SOME REMARKS ON THE INTEGRATION OF THE POISSON ALGEBRA

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Abstract

We prove that the Poisson algebra of certain non compact symplectic manifolds is isomorphic to a Lie algebra of vector fields on a smooth manifold. We also prove the integrability of the Poisson algebra of functions with compact supports. Finally we discuss and extend the notion of integrability of infinite dimensional Lie algebras.

Key words : Integrability, Poisson Algebra, Infinite dimensional Lie algebra, Calabi homomorphism.

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Introduction.

The so-called third Lie's theorem asserts that any finite dimensional Lie algebra is isomorphic to the Lie algebra of left invariant vector fields on a Lie group.

This assertion does not hold in some infinite dimensional situations, as examples we refer to some Banach Lie algebras [15].

A topological Lie algebra l is said to be *integrable* if there exists a group G which is modeled on a Lie algebra isomorphic to l. In such a case l will be said to be *integrated* by G. One of the main example is the Lie algebra Vect(M) of vector fields on a compact manifold M which is integrated by the group Diff(M) of all diffeomorphisms [11], both spaces being endowed with the C^∞ topology. In fact Leslie proved that on a compact manifold, all the infinite dimensional Lie algebras of Cartan (which include Vect(M)) are integrable[10]. However there are Lie subalgebras of Vect(M) which may not be integrated by any subgroup of Diff(M) or by any other group. We refer to [10] where sufficient conditions for the integration of Lie subalgebras of Vect(M) are given.

An ubiquitous example of infinite dimensional Lie algebra is the Poisson algebra of a symplectic manifold. Let us recall some definitions: if (M,Ω) is a symplectic manifold, the space $C^\infty(M)$ of real valued functions on M is equipped by the *Poisson bracket*. Every $f \in C^\infty(M)$ defines a vector field X_f given by the equation:

$$i_{X_f}\Omega = df$$

up to sign which varies from an author to another. The Poisson bracket of two functions is then defined by:

$$\{f,g\} = \Omega(X_f, X_g).$$

It may be checked that $(C^\infty(M), \{,\})$ is an infinite dimensional Lie algebra which we denote $P(M,\Omega)$ and call the Poisson Lie algebra of (M,Ω); it plays a major role in several parts of Mathematics and Mechanics.

It is a well known fact (e.g. following Souriau [13] or Kostant [6]) that if the period group of Ω, denoted by $Per(\Omega)$, is discrete then $P(M,\Omega)$

is isomorphic to a Lie algebra of vector fields. Indeed there then exists a principal circle-bundle $\pi: Y \to M$ carrying a connexion form ω such that $\pi^*\Omega = d\omega$, $P(M,\Omega)$ is then isomorphic to the Lie algebra of vector fields which strictly preserve ω. If M is compact, $P(M,\Omega)$ is then integrated by Diff$(Y)_\omega$, the group of diffeomorphisms preserving ω.

More generally, for M compact, Ratiu-Schmid proved the integrability of $P(M,\Omega)$ by $[\mathrm{Diff}(M)_\Omega, \mathrm{Diff}(M)_\Omega] \times S^1$; here the brackets denote the commutator subgroup which is also modeled on the globally hamiltonian vector fields of M [1][12]. Hence the discreteness of the period group is not necessary to realize $P(M,\Omega)$ as the Lie algebra of some group of diffeomorphisms.

Unfortunately in the non compact case the integrability as defined above, runs into several difficulties due to the lack of a reasonable topology on the considered spaces. Even for Vect$_c(M)$, the compactly supported vector fields, the topological integration fails, since we don't know if the group of compactly supported diffeomorphisms is modeled on Vect$_c(M)$. However the Kostant-Souriau construction which is independant of the compactness of M indicates that the definition of integrability shou ld be weakened. At a first step we might simply ask when the Poisson algebra of a non compact symplectic manifold is isomorphic to a Lie algebra of vector fields? We prove here that this may be realized whenever $H^2(\widehat{M}) = 0$, $\widehat{M}$ is the universal covering of M, and whithout any additionnal hypothesis on $Per(\Omega)$. More precisely we prove the following result:

Theorem 1. *Given a symplectic manifold* (M,Ω), *if the pullback of* Ω *to the universal covering of* M *is exact, then* $P(M,\Omega)$ *is isomorphic to a Lie algebra of vector fields.*

All surfaces of genus $g > 0$ enter in this case, also all finite dimensional Lie groups equipped of a symplectic form.

