbative effects. To solve the system the simplest approximation is to neglect all higher order correlators. We postpone discussion of this approximation till the last section, and now present the resulting equations for $<\varphi>, <\varphi\eta>, <\varphi\varphi>$ which we call a "Gaussian approximation" for obvious reasons.

$$<\varphi(x,\tau)>= \int_0^\tau d\tau' dy (Dz)_{xy} K(\tau,\tau') \int_{\tau'}^{\tau} d\lambda g <\eta(y,\tau')\varphi(z(\lambda),\lambda)><F(\tau,\tau')> \quad (16)$$

$$<\varphi(x,\tau)\eta(u,t)>= 2\int (Dz)_{xu} K(\tau,t) <F(\tau,t)> + \int_0^\tau d\tau' dy (Dz)_{xy} K(\tau,\tau') \times \int d\lambda g <\eta(u,t)\varphi(z(\lambda),\lambda)> \int_{\tau'}^{\tau} d\lambda' g <\eta(y,\tau')\varphi(z(\lambda'),\lambda')><F(\tau,\tau')> \quad (17)$$

$$\begin{aligned} &<\varphi(x,\tau)\varphi(\bar{x},\bar{\tau})>= \\ &\int_0^\tau d\tau' \int_0^{\bar{\tau}} d\bar{\tau}' dy d\bar{y} (Dz)_{xy} (D\bar{z})_{\bar{x}\bar{y}} K(\tau,\tau') K(\bar{\tau},\bar{\tau'}) \times \\ &g^2 [<\eta(y,\tau')(\int_{\tau'}^{\tau} \varphi(z(\lambda),\lambda) d\lambda + \int_{\bar{\tau}'}^{\bar{\tau}} \varphi(\bar{z}(\lambda'),\lambda') d\lambda')> + \quad (18) \\ &<\eta(\bar{y},\bar{\tau'})(\int_{\tau'}^{\tau} \varphi(z(\lambda),\lambda) d\lambda + \int_{\bar{\tau}'}^{\bar{\tau}} \varphi(\bar{z}(\lambda'),\lambda') d\lambda')>] <F(\tau,\tau')F(\bar{\tau},\bar{\tau'})> + \\ &2\int d\tau' K(\tau,\tau') K(\bar{\tau},\tau') dy (Dz)_{xy} (D\bar{z})_{\bar{x}y} <F(\tau,\tau')F(\bar{\tau},\tau')> \end{aligned}$$

where K, F are defined in (10) and we have for $<F>, <FF>$ in the Gaussian approximation (i.e. omitting all higher order correlators)

$$\begin{aligned} &<F(\tau,\tau')>= exp[g \int_{\tau'}^{\tau} <\varphi(z(\lambda),\lambda)> d\lambda + \\ &\frac{g^2}{2} \int_{\tau'}^{\tau} d\lambda \int_{\tau'}^{\tau} d\lambda' \ll \varphi(z(\lambda),\lambda)\varphi(z(\lambda'),\lambda') \gg] \end{aligned} \quad (19)$$

and

$$< F(\tau,\tau')F(\bar{\tau},\tau') >=< F(\tau,\tau') >< F(\bar{\tau},\tau') >$$
$$exp\{g^2 \int_{\tau'}^{\tau} d\lambda \int_{\tau'}^{\bar{\tau}} d\lambda' \ll \varphi(z(\lambda),\lambda) > \varphi(\bar{z}(\lambda'),\lambda') \gg\} \tag{20}$$

where for $< F(\bar{\tau},\tau') >$ one should replace in (19) $z(\lambda) \to \bar{z}(\lambda)$.

Equations (16-18) with definitions (19-20) are a minimal closed set of equations for correlators $< \varphi >, < \varphi\eta >, < \varphi\varphi >$.

To conclude this section we simplify the last term on the r.h.s. of (18), using the fact that it can be written as

$$\int dy < x \mid KF \mid y >< y \mid KF \mid \bar{x} >=< x \mid KFKF \mid \bar{x} >$$

For $\tau = \bar{\tau}$ one obtains for this term

$$\int_0^{2\tau} d\tau_1 (Dz)_{x\bar{x}} K(\tau_1, 0) < \tilde{F}(\tau_1, 0) > \tag{21}$$

where we have defined

$$\tilde{F}(\tau_1,0) = expg[\int_0^{\tau_1/2} \varphi(z(\tilde{\lambda}),\tilde{\lambda}+\tau-\frac{\tau_1}{2})d\tilde{\lambda} + \int_{\tau_1/2}^{\tau_1} \varphi(z(\tilde{\lambda}),\tilde{\lambda}+\tau-\tau_1)d\tilde{\lambda}] \tag{22}$$

Note that in the asymptotical regime, when one drops dependence on λ in $\varphi(z,\lambda)$, we have $\tilde{F} \to F$.

