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A thermodynamical approach to hadron production in e^+e^- collisions

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Abstract

The hadron production in e^+e^- collisions is studied assuming that particles originate in a hadron gas at thermal and chemical equilibrium. The parameters of the hadron gas are determined with a fit to the average multiplicities of various hadron species measured at LEP and PEP-PETRA. A strikingly good agreement is found between the predictions of this model and data for almost all particles over a range of production rate of four orders of magnitude.
The methods and assumptions of the fit are described and discussed in detail.
The temperature of the hadron gas estimated from the fit is around 166 MeV at $\sqrt{s} = 91.2$ GeV and 180 MeV at $\sqrt{s} = 29 \div 35$ GeV.

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1 Introduction

Hadron production in e^+e^- collisions at high energy is generally believed to be the result of a two-stage process: a parton shower generated by the $q\bar{q}$ pair emerging from the annihilation, and a fragmentation of the partons into observable hadrons. The former process is hard and can be described by perturbative QCD, whereas the latter is soft and not calculable with a perturbative approach. Therefore, several phenomenological models aimed at describing quantitatively the fragmentation process have been developed. Amongst them, the most popular are those implemented in the Monte Carlo generators JETSET [1] and HERWIG [2], using the concepts of string and cluster fragmentation respectively. The main unsatisfactory feature of these models, as far as the relative production rates of the hadron species is concerned, is the large number of free parameters required in order to correctly reproduce experimental data. As a consequence, those models have a rather poor predictive power. A first empirical regularity has been observed in ref. [3] although some quite artificial assumptions were made about meson quantum numbers and masses. In this paper we discuss a thermodynamical approach to the problem of hadronization, postulating the existence of a hadron gas in thermodynamical equilibrium before the hadrons themselves decouple (freezing-out) and decay giving rise to observable particles in the detector. We show that this model is able to fit almost all inclusive rates measured so far at LEP and PEP-PETRA colliders in a natural way by using only three parameters. Two of them are the basic parameters of the hadron gas description, namely its temperature T and its volume V; the third one, γ_s is a parameter describing the partial strangeness chemical equilibrium, which has been used already in some analysis of hadron production in heavy-ion collisions [4]. The only required inputs to determine the particle yields are the mass, the spin and the quantum numbers such as baryon number, strangeness, charm and beauty.

2 Hadron gas scheme

Most hadronic events in high energy e^+e^- annihilations are two-jet events. Here we assume that each jet represents an independent phase in complete thermodynamical equilibrium just before the freezing-out time and that the number of such independent phases in hadronic events is always two, which corresponds to neglecting hard gluon jets. The previous statement implies that one can describe a jet as an object defined by thermodynamical and mechanical quantities such as the temperature, the volume in its rest frame and the Lorentz-boost factor γ. As far as chemical equilibrium is concerned, we assume that, in general, a jet keeps the quantum numbers of the quark from which it originated throughout its evolution. On the other hand, as the two jets must be colourless, they must interact at some stage of the hadronization process through the creation of one or several $q\bar{q}$ pairs whose members are shared. This means that jets do not conserve all the quantum numbers of the parent quark, in particular they do not have baryon number 1/3. We use two different schemes for the jet quantum numbers. In the first one, that we define as the uncorrelated jet scheme, we assume that the shared pairs are of either $u\bar{u}$ or $d\bar{d}$ type, so that the strangeness, charm and beauty of the original quark are conserved, unlike upness and downness. Furthermore, we assume that the baryon number of a jet is always zero. In the second one, that we define as the correlated jet scheme, we allow $s\bar{s}$ sharing in the inter-jet interaction and non-vanishing jet baryon number, provided that the total strangeness and baryon number of the whole system are zero.
Adequate tools to deal with such a problem in a statistical mechanics framework are the canon-

ical partition functions of systems with internal symmetries. The partition function of a system which transforms under the irreducible representation D^ν of a symmetry group G can be expressed as [5,6]:

$$Z = \frac{d_\nu}{M(G)} \int d\mu(g)\, \chi_\nu^{-1}(g)\, \mathcal{Z}(g) \;, \tag{1}$$

where $\mu(g)$ is the group measure, $M(G) = \int d\mu(g)$, d_ν is the dimension of D^ν and $\chi_\nu(g)$ is the character of D^ν, namely $\chi_\nu(g) = \mathrm{tr}\,(D^\nu(g))$. The function $\mathcal{Z}$ is defined as:

$$\mathcal{Z}(g) = \mathrm{tr}\,(e^{-\beta\cdot P} e^{-i\sum \phi_i Q_i}) \;, \tag{2}$$

where the ϕ_i's are the parameters of the group and Q_i its generators. The exponential factor is the relativistic expression of the usual canonical distribution with P as the four-momentum and β as the inverse temperature four-vector.
In the present case the symmetry group is $U(1)^4$, each $U(1)$ corresponding to the conservation of baryon number N, strangeness S, charm C and beauty B respectively. The equation (1) becomes:

$$Z(\mathbf{Q}) = \frac{1}{(2\pi)^4} \int_0^{2\pi}\!\!\int_0^{2\pi}\!\!\int_0^{2\pi}\!\!\int_0^{2\pi} d^4\phi\; e^{i\mathbf{Q}\cdot\phi}\, \mathcal{Z}(\phi) \;. \tag{3}$$

$\mathbf{Q} = (N, S, C, B)$ is a four dimensional vector with integer components representing the quantum numbers of a jet; and $\phi = (\phi_1, \phi_2, \phi_3, \phi_4)$, each ϕ_i being the parameter of a $U(1)$ group. For a gas of N_B boson species and N_F fermion species one can easily show that from equations (2) and (3) follows

$$\begin{aligned} Z(\mathbf{Q}) &= \frac{1}{(2\pi)^4} \int d^4\phi\; e^{i\mathbf{Q}\cdot\phi} \exp\{\sum_{j=1}^{N_B}\sum_k \log(1 - e^{-\beta\cdot p_k - i\mathbf{q}_j\cdot\phi})^{-1} + \\ &+ \sum_{j=1}^{N_F}\sum_k \log(1 + e^{-\beta\cdot p_k - i\mathbf{q}_j\cdot\phi})\} \;, \end{aligned} \tag{4}$$

where k labels all available states of phase space for the j^{th} particle and $\mathbf{q}_j = (N_j, S_j, C_j, B_j)$. Since the total number of each hadron in the gas is not determined, all chemical potentials must vanish. From equation (4) we are able to estimate the average number per jet of any hadronic particle assigning each of them a fictitious fugacity λ_i which multiplies the $e^{-\beta\cdot p_k}$ factor and deriving

$$< n_i > = \lambda_i \frac{\partial \log Z}{\partial \lambda_i}|_{\lambda_i = 1} \;. \tag{5}$$

Hence

$$\begin{aligned} < n_i > &= \frac{1}{Z}\frac{1}{(2\pi)^4} \int d^4\phi\; e^{i\mathbf{Q}\cdot\phi} \exp\{\sum_{j=1}^{N_B}\sum_k \log(1 - e^{-\beta\cdot p_k - i\mathbf{q}_j\cdot\phi})^{-1} + \\ &+ \sum_{j=1}^{N_F}\sum_k \log(1 + e^{-\beta\cdot p_k - i\mathbf{q}_j\cdot\phi})\} \sum_k \frac{1}{e^{\beta\cdot p_k - i\mathbf{q}_i\cdot\phi} \pm 1} \;, \end{aligned} \tag{6}$$

where the sign $+$ is for fermions and $-$ is for bosons. The sum over the phase space, for a continuous level density becomes:

$$\sum_k \longrightarrow (2J+1)\frac{V}{(2\pi)^3}\int d^3p \ , \tag{7}$$

where J is the spin of the particle.
In order to evaluate the integrals in the equation (6) we choose the rest frame of the system, where

$$\beta = (\frac{1}{T},0,0,0) \ , \tag{8}$$

so that we have to compute the integrals

$$\begin{aligned} &(2J_i+1)\frac{V}{(2\pi)^3}\int d^3p \ \log\left(1 \pm e^{-\frac{\sqrt{p^2+m_i^2}}{T} - i\mathrm{q}_i\cdot\phi}\right)^{\pm 1} \\ &(2J_i+1)\frac{V}{(2\pi)^3}\int d^3p \ \frac{1}{e^{\frac{\sqrt{p^2+m_i^2}}{T} - i\mathrm{q}_i\cdot\phi} \pm 1} \ . \end{aligned} \tag{9}$$

