STRING THEORY IN COSMOLOGY

N. SANCHEZ
Observatoire de Paris
Section de Meudon, Demi-m"m
5, Place Jules Janssen
92195 Meudon Principal Cedex
FRANCE

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ABSTRACT: In this lecture, I will describe those aspects of string theory which are relevant for Cosmology, with emphasis, for the purposes of this meeting, on the problem of connecting string theory to observational reality. I will talk on:

- fundamental strings
- cosmic strings
- the possible connection among them

1. Fundamental Strings

There exists only three fundamental dimensional magnitudes (length, time and energy) and so three fundamental dimensional constants

\( (c, \hbar, G) \).

All other physical parameters being dimensionless, they must be calculable in an unified quantum theory of all interactions including gravity ("theory of everything"). The present interest in the theory of fundamental strings comes largely from the hope that it will provide such a theory. (There is no hope to construct a consistent quantum theory of gravity in the context of point particle field theory). As it is known, fundamental strings cannot exist in arbitrary space time and in particular they are consistent only in well determined (critical) dimensions \( D_c > 4 \). The unification of all interactions described by these theories takes place at the critical
dimensions $D = D_c > 4$ where the characteristic unification scale is the Planck scale:

\[
\ell_{Pl} = (2G\hbar/c^3)^{1/2} = 10^{-33} \text{ cm}, \quad m_{Pl} = (\hbar c/2G)^{1/2} = 10^{-5} \text{ g},
\]
corresponding to energies of order $E_{Pl} = 10^{19} \text{ GeV}$. Here $\mu = 1/\alpha'$, ($\alpha'$ is the characteristic string slope parameter with dimension of (length)$^2$ and $(\mu)^{1/2}$ characterizes the energy of string excitations. The problem of connecting string theory to reality is that of relating the theory in $D > 4$ dimensions where:

\[
(G\mu)_{D} \approx 0 \quad (1),
\]
to the real world where $D = 4$ and:

\[
(G\mu)_{D=4} \ll 1,
\]
problem currently handled within the so-called compactification schemes (or alternatively by the "four dimensional models").(see for example refs.1 and 2). The effective low energy point particle field theory ( containing GUTs and classical General Relativity ) obtained by compactification is governed by physical couplings expressed in terms of $\mu$ via dimensionless numbers. (These numbers are vacuum expectation values of the different fields of the theory). The gauge ($g$) and gravitational couplings are (at the tree level) related by the Kaluza Klein ( heterotic type ) relation

\[
g^2 = G\mu \quad \text{ in } \quad D = 4
\]

The four and $D$ dimensional couplings are related by

\[
g_D^2 = g_4^2 V_{D-4} = (G\mu)_4 V_{D-4}
\]

where $V_{D-4}$ is the volume of the $(D - 4)$ dimensional compact manifold $K_{D-4}$ of size $\ell_{Pl}$ (assuming the ground state of the metric to be $R^4 \times K_{D-4}$). It is interesting here to plot the dimensionless parameter $G\mu$ against $M_{GUT}^2/\mu$ as required by string unification constraints \[3\] on the
renormalization group equations. Any perturbative compactification scheme leads to the non shaded triangle delimited in fig.1 (double logarithmic plot).

It is natural to require $M_{\text{GUT}} < (\mu)^{1/2}$ which excludes the upper shaded region. It is also natural to require that loop string corrections be under control, that is the ratio $(\text{loops/trees}) \ll 1$, which excludes the lower shaded region. The left shaded vertical region is also excluded in order to reproduce the value of the fine structure constant $g^{\text{em}} = 1/137$. This constraint is relevant here. The renormalization group equations:

$$M \left( \frac{dg^2}{dM} \right) = \frac{2}{3\pi} (g^2)^2$$

yield

$$g^2 (M_0) = \frac{g^2(M)}{1 + (2/3\pi) g^2(M) \ln \left( \frac{M}{M_0} \right)}$$

(3)

For $M_F < M < M_{\text{GUT}}$, the coupling constant $g(M)$ increases with $M$ as it should be (QED is infrared stable), and with $M_0 = M_F = 200$ GeV, $M = M_{\text{GUT}} = 10^{16}$ GeV, we see:

$$1/g^2 (M_{\text{GUT}}) = 1/g^2 (M_F) - 6.35,$$

i.e.:

$$g^2 (M_{\text{GUT}}) > g^2 (M_F) = 1/137.$$

Since the string unification relation eq (1), one must have:

$$G\mu > 1/137 \approx 10^{-2}.$$  

(4)