3. Perturbative expansion of Equations (16-18)

We expand systematically in powers of g the r.h.s. of eqs. (16-18). To the lowest order we get

$$< \varphi >_0 = 0; < \varphi\eta >_0 = \int (Dz)_{xu} K(\tau, t), \tag{23}$$

and using (22),

$$< \varphi(x,\tau)\varphi(\bar{x},\tau) >_0 = \int_0^{2\tau} d\tau_1 (Dz)_{x\bar{x}} K(\tau_1, 0). \tag{24}$$

A simple calculation yields:

$$G_0 \equiv (Dz)_{xu} K(\tau, t) = \int \frac{d^4p}{(2\pi)^4} e^{ip(x-u)-(p^2+m^2)(\tau-t)} \tag{25}$$

and hence

$$< \varphi(x,\tau)\varphi(\bar{x},\tau) >_0 = \int \frac{d^4p}{(2\pi)^4} \frac{e^{ip(x-\bar{x})}(1-e^{-2\tau(p^2+m^2)})}{p^2+m^2} \tag{26}$$

The last expression tends to the free Green's function in the limit $\tau \to \infty$ as usual in the stochastic quantization method [7].

In the next order we have

$$\begin{aligned} < \varphi >_1 = 2g \int_0^\tau d\tau' dy (Dz)_{xy} K(\tau, \tau') \cdot \int_\tau^{\tau'} (Dz)_{yz} d\lambda = \\ = g \int \frac{d^4p}{(2\pi)^4} \frac{1}{m^2(m^2+p^2)} + 0(e^{-m^2\tau}) \end{aligned} \tag{27}$$

where we have used a simple decomposition

$$(Dz)_{xy} = (Dz)_{xz(\lambda)} \cdot dz(\lambda) \cdot (Dz)_{z(\lambda)y} \tag{28}$$

Thus the limit of $\tau \to \infty$ of (27) corresponds to the lowest order tadpole diagram.

Keeping only $< \varphi >_1$ in the last term on the r.h.s. of (18) we obtain

$$\begin{aligned} < \varphi(x,\tau)\varphi(\bar{x},\tau) > = \int_0^{2\tau} d\tau_1 \int \frac{d^4p}{(2\pi)^4} e^{ip(x-\bar{x})-(p^2+m^2-<\varphi>_1)\tau_1} = \\ = \int \frac{d^4p}{(2\pi)^4} \frac{e^{ip(x-\bar{x})}}{p^2+m^2-g<\varphi>_1} + 0(e^{-2m^2\tau+2g<\varphi>_1)\tau}) \end{aligned} \tag{29}$$

Expanding last equation in powers of g we obtain an infinite set of diagrams.

Note that the instability of the vacuum in our model leads to the divergence of $< \varphi >$, $g < \varphi >> m^2$, which in turn leads to the divergence of the limit $\tau \to \infty$ for the correlator (29), so that the stochastic process has no limiting equilibrium.

4. The case of gluodynamics

The Langevin equation in this case is

$$\dot{A}_\mu(x,\tau) = D_\lambda F_{\lambda\mu}(x,\tau) + \eta_\mu(x,\tau) =$$
$$= (D^2_\lambda \delta_{\mu\nu} - D_\nu D_\mu)A_\nu + \eta_\mu(x,\tau) \equiv K_{\mu\nu}A_\nu + \eta_\mu \quad (30)$$

where $\eta_\mu = \eta^a_\mu t^a$, $tr t^a t^b = \frac{1}{2}\delta_{ab}$, satisfy conditions

$$< \eta^a_\mu(x,\tau)\eta^b_\nu(y,\tau') >= 2\delta_{\mu\nu}\delta_{ab}\delta(x-y)\delta(\tau-\tau') \quad (31)$$

one can define the "Green's function" $G^{ab}_{\mu\nu}$, satisfying an equation

$$[\frac{\partial}{\partial\tau}\delta_{ca}\delta_{\mu\nu'} - K^{ca}_{\mu\nu'}(x,\tau)]G^{ab}_{\nu'\nu}(x,y,\tau) = \delta_{\mu\nu}\delta(\tau)\delta(x-y)\delta_{bc} \quad (32)$$

A formal solution of (32) looks like

$$G^{ab}_{\nu\lambda}(x,y,\tau) = \Theta(\tau) < x \mid e^{\int_0^\tau d\lambda \hat{K}(\lambda)} \mid y >_{\nu\lambda,ab}, \quad (33)$$

and the solution of (30) satisfying a boundary condition $A_\mu(x,\tau<0) = 0$ is

$$A^a_\mu(x,\tau) = +\int_0^\tau d\tau' < x \mid e^{\int_{\tau'}^\tau d\lambda \hat{K}(\lambda)} \mid y >_{\mu\nu,ab} \eta^b_\nu(y)dy \quad (34)$$

one can represent $K_{\mu\nu}$ as

$$K_{\mu\nu} = D^2_\lambda\delta_{\mu\nu} - igF_{\mu\nu} - D_\mu D_\nu \quad (35)$$