If $T \sim \mathcal{O}(100)$ MeV, as we can argue from the QCD soft scale, the exponential factors are expected to be very small for all particles but pions, so that

$$\begin{aligned} &\log\left(1 \pm e^{-\frac{\sqrt{p^2+m_i^2}}{T} - i\mathrm{q}_i\cdot\phi}\right)^{\pm 1} \simeq e^{-\frac{\sqrt{p^2+m_i^2}}{T} - i\mathrm{q}_i\cdot\phi} \\ &\frac{1}{e^{-\frac{\sqrt{p^2+m_i^2}}{T} - i\mathrm{q}_i\cdot\phi} \pm 1} \simeq e^{-\frac{\sqrt{p^2+m_i^2}}{T} - i\mathrm{q}_i\cdot\phi} \end{aligned} \tag{10}$$

are very good approximations. This corresponds to the Boltzmann limit of Fermi and Bose statistics. Inserting (10) into (9) and using the replacement (7), the (4) becomes:

$$Z(\mathrm{Q}) = \frac{F_\pi}{(2\pi)^4}\int d^4\phi \ e^{i\mathrm{Q}\cdot\phi} \exp\{\sum_i z_i e^{-i\mathrm{q}_i\cdot\phi}\} \ , \tag{11}$$

where

$$z_i = (2J_i+1)\frac{V}{(2\pi)^3}\int d^3p \ e^{-\frac{\sqrt{p^2+m_i^2}}{T}} = (2J_i+1)\frac{VT}{2\pi^2}m_i^2 K_2(\frac{m_i}{T}) \tag{12}$$

and

$$F_\pi = \exp\{\sum_{i=1}^{3}\frac{V}{(2\pi^3)}\int d^3p \ \log\left(1 + e^{-\frac{\sqrt{p^2+m_i^2}}{T}}\right)\} \tag{13}$$

is the exponential of the sum over the three pion states, whose relevant quantum numbers are zero in this scheme. The function K_2 in (12) is the modified Bessel's function of order 2, whose asymptotic behaviour for $m \gg T$ is

$$K_2(\frac{m}{T}) \approx \sqrt{\frac{\pi T}{2m}}e^{-m/T} \ . \tag{14}$$

The sum in (11) now runs over all hadrons except pions. We can get the mean number of particles i produced per jet by using (5) and (11)

$$< n_i >= z_i \frac{Z(\mathbf{Q} - \mathbf{q}_i)}{Z(\mathbf{Q})} , \tag{15}$$

whereas for pions

$$< n_i >= \frac{V}{(2\pi)^3} \int d^3p \, \frac{1}{e^{\frac{\sqrt{p^2+m_i^2}}{T}} - 1} , \tag{16}$$

Therefore, in order to calculate the $< n_i >$ one has to evaluate complex four-dimensional integrals over an hypercube with side 2π. Nevertheless, the extent of this problem can be reduced by observing that the production rate of charm and bottom hadrons in the annihilations of e^+e^- into light quarks is expected to be negligibly small. This means that their functions z_i must be much smaller than one, otherwise, as stated by (15), there should be a detectable signal of hadrons like B or D even in $e^+e^- \rightarrow u\bar{u}$, $e^+e^- \rightarrow d\bar{d}$ events. Thus, assuming that $z_i \ll 1$ for charm and bottom hadrons, we can expand $\exp\{z_i e^{-i\mathbf{q}_i\cdot\phi}\}$ in powers of z_i up to first order

$$\exp\{z_i e^{-i\mathbf{q}_i\cdot\phi}\} \simeq 1 + z_i e^{-i\mathbf{q}_i\cdot\phi} \tag{17}$$

Inserting the previous expansion in equation (11) we obtain a formula much easier to handle as far as numerical integration is concerned

$$\begin{aligned} Z(\mathbf{Q}) \;\cong\; & \frac{F_\pi}{(2\pi)^2} \int d^2\phi \; e^{i\mathbf{Q}\cdot\phi} e^{f(\phi)} \, \delta_{C,0}\delta_{B,0} + \\ + \; & \sum_{i_c} z_{i_c} \frac{F_\pi}{(2\pi)^2} \int d^2\phi \; e^{i(\mathbf{Q}-\mathbf{q}_{i_c})\cdot\phi} e^{f(\phi)} \, \delta_{C,C_{i_c}} \delta_{B,0} + \\ + \; & \sum_{i_b} z_{i_b} \frac{F_\pi}{(2\pi)^2} \int d^2\phi \; e^{i(\mathbf{Q}-\mathbf{q}_{i_b})\cdot\phi} e^{f(\phi)} \, \delta_{C,0}\delta_{B,B_{i_b}} + \\ + \; & \sum_{i_c}\sum_{i_b} z_{i_c} z_{i_b} \frac{F_\pi}{(2\pi)^2} \int d^2\phi \; e^{i(\mathbf{Q}-\mathbf{q}_{i_c}-\mathbf{q}_{i_b})\cdot\phi} e^{f(\phi)} \, \delta_{C,C_{i_c}} \delta_{B,B_{i_b}} \, . \end{aligned} \tag{18}$$

In the formula above $\mathbf{Q}$ and $\mathbf{q}_i$ are now two-dimensional integer vectors having as components the baryon number and the strangeness of jet and particles respectively, whilst the charm C and beauty B appear just in the Kronecker δs. The index i_c runs over all charm hadrons, i_b runs over all bottom hadrons. The function $f(\phi)$ is defined as:

$$f(\phi) = \sum_i z_i e^{-i\mathbf{q}_i\cdot\phi} , \tag{19}$$

where the index i runs over all hadrons except heavy flavoured ones. The behaviour of Z as a function of N, S is shown in figure 1. The yields of charm (bottom) hadrons in $e^+e^- \rightarrow c\bar{c}$ ($e^+e^- \rightarrow b\bar{b}$) events can be obtained by using (15) and (18) and have a quite simple expression

$$< n_i >= \frac{z_i}{\sum_j z_j} \, . \tag{20}$$

Since the sum is extended to all charm (bottom) hadrons, it turns out that the overall number of such particles per jet is one, as one may reasonably expect.

3 Fit of LEP data

In section 1 we mentioned that jets are considered as phases in thermodynamical equilibrium before hadronic decoupling and decays take place. We have assumed that there are just two such phases per hadronic event and that they have definite quantum numbers related to the quarks from which they originated. In the following we use the uncorrelated jet scheme, in which the partition function $\hat{Z}$ of the whole system reads:

$$\hat{Z} = Z(\mathbf{Q})Z(-\mathbf{Q}) \; , \tag{21}$$

where $\mathbf{Q} = (N, S, C, B)$ with baryon number $N = 0$ and S, C, B are the strangeness, charm and beauty of the primary quark. The correlated jet scheme will be treated in section 4. The volume and the temperature of the two phases at the decoupling time, which are to be determined with a fit to the available data on hadron inclusive production, are assumed to be equal and also equal for different kinds of primary quarks. An estimation of possible effects due to different volumes for light and heavy quarks Z decays will be given in section 4. A parameter $\gamma_s < 1$ is introduced in order to take into account a non-complete strange chemical equilibrium; if a hadron contains n strange valence quarks, its production is reduced by a factor γ_s^n. This reduction applies also to neutral mesons such as η, η', ϕ, ω according to the fraction of $s\bar{s}$ content in the meson itself. For this purpose we used mixing formulas quoted in the Review of Particles Properties [7] with angles $\theta = -10°$ and $\theta = 39°$ for the (η, η') system and for (ω, ϕ) system respectively.

The hadron rates calculation basically proceeds through two steps; in the first one a primary number of particles is calculated by using (5) and (18) (which yields simply the sum of the average numbers for each jet as expressed by (15)) and multiplied by γ_s^n according to the strange quark content; in the second, all decays are performed according to decay modes and branching ratios as implemented in the program JETSET 7.4, which are basically the same as in ref. [7]. The overall production of each particle is then computed by adding up the fraction stemming from decays of heavier particles to the primary one. The decay chain stops when π, K, K^0_L, μ or stable particles are reached, in order to match the numbers of production rates provided by LEP experiments, including all decay products of particles with $c\tau < 10$ cm.

All hadrons included in JETSET 7.4 tables have been considered, with the quoted masses and widths. All other light flavoured resonances up to a mass of 1.7 GeV have been included with masses, widths and branching ratios quoted in ref. [7]. The mass of resonances with $\Gamma > 1$ MeV has been distributed according to a relativistic Breit-Wigner function within $\pm 2\Gamma$ from the central value.