Fig.1. The elementary unshaded triangle restricting the values of $G\mu$ according to current compactification schemes (log-log plot as in ref.[3]).
Let us recall now that cosmic strings are macroscopic objects, finite energy solutions (vortices) of gauge theories coupled to Higgs fields. They have a mass per unit length $\mu$:

$$G\mu = \left( \langle \Phi \rangle / m_{\text{pl}} \right)^2 ,$$

and a radius $\delta \sim 1/M_{\text{UNIF}}$, where $\langle \Phi \rangle$ is the vacuum expectation value of the Higgs field. For grand unified theories:

$$\langle \Phi \rangle_{\text{GUT}} = 10^{-3} m_{\text{pl}} ,$$

and thus:

$$G\mu = 10^{-6}$$

These thin strings are described by the Nambu equations.

On the other hand, cosmic strings should be derived from fundamental strings. The thermodynamical model of strings (ideal gas of free strings at finite temperature) predicts the existence of a phase transition at the Hagedorn [4] critical temperature $\approx (\mu)^{-1/2}$ to a finite energy state in which all the energy is concentrated in a very long string [5]. When applied to the primordial universe, (at $t \approx 10^{-32}$ s) this transition can be interpreted as the condensation of fundamental strings into macroscopic cosmic strings and this stringy phase could be the ancestor of the adiabatic era [6,7]. But if cosmic strings are created in this way, they must satisfy the constraint eq. (4).

Timing of millisecond pulsars as well as cosmic microwave radiation data on $\Delta T/T$ (see below Section 2) sets up an upper limit to the string parameter $G\mu$ such that $G\mu < 10^{-6} - 10^{-7}$.

At present, we have two ways of connecting string theory to observational reality and thus of limiting the string parameter $G\mu$ [8]:

(i) radioastronomy observations (millisecond pulsar timing; also microwave background data) which puts:

$$G\mu < 10^{-7} - 10^{-6} ;$$

(ii) elementary particle phenomenology (compactification schemes; also "four dimensional models") which require:

$$G\mu > 10^{-2} .$$
Cosmic strings derived from grand unified theories agree with (i). For cosmic strings derived from fundamental strings or for fundamental strings themselves there is contradiction between the possibilities (i) and (ii). One of these scenarii connecting string theories to reality must be revised (or the connection between fundamental and cosmic strings rejected). Meanwhile, FIRAS/COBE data could help to solve the problem, select one scenario or reject both of them.

1.1 FUNDAMENTAL STRINGS IN COSMOLOGICAL BACKGROUNDS

We have first studied quantum string propagation in de Sitter space [9]. We have found the mass spectrum and vertex operator and found an string instability in de Sitter space. The lower mass states are the same as in flat space-time but heavy states deviate significantly from the linear Regge trajectories. We found that there exists a maximum (very large) value of order 1/g^2 for the quantum number N and spin J of particles. There exists real mass solutions only for:

\[ N < N_{\text{max}} = \frac{\pi}{2g^2} + O(\frac{g^{-2}}{g^2}) \quad , \quad g = 10^{-61} \]

Moreover, for states in the leading Regge trajectory, the mass monotonously increases with J up to the value

\[ J_{\text{max}} = \frac{1}{g^2} + O(1) \]

corresponding to the maximal mass \( m_{\text{max}}^2 = 0.76 + O(g^2) \). Beyond \( J_{\text{max}} \) the mass becomes complex. These complex solutions correspond to unstable states already present here at the tree (zero handle) level.

From the analysis of the mass spectrum, we find that the critical dimension for bosonic strings in de Sitter space-time is \( D=25 \) instead of 26 in Minkowski space-time). This result is confirmed by an independent calculation of the critical dimension from the path integral Polyakov's formulation, using heat-kernel techniques: we find that the dilaton \( \beta \)-function in D-dimensional de Sitter space-time must be:

\[ \beta \phi = \left( D + 1 - 26 \right) / \left( \alpha'48\pi^2 \right) + O(1) \]
It is a general feature of de Sitter space-time to lower the critical dimensions in one unit. For fermionic strings we find $D=9$ instead of the flat value $D=10$.