In the second part of we prove the following result:

Theorem 2. *The Poisson algebra* $P_c(M,\Omega)$ *of compactly supported functions is integrable.*

Our proof generalizes and gives a new demonstration of Ratiu-Schmid's result subject to mild restriction.

1. The prequantization framework.

In this paper differentiability is assumed to be C^∞

Let M be a differentiable manifold. The integral of a closed two form Ω over a two cycle, defines a groups morphism from $H_2(M,\mathbf{R})$ to the real line, only depending on the class a of Ω in $H^2(M,\mathbf{R})$. The group of periods $Per(\Omega)$ is then the image of this morphism. If $a \in H^2(M,\mathbf{R})$ has a discrete group of periods one may then define a circle fibre bundle $p: E \to M$, equipped of a connection form ω such that $p^*\Omega = d\omega$, the Chern class of the fiber bundle is precisely a. Moreover if Ω is symplectic, ω is then a contact form; in this case this procedure is called the Kostant-Souriau prequantization of (M,Ω) (we refer to [6] or [13] for a detailled treatement of such questions). Moreover, there exists a Lie algebra isomorphism between $P(M,\Omega)$ and $\mathcal{L}_\omega(E) = \{ X \in \mathrm{Vect}(E) \ / \ L_X\omega = 0 \}$ the vector fields which preserve ω. This isomorphism is explicitly given by the following procedure:

for all $f \in C^\infty(M)$ we denote by $Y_f \in \mathrm{Vect}(E)$ the unique vector field characterized by

$$i(Y_f)\omega = f o\pi \quad \text{and} \quad i(Y_f)d\omega = -d(f o\pi)$$

one proves that $Y_f \in \mathcal{L}_\omega(E)$ and that $f \mapsto Y_f$ realizes a Lie algebra isomorphisms between $P(M,\Omega)$ and $\mathcal{L}_\omega(E)$.

Remark: in the case Ω is exact, i.e. $\Omega = d\alpha$ (M is then necessarly non compact) then the fibre bundle is trivial: $E = M \times S^1$ and $\omega = \alpha + \frac{dz}{iz}$. By notation abuse, α and $\frac{dz}{iz}$ denote the pull back by the natural projections of α and of the standard form on the circle.

2. A group cocycle and proof of theorem 1.

We denote by $p: \widetilde{M} \to M$ the universal covering of M, then $\widetilde{\Omega} = p^*\Omega$ is symplectic on $\widetilde{M}$. The fundamental group $\pi_1(M)$ acts on $\widetilde{M}$ by deck transformations. Moreover $\pi_1(M)$ acts as a subgroup of $\mathrm{Diff}_{\widetilde{\Omega}}(\widetilde{M})$ the group of symplectic diffeomorphisms of the covering; indeed we have:

$\forall c \in \pi_1(M)$, $c^*\widetilde{\Omega} = c^*p^*\Omega = p^*\Omega = \widetilde{\Omega}$ since $poc = p$.

We suppose now that $\widetilde{\Omega}$ is exact, we denote by α a potential: $\widetilde{\Omega} = d\alpha$.

Let us consider $Y = \widetilde{M} \times S^1$ on which we define the one form:

$$\omega = \alpha + \frac{dz}{iz}$$

Lemma 1. *$\pi_1(M)$ admits a real central extension which acts on Y as a subgroup of the ω-preserving diffeomorphisms.*

Proof:

The relation $c^*\widetilde{\Omega} = \widetilde{\Omega}$, $c \in \pi_1(M)$ implies $d(c^*\alpha - \alpha) = 0$. Since $\widetilde{M}$ is simply connected, there exists a unique function $f_c \in C^\infty(\widetilde{M})$, such that $c^*\alpha - \alpha = df_c$ and $f_c(x_0) = 0$ for a fixed base point $x_0 \in \widetilde{M}$.

Given c_1 and c_2 in $\pi_1(M)$ we have

$$\begin{aligned} d(f_{c_1c_2}) &= (c_1oc_2)^*\alpha - \alpha \\ &= c_2^*(c_1^*\alpha - \alpha) + c_2^*\alpha - \alpha \\ &= c_2^*(df_{c_1}) + df_{c_2} = d(f_{c_1}oc_2 + f_{c_2}) \end{aligned}$$

Therefore

$$f_{c_1c_2} = f_{c_1}oc_2 + f_{c_2} + w(c_1,c_2)$$

here $w(c_1,c_2)$ is constant. We claim that w is a two cocycle of $\pi_1(M)$ with real values. Indeed for any three elements in $\pi_1(M)$, we have

$$f_{c_1c_2c_3} = f_{c_1c_2}oc_3 + f_{c_3} + w(c_1c_2,c_3)$$

on the other hand we also may write

$$f_{c_1c_2c_3} = f_{c_1}oc_2c_3 + f_{c_2c_3} + w(c_1,c_2c_3)$$

developping $f_{c_1c_2}$ and $f_{c_2c_3}$ in each expression, leads after simplification to a 2-cocycle relation

$$w(c_1,c_2) + w(c_1c_2,c_3) = w(c_2,c_3) + w(c_1,c_2c_3)$$

We pointed out that this cocycle has generally non trivial cohomology class. It determines a central extension of $\pi_1(M)$ by $\mathbf{R}$, i.e. a group structure on the cartesian product $\pi_1(M) \times \mathbf{R}$ with a modified product law

$$(c_1,r)(c_2,t) = (c_1c_2, r + t + w(c_1,c_2)).$$

We shall denote by $\pi_1(M)\rtimes\mathbf{R}$ this extension. For all $(c,r)\in\pi_1(M)\rtimes\mathbf{R}$ we define

$$(c,r):\widetilde{M}\times S^1\to\widetilde{M}\times S^1 \quad\text{by}\quad (c,r)(x,z)=(c(x),ze^{-if_c(x)}e^{ir})$$

We have now an action of the central extension on Y, indeed:

$$\begin{aligned}(c_1,r)\big[(c_2,t)(x,z)\big]&=(c_1,r)(c_2(x),ze^{-if_{c_2}(x)}e^{it})\\&=(c_1(c_2(x)),ze^{-if_{c_1}(c_2(x))}e^{-if_{c_2}(x)}e^{it}e^{ir})\\&=(c_1\circ c_2(x),ze^{-i[f_{c_1}\circ c_2+f_{c_2}](x)}e^{i(t+r)})\\&=(c_1\circ c_2(x),ze^{-if_{c_1c_2}(x)}e^{i(t+r+w(c_1,c_2))})\\&=(c_1c_2,r+t+w(c_1,c_2))(x,z)\\&=(c_1,r)(c_2,t)(x,z)\end{aligned}$$

It is clear that this action lifts the one of $\pi_1(M)$ on $\widetilde{Y}$.

Let us now compute $(c,r)^*\omega$. We have:

$$(c,r)^*\omega=(c,r)^*(\alpha+\frac{dz}{iz})=\alpha+df_c+(c,r)^*(\frac{dz}{iz})$$

but

$$(c,r)^*dz=d(ze^{-if_c(x)}e^{ir})=(dz-izdf_c)e^{-if_c(x)}e^{ir}$$

hence

$$(c,r)^*\frac{dz}{iz}=\frac{dz}{iz}-df_c$$

finally

$$(c,r)^*\omega=\alpha+df_c+\frac{dz}{iz}-df_c=\omega$$

□

We can now adopt the prequantization arguments to complete the proof of the anounced theorem. Let

$$\mathcal{L}=\{X\in\mathrm{Vect}(Y)\ /\ \forall(c,r)\in\pi_1(M)\rtimes\mathbf{R}\ \ (c,r)_*X=X\ \text{ and }\ L_X\omega=0\}$$

We denote by $\sigma=p\circ\pi$ the natural projection from Y onto M.

For all $f\in C^\infty(M)$ one shall define $X_f\in\mathrm{Vect}(Y)$ characterized by the relations

$$i(X_f)\omega=f\circ\sigma\quad\text{and}\quad i(X_f)d\omega=-d(f\circ\sigma)$$

As ω is contact X_f is well defined. And if ξ denotes the Reeb field of ω then $\xi(f\circ\sigma)=0$. It clearly results from Cartan's formula that

$$L_{X_f}\omega=0.$$

For convenience we put $G=\pi_1(M)\rtimes\mathbf{R}$. We have now to prove that X_f is G-invariant. It suffices to evaluate separately $i(g_*X_f)\omega$ and $i(g_*X_f)d\omega$, for all $g\in G$. We have:

$$\begin{aligned}i(g_*X_f)\omega&=g^{-1*}[i(X_f)(g^*\omega)]\\&=g^{-1*}[i(X_f)\omega]\\&=g^{-1*}(f\circ\sigma)=f\circ\sigma\circ g^{-1}\end{aligned}$$

since

$$\sigma\circ g^{-1}=\sigma$$

we get

$$i(g_*X_f)\omega=f\circ\sigma.$$

Similar computations leads to

$$i(g_*X_f)d\omega=-d(f\circ\sigma).$$

Hence $g_*X_f=X_f$ so $X_f\in\mathcal{L}$. Conversly any $X\in\mathcal{L}$ defines a function $f=i(X)\omega\in C^\infty(Y,\mathbf{R})$. As

$$L_X\omega=i(X)d\omega+d(i(X)\omega)=0$$

we get

$$\begin{aligned}\xi.f&=i(\xi)df\\&=i(\xi)d(i(X)\omega)\\&=-i(\xi)i(X)d\omega=i(X)i(\xi)d\omega=0\end{aligned}$$

Hence f is basic for the projection π. Moreover, if $g\in G$ we have

$$\begin{aligned}g^*f&=f\circ g\\&=g^*(i(X)\omega)\\&=i(g_*^{-1}X)g^*\omega\\&=i(X)\omega=f\qquad(\text{ since }\ g_*X=X)\end{aligned}$$

Finally f is basic for $p\circ\pi$. Obviously $f \mapsto X_f$ is a linear isomorphism from $P(M,\Omega)$ onto $\mathcal{L}$. The verification of the relation

$$X_{\{f,g\}} = [X_f, X_g]$$

is straightforward and reproduces word by word the prequantized proof. That completes the proof of theorem 1.□

Examples:

1. If (M,Ω) is riemanian with non positive curvature, it is a classical fact (Cartan) that $\widetilde{M}$ is diffeomorphic to an euclidian space, our theorem applies then is this case.

2. If G is a finite dimensional Lie group; its universal covering $\widetilde{G}$ satisfies $H^2(\widetilde{G},\mathbf{R}) = 0$ for the second homotopy group of any finite dimensional Lie group is trivial [15]. Hence a Lie group (G,Ω) equipped with a symplectic form satisfies the hypothesis of our theorem.

3. Integration of the compactly supported Poisson algebra. Remarks on Ratiu-Schmid theorem.

We first precise some topological definitions. If (M,Ω) is a symplectic manifold, we denote by $C_c^\infty(M)$, $Ham_c(M)$ $\mathrm{Diff}_{\Omega,c}(M)$ respectively the space of smooth compactly supported real functions on M, the globally hamiltonian compactly supported vector fields and the group of compactly supported symplectic diffeomorphisms of M. $\mathrm{Diff}_{\Omega,c}(M)$ will be endowed with inductive limit topology:

$$\mathrm{Diff}_{\Omega,c}(M) = \varinjlim_K \mathrm{Diff}_\Omega(M)_K$$

where K runs over all compact subsets of M and $\mathrm{Diff}_\Omega(M)_K$ is the group of symplectic diffeomorphisms supported in K with the C^∞ topology.

Let $G_\Omega(M)$ be the identity component in $\mathrm{Diff}_{\Omega,c}(M)$ and let

$\widetilde{S}: \widetilde{G_\Omega(M)} \to H_c^1(M,\mathbf{R})$

be the Calabi homomorphism from the universal covering $\widetilde{G_\Omega(M)}$ of $G_\Omega(M)$ to the first De Rham cohomology group with compact support. Finally let $S: G_\Omega(M) \to H_c^1(M,\mathbf{R})/\Gamma$, where $\Gamma = \widetilde{S}(\pi_1(G_\Omega(M)))$, be the induced homomorphism (see [2] for the definitions of S and $\widetilde{S}$). We have the following result:

Theorem2. *If Γ is discrete, then $C_c^\infty(M)$ is integrated by the kernel $\ker S$. In particular, if M is simply connected $C_c^\infty(M)$ is integrated by $G_\Omega(M)$.*

Proof:

According to Weinstein[16], there is a C^∞ one-to-one correspondance $w: \mathcal{U} \to Z_c^1(M)$ from a neighborhood $\mathcal{U} \subset \mathrm{Diff}_{\Omega,c}(M)$ of the identity with a neighborhood of zero in the space of closed one-forms with compact supports. If $h \in \mathcal{U}$ we let $[w(h)] \in H_c^1(M,\mathbf{R})$ be the cohomology class of $w(h)$. Let $h_t^c = w^{-1}(t.w(h))$ be the "canonical" isotopy from h to the identity, then $\widetilde{S}(\{h_t^c\}) = [w(h)]$ [3].

Suppose now $h \in \ker S$, then $\widetilde{S}(h_t^c) \in \Gamma$. Since h_t^c is close to the identity mapping for all t then $\widetilde{S}(h_t^c) = 0$ if Γ is discrete. Therefore we see that Weinstein'charts induces a 1-1 correspondance between $\mathcal{V} = \mathcal{U} \cap \ker S$ and $B_c^1(M)$, the space of exact 1-forms with compact supports. The isomorphims $X \mapsto i(X)\Omega$ from the vector space $\mathcal{L}_\Omega(M)_c$ of symplectic vector fields to $Z_c^1(M)$, induces an isomorphism $Ham_c(M) \simeq B_c^1(M)$. Also, taking the symplectic gradient induces an isomorphism $C_c^\infty(M) \simeq Ham_c(M)$.

Finally, we get a C^∞ 1-1 correspondance between a neighborhood $\mathcal{V}$ of the identity in $\ker S$ with a neighborhood of zero in $C_c^\infty(M)$. □

Remarks:

We have been vague on purpose on the topologies involved, but we reached to integrate. Indeed it is clear that for any $f \in C_c^\infty(M)$, then the corresponding element $X_f \in Ham_c(M)$ integrates to an element of $\ker S$. On the other hand, according to [2] (proposition II.3.1 p 189) for any $g \in \ker S$, there exists an isotopy $g_t \in \ker S$ such that the corresponding family of vector fields $\dot{g}_t \in Ham_c(M)$.

In the case M is compact, we get Ratiu-Schmid result by a similar argument: indeed $\ker S$ is now modeled on $B^1(M)$ the space of exact 1-forms, which is isomorphic to $Ham(M)$. But the extension:

$$0 \to \mathbf{R} \to C^\infty(M) \to Ham(M) \to 0$$

is trivial, and hence $C^\infty(M) = Ham(M) \times \mathbf{R}$ as Lie algebras. Actually Dumortier-Takens [5] proved that the compacity of a symplectic manifold is equivalent to the triviality of the above extension! Therefore, because $Ham(M)$ is integrated by $\ker S$, $C^\infty(M) = Ham(M) \times \mathbf{R}$ is integrated by $\ker S \times S^1$. But for M compact $\ker S$ is equal to the commutator subgroup $[G_\Omega(M), G_\Omega(M)]$ [2]. Therefore $[G_\Omega(M), G_\Omega(M)] \times S^1$ integrates $C^\infty(M)$.

The existence of Weinstein's map was crucial in this proof. It seems hard via topology to modelize $\mathrm{Diff}_c(M)$ on $\mathrm{Vect}_c(M)$; this will lead us to the following remarks.

4. More remarks on integration of infinite dimensional Lie algebras.

Let $\mathrm{Diff}(M)$ denote the group of all diffeomorphisms of a non compact manifold M. This group may be equipped of two natural topologies: the Whitney C^∞-topology or the C^∞-compactly supported one. Neither one nor the other turns $\mathrm{Diff}(M)$ into a topological group. The same remark holds for $\mathrm{Diff}_c(M)$ the compact supported diffeomorphisms. So any tentative to get a Lie third theorem for the "natural" corresponding Lie algebras (we mean here $\mathrm{Vect}(M)$ and $\mathrm{Vect}_c(M)$) fails. In order to avoid topological constraints, we propose an extension of the notion of Lie algebra of a group:

The expression "group of diffeomorphisms" will refers to any subgroup of the group $\mathrm{Diff}(M)$ for a certain manifold.

Given a real interval I, a path $I \ni t \mapsto \gamma(t) \in G$ is said to be *differentiable* if it defines a differentiable isotopy, namely: $(t,x) \mapsto \gamma(t)(x)$ is differentiable as a map from $I \times M$ to M.

To any differentiable path such that $\gamma(0) = I_M$ corresponds a vector field $X_\gamma(x) = \frac{d}{dt}\gamma(t)(x)_{|t=0}$.

Definition. *The* Lie algebra *of a group of diffeomorphisms G is the Lie algebra of vector fields $\mathfrak{g}$ generated by the following set*

$$\mathcal{D} = \left\{X \in \mathrm{Vect}(M) / X = \frac{d}{dt}\gamma(t)(x)_{|t=0} \text{ for some differentiable path } \gamma \textit{ in } G\right\}$$

$\mathcal{D}$ is a vector space, we have in fact: if γ and μ are differentiable paths passing through I_M at time zero, $\gamma\mu$ being $t \mapsto \gamma(t)\mu(t)$, it is then easy to check that:

$$X_\gamma + X_\mu = X_{\gamma\mu}.$$

$$aX_\gamma = \frac{d}{dt}\gamma(at)_{|t=0} \text{ for } a \in \mathbf{R}.$$

Moreover

$$[X_\gamma, X_\mu] = \frac{d^2}{dsdt}\gamma(s)\mu(t)\gamma(s)^{-1}\mu(t)^{-1}|_{s=t=0}.$$

The naturality of this definition holds on several reasons: first it coincides with the classical one in all finite dimensional cases; second as any compactly supported vector field has complete flow, $\mathrm{Vect}_c(M)$ is the Lie algebra of $\mathrm{Diff}_c(M)$ in this sense. We point out that this notion of integrability has a natural setting: the Diffeological Category following Chen [4], Souriau[14] or Haefliger[7].

As an epilog:

let $[exp'](M)$ denotes the Lie algebra generated by the complete vector fields on a non compact manifold M. One has the obvious inclusions:

$$[exp'](M) \subset \mathfrak{g} \subset \mathrm{Vect}(M)$$

where $\mathfrak{g}$ is the Lie algebra of $\mathrm{Diff}(M)$ previously defined. Which of these inclusions is strict? As example, one may check that the constant vector field on an open bounded interval I, which is of course not complete, belongs to $\mathfrak{g}_I$. We have the deep conviction that more comprehension about the infinite dimensional status of Lie's third theorem will come from lighting this inclusions.

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