All available measurements about the production rate of $\pi^\pm$ [8], π^0 [9], $K^\pm$ [8,10], K^0 [9,11–13], η [9,14], ρ^0 [12],$K^{*\pm}$ [12,15], K^{*0} [16,17] η' [14], p [8,10], ϕ [17], Λ [9,11,18,19], $\Sigma^{*\pm}$ [18,19], Ξ^- [18,19], Ξ^{*0} [18,19], Ω [18] at LEP at $\sqrt{s} = 91.2$ GeV have been used. In case of several measurements of the same hadron from the same experiment, we used only the most recent value. For each particle we averaged the measurements of the various experiments weighting by their quoted errors. We did not take into account possible correlations between measurements of different experiments, so that errors might be slightly underestimated. The ALEPH values of η and η' production [14] have been extracted from ref. [3], in which an extrapolation is performed according to JETSET and HERWIG predicted momentum spectrum. We did not use as data points any heavy resonance like a_0, f_0 and f_2 because of the large uncertainty on the width, which can heavily affect the rate calculation.

In order to perform the fit the ratios $R_q = \Gamma(Z \to q\bar{q})/\Gamma(Z \to hadrons)$, for each flavour q, are

needed. We used values of $R_c = 0.17$, $R_b = 0.22$ and $R_s = 0.22$, according to recent Standard Model fits [20]. The $(R_u + R_d)$ is then determined by subtracting $(R_s + R_c + R_b)$ from 1.
In order to estimate properly the parameters T, V and γ_s from the fit it is also necessary to take into account the uncertainties on all input data such as branching ratios, masses and widths of all particles involved in this fit. Among those parameters we chose the most relevant potentially affecting the production of particles through the decay chains. We then varied them by their errors quoted in ref. [7] to compute new hadrons yields keeping T, V and γ_s fixed. All uncertainties concerned with mass, width and branching ratios of light flavoured resonances heavier than 0.9 GeV have been taken into account, whilst no variation of any heavy flavoured hadron parameter has been performed. The differences between the old and the new predicted yields have been considered as additional systematic errors to be added in quadrature to the experimental errors. The fit has been then repeated inserting the overall errors so as to obtain final values for the fit parameters and for the hadron rates.
The results of the fit in the uncorrelated jet scheme are summarized in tables 1, 2 and figure 2. There is a good agreement between calculated rates and measured ones with the exception of ϕ and Σ^*, whose predicted values are larger than the experimental ones by a factor $\simeq 1.7$. A possible explanation of this disagreement could be a major discrepancy between the real and the JETSET momentum spectrum in the unobserved low-momentum region, i.e. $p < 1.6$ GeV for the ϕ and $p < 2.3$ GeV for the Σ^*. In this case the extrapolation of the experimental spectrum by means of JETSET, which is the technique used by all LEP experiments to get the total multiplicity, would have led to a wrong result. If ϕ and Σ^* are excluded from the data set, a much better fit is obtained with a $\chi^2/dof = 15.8/11$ (see tables 1 and 2). The discrepancy between the expected and measured value for Λ in the first fit is essentially due to insertion of Σ^* (which always produces a Λ) in the data set, because of the pull towards the measured Σ^* value.
The mean charged multiplicity turns out to be $\bar{n}_{ch} = 20.6$ (π, K, p, e and μ are included), which is in agreement with LEP data [21]. The mean number of primary hadrons is 16.3 (see also figure 7), implying a hadron density of 0.44 Fm^{-3} at the freezing-out time. The values of the z functions of charm hadrons and bottom hadrons are $\sim 10^{-4}$ and $\sim 10^{-11}$ respectively; this confirms the validity of the approximation (17). The error on the multiplicity caused by using Boltzmann statistics instead of quantum statistics, turns out to be $< 1\%$ for K and it is negligible for all other particles.
The model also yields a definite prediction for the relative production rate of heavy flavoured hadrons once R_c and R_b are known. We did not use available measurements as input data for the fit as they turn out to be affected from the poor knowledge of masses and branching ratios of most such hadrons, in particular heavy meson resonances such as D^{**}, D_1, D_2, B_1 etc. and almost all baryon states. In table 3 the multiplicities obtained for some of the lightest heavy flavoured hadrons are compared with LEP data [22].

4 Fit variation and other checks

In order to prove that the thermal shape of the hadron production function does not depend on the assumptions made, we changed some of them and we also varied some major parameters involved in the fit.
The independence of T from the part of the hadronic mass spectrum that we neglected, i.e. light-flavoured resonances with mass > 1.7 GeV, has been tested by moving the cut-off point down to 1.3 GeV by steps of 0.1 GeV. The new values found are consistent with the main

quoted values within the fit error, as shown in figure 6. This indicates that missing heavy resonances should not affect significantly the fit.
The possibility of different average values of V as a function of the primary quark mass has been taken into account introducing two dimensionless variables $x_c, x_b \in [0,1]$ such that $x_c V$ is the volume in $Z \to c\bar{c}$ and $x_b V$ in $Z \to b\bar{b}$ events, whereas $V(1 - x_c R_c - x_b R_b)/(R_u + R_d + R_s)$ is the volume for $Z \to q\bar{q}$ where q is a light quark. The mesurement of the mean charged multiplicity in $Z \to c\bar{c}$ and $Z \to b\bar{b}$ by DELPHI [23] and OPAL [24] determine $x_c = 0.96$ and $x_b = 0.74$. Inserting those values in the fit, it is found that new fitted parameters are almost unchanged (see table 4).
The fit has been repeated also varying the Z hadronic branching fraction R_c by ± 0.2 and correspondingly $(R_u + R_d)$ in order to keep the unitarity; a very small variation of the fitted parameters has been found (see table 4).
Finally, we turned to the correlated jet scheme allowing the occurrence of pairs of jets with net strangeness up to 2 and net baryon number up to 1. (in $Z \to s\bar{s}$ events allowance is made only for $S = 1, 2$). The partition function of the whole system can be written then as:

$$\hat{Z} = \sum_{N=0}^{1} \sum_{S=-2}^{2} Z(\mathbf{Q}) Z(-\mathbf{Q}) \,, \tag{22}$$

where $Z(\mathbf{Q})$ are the partition functions defined in section 2 and $\mathbf{Q} = (N, S, C, B)$. Thus, the probability of occurrence of jets having net baryon number N and net strangeness S, once charm C and beauty B are given, is (see also figure 8):

$$p(N,S) = \frac{Z(N,S,C,B) Z(-N,-S,-C,-B)}{\hat{Z}} \,. \tag{23}$$

The results of the fit performed in the correlated jet scheme are shown in tables 5, 6 and figure 3. It is worth mentioning that the computed ϕ production rate is closer to measured one in this scheme rather than in the original one, while Σ^* is still far from the measured value. Excluding Σ^* from data set we obtain a better fit and a Λ rate closer to data.

5 Fit of PETRA and PEP data

Hadron rates measured at PEP and PETRA e^+e^- colliders, at a center of mass energy between 29 and 35 GeV, have been fitted according to the same procedure described for LEP. Data have been extracted from ref. [7] where averages of various experimental measurements are quoted. The ratios

$$R_q = \frac{\sigma(e^+e^- \to qq)}{\sigma(e^+e^- \to hadrons)} \tag{24}$$

for each flavour q have been estimated using the tree-level expression:

$$R_q = \frac{Q_q^2[\beta_q(3 - \beta_q^2)]}{\sum_{i=1}^{5} Q_i^2[\beta_i(3 - \beta_i^2)]} \,. \tag{25}$$

Both uncorrelated and correlated jet schemes have been used. The fit results are shown in tables 7, 8 and figures 4, 5. The T and γ_s fitted values are close to those found at 91.2 GeV indicating that these quantities do not seemingly depend on $\sqrt{s}$. The ϕ calculated rate is in better agreement with data in the correlated jet scheme, as it is found for LEP data. Moreover, unlike at $\sqrt{s} = 91.2$ GeV, the Σ^* computed production value agrees with data. It is worth remarking that also JETSET succeeds in reproducing the Σ^* rate at PEP-PETRA and fails at LEP by using the same value of the spin 1 diquark suppression factor ($\alpha = 0.05$) [25,26] which mainly governs the Σ^* production. All other decuplet baryon rates are fairly well reproduced. Amongst other hadronization models, HERWIG fails to reproduce all decuplet baryon rates [26] and the UCLA model [27], which has only 5 major tunable parameters, reproduces succesfully all hadron rates at LEP [3] except the Σ^* one.