We have found that for the first order amplitude $\eta^i(\sigma, \tau), (i = 1,...,D-1$ refers to the spatial components), the oscillation frequency is:

$$\omega_n = \left[ n^2 - (\alpha'mH)^2 \right]^{1/2},$$

instead of $n$, where $H$ is the Hubble constant. For high modes $n \gg \alpha'mH$, the frequencies $\omega_n \approx n$ are real. The string shrinks as the universe expands. This shrinking of the string cancels precisely the expanding exponential factor of the metric and the invariant spatial distance does not blow up. Quantum mechanically, these are states with real masses $(m^2H^2<1)$. This corresponds to an expansion time $H^{-1}$ very much bigger than the string period $2\pi/n$, that is, many string oscillations take place in an expansion period $H^{-1}$ (in only one oscillation the string does not see the expansion).

For low modes $n < \alpha'mH$, the frequencies become imaginary. This corresponds to an expansion time very short with respect to the oscillation time $2\pi/n$ ("sudden" expansion, that is the string "does not have time" to oscillate in one time $H^{-1}$). These unstable modes are analyzed as follows. The $n=0$ mode describes just small deformations of the center of mass motion and it is therefore a physically irrelevant solution. When $\alpha'mH > 1$ relevant unstable modes appear. Then, the $n=1$ mode dominates $\eta^i(\sigma, \tau)$ for large $\tau$. Hence, if $\alpha'mH > (2)^{1/2}$, $\eta^i$ diverges for large $\tau$, that is fluctuations become larger than the zero order and the expansion breaks down. However, the presence of the above instability is a true feature as it has been confirmed later by further analysis [10].

The physical meaning of this instability is that the string grows driven by the inflationary expansion of the universe. That is, the string modes couple with the universe expansion in such a way that the string inflates together with the universe itself. This happens for inflationary (i.e., accelerated expanding) backgrounds. In ref. [10] we have studied the string propagation in Friedman-Robertson-Walker (FRW) backgrounds (in radiation as well as matter dominated regimes) and interpreted the instability above discussed as Jeans-like instabilities. We have also determined under which conditions the universe expands, when distances are measured by stringy rods.
It is convenient to introduce the *proper amplitude* $\chi^i = C \eta^i$, where $C$ is the expansion factor of the metric. Then, $\chi^i$ satisfies the equation:

$$\ddot{\chi}^i + \left[ n^2 - \frac{\ddot{C}}{C} \right] \chi^i = 0$$

Here dot means $\tau$-derivative. Obviously, any particular (non-zero) mode oscillates in time as long as $\frac{\dot{C}}{C}$ remains $< 1$ and, in particular, when $\frac{\dot{C}}{C} < 0$. A time-independent amplitude for $\chi$ is obviously equivalent to a fixed proper (invariant) size of the string. In this case, the behaviour of strings is stable and the amplitudes $\eta$ shrink (like $1/C$).

It must be noticed that the time component, $\chi^0$ or $\eta^0$, is always well behaved and no possibility of instability arises for it. That is the string time is well defined in these backgrounds.

i) For non-accelerated expansions (e.g. for radiation or matter dominated FRW cosmologies) or for the high modes $n >> \alpha \Omega_m$ in de Sitter cosmology, string instabilities do not develop (the frequencies $\omega_n \approx n$ are real). Strings behave very much like point particles: the centre of mass of the string follows a geodesic path, the harmonic-oscillator amplitudes $\eta$ shrink as the universe expands. in such a way to keep the string's proper size constant. As expected, the distance between two strings increase with time, relative to its own size, just like the metric scale factor $C$.

ii) For inflationary metrics (e.g. de Sitter with large enough Hubble constant), the proper size of the strings grow (like the scale factor $C$) while the co-moving amplitude $\eta$ remains fixed ("frozen"), i.e. $\eta \approx \eta(0)$.