It is convenient to transform the kernel (35) using a τ–dependent gauge transformation [11, 7]

$$B_\mu = U^+(\tau,x)(A_\mu + \frac{1}{g}\partial_\mu)U(\tau,x), \frac{\partial U}{\partial\tau} = g\dot{\Lambda}U \quad (36)$$

Using (30) and choosing

$$\dot{\Lambda} = -igD_\nu B_\nu \quad (37)$$

we arrive at the equation, containing only field B_μ

$$\dot{B}_\mu = (D^2\delta_{\mu\nu} - igF_{\mu\nu})B_\nu + \eta_\mu \quad (38)$$

with η_μ again satisfying (31). One can generalize the Feynman-Schwinger representation (6) to the nonabelian case [12]

$$< x \mid exp \int d\lambda (D^2 \delta_{\mu\nu} - igF_{\mu\nu}) \mid y >= (Dz)_{xy} P_A P_F \times$$
$$\times exp(-\frac{1}{4} \int \dot{z}^2 d\lambda \cdot \delta_{\mu\nu} + ig \int_y^x \hat{A}_\alpha dz_\alpha \delta_{\mu\nu} - ig \int \hat{F}_{\mu\nu} d\lambda) \qquad (39)$$

Here P_A is the ordering operator for the vector potential $\hat{A}_\alpha$ in the adjoint representation, while P_F keeps ordering of the adjoint operator $\hat{F}_{\mu\nu}$. The meaning of this ordering was discussed elsewhere [12,13].

Insertion of Eq. (39) into (34) (with all A_μ replaced by B_μ) yields

$$B_\mu^a(x,\tau) = \int_0^\tau d\tau' (Dz)_{xy} dy K_0(\tau,\tau') \Psi_{\mu\nu}^{ab}(x,y;\tau,\tau') \eta_\nu^b(y) \qquad (40)$$

where K_0 is obtained from K in eq. (10) putting $m = 0$,

$$\Psi_{\mu\nu}(x,y,\tau,\tau') \equiv P_A P_F exp(ig \int_y^x \hat{B}_\sigma dz_\sigma \cdot \delta_{\mu\nu}) exp(-ig \int_{\tau'}^\tau \hat{F}_{\mu\nu} d\lambda), \qquad (41)$$

The advantage of (40) as compared to (35) is that it contains in the kernel only fields $A_\mu, F_{\mu\nu}$ and not differential operators D_μ. Moreover, all components of B_μ, in contrast to A_μ, tend to stochastic equilibrium at $\tau \to \infty$ and do not explode in this limit [11,7].

One can check, at least for infinitesimal τ-independent transformations that $B_\mu$is invariant in the limit $\tau \to \infty$ [7].

The stochastic gauge constraint, associated with (33), corresponds to the covariant gauge in the perturbation expansion of (40). One can expand (40) in powers of g for $\bar{B} = 0$ and obtains automatically a series in powers of η. Using (32) one recovers the usual perturbation theory for the gluon propagator. This is similar to a direct perturbative solution of (38) or (30) , as was done e.g. in [7]. We shall present this analysis elsewhere.

Now we turn to the derivation of equations for vacuum correlators, as we have done it in the previous section for the φ^3 theory. In what follows we only sketch the derivation, leaving final equations for the next publication. The main idea is to form on the l.h.s. gauge-invariant combinations like $(F_{\mu\nu}(x_1)\Phi(x_1 x_2) F_{\rho\sigma}(x2)\Phi(x_2,x_1))$ where Φ is a parallel transporter

$$\Phi(xy) = Pexpig \int_y^x B_\mu dz_\mu \qquad (42)$$

As a result on the r.h.s. there appears a closed contour of $\Phi_{\mu_i \nu_i}$ with insertions of $\eta(y_i, \tau)$. As in the previous case of the φ^3 theory one defines a generating functional, which obtains when one replaces in the mentioned above closed contour $\eta(y,\tau) \rightarrow exp \int J(y,\tau)\eta(y,\tau) dy d\tau$. Next one has to calculate an average of the exponential operator

$$exp ig \int (B_\mu dz_\mu - igF d\tau + J\eta dy d\tau) \tag{43}$$

This is done using the cluster expansion theorem [8] now for the non-abelian matrix-valued operators [10,14] which results in the exponential of the cumulants of the generic structure

$$\ll F(1)F(2)...\eta(k_1)F(k_2)...\eta(k_3) \gg \tag{44}$$

Thus one again obtains an infinite system of integral equations for the cumulants (44). In the Gaussian approximation, i.e. neglecting all triple and higher-order correlators of $< F(1)...F(n)\eta >$, one has coupled equations for $< F(1)\eta(2) >, < F(1)F(2) >$. We shall present an explicit derivation of these equations elsewhere.