6 Conclusions

The problem of the relative production of hadrons in e^+e^- collisions is treated with a thermodynamical approach postulating that hadronic jets must be identified with hadron gas phases in thermodynamical equilibrium before the hadrons themselves decouple and subsequently decay into lighter particles. The basic assumption is that only two jets exist as thermodynamical phases. Nevertheless, since the particle production depends almost linearly on the volume, the occurrence of additional jets, i.e. phases, in the same event should affect the relative hadron production rates very mildly provided that the temperature of the additional jets is the same. Two different schemes have been tested: in the first one it has been assumed that jets conserve the strangeness, charm and beauty of their parent quark and that their baryon number vanishes; in the second, allowance is made of pairs of jets with opposite baryon number up to 1 and opposite strangeness up to 2. It has been shown that this model is able to fit impressively well the average multiplicities of light hadrons per hadronic event observed at LEP ($\sqrt{s}$=91.2 GeV) and PEP-PETRA ($\sqrt{s} = 29 \div 35$ GeV) for a variation of production rate by four orders of magnitude, from π to Ω.
Provided that decay modes and masses of heavy flavoured resonances are essentially correct, the predicted rates of D, D^*, B, B^*, Λ_c at LEP are also in good agreement with data. Only three parameters are required in order to reproduce data correctly, namely the temperature and the volume of the hadron gas and a parameter $\gamma_s \in [0,1]$ allowing an incomplete strange chemical equilibrium. The γ_s fitted value is $\approx 0.7 \div 0.8$ both at LEP and PEP-PETRA energies. The temperature values at $\sqrt{s}$=91.2 GeV and $\sqrt{s} = 29 \div 35$ GeV, determined with the fit, are very similar and both close to the basic QCD parameter Λ_{QCD}, which is an interesting subject of reflection. The thermalization of the system could be a characteristic of the quark-hadron transition, brought about by strong interactions.

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Parameters	All data points	ϕ,Σ^* excluded
Temperature(MeV)	165.65±2.2	170.3±2.7
Volume(Fm^3)	18.3±1.8	15.8±1.8
γ_s	0.699±0.044	0.672±0.053
χ^2/dof	103/13	15.8/11

Table 1: Values of fitted parameters at $\sqrt{s}$ = 91.2 GeV in the uncorrelated jet scheme.

Hadron	Fit	Excluding ϕ, Σ^*	Measured	Exp. Error	Total error
π^+	8.59	8.80	8.53	0.22	0.40
π^0	9.73	9.96	9.18	0.73	0.82
K^+	1.07	1.08	1.18	0.05	0.52
K^0	1.01	1.02	1.015	0.018	0.022
η	0.952	0.977	0.934	0.077	0.12
ρ^0	1.14	1.18	1.21	0.15	0.22
K^{*+}	0.348	0.362	0.357	0.026	0.028
K^{*0}	0.342	0.356	0.372	0.025	0.027
η'	0.107	0.111	0.13	0.05	0.05
p	0.459	0.531	0.488	0.043	0.058
ϕ	0.166	0.171	0.010	0.009	0.009
Λ	0.148	0.171	0.185	0.005	0.009
$(\Sigma^{*+} + \Sigma^{*-})/2$	0.0172	0.0201	0.0094	0.0009	0.0014
Ξ^-	0.0107	0.0123	0.0122	0.0007	0.00082
Ξ^{*0}	0.00379	0.00444	0.0033	0.00046	0.00047
Ω	0.000757	0.000887	0.0014	0.00045	0.00046
ω	0.787	0.818	-	-	0.013
Σ^+	0.0358	0.0414	-	-	0.004
Δ^{++}	0.0697	0.0813	-	-	0.0064

Table 2: Measured average multiplicities of hadrons at $\sqrt{s}$=91.2 GeV per hadronic event, experimental errors and total errors (computed as described in the text) compared to the fitted values in the uncorrelated jet scheme. The measured multiplicities are the weighted averages of all available LEP measurements. Predictions are shown for ω, Δ^{++} and Σ^+ rates with errors due to uncertainties on experimental values of various masses, widths and branching ratios used in the fit.

Hadron	Uncor. jets fit	Corr. jets fit	Measured	Exp. Error
D^+	0.0944	0.0913	0.114	0.011
D^0	0.239	0.230	0.253	0.02
D^{*+}	0.110	0.106	0.0945	0.009
B	0.0927	0.0891	0.0965	0.026
Λ_c^+	0.0237	0.0304	0.0365	0.0085
B^*/B	0.697	0.692	0.74	0.058

Table 3: Measured average multiplicities of heavy flavoured hadrons at $\sqrt{s}$=91.2 GeV per hadronic event and experimental errors compared to the predictions of the fit. The measured multiplicities are the weighted averages of all available LEP measurements with $R_c = 0.17$ and $R_b = 0.22$. The Λ_c^+ rate has been obtained using the $\Lambda_c^+ \rightarrow p\pi^+ K^-$ branching ratio quoted in ref. [7]

Variation	ΔT(MeV)	ΔV(Fm^3)	$\Delta\gamma_s$
Heavy flavour volume	0.08	0.063	0.006
Exclusion of ϕ,Σ^*	4.6	2.52	0.027
Z branching ratios	0.47	0.47	0.005

Table 4: Summary of variations of fitted parameters.

Parameter	All data points	Excluding Σ^*
Temperature(MeV)	156.5±1.8	161.2±2.0
Volume(Fm^3)	28.1±2.5	24.03±2.1
γ_s	0.690±0.032	0.649±0.029
χ^2/dof	56.4/13	24.4/12

Table 5: Values of fitted parameters at $\sqrt{s}$=91.2 GeV in the correlated jet scheme.

Hadron	Fit	Fit without Σ^*	Measured	Error
π^+	8.83	8.98	8.53	0.4
π^0	9.96	10.15	9.18	0.82
K^+	1.08	1.07	1.18	0.52
K^0	1.03	1.01	1.015	0.022
η	0.957	0.985	0.934	0.12
ρ^0	1.17	1.22	1.21	0.22
K^{*+}	0.341	0.349	0.357	0.028
K^{*0}	0.335	0.343	0.372	0.027
η'	0.0966	0.0991	0.13	0.05
p	0.477	0.554	0.488	0.058
ϕ	0.117	0.114	0.10	0.009
Λ	0.146	0.165	0.185	0.009
$(\Sigma^{*+}+\Sigma^{*-})/2$	0.0159	0.0183	0.0094	0.0014
Ξ^-	0.0116	0.0127	0.0122	0.00082
Ξ^{*0}	0.00387	0.00436	0.0033	0.00047
Ω	0.000906	0.000998	0.0014	0.00046
ω	0.640	0.656	-	0.013
Σ^+	0.0347	0.0392	-	0.004
Δ^{++}	0.0729	0.0862	-	0.0064

Table 6: Measured average multiplicities of hadrons at $\sqrt{s}$ = 91.2 GeV per hadronic event compared to the fitted values in the correlated jet scheme, with and without including Σ^* into the data set.

Parameter	Uncorr. jets fit	Corr. jets fit
Temperature(MeV)	181.9±5.2	167.0±3.0
Volume(Fm^3)	6.04±1.3	10.41±1.5
γ_s	0.817±0.073	0.769±0.05
χ^2/dof	52.27/12	26.9/12

Table 7: Values of fitted parameters at $\sqrt{s} = 29 \div 35$ GeV.

Hadron	Uncorr.jets fit	Corr.jets fit	Measured	Exp. Error	Total error
π^+	5.48	5.38	5.15	0.15	0.28
π^0	6.32	6.18	5.6	0.3	0.36
K^+	0.724	0.751	0.74	0.045	0.046
K^0	0.673	0.706	0.74	0.035	0.036
η	0.593	0.574	0.61	0.07	0.08
ρ^0	0.733	0.708	0.81	0.08	0.13
K^{*+}	0.239	0.244	0.32	0.025	0.026
K^{*0}	0.236	0.240	0.28	0.03	0.030
η'	0.0767	0.0698	0.26	0.1	0.1
p	0.291	0.291	0.32	0.025	0.034
ϕ	0.132	0.0999	0.085	0.011	0.011
Λ	0.102	0.104	0.103	0.005	0.007
$(\Sigma^{*+} + \Sigma^{*-})/2$	0.0116	0.0108	0.0085	0.002	0.0023
Ξ^-	0.00814	0.00924	0.0088	0.00135	0.0014
Ω	0.000616	0.000741	0.007	0.0035	0.0035
ω	0.538	0.429	-	-	0.0092
Σ^+	0.0246	0.0245	-	-	0.0027
Δ^{++}	0.0438	0.0432	-	-	0.0028
Ξ^{*0}	0.00243	0.00279	-	-	0.00005

Table 8: Measured average multiplicities of hadrons at $\sqrt{s} = 29 \div 35$ GeV per hadronic event, experimental errors and total errors (computed as described in the text), compared to the fitted values in the uncorrelated and correlated jet schemes. Predictions are shown for ω, Δ^{++}, Σ^+ and Ξ^{*0} rates with errors due to uncertainties on experimental values of various masses, widths and branching ratios used in the fit.

Figure captions

Figure 1 Behaviour of the jet partition function Z as a function of baryon number N for $(S, C, B) = 0$ (left) and as a function of the strangeness S for $(N, C, B) = 0$ (right), for $T = 170$ MeV and $V = 17$ Fm^3. The suppression factors $Z(\mathbf{Q} - \mathbf{q}_i)/Z(\mathbf{Q})$ are smaller for increasing N and S.