Although the methods of references [9] and [10] allow to detect the onset of instabilities, they are not adequate for a quantitative description of the high unstable (and non-linear) regime. In ref. [11] we have developed a new quantitative and systematic description of the high unstable regime. We have been able to construct a solution to both the non-linear equations of motion and the constraints in the form of a systematic asymptotic expansion in the large $C$ limit, and to classify the (spatially flat) Friedman-Robertson Walker (FRW) geometries according to their compatibility with stable and/or unstable string behaviour.
An interesting feature of our solution is that it implies an asymptotic proportionality between the world sheet time \( t \) and the *conformal time* \( T \) of the background manifold. This is to be contrasted with the stable (point-like) regime which is characterized by a proportionality between \( t \) and the *cosmic time*. Indeed, the conformal time (or \( \tau \)) will be the small expansion parameter of the solution: the asymptotic regime (small \( \tau \) limit) thus corresponds to the large \( C \) limit only if the background geometry is of the inflationary type. The non-linear, high unstable regime is characterized by string configurations such that:

\[
X^0 \ll |\dot{X}^0| , \quad |\dot{X}^i| \ll |X^i|
\]

with:

\[
X^0(\sigma, \tau) = C L(\sigma), \quad L(\sigma) = (\delta_{ij} X^i X^j)^{1/2}
\]

\[
X^i(\sigma, \tau) = A^i(\sigma) + \tau^2 D^i(\sigma)/2 + \tau^{1+2\alpha} F^i(\sigma)
\]

where \( A^i, D^i \) and \( F^i \) are functions determined completely by the constraints, and \( \alpha \) is the time exponent of the scale factor of the metric: \( C = \tau L^{-\alpha} \).

For power-law inflation: \( 1 < \alpha < \infty \), \( X^0 = \tau L^{1-\alpha} \).

For de Sitter inflation: \( \alpha = 1 \), \( X^0 = \ln(-\tau HL) \).

For Super-inflation: \( 0 < \alpha < 1 \), \( X^0 = \tau L^{1-\alpha} + \text{const} \).

Asymptotically, for large radius \( C \to \infty \), this solution describes string configurations with expanding proper amplitude.

These highly unstable strings contribute with a term of negative pressure to the energy-momentum tensor of the strings. The energy momentum tensor of these highly unstable strings (in a perfect fluid approximation) yields to the state equation \( \rho = -P (D-1) \), \( \rho \) being the energy density and \( P \) the pressure (\( P < 0 \)). This description corresponds to large radius \( C \to \infty \) of the universe.

For small radius of the universe, highly unstable string configurations are characterized by the properties

\[
|\dot{X}^0| \gg |X^0| , \quad |\dot{X}^i| \ll |\dot{X}^i|
\]

The solution for \( X^i \) admits an expansion in \( \tau \) similar to that of the large radius regime. The solution for \( X^0 \) is given by \( L/C \), which corresponds to small radius \( C \to 0 \), and thus to small \( \tau \).
This solution describes, in this limit, string configurations with shrinking proper amplitude, for which $C^{X^i}$ behaves asymptotically like $C$, while $C^{X^i}$ behaves like $C^{-1}$. Moreover, for an ideal gas of these string configurations, we found:

$$\rho = P(D-1),$$

with positive pressure which is just the equation of state for a gas of massless particles.

More recently \[12\], these solutions have been applied to the problem in which strings became a dominant source of gravity. In other words, we have searched for solutions of the Einstein plus string equations. We have shown, that an ideal gas of fundamental strings is not able to sustain, alone, a phase of isotropic inflation. Fundamental strings can sustain, instead, a phase of anisotropic inflation, in, which four dimensions inflate and simultaneously, the remaining extra (internal) dimensions contract. Thus, fundamental strings can sustain, simultaneously, inflation and dimensional reduction. In ref. \[12\] we derived the conditions to be met for the existence of such a solution to the Einstein and string equations and discussed the possibility of a successful resolution of the standard cosmological problems in the context of this model.

2. Cosmic Strings : Observational Tests

In January 1990) \[13\], the spectrum of the cosmic microwave background radiation (MBR) between 1cm and 500mm from regions near the North Galactic pole, observed by the FIRAS (Far Infrared Absolute Spectrophotometer) experiment on COBE satellite, has been reported. It is a "pure" spectrum of a black body with a temperature of $2.735\pm0.06$ K: the deviation from a black body is less than 1% of the peak intensity over the range 1 cm to 500mm; this COBE spectrum thus disclaims the Berkeley-Nagoya excess reported by Matsumoto et al (1988)\[14\].

A stochastic background of gravitational waves can be detected by means of pulsar timing observations \[15\]. The stable rotation and sharp radio pulses of PSR 1937+21 make this millisecond pulsar a clock whose frequency stability may exceed that of the best atomic clocks.
Fig. 2. Schematic logarithmic spectrum \( \Omega_g(P) \) as a function of the present period \( P = 2 \pi / \omega \) of the gravitational waves generated by cosmic strings. \( P_1 = 10^4 \) yrs, \( P_2 = 10^6 \) yrs, \( P_0 = 10^{10} \) yrs. The observational time span is \( T \approx 5 \) yrs for millisecond pulsars; \( T \approx 10^{-10^6} \) yrs for the binary pulsar. The main effects of distortions in the cosmic microwave background due to electromagnetic and/or gravitational waves in each of the three regions are also indicated.

