Optimizing the Reconstruction of Primordial Density Fluctuations by Choice of Smoothing Windows

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Abstract

Recently, considerable attention has been focused on improving algorithms for restoring primordial density fluctuations in the universe by going to higher-order approximations relating nonlinear states to their initial conditions in the early universe. I present evidence that the universally used Gaussian smoothing window is far from optimal. A sharp truncation $P(k) = 0$ for $k > k_c$ leads to a much more direct connection to initial conditions for either "Gaussianization" methods (e.g. Weinberg 1992) or use of the Zel'dovich approximation (e.g. Nusser and Dekel 1993).

Subject headings: cosmology:theory – dark matter – gravitation large-scale structure of the universe – methods: numerical
I. Introduction

The formation of large-scale structure is one of the most interesting problems in cosmology. The continual increase of data from redshift surveys provides improved information about our sample of the universe, and microwave background experiments offer a look at hopefully primordial variations caused by density fluctuations in the smooth early universe. The gravitational instability hypothesis connects the two, describing how small initial fluctuations may have evolved into the structure we see today. The amplitude agreement between theory and data is rather good, at least if nonbaryonic dark matter dominates.

The great improvement in data has stimulated attempts to reconstruct initial conditions from the present highly nonlinear state. There are two main venues in this effort: Gaussianization and use of the Zel'dovich approximation.

II. Major Approaches Used

In the Gaussianization method (Weinberg 1992) one uses a smoothed version of the evolved density field. Using the tendency of linear and mildly non-linear gravitational instability to preserve the rank order of density fluctuations, one can use the known properties of Gaussian distributions to go from rank order directly to an initial fluctuation \( \delta \) (modulo a normalization constant). This method of course relies on the assumption that the initial conditions were Gaussian. This assumption seems to be reasonable based on present data, but might be unduly restrictive.

Another approach involves the use of the Zel’dovich (1970) approximation to extrapolate backward from the present density or velocity field to the initial conditions (Bertschinger and Dekel 1989; Dekel, Bertschinger, and Faber 1990; Nusser, et al. 1991; Nusser and Dekel 1993; Gramann 1993a,b). This procedure does not depend on any assumptions of Gaussianity. Independent restoration of velocity and galaxy density contrast fields can be used to infer \( \Omega \), the cosmological mean density (e.g. Nusser and Dekel 1993). (All of this is of course intimately related to the procedure of using present velocities to infer present mass density. This can then be compared with the directly observed galaxy density and infer the relation between the two. Here I focus on the issue of restoring initial conditions). This second approach also uses smoothing to reach a mildly nonlinear state if density is the basis, or to create an irrotational velocity field which can be used to infer a potential. Gramann (1993a,b) has proposed improvements on the reconstruction method based on going to higher order in perturbation theory.
What has not been questioned in all these approaches is the choice of smoothing window. A Gaussian convolution is typically used, probably because theorists developed these approaches and Gaussian windows have nice properties for doing calculations. But other choices are possible. Observers often use a tophat smoothing window, in which the density at a point is set equal to the mean density within a sphere of radius \( \lambda \) centered at that point. This has the advantage that in a real data sample boundary effects are confined to a layer at the surface of the sample. Nonlinear dynamics suggest that \( k \)-truncation might be preferable (setting the fluctuation power \( P(k) = 0 \) for \( k > k_c \)). This could suppress fully the unwanted nonlinearities, while not affecting fluctuations on larger scales, as a Gaussian convolution will. Coles et al. (1993) have found, that such a truncation in the initial conditions greatly enhances the validity of the Zel'dovich approximation for a variety of initial conditions.

Systematic testing is needed to establish one weak link in all these procedures. We need to know which type of smoothing window allows us to remove undesirable nonlinearities that cause trouble for extrapolation schemes, while preserving the most information. In other words, which is the best window into mildly nonlinear dynamics?

I investigate this question by using sets of \( PM \) type numerical clustering simulations. For this Letter I use simulations with \( 128^3 \) particles, with power-law initial spectra \( P(k) \propto k^n \). I show results for two realizations each of \( n = +1 \) and \( n = -1 \), which spans the likely spectra on scales of cosmological interest. I have used only Gaussian initial conditions. The simulations are described more fully in Melott and Shandarin (1993).

### III. Optimizing Gaussianization

Weinberg (1992) smooths the nonlinear distribution in a CDM model at moments \( \sigma_8 = 0.5 \) and \( \sigma_8 = 1.0 \) with a \( 3 \, h^{-1} \) Mpc Gaussian window. He then reassigns each pixel a new density, maintaining the overall variance and the rank order of that pixel. He observes that his following reconstruction scheme works reasonably well. He does find that fairly large ad hoc correction factors are also needed to restore the initial amplitude at modest wavenumbers, corrections as large as 2 to 3.