Figure 2 Fit of hadron production measured at LEP with exclusion of Σ^* and ϕ from data set in the uncorrelated jet scheme. Above: the dashed line connects fitted values while data are shown as black dots. Below: fluctuations of measured points from the fitted values (solid line) in standard deviation units; the Σ^* and ϕ points are out of range.

Figure 3 Fit of hadron production measured at LEP for the correlated jet scheme without and with exclusion of Σ^* from data set. Below: fluctuations of measured points from the fitted values (solid line) in standard deviation units. The Σ^* point is out of range.

Figure 4 Fit of hadron production at PEP and PETRA in the uncorrelated jet scheme.

Figure 5 Fit of hadron production at PEP and PETRA in the correlated jet scheme.

Figure 6 Fitted temperature, volume and γ_s parameter at $\sqrt{s} = 91.2$ GeV as a function of the light flavored resonance cut-off mass. The temperature and γ_s values are not sensitive to it. The volume tends to increase in the low mass region in order to conserve the overall normalization.

Figure 7 Fraction of primary production for each particle at LEP and PEP-PETRA predicted using T, V and γ_s estimated from the fit.

Figure 8 Probability of occurrence of jet pairs with given baryon number and strangeness at LEP and PEP-PETRA in the correlated jet scheme.

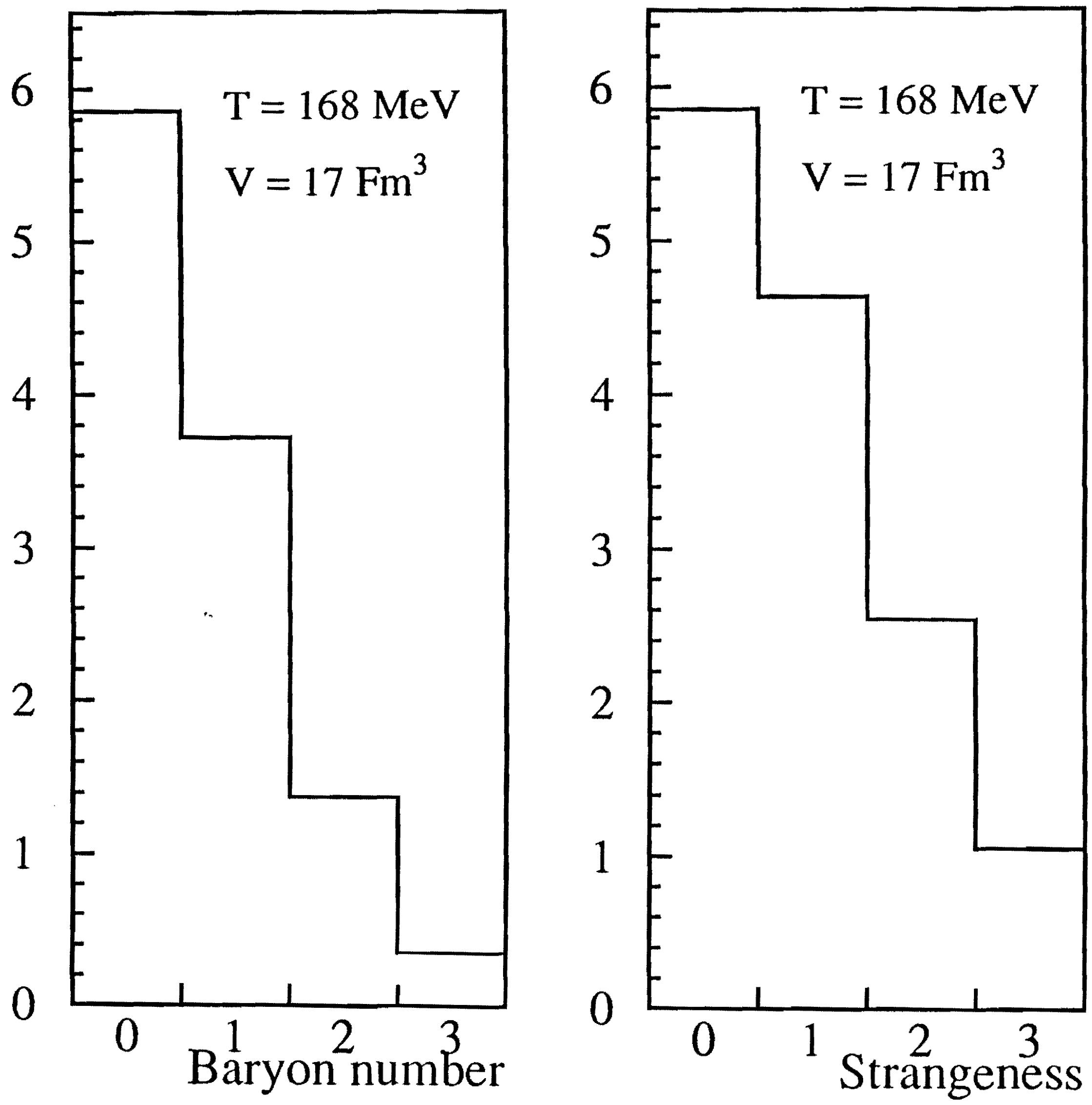


Figure 1:

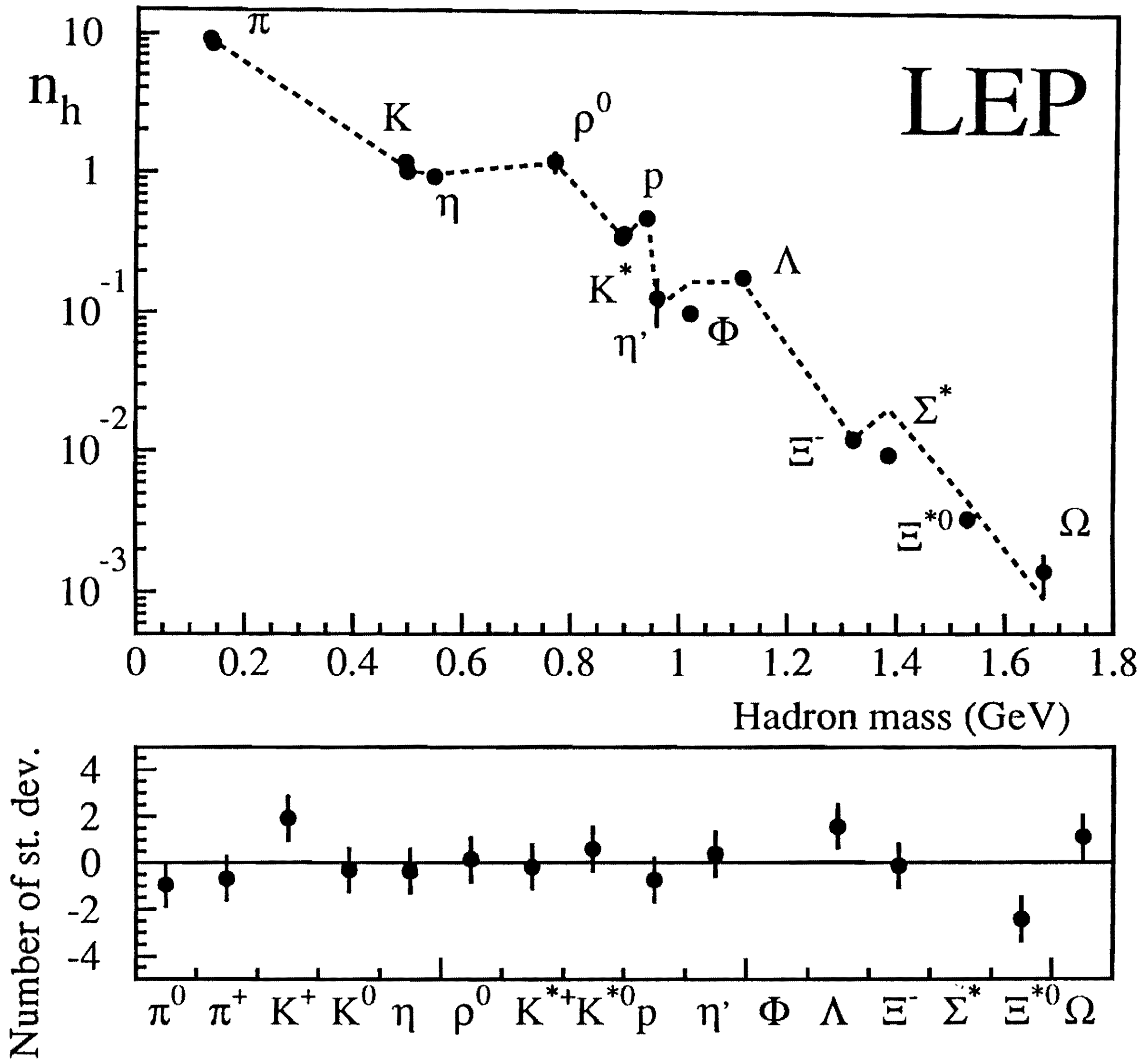

LEP
n_h
10
1
10^{-1}
10^{-2}
10^{-3}
π
K
η
ρ^0
p
K^*
η'
Φ
Λ
Ξ^-
Σ^*
Ξ^{*0}
Ω
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
Hadron mass (GeV)
Number of st. dev.
4
2
0
-2
-4
π^0 π^+ K^+ K^0 η ρ^0 K^{*+} K^{*0} p η' Φ Λ Ξ^- Σ^* Ξ^{*0} Ω