From the compatibility among these measurements, including the absence of distortion in the MBR spectrum recently reported by the COBE-FIRAS experiment, we obtained: (a) the coefficient \( f \) relating the electromagnetic and gravitational radiation rates released by SCS, (b) the chemical potential \( \mu_{\text{SCS}} \) produced by SCS, (c) the string loop evolution parameters \( (\alpha, \beta, \gamma) \), and (d) the string parameter \( G \mu \). Here \( \alpha \) measures the initial loop size relative to the horizon size, \( \beta \) accounts for the loop formation rate and \( \gamma \) for the total power energy emitted by each loop.
Up to now, the data yield a firm upper limit $\rho_g < 3.5 \times 10^{-36}$ g/cm$^3$ for the energy density of a cosmic background of gravitational radiation at the frequency of about $10^{-8}$ Hz [16]. This limit corresponds to approximately $2 \times 10^{-7}$ of the density required to close the Universe. On the other hand, accurate time of arrival measurements of pulses from the binary pulsar PSR 1913+16 over the last 14 years have led in particular to an accurate observed orbital decay rate $P_b$. The difference between predicted and observed orbital decay rates $P_b$ of PSR 1913+16 [17] can be used to place the best available limit on the total energy density of a cosmic gravitational wave background $\Omega_{\text{total}} < 4 \times 10^{-2} h^{-2} (h_0 = 0.7)$ at the ultra low frequencies of $10^{-9}$ to $10^{-13}$ Hz.

Cosmic superconducting strings (SCS) attract current considerable interest in astroparticle physics [18,19]. If SCS exist, loops formed as a consequence of string interactions, decay during the expansion of the universe, by emitting gravitational and electromagnetic waves. The radiation emitted by loops at different epochs, from their formation at $t \sim 10^{-32}$ s till now, adds up into a stochastic gravitational wave background characterized by the (dimensionless energy density) spectrum $\Omega_g(\omega)$, known on a wide range of frequencies in three different regions [20] (as it is shown in Fig.2):

I: $\omega >> 10^{-15}$ Hz, II: $10^{-13}$ Hz $< \omega < \omega_1$, and III: $\omega < \omega_2$. In ref [21] we have studied the constraints which can be placed on SCS through the results of five different measurements:

(i) $\Omega_{gI}(\omega) = (\omega/\rho_{\text{crit}}) d\rho_{gI}/d\omega$, $\rho_{\text{crit}} = 4 \times 10^{-29}$ g cm$^{-3}$;

(ii) $\Omega_{g\text{total}}(\omega) = \int_{10^{-15} h_0}^{10^{-8} h_0} d\omega \Omega_g(\omega)/\omega$, through the binary pulsar residual $R_b$.

(iii) MBR temperature fluctuation $\Delta T/T$ for angular separation of about $1^\circ$, (region III);

(iv) chemical potential $\mu_o$ (region I) and

(v) Comptonization parameter $y$ (region II), characterizing the MBR spectral distortions.
Recently, the measurable quantities (i)-(v) above referred have been expressed explicitly in terms of $G\mu$, the set $(\alpha, \beta, \gamma)$ and $f$ covering the whole spectrum and SCS evolution [22]:

$$\Omega_{gl} = 1.8 \times 10^{-6} [G\mu/10^{-6}]^{1/2} (\alpha^{3/2} \beta)^{-1/2} (1+f)^{-3/2}$$

$$\Omega_{total} = 4.6 \times 10^{-4} [G\mu/10^{-6}]^{1/2} (\alpha^{3/2} \beta)^{-1/2} (1+f)^{-3/2}$$

$$\Delta T/T \leq 2.5 \times 10^{-6} [G\mu/10^{-6}] (\alpha \beta \gamma)^{1/2} (1+f)^{-3/4}$$

$$\mu_0 = 0.2 [G\mu/10^{-6}]^{1/2} (\alpha^{3/2} \beta)^{-1/2} f (1+f)^{-3/2}$$

$$y = 0.4 [G\mu/10^{-6}]^{1/2} (\alpha^{3/2} \beta)^{-1/2} f (1+f)^{-3/2}$$

At the present time, observations place the following constraints:

(i) millisecond pulsar timing measurements lead to:

$$\Omega_g (\omega) < 2 \times 10^{-7} \text{ for } (\omega/2\pi) \sim 0.2 \text{ cy/yr} \quad (6)$$

(Taylor, 1989 [16]), but more generally:

$$\Omega_g (\omega) < 4 \times 10^{-7} (2\pi/T)^4 R^2 \mu \text{ for } \omega > 2\pi/T \quad (7)$$

where $T$ is the observation time-span in years and $R\mu$ the residual in $\mu$s.

(ii) binary pulsar timing measurements lead to:

$$\Omega_{total} < 4 \times 10^{-2} \text{ h}^{-2} \text{ for } 10^{-12} \text{ Hz} < \omega < 10^{-9} \text{ Hz} \quad (8)$$
From the latest millisecond pulsar timing and from the preliminary COBE data, we obtain the following upper limits on $G\mu$:

<table>
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<tr>
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<th>generic values</th>
<th>latest numerical simulation values</th>
<th>kinky strings</th>
</tr>
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<tbody>
<tr>
<td>$\alpha \sim \beta \sim 1$</td>
<td>$\gamma \sim 100$</td>
<td>$\beta \sim 850, \gamma \sim 100$</td>
<td>$\beta \sim 750, \gamma \sim 100$</td>
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</tbody>
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<table>
<thead>
<tr>
<th></th>
<th>$G\mu &lt; 1.2 \times 10^{-6}$</th>
<th>0.7 $\times 10^{-7}$</th>
<th>0.2 $\times 10^{-7}$</th>
</tr>
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<tbody>
<tr>
<td>PSR 1937 + 21</td>
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<tr>
<td>COBE</td>
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The first set of values for $(\alpha, \beta, \gamma)$ correspond to generic values [25], the second set correspond to latest numerical simulation results [26,27] and the third set to "kinky" string values [28].

We see that the upper limits placed by COBE on $G\mu$ converge to those placed by the latest PSR 1937+21 timing measurements and that generic values of $(\alpha, \beta, \gamma)$ lead to $G\mu \sim 10^{-6}$ whereas numerical values yield $G\mu \sim 10^{-7}$.

We have placed new constraints on SCS and on the parameters governing their evolution by requiring that electromagnetic and gravitational energies released by loops, be compatible with the preliminary data on the MBR spectrum from COBE's - FIRAS experiment.
(Taylor and Weisberg, 1989 [17]), and more generally:

$$\Omega_{\text{total}} < \left( \frac{2H^2}{\Omega_b} \right)^2 \left( \frac{R_b}{T_b} \right)^2$$  \hspace{1cm} (9)

where \( H = h \times 100 \text{ km s}^{-1} \text{ Mpc}^{-1} \) is the Hubble constant, \( (h \approx 0.7) \).

(iii) angular temperature anisotropy measurements yield:

$$\frac{\Delta T}{T} < 6 \times 10^{-5} \quad \text{for angular separations} \quad \theta \sim 1^\circ$$  \hspace{1cm} (10)

(Wilkinson 1986, Partridge 1988) [23].

(iv) latest Rayleigh-Jeans MBR spectrum measurements yield:

$$\mu_0 < 10^{-2} \quad \text{with} \quad T_{\text{RJ}} \approx 2.74 \pm 0.02 \text{ K}$$  \hspace{1cm} (11)

(Smoot et al 1987) [24].

The results from COBE [13] show a pure Black-Body spectrum at \( 2.735 \pm 0.06 \text{ K} \) within 1% of the peak intensity over the range 1 cm to 500 mm. Using their conservative 1% error bands, Mather et al [13] set a 3σ-upper limit on the Comptonization \( y \)-parameter of 0.001, and then:

$$y < 10^{-3} \quad \text{for} \quad T \sim 2.735 \pm 0.06 \text{ K}$$  \hspace{1cm} (12)

From a fit to a pure Bose-Einstein spectrum with a chemical potential \( \mu_0 \) independent of frequency, Mather et al. gave also the 3σ-upper limit:

$$\mu_0 < 0.9 \times 10^{-2}$$  \hspace{1cm} (13)

Notice that this preliminary value is about an order of magnitude greater than the value (12) on \( y \).