The smoothing is supposed to restore the pixel rank order that the linear primordial Gaussian field would have if smoothed in the same way. I have tested this assumption by measuring the RMS error \( E_r \) in the rank order of the pixel assignment after smoothing and
assigning a rank $n_f$ out of $N$ pixels, compared with the rank order $n_i$ in the initial conditions given similar smoothing

$$E_r = \frac{\left< (n_f - n_i)^2 \right>^{1/2}}{N}$$

(1)

as a diagnostic of the validity of the central assumption of the procedure. I measure $E_r$ for Gaussian (G) windows Tophat smoothing (TH), and $k$–space truncation (KT) for a variety of smoothing scales applied to our 2 power–law simulations at various stages of nonlinearity. For obvious reasons we do not probe any strongly nonlinear regime.

The results are shown in Figure 1(a) for $n = -1$ initial conditions and 1(b) for $n = +1$ initial conditions, as a function of $\sigma_p$, the rms density fluctuation of the evolved $n$–body model after smoothing.

It is clear that KT has significantly less error in the rank–order assignment than either of the other two smoothing methods. As rank order is improved, the accuracy of this method can only improve.

IV. Optimizing use of the Zel’dovich Approximation

To test this, we compare density distributions in the $n$–body simulations with $n$–body distributions constructed by using Zel’dovich approximation extrapolations of the initial conditions. These extrapolations are based on the Coles et al. (1993) finding that agreement is improved if power spectra in the initial conditions are set to zero for $k > k_c$ before applying the Zel’dovich approximation. This does not mean there is negligible small–scale power. The operations of smoothing and application of the Zel’dovich approximation do not commute; the approximation generates lots of small-scale power (see Coles et al. Fig. 4a,b, and c). One might worry that the use of a sharp truncation of in the initial conditions predisposes a sharp truncation of the evolved state to work best. I checked that the result reported below also holds when the initial amplitudes have a Gaussian cutoff before applying the Zel’dovich approximation. But it does mean that a great deal of noise has been removed from the approximation. Because mode coupling only works robustly from small $k$ to large $k$, either kind of cutoff improves the probability that the extrapolated field will have the right phases and amplitudes, and will imply a more correct velocity potential as well. In fact agreement of density fields is a more stringent test than agreement of velocity potential.

If we wish to restore the initial field by use of the Zel’dovich approximation, then a field created with that approximation must agree with our numerical simulation to the extent
possible. To test this agreement we measure the cross-correlation

$$S_{12} = \frac{\langle \delta_1 \delta_2 \rangle}{\sigma_1 \sigma_2}$$

(2)

where $\delta$ is the local density contrast and $\sigma$ its RMS, after Coles et al. (1993). Figure 2,a,b $S_{12}$ is plotted against $\sigma\rho$ again for the smoothing windows applied in the same way to the simulation and the Zel'dovich approximation to it.

Again $k$-space truncation stands out as better than the other two. This means that the Zel'dovich approximation link is noticeably more reliable when KT is used on the evolved state. If this link is more reliable for densities, it is by implication more reliable for the velocity potential.

V. Discussion

I have posed the following question for the two leading schemes for reconstructing the initial conditions of the gravitational clustering in the universe: If we wish to construct a representation of the initial conditions with our scheme, and describe that representation through the same smoothing window we have used on the present nonlinear universe, which choice of window (tophat, Gaussian, or $k$-space truncation) works best? I have used a symmetric set of windows (a Gaussian is its own Fourier Transform, and the other two are similar operations in $k$ or $\omega$) and all 3 are in use. However, only Gaussian has so far been used for reconstruction. I have used a pragmatic definition of "works best", using agreement in the rank order in one case or agreement in the full density field (and by implication the velocity potential) in the other. If these measures improve, then reconstruction of the initial conditions must also improve. The exact amount of improvement in realistic data sets is not known, and is outside the scope of this Letter. However, it is likely to be substantial, especially for schemes based on the Zel'dovich approximations.

It is important to question our assumptions, in this case the routine use of Gaussian smoothing. The superiority of $k$-truncation is likely based on a fairly sudden transition from the regime in which multistream (phase mixing) is unimportant to where it dominates. A sharp bandpass filter seems to be more effective in salvaging what we can of the initial conditions in the universe, rather like the approach used in cleaning up old 78 RPM Hank Williams records to eliminate noise. It appears that for purposes involving gravitational instability, there is a clear choice.
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References


Figure Captions

Figure 1. The rms error in rank ordering between the initial conditions and the evolved simulation, each smoothed with the same size and type of smoothing window, is plotted against the rms density fluctuation in the smoothed nonlinear field for power-law initial spectra as specified in the Figure. Open circles: K-truncation smoothing; Filled circles: Tophat smoothing; X’s: Gaussian smoothing.

Figure 2. The cross-correlation between the nonlinear field and the Zeldovich approximation to that field, is plotted in the same way as for Figure 1.
Fig. 1
Fig. 2

(a) $n = -1$

(b) $n = +1$