5. Conclusions

Combining the stochastic quantization method Parisi and Wu [7] with the cluster expansion and the Feynman-Schwinger representation [8] we were able to derive a full set of coupled integral equations for vacuum correlators. It is expected that by construction these equations contain both P and NP contributions. A perturbative expansion of these equations yields few first standard Feynman amplitudes, the full analysis is however missing. The short-range divergencies can be regulated modifying noise correlators (3), (31), e.g. by smearing the stochastic time δ-function [7,15], and since the P contents seems to be standard, one expects the usual renormalization scheme to be valid.

A crucial problem for the usefulness of the obtained integral equations is the convergence of the cluster expansion for quantities like $F(\tau, \tau')$ in (16-18) or products of $\Psi_{\mu\nu}$, Eq. (41). For QCD the Gaussian approximation is compatible with Monte-Carlo calculations for quarks in higher representations (see [16] for a discussion), and one may hope that for the quark interactions the lowest cumulant yields the dominant contribution. At the same time topological objects are known to destroy the convergence of cluster expansion [17] and the latter cannot be used e.g. to study chiral symmetry breaking and spin interactions in the pseudoscalar channel [18] . Happily enough, topological objects like instantons and magnetic monopoles are usually mostly classical solutions and these can be separated out in the stochastic quantization method and in the resulting integral equations. The detailed derivation and discussion of those will be given elsewhere.

The main hope of our approach is the possibility of extracting NP contents of vacuum correlators. As explained in Introduction, we have few tools in field theory to study NP effects. Besides Monte-Carlo simulations, one has Dyson–Schwinger equations which however are inapplicable to QCD in the confining regime, and Migdal-Makeenko loop equations for large N_c [2].

The new integral equations suggest a new possibility to study NP contents of physical gauge-invariant correlators.

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APPENDIX

The generating functional for vacuum averages.

$\Phi(J)$ is defined as in (13) and (14). We start from the simplest average, entering the r.h.s. of (9)

$$< \eta(y,\tau')F(\tau,\tau') >= \frac{\delta\Phi(J)}{\delta J(y,\tau')} |_{J=0,F(\bar\tau,\bar\tau')=1} \qquad (A1.1)$$

From (14) one readily obtains

$$(A1.1) = \sum_{n=2}^{\infty} \frac{1}{n!}\frac{\delta}{\delta J} \ll (g\int \varphi d\lambda + \int J\eta)^n \gg \cdot < F(\tau,\tau') >=$$

$$= \{< g\int \varphi d\lambda \cdot \eta(y,\tau') > +\frac{g^2}{2} < \int \varphi d\lambda \int \varphi d\lambda' \eta(y,\tau') > -$$

$$-g^2 < \int \varphi d\lambda \eta(y,\tau') >< \int \varphi d\lambda > +\frac{g^3}{6} < (\int \varphi d\lambda)^3 \eta(y,\tau') > -...\} < F(\tau,\tau') >= (A1.2)$$

$$(A1.3)$$

$$= \{\sum_{n=1} \frac{1}{(n-1)!} \ll (g\int \varphi d\lambda)^{n-1}\eta(y,\tau') \gg\} \cdot < F(\tau,\tau') >$$

In analologous way we get

$$< \eta(y,\tau')\eta(u,t)F(\tau,\tau') >= \frac{\delta\delta\Phi(J)}{\delta J(y,\tau')\delta J(u,t)} |_{J=0,F(\bar\tau,\bar\tau'=1}=$$

$$= \{\sum_{n=2}^{\infty} \frac{1}{n-2)!} \ll (g\int \varphi d\lambda)^{n-2}\eta(y,\tau')\eta(u,t) \gg\} < F(\tau,\tau') > + \quad (A1.4)$$

$$+(\sum_{n=1}^{\infty} \frac{1}{n-1)!} \ll (g\int \varphi d\lambda)^{n-1}\eta(y,\tau') \gg (\sum_{k=1}^{\infty} \frac{1}{k-1)!} \ll (g\int \varphi d\lambda)^{k-1}\eta(u,t) \gg\} < F(\tau,\tau')$$

Finally, the average in Eq.(12) is readily obtained from (A 1.3) by substitution $F(\tau,\tau') \to F(\tau,\tau')F(\bar\tau,\bar\tau'), g\int \varphi d\lambda \to g\int \varphi d\lambda + g\int \varphi d\bar\lambda$.

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