Now, we can express the string parameter \( G\mu \) in terms of these measurable quantities with their present upper limits:
\[ G\mu = 1.2 \times 10^{-8} \left( \frac{\Omega_{gI}}{2} \times 10^{-7} \right)^2 \left( \alpha^{3/2} \beta \right)^{-2} \gamma (1+f)^3 \]  

(14)

\[ G\mu = 0.4 \times 10^{-7} \left( \frac{\Omega_{\text{total}}}{10^{-4}} \right)^2 \left( \alpha^{3/2} \beta \right)^{-2} \gamma (1+f)^3 \]  

(15)

\[ G\mu = 2.4 \times 10^{-5} \left( \frac{[\Delta T/T]}{6} \times 10^{-5} \right) \left( \alpha \beta \gamma \right)^{-1/2} (1+f)^{3/4} \]  

(16)

\[ G\mu = 2 \times 10^{-9} \left( \frac{\mu_c/0.9}{10^{-2}} \right)^2 \left( \alpha^{3/2} \beta \right)^{-2} \gamma (1+f)^3 f^{-2} \]  

(17)

\[ G\mu = 6.2 \times 10^{-12} \left( \frac{y/10^{-3}}{2} \right)^2 \left( \alpha^{3/2} \beta \right)^{-2} \gamma (1+f)^3 f^{-2} \]  

(18)

Note that in eq.(14), the quoted upper limit $10^{-4}$ for $\Omega_{\text{total}}$ is not yet reached. From eqs. (14) and (16) and on the other hand, from eqs. (18) and (16) we get the following relations for the string evolution parameters and the ratio between electromagnetic and gravitational radiation rates:

\[ \alpha^5 \left( \frac{\beta}{\gamma} \right)^3 = 6.8 \times 10^{-14} (1+f)^{9/2} f^{-4} \left( \frac{y/10^{-3}}{2} \right)^4 \left( \frac{[\Delta T/T]}{6} \right)^{-2} \]  

(19)

\[ \alpha^5 \left( \frac{\beta}{\gamma} \right)^3 = 0.25 \times 10^{-6} (1+f)^{9/2} \left( \frac{\Omega_{gI}}{2} \times 10^{-7} \right)^4 \left( \frac{[\Delta T/T]}{6} \right)^{-2} \]  

(20)

\[ f = 2.3 \times 10^{-2} \left( \frac{y/10^{-3}}{2} \right) \left( \frac{\Omega_{gI}}{2} \times 10^{-7} \right)^{-1} \]  

Finally, with the latest upper limits on $\Omega_{gI}(\omega)$, $\Delta T/T$ at 1°, $y$ and the value $\mathcal{O}_L^2$ for $f$, from eqs. (14), (16), (18) we have for the string parameter:
with the already performed MBR angular $\Delta T/T$ measurements (Sachs-Wolf effect) for $\theta \sim 1^\circ$, and with the latest millisecond pulsar timing measurements. These limits continue to descend but the breaking point is not yet reached. The absence of distortion lies within 1% of the peak intensity, but the FIRAS sensitivity for each spectral element is expected to be $1/1000$ of the peak of the 2.735 K spectrum. In addition, new constraints are expected to be placed from limits on MBR anisotropies: from the anisotropy experiment$^{[29]}$ (Differential Microwave Radiometers) on COBE satellite, the sensitivity on $\Delta T/T$ in a field of view of about $7^\circ$ is expected to be increased in an order of magnitude with respect to previous experiments. This will set up stringent limits on $G\mu$ for large straight moving SCS which produce $\Delta T/T$ fluctuations through lensing type effects.

3. Connection between Fundamental Strings and Cosmic Strings. Macroscopic Behaviour of Fundamental Strings

Fundamental strings exhibit macroscopic behaviour which appears quite similar to those of topological cosmic strings. These include:
(i) the reconnection or self-intersection processes,
(ii) the decay rate of fundamental strings into gravitational radiation,
(iii) the thermodynamical behaviour of strings at high temperatures.