Figure 2:

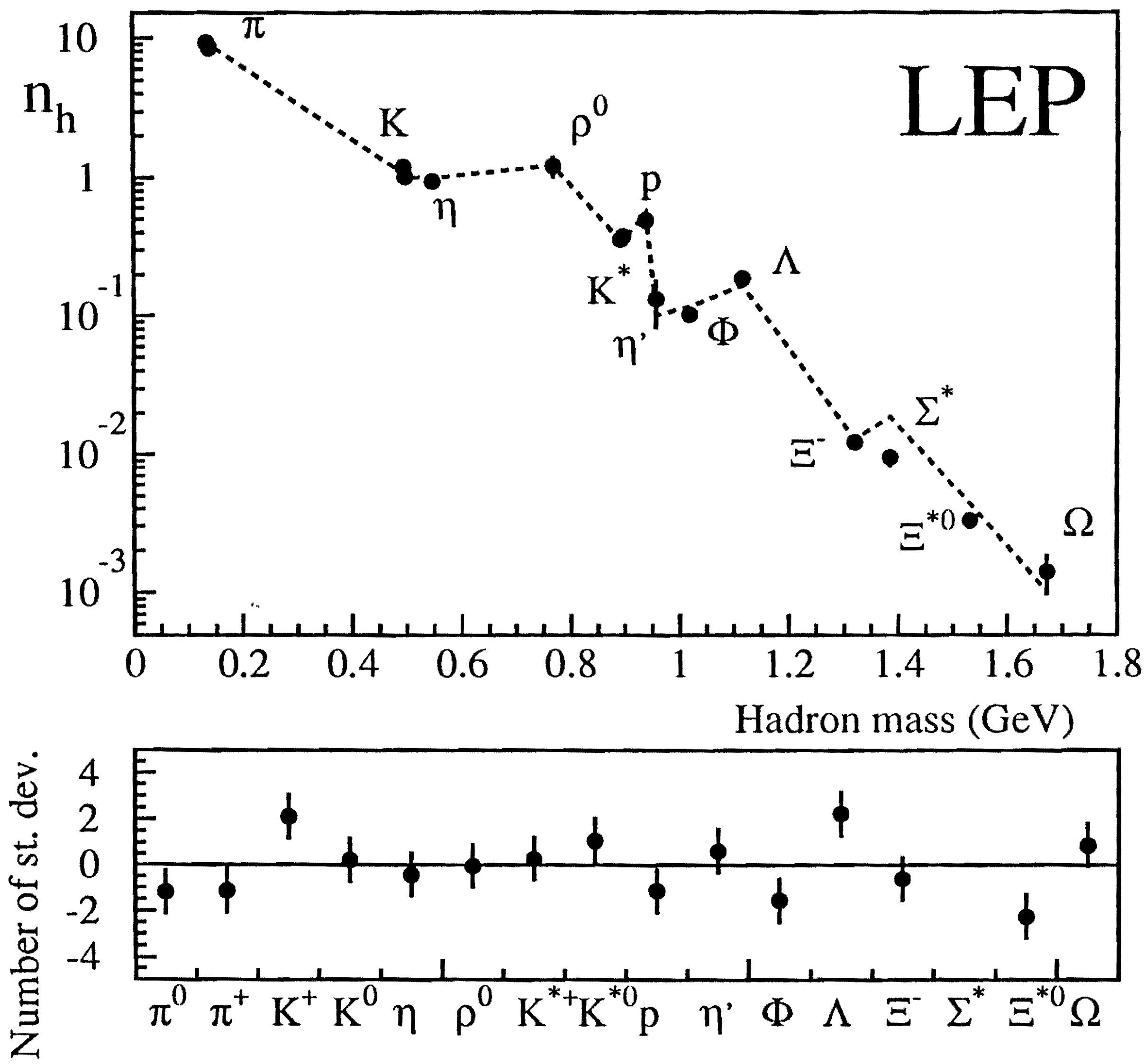

LEP
n_h
10
1
10^-1
10^-2
10^-3
π
K
η
ρ^0
K*
p
η'
Φ
Λ
Ξ^-
Σ*
Ξ*0
Ω
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
Hadron mass (GeV)
Number of st. dev.
4
2
0
-2
-4
π^0 π^+ K^+ K^0 η ρ^0 K*+ K*0 p η' Φ Λ Ξ^- Σ* Ξ*0 Ω

Figure 3:

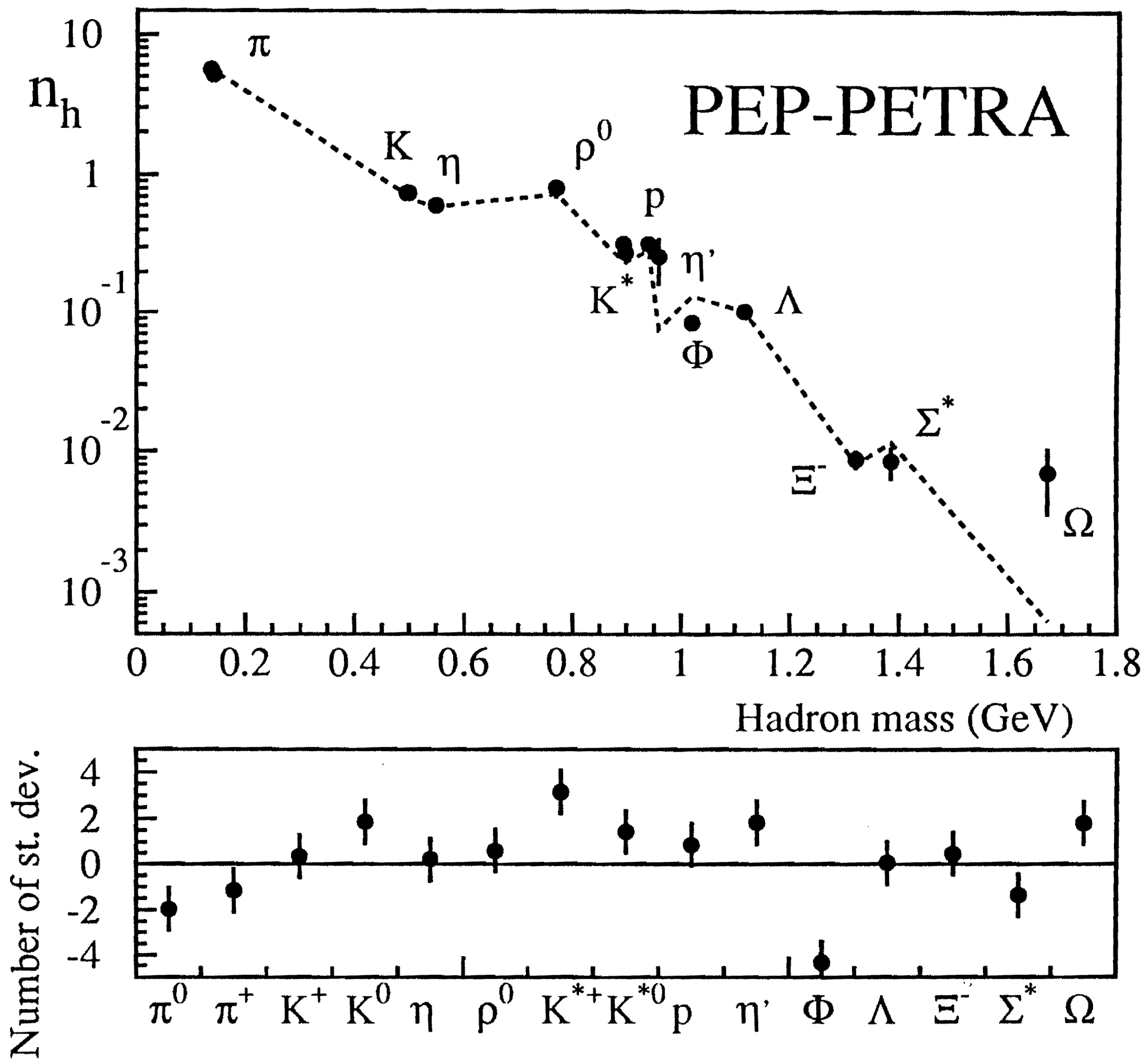
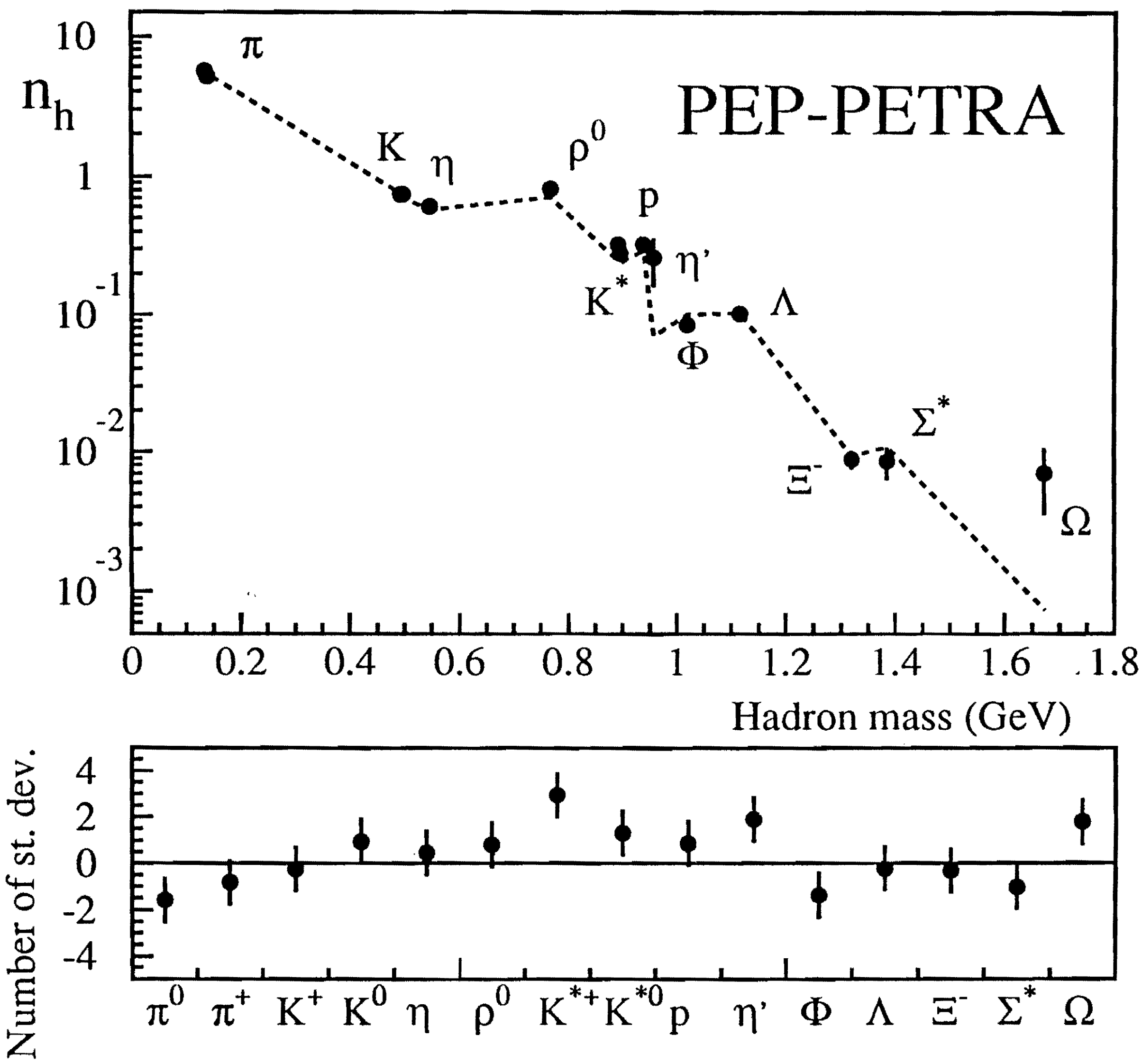

PEP-PETRA
n_h
10
1
10^{-1}
10^{-2}
10^{-3}
π
K
η
ρ^0
p
η'
K^*
Λ
Φ
Σ^*
Ξ^-
Ω
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
Hadron mass (GeV)
Number of st. dev.
4
2
0
-2
-4
π^0 π^+ K^+ K^0 η ρ^0 K^{*+} K^{*0} p η' Φ Λ Ξ^- Σ^* Ω

Figure 4:



PEP-PETRA
n_h
10
1
10^{-1}
10^{-2}
10^{-3}
π
K
η
ρ^0
p
η'
K^*
Λ
Φ
Σ^*
Ξ^-
Ω
0 0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8
Hadron mass (GeV)
Number of st. dev.
4 2 0 -2 -4
π^0 π^+ K^+ K^0 η ρ^0 K^{*+} K^{*0} p η' Φ Λ Ξ^- Σ^* Ω

Figure 5:

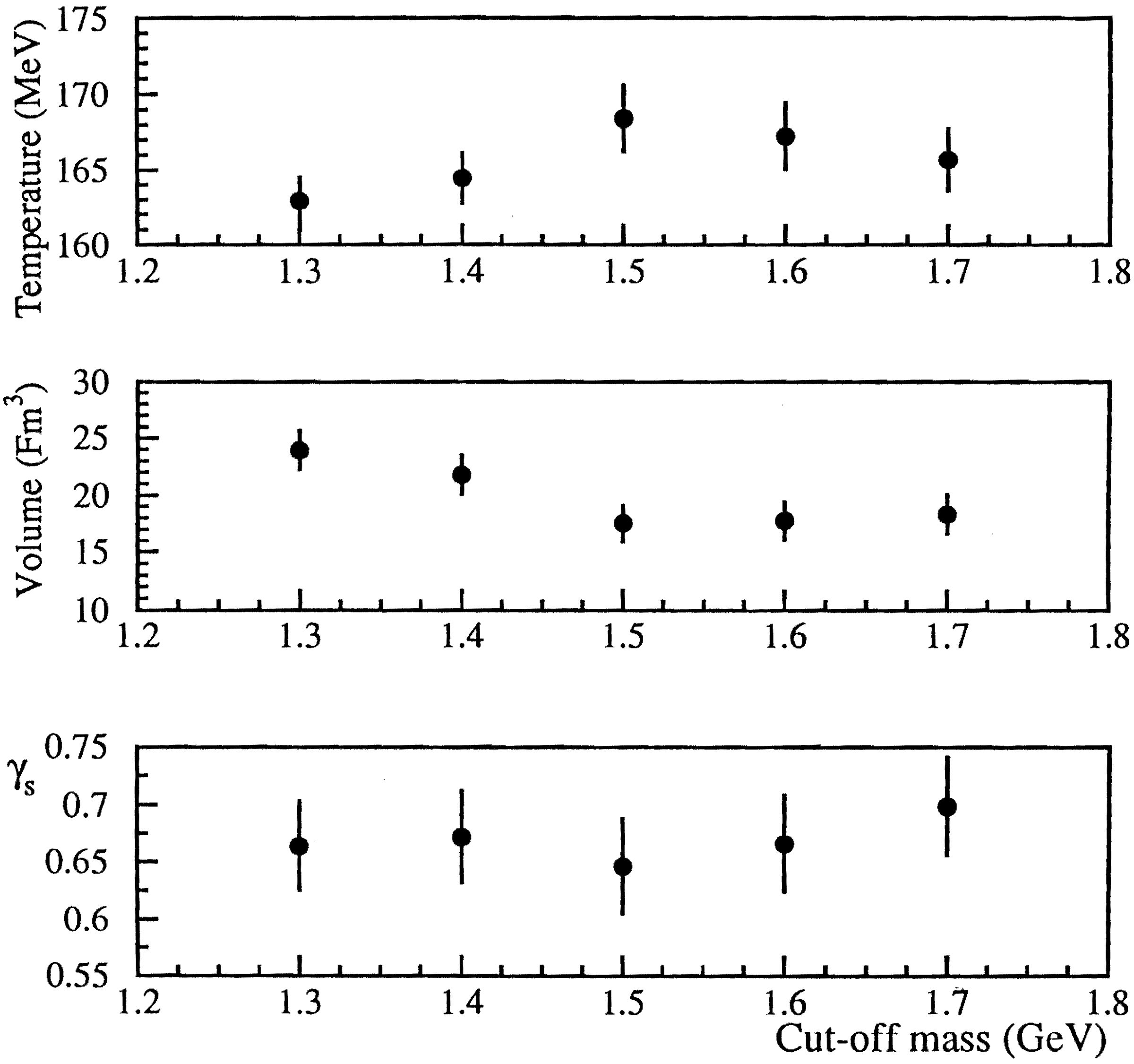
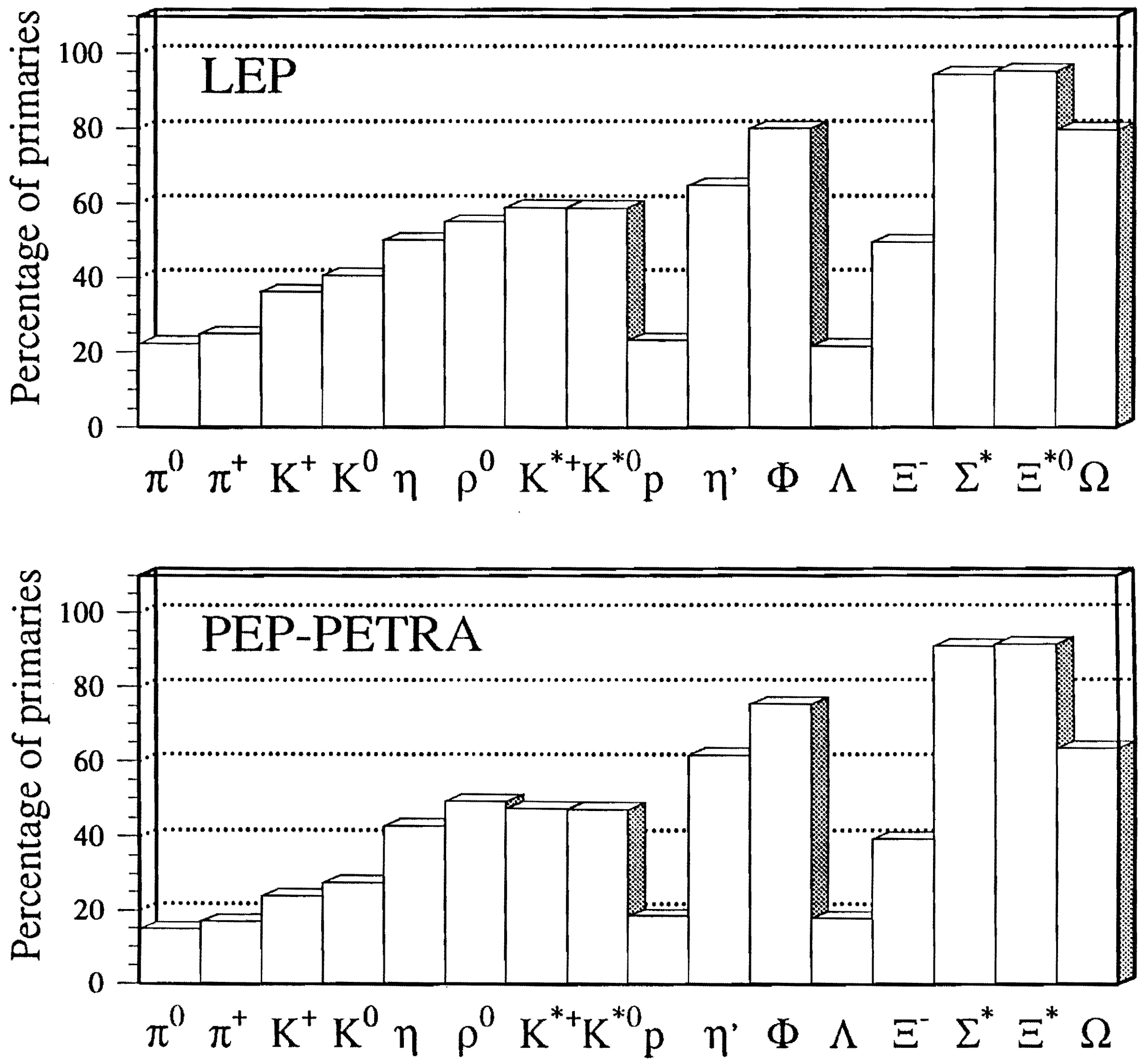
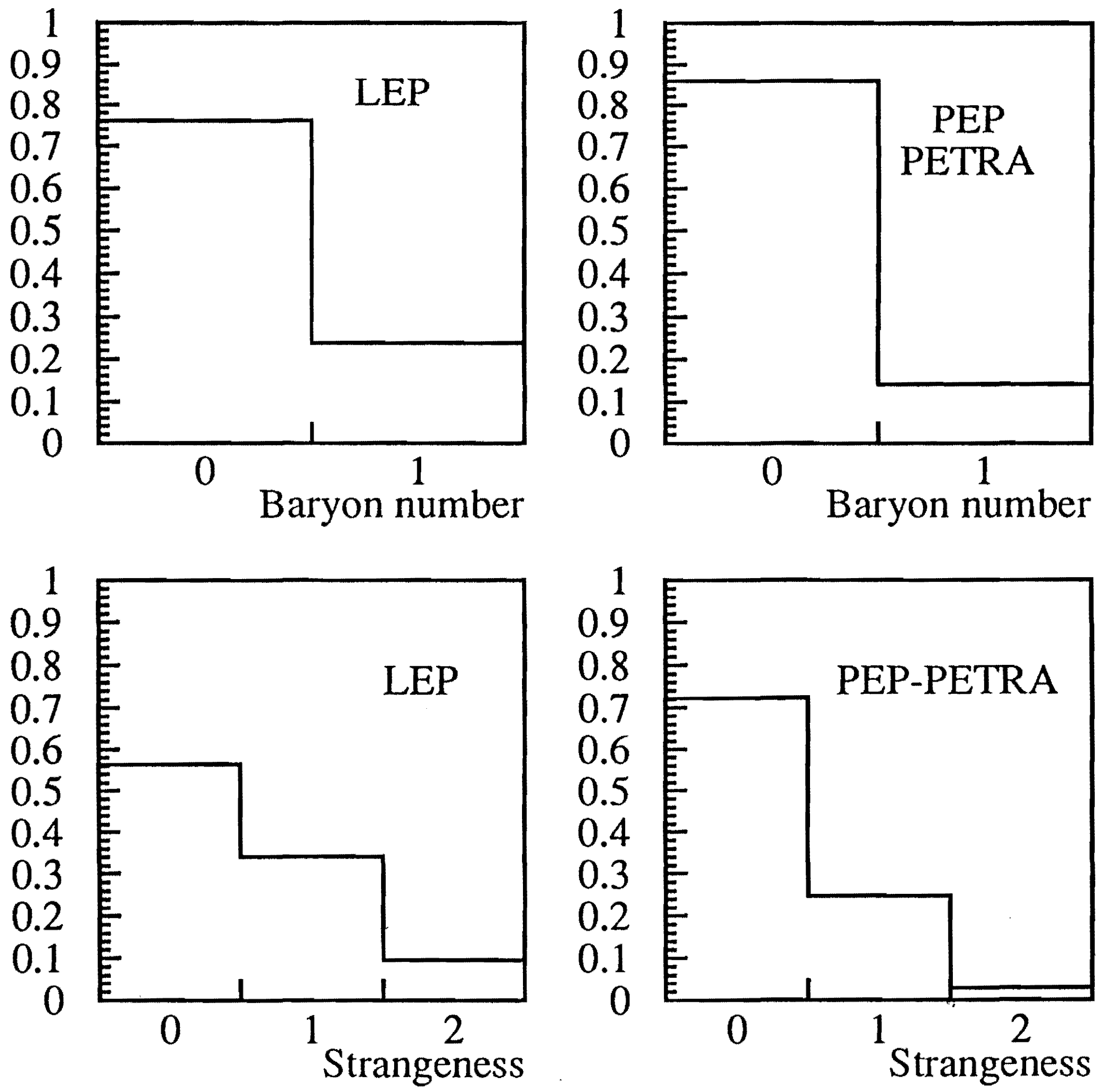

Temperature (MeV)
175
170
165
160
1.2
1.3
1.4
1.5
1.6
1.7
1.8
Volume (Fm^3)
30
25
20
15
10
γ_s
0.75
0.7
0.65
0.6
0.55
Cut-off mass (GeV)

Figure 6:



LEP
Percentage of primaries
0
20
40
60
80
100
π0 π+ K+ K0 η ρ0 K*+ K*0 p η' Φ Λ Ξ- Σ* Ξ*0 Ω
PEP-PETRA
Percentage of primaries
0
20
40
60
80
100
π0 π+ K+ K0 η ρ0 K*+ K*0 p η' Φ Λ Ξ- Σ* Ξ* Ω

Figure 7:



LEP
PEP
PETRA
LEP
PEP-PETRA
Baryon number
Baryon number
Strangeness
Strangeness

Figure 8: