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Shape-Independent Expansion for the $^3S_1 - ^3D_1$ Mixing Parameter

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Abstract

A low - energy shape - independent expansion is suggested for the function $[tan(2\epsilon_{BB})/(2k^2)]$, where ϵ_{BB} is the Blatt-Biedenharn mixing parameter for the $^3S_1 - ^3D_1$ channel. This expansion allows an evaluation of the mixing parameter ϵ_{BB} from a knowledge of the deuteron asymptotic D to S ratio, pion mass and other low-energy observables, such as the scattering lengths, deuteron binding etc., of the nucleon-nucleon system. We demonstrate that the correct long range behavior of the tensor potential is essential for a realistic reproduction of ϵ_{BB}.

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The following effective-range expansion has been extremely useful for low-energy phase-shift analysis[1, 2]

$$k^{2l+1}cot\delta_l = -\frac{1}{a} + \frac{1}{2}r_0k^2 + ..., \tag{1}$$

where a is the scattering length, r_0 is the effective range, k^2 is the c.m. energy and δ_l is the phase-shift. We employ units $\hbar = m_n = 1$, where m_n is the nucleon mass. The parameters a and r_0 are determined by some average properties of the potential and are independent of detailed shape of the potential. This is why these expansions are termed shape independent. In the coupled ${}^3S_1 - {}^3D_1$ channel, apart from the phase shifts, one needs the mixing parameter for a complete analysis. Here we provide an effective-range-type shape independent expansion for the mixing parameter.

There are two commonly used mixing parameters, the Stapp-Ypsilantis[3] mixing parameter ϵ_1 and the Blatt-Biedenharn[4] mixing parameter ϵ_{BB}. In the present treatment we shall only consider the Blatt-Biedenharn mixing parameter as it is convenient to define an analytic function appropriate for making the desired expansion involving this parameter. This expansion involves the deuteron D state to S state ratio, η_d, pion mass, m_π, and other on-shell quantities such as, the nucleon-nucleon (NN) binding energies and scattering lengths and allows one to evaluate the mixing parameter ϵ_{BB} at low energies in a model independent way. Lacking precise measurements for ϵ_{BB}, we illustrate the present approach using the theoretically calculated mixing parameters of model NN potentials.

It is interesting to point out that there have been various studies of model-independent correlations involving D state observables of the NN system. For example, there have been studies of correlations among η_d and the deuteron quadrupole moment Q,[5] and among ϵ and Q.[6]

Ever since Amado and coworkers[7] pointed out the importance of η_d, in measuring the strength of the D state, there has been lot of theoretical and experimental activities[5, 8] in measuring η_d. The present expansion could be used for a evaluation of η_d once accurate low-energy mixing parameters ϵ_{BB} are available.

It is convenient to consider the following 'effective-range-type' function appropriate for making expansion[4]

$$\frac{tan(2\epsilon_{BB})}{2k^2} \equiv \frac{\hat{K}_{02}/k^2}{\hat{K}_{00} - \hat{K}_{22}}, \tag{2}$$

which is analytic in k^2 in the scattering region. We use notation $\hat{K}_{ll'} \equiv K_{ll'}(k^2)$, where $K_{ll'}(k^2) \equiv K_{ll'}(k,k;k^2)$ is the on-shell NN K matrix element for the ll' channel. This function satisfies the two following limits:

$$\lim_{k\to 0} \frac{tan(2\epsilon_{BB})}{2k^2} = \lim_{k\to 0} \frac{\hat{K}_{02}/k^2}{\hat{K}_{00} - \hat{K}_{22}} \equiv \frac{a_{02}}{a_t}, \tag{3}$$

and

$$\lim_{k\to i\alpha} \frac{tan(2\epsilon_{BB})}{2k^2} = \lim_{k\to i\alpha} \frac{\hat{K}_{02}/k^2}{\hat{K}_{00} - \hat{K}_{22}} \simeq \frac{\eta_d}{\alpha^2}, \tag{4}$$

where a_t is the triplet scattering length for the (00) channel in units of fm, a_{02} is the scattering length for the (02) channel in units of fm^3 defined by $\lim_{k\to 0} \hat{K}_{02}/k^2 \equiv a_{02}$ and α^2 is the deuteron binding energy in units of fm^{-2}. At the deuteron bound-state pole the approximation, $\lim_{k\to i\alpha} \hat{K}_{02}/[\hat{K}_{00} - \hat{K}_{22}] = -\eta_d$, holds within an estimated error of less than 0.1%, in place of the exact relation $\lim_{k\to i\alpha} \hat{K}_{02}/\hat{K}_{00} = -\eta_d$, so for all practical purposes Eq. (4) is taken to be exact. Both η_d and a_{02} are to be taken as measures of the strength of the tensor force.

In realistic situations the two limits of the function $tan(2\epsilon_{BB})/(2k^2)$ given by Eqs. (3) and (4) are quite different and this is a consequence of including a realistic long range behavior of the NN interaction. These two limits are essential for a correct reproduction of the mixing parameters. In the case of the Yamaguchi[9] tensor potential these two limits are practically the same because of the slowly varying form factors of this potential. This is why the Yamaguchi tensor potential usually fails to reproduce even qualitatively the mixing parameters.

We consider the following function $B(k^2)$ for our purpose

$$B(k^2) \equiv -\frac{\eta_d}{\alpha^2} + \frac{\hat{K}_{02}/k^2}{\hat{K}_{00} - \hat{K}_{22}}. \tag{5}$$

The K matrix elements are analytic functions of k^2 for positive energies and have the left-hand potential cuts. Hence the function $B(k^2)$ also has similar analytic properties. Particularly, it is analytic in the whole complex energy plane except the left-hand meson-exchange cuts at negative energies for $-\infty < k^2 < -m_\pi^2/4$, where m_π is the pion mass. Equations (3) and (4) imply

$$B(-\alpha^2) = 0 \tag{6}$$

and

$$B(0) = \frac{a_{02}}{a_t} - \frac{\eta_d}{\alpha^2}. \tag{7}$$

Using the above properties we suggest a convenient way of parametrizing the function $B(k^2)$. The left-hand cut of this function is essential for a correct reproduction of the mixing parameters. It is well-known that in region remote from the cuts the function $B(k^2)$ could be parametrized by approximating the nearest branch-point by a pole, e.g., at $k^2 = -m_\pi^2/4$, which corresponds to the exchange of a pion. Recalling, also, that $B(-\alpha^2) = 0$, one has the following Taylor-series-type expansion

$$B(k^2) = \frac{k^2 + \alpha^2}{k^2 + m_\pi^2/4}[c_1 + c_2(k^2 + \alpha^2) + ...], \tag{8}$$

where c_1, c_2 etc. are parameters essentially determined by some average properties of the tensor force and low-energy on-shell NN observables. For the usual effective-range function (1) the effect of the left-hand cut is less prominent. Hence simple NN potentials, such as, Yamaguchi and square-well potentials, provide a good description of the phase shifts, but not the mixing parameters.

At low energies, the parameter c_1 alone is supposed to contribute to $B(k^2)$ and $tan(2\epsilon_{BB})$. Consequently, setting $k = 0$, keeping only the term involving c_1 in Eq. (8) and using Eq. (7), one has the following approximate relation

$$c_1 \simeq \frac{m_\pi^2}{4\alpha^2} B(0) = \frac{m_\pi^2}{4\alpha^2}\left(\frac{a_{02}}{a_t} - \frac{\eta_d}{\alpha^2}\right), \tag{9}$$

and $c_2 = 0$. At higher energies, however, both c_1 and c_2 are supposed to contribute.

From Eqs. (2), (5) and (8) we obtain the following expansion for the parameter ϵ_{BB}

$$\frac{tan(2\epsilon_{BB})}{2k^2} = \frac{\eta_d}{\alpha^2} + \frac{k^2 + \alpha^2}{k^2 + m_\pi^2/4}[c_1 + c_2(k^2 + \alpha^2) + ...], \tag{10}$$

which reduces at low energies to the following shape independent approximation

$$\frac{tan(2\epsilon_{BB})}{2k^2} = \frac{\eta_d}{\alpha^2} + \frac{m_\pi^2}{4\alpha^2}\left(\frac{a_{02}}{a_t} - \frac{\eta_d}{\alpha^2}\right)\frac{k^2 + \alpha^2}{k^2 + m_\pi^2/4}. \tag{11}$$

Expansions (10) and (11) are the desired expansions. In Eq. (11) all the quantities are zero- or negative-energy on-shell observables. These observables are determined by some average properties of the potential and not by its full intricacy. The new on-shell parameter of interest in this case is a_{02}. Alternatively, this parameter is related to c_1 and $B(0)$ of Eq. (9).

We use expansions (10) and (11) for predicting the mixing parameters ϵ_{BB}. In the absence of accurate experimental mixing parameters[10] we illustrate the present approach using the

numerical results of the Peiper modified Reid soft core (PRSC)[11] and the momentum space one boson exchange Bonn (OBE)[12] potentials for the $^3S_1 - ^3D_1$ channel. For the PRSC potential, deuteron binding is 2.2298 MeV, $\eta_d = 0.02636$, $a_t = 5.386$ fm, and $a_{02} = 1.62$ fm^3. The pion mass m_π is taken to be 0.7 fm^{-1}. Using these parameters in Eq. (9) we obtain $B(0)$=-0.190 fm^2 and $c_1 = -0.432$ fm^2. For the OBE potential[12] deuteron binding is 2.2245 MeV, $a_t = 5.4218$ fm, $\eta_d = 0.0267$, $a_{02} = 1.71$ fm^3. Using these parameters in Eq. (9) we obtain $B(0)$=-0.182 fm^2 and $c_1 = -0.416$ fm^2. If we use these theoretical parameters in Eq. (11) we have an excellent fit to the numerically calculated ϵ_{BB} for both these potentials at low energies: $E_{lab} < 4$ MeV. However, at higher energies ($E_{lab} < 100$ MeV) we need to modify the numerical value of these parameters in order to obtain a good fit. In Fig. 1 we plot the numerically calculated mixing parameters ϵ_{BB} ($\diamond-$ PRSC; $\times-$ OBE) and the present fit via Eq. (11) (full line – PRSC; broken line – OBE) versus E_{lab}. For PRSC (OBE) potentials we used $B(0) = -0.191$ fm^2 (-0.180 fm^2), $c_1 = -0.458$ fm^2 ($=-0.482$ fm^2). The small deviation of c_1 at higher energies from the theoretically calculated value is due to the approximate nature of Eq. (9), which is strictly valid for low energies. For still higher energies ($E_{lab} < 300$ MeV) we need both c_1 and c_2 for a correct reproduction of ϵ_{BB}. In Fig. 2 we exhibit the numerically calculated ϵ_{BB} for PRSC (OBE) potentials and the present fit via Eq. (10) with $c_1 = -0.468$ fm^2 ($=-0.492$ fm^2), and $c_2 = 0.014$ fm^4 ($= 0.008$ fm^4). The agreement between the theoretical mixing parameters and those obtained with the shape independent expansion is good in all cases.

The value of $B(0)$ of Eq. (7) is particularly interesting because it essentially determines the low-energy behavior of $tan(2\epsilon_{BB})/2$ given by Eq. (11). We provide an approximation to $B(0)$ which we use to show that this parameter is essentially determined by the long-range part of the tensor force and on-shell S wave NN observables. The present approximation to $B(0)$ could be obtained by dividing the explicit on-shell Lippmann-Schwinger equation for K_{02} by $(k^2\hat{K}_{00})$:

$$\frac{\hat{K}_{02}}{\hat{K}_{00}k^2} = \frac{V_{02}(k,k)}{\hat{K}_{00}k^2} + \frac{2}{\pi}\mathcal{P}\int_0^\infty \frac{dqq^2K_{00}(k,q;k^2)V_{02}(q,k)}{\hat{K}_{00}k^2(k^2-q^2)} + \frac{< k \mid K_{02}G_0V_{22} \mid k >}{\hat{K}_{00}k^2}, \qquad (12)$$

where $\mathcal{P}$ denotes the principal value prescription. Subtracting Eq. (12) at $k = i\alpha$ from Eq.

(12) at $k = 0$ one gets the following approximate relation

$$B(0) \simeq \frac{1}{a_t} \lim_{k\to 0} \left(\frac{V_{02}(k,k)}{k^2} \right) - \frac{2}{\pi}\alpha^2 \int_0^\infty dq \frac{K_{00}(0,q;0)}{a_t(\alpha^2+q^2)} \lim_{k\to 0} \left(\frac{V_{02}(q,k)}{k^2} \right). \tag{13}$$

In deriving Eq. (13) we have ignored the difference between the higher order terms of the scattering equation (12) involving V_{22} and made approximations, such as $lim_{k\to 0} V_{02}(q,k)/k^2 \simeq -V_{02}(q,i\alpha)/\alpha^2$.

In Eq. (13) the low-q values of the tensor potential $V_{02}(q,k)$, and the K matrix elements dominate the integral because of additional factors of q^2 in the denominator for large q. It is well known that the half-shell function $g(q) \equiv K(0,q;0)/a_t$ is a universal function independent of potential models.[13] We find that any reasonable approximation to this quantity in Eq. (13) leads to the same result to less than an estimated error of 1 %. We calculated $B(0)$ with the PRSC NN tensor potentials V_{02}[11]; for $g(q)$ we used the exact results for the PRSC potential and the Yamaguchi potential: $g(q) = \beta^2/(q^2+\beta^2), \beta = 1.4$ fm^{-1}. We obtained from Eq. (13) $B(0) = -0.184$ fm^2, using PRSC half shell functions; and $B(0) = -0.183$ fm^2, with the Yamaguchi form-factor. This shows that Eq. (13) provides a very good approximation to the exact $B(0)$, given by Eq. (7), which in this case is -0.190 fm^2.

The approximation (13) to $B(0)$ can also be used to demonstrate the importance of the long-range behavior of the tensor potential. The PRSC tensor potential[11] V_{02} is a superposition of four Yukawa type potentials, with the longest range component simulating the exchange of a pion with a range parameter of $\mu = 0.7$ fm^{-1}. The three other components of this potential simulates exchange of mesons of masses $2m_\pi$, $4m_\pi$, and $6m_\pi$. We evaluated $B(0)$ using Eq. (13) and setting the strengths of the components of the PRSC tensor potential V_{02} corresponding to exchanged mesons of masses $4m_\pi$, and $6m_\pi$ to zero, using both Yamaguchi and PRSC half shell functions. We obtained for $B(0)$ essentially the previous value $B(0) = -0.184$ fm^2 in both cases. This shows that $B(0)$ is insensitive to the short range part of the tensor potential. Finally, we calculated $B(0)$ with only the longest range part of tensor PRSC potential V_{02} corresponding to the exchanged pion. With both types of half-shell functions we obtained in this case $B(0) = -0.162$ fm^2. This corresponds to a percentage error of only 12% in relation to the full potential: $B(0) = -0.184$ fm^2. Hence only the correct pion exchange tail of the potential approximately reproduces $B(0)$. The Yamaguchi tensor potential, which does not have this long range behavior yields for $B(0)$ of Eq. (7) the very small value $B(0) = 0.02$ fm^2, and

fails to reproduce even approximately the realistic mixing parameters. This simple calculation demonstrates the importance of the correct long-range behavior of the tensor potential V_{02} in reproducing the exact mixing parameters.

In short, we have presented an effective-range-type expansion for the on-shell quantity $tan(2\epsilon_{BB})/(2k^2) \equiv (\hat{K}_{02}/k^2)/[\hat{K}_{00} - \hat{K}_{22}]$ valid at low energies, which can be used for predicting the mixing parameter ϵ_{BB} using the experimental η_d, deuteron binding, NN scattering lengths a_t and a_{02}, pion mass m_π. The parameter a_{02} (or $B(0)$) is essentially determined by the correct long-range behavior of the NN tensor potential, which is demonstrated to be essential for a realistic prediction of the mixing parameter ϵ_{BB}.

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References

[1] H. A. Bethe, Phys. Rev. 76 (1949) 38.

[2] J. M. Blatt and V. F. Weisskopf, Theoretical Nuclear Physics, (Wiley, New York, 1952).

[3] H. P. Stapp, T. Ypsilantis, and N. Metropoulis, Phys. Rev. 105 (1957) 302.

[4] J. M. Blatt and L. C. Biedenharn, Phys. Rev. 86 (1952) 399; L. C. Biedenharn and J. M. Blatt, ibid. 93 (1954) 1387.

[5] T. E. O. Ericson and M. P. Rosa Clot, Ann. Rev. Nucl. Part. Sci. 35 (1985) 271.

[6] M. M. Mustafa, Phys. Rev. C 47 (1993) 473; D. W. L. Sprung, H. Wu, and J. Martorell, ibid. 42 (1990) 863; W. van Dijk, M. W. Kermode, D.-C. Zheng, ibid. 47 (1993) 1898.

[7] R. D. Amado, Phys. Rev. C 19 (1974) 1473; R. D. Amado, M. P. Locher, and M. Simonius, Phys. Rev. C 17 (1978) 403.

[8] J. T. Londergan, C. E. Price, and E. J. Stephenson, Phys. Rev. C 35 (1987) 902; L. D. Knutson, P. C. Colby, and B. P. Hichwa, Phys. Rev. C 24 (1981) 411; L. D. Knutson, B. P. Hichwa, A. Barrosso, A. M. Eiro, F. D. Santos, and R. C. Johnson, Phys. Rev. Lett. 35 (1975) 1570.

[9] Y. Yamaguchi and Y. Yamaguchi, Phys. Rev. 95 (1954) 1635.

[10] A. R. Arndt, I. D. Roper, R. A. Bryan, R. B. Clark, B. J. VerWest, and P. Signell, Phys. Rev. D 28 (1983) 97.

[11] S. C. Peiper, Phys. Rev. C 9 (1974) 883; R. V. Reid, Ann. Phys. (N.Y.) 50 (1968) 411.

[12] R. Machleidt, K. Holinde, and Ch. Elster, Phys. Rep. 149 (1987) 1.

[13] E. F. Redish and K. Stricker Bauer, Phys. Rev. C. 36 (1987) 513; K. Amos et al., ibid. 37 (1988) 934.

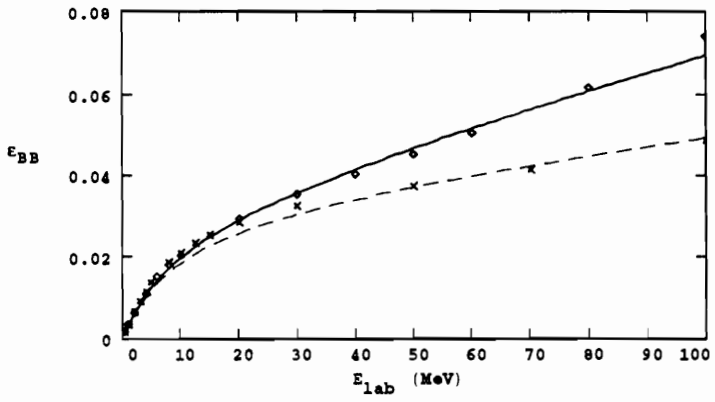

Fig.1 The numerically calculated Blatt-Biedenharn mixing parameters in radians for the PRSC[11] ($\diamond$) and the OBE[12] ($\times$) potentials and the present fit, with $c_1 = -0.458$ fm^2 (PRSC - full line), $= -0.482$ fm^2 (OBE - broken line) and $c_2 = 0$, for E_{lab} upto 100 MeV.

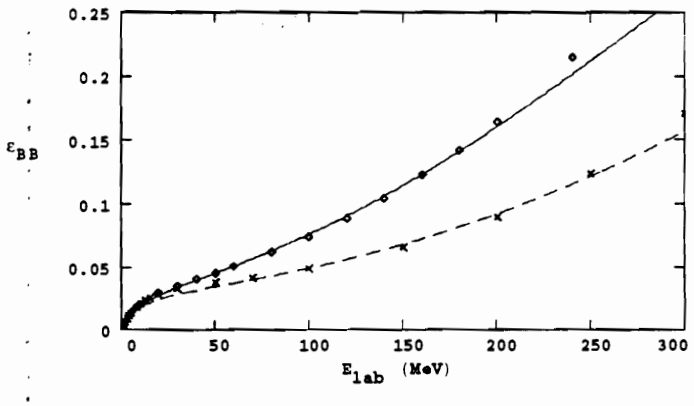

Fig.2 Same as Fig. 1 with $c_1 = -0.468$ fm^2, $c_2 = 0.014$ fm^4 (PRSC $-$ full line), $c_1 = -0.492$ fm^2 and $c_2 = 0.008$ fm^4 (OBE $-$ broken line), for E_{lab} upto 300 MeV.