It is important to know whether highly excited fundamental strings behave in a classical way. A quantity which is interesting and which can be obtained analytically is the decay rate of highly excited (open or closed) strings. One can compare then the gravity wave emission computed from the decay rate of a quantized closed bosonic string with the classical calculations for gravitational radiation already done for cosmic strings. Recently $^{[30]}$, the vertex operator for the absorption and emission of states from a closed string has been computed to obtain the one-loop scattering amplitude whose imaginary part gives the total decay rate $\Gamma$ (total means summing over all the decay products). The computation is more easily performed for the states lying on the leading Regge trajectory corresponding to spin $J$ massive particles with squared mass $m^2 = 4(J-2)$. Classically, these states correspond to classical trajectories which are rotating pairs of straight strings joined at the ends (with the ends moving at the speed of light); the quantum state (which is an occupation number eigenstate) is the superposition of such classical trajectories.
The decays in which one of the products is massless form the largest class. The amplitude $\Gamma$ can be converted into a rate of radiation which gives the quantum state (which is an occupation number eigenstate) is the superposition of such classical trajectories. The decays in which one of the products is massless form the largest class. The amplitude $\Gamma$ can be converted into a rate of radiation which gives:

$$P = \Gamma E \sim 360 \, G\mu^2,$$

$E$ being the energy of the radiated massless particles.

Let us recall the gravitational radiation rate from cosmic strings, evaluated classically for some particular string trajectories, using the quadrupole formula:

$$P = \gamma \mu^2 \text{ with } \gamma \sim 100.$$

We see that a highly excited fundamental string exhibits a classical behavior by emitting as it would be a classical oscillating quadrupole. It is very interesting that the purely quantum mechanical calculation of decaying fundamental strings into gravitons, agrees with the classical results of cosmic strings. For fundamental strings, the value of $\gamma$ is larger than that obtained from cosmic strings, but this is due to the particular chosen trajectory—the leading Regge trajectory state corresponding classically to a rigid rotating rod for which the mass is proportional to the length.

The value of $\gamma$ depends on the particular trajectory. It would be interesting to compute the rate $\Gamma$ for other states (different from the leading Regge trajectory) and to compare with the values obtained for cosmic strings.

It must be noticed that besides the graviton, the rate $\Gamma$ contains decays into the massless dilaton and antisymmetric tensor fields, and that, in general, the expression of $\Gamma$ in terms of the string parameter $\mu$ is highly dependent on the compactification scheme. The results in $D=4$ and $D=26$ are rather different. (For instance, in $D=26$ $\Gamma$ is inversely proportional to the string length, and decays into massive states are not suppressed as it is the case in $D=4$).
But in spite of these connections, important questions remain to be solved or to be revised and we are not still in a position to make contact with the precise results of observations, as has been done for cosmic strings:

a) we would like to know not only the parameter $\gamma$, but also the analogues of the $\alpha$ and $\beta$ evolution parameters of cosmic string interactions, from a quantum fundamental string computation.

b) it would be interesting to compute the rate $\Gamma$ of fundamental strings for other states (different from the leading Regge trajectory) and to compare with the values obtained for cosmic strings.

c) and more important, to derive the value of $(G\mu)$ in $D=4$ of fundamental strings, for which compactification and perturbative renormalization group techniques are in conflict with the radioastronomical constraints on cosmic strings.

4. Recent Developments

Interestingly, the complete integrability of the string propagation in $D$-dimensional De Sitter space time has been shown [30]. The string equations of motion -which correspond to a non-compact $O(D,1)$ symmetric sigma model- plus the string constraints- are equivalent to a generalized Sinh-Gordon equation. In $D=2$, this is the Liouville equation, in $D=3$, this is the standard Sinh-Gordon equation and in $D=4$, this equation is related to the $B_2$ Toda model. We show that the presence of instability is a general exact feature of strings in the De Sitter space, as a direct consequence of the strong instability of the generalized Sinh Gordon Hamiltonian (which is unbounded from below), irrespective of any approximative scheme. We find all the classical solutions in $D=2$ and physically analyze them: they correspond to a string winding $n$ times around de Sitter space (here the circle $S^1$) and evolving with it, the string inflates (or shrink) following the expansion (or contraction) rate.

More recently[31], exact and explicit string solutions propagating in 2+1 dimensional De Sitter spacetime were found. In all these solutions the strings generically tend to inflate or either to collapse.
The world sheet time \( \tau \) interpolates between the cosmic time (\( \tau \to \pm \infty \)) and conformal (\( \tau \to 0 \)) time. For \( \tau \to 0 \), the typical string instability is found, while for \( \tau \to +\infty \), a new string behaviour appears. In that regime, the string expands (or contracts) but not with the same rate as the universe does.

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