In Search of V_{ub} and CP Noninvariance in Heavy Quark Decays using the Quark Diagram Scheme

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ABSTRACT

Using the model-independent Quark Diagram Scheme some specific beauty particle nonleptonic decays are pointed out for experimental measurements to establish the nonzeroness of $|V_{ub}|$, and to measure $|V_{ub}/V_{cb}|$. General properties of partial decay rate differences in charm, beauty and t particles are discussed, also using the Quark Diagram Scheme. Some interesting $B^{\pm}$ decays with all charged particles in the final states are given for the search of CP noninvariance in the decay amplitudes. Some highlights are pointed out in the event that higher than three generations of quarks exist.

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Introduction

Since the proposal of Kobayashi-Maskawa (KM) scheme of quark mixing for CP noninvariance[1] and the discovery of the beauty particles,[2] much progress has been made in measuring the quark mixing matrix in the case of three generations of quarks.[3] Besides the urgently needed observation of the t quark and its decays to measure V_{tb}, V_{ts}, V_{td}, presently the measurement of V_{ub} and the verification of its nonzeroness is most urgent. If $|V_{ub}|$ is too small or zero, higher than three generations of quarks will be needed if we still want to describe CP noninvariance through the quark mixing phenomena. Theoretically the neatest way to measure $|V_{ub}|$ is to observe $B_u \to \tau\nu_\tau$. Unfortunately the branching ratio of such decay is very small $Br(B_u \to \tau\nu_\tau) \lesssim 10^{-5}$, using the current bound on $|V_{ub}/V_{cb}|^2 < (\frac{1}{5})^2$. It is therefore important to find other decay channels which are nonzero only when $|V_{ub}|$ is not zero. Certainly the observation of charmless semileptonic decay $b \to \ell^- \overline{\nu}_\ell X_{c=0}$ inclusively or exclusively will establish $|V_{ub}| \neq 0$. Here I would like to point out some pure nonleptic decays which are non-zero only if $|V_{ub}| \neq 0$, and some ratios of nonleptonic decays that are proportional to $|V_{ub}/V_{cb}|$. Such a selection is possible based upon the general model-independent Quark-Diagram Scheme.

It has been shown[4] that all meson decays can be expressed model-independently in terms of six quark diagrams (shown in Fig. 1): $\mathcal{A}$, the external W-emission diagram; $\mathcal{B}$, the internal W-emission diagram; $\mathcal{C}$, the W-exchange diagram; $\mathcal{D}$, the W-annihilation diagram, these we call the W tree diagrams; $\mathcal{E}$, the horizontal W-loop diagram; $\mathcal{F}$, the vertical W-loop diagram. These quark diagrams are specific, well defined physical quantities. They are classified according to the topology of weak interactions with only the quark and intermediate bosons explicitly shown. All QCD strong-interaction effects are there though not explicitly shown. QCD effects can also be treated perturbatively, e.g., the one-gluon-exchange diagram of the $\mathcal{E}$ graph is the so called Penguin diagram. These diagrams keep their identities under all QCD effects. Only so such classification is useful. It has recently been successfully used[5] to analyze two-body charm decay data, to compare experimental data with various theoretical models to check their validity, and further to make predictions and to suggest new experimental measurements. This scheme has also provided a model independent way to map out decays of heavy quark states which are possible to have CP noninvariance of partial decay rate differences, which I shall elaborate in Section II.

In section I, I discuss how this scheme can provide a model-independent way to pick out nonleptonic decays of beauty particles, which are nonzero only if $|V_{ub}|$ is

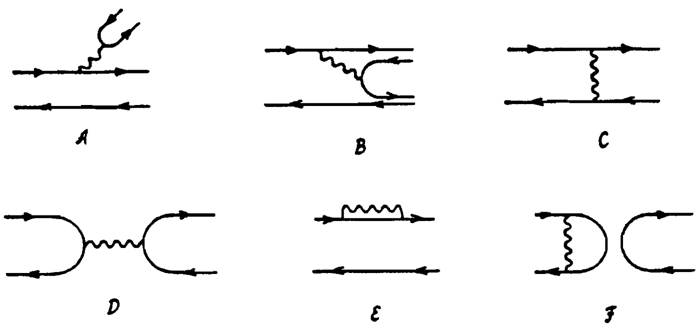

Fig. I.a. The six quark diagrams for inclusive meson decay.

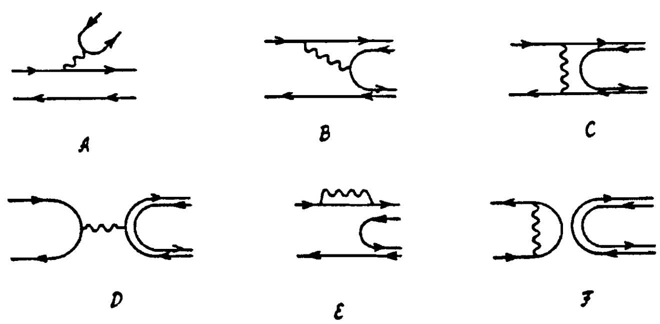

Fig. I.b. The six quark diagrams for a meson → two mesons.

nonzero. The result is that the following decay channels are purely proportional to $|V_{ub}|$:

$B_{d,u} \to F^+ X_{c=0,s=0}$ or $D X_{c=0,s=1}$,
which include exclusive channels like:
$B_u^+ \to F^+\pi^0,\ F^+\rho^0,\ F^+\pi^+\pi^-,\ F^+\phi,\ F^+K^+K^-,$
$F^+\eta_8,\ F^+\eta_0,\ F^+\eta_c,\ F^+J/\psi,$ etc; $D^0K^+,$
$D^+K^0,\ D^{*0}K^+,\ D^0K^{*+},\ D^{*0}K^{*+},$ etc.
$B_d^0 \to F^+\pi^-,\ F^+\rho^-,\ F^+\pi^-\pi^0,$ *etc.*; $D^0K^0,\ D^{*0}K^0,$
$D^0K^+\pi^-,\ D^{*0}K^{0*},\ D^{*0}K^{0*},$ *etc.*
$B_s \to F X_{c=0,s=-1}$, or $D X_{c=0,s=0}$,
which include exclusive channels like:
$B_s^0 \to F^+K^-,\ F^+K^{*-},\ F^+\overline{K}^0\pi^-,$ *etc.*;
$D^0\pi^0,\ D^0\eta_8,\ D^0\eta_0, D^0\phi,\ D^{*0}\phi,\ D^{*0}\eta_8,$
$D^{*0}\eta_0,\ D^0\eta_c,\ D^0J/\psi,$ *etc.*; $D^+\pi^-,\ D^+\rho^-,$
$D^+\pi^-\pi^0,\ D^{*+}\pi^-,\ D^{*+}\rho^-,$ *etc.*

They typically have a branching ratio of 10^{-4} within the reach of current experimental capability. Note that all these channels have a charm particle in the final states.

Recently bounds on charmless nonleptonic decays like $B^0 \to \pi^+\pi^-$ have been used as direct measurement of bound on $|V_{ub}|$. Caution must be given in such a procedure. Observation of charmless beauty particle decays does not necessarily mean $|V_{ub}| \neq 0$. The important point is that those charmless decays do not have to be zero if $|V_{ub}| = 0$ according to the general Quark Diagram Scheme since they have W-loop diagram contributions with the dominant coefficient V_{cb}. Only if we can establish experimentally that the W-loop-diagram contributions are negligibly small comparing to the tree-diagram contributions then charmless decays can give indications on $|V_{ub}|$. To find ways to study the contributions of the W-loop diagram, again the Quark-Diagram Scheme is useful. It can be shown that the following decays can have only W-loop-diagram contributions, even with the dominant coefficient $V_{cs}V_{cb}^*$:

$$B_d^0 \to K^0\phi,\ K^{*0}\phi;\ B_s^0 \to \overline{K}^0\phi,\ \overline{K}^{*0}\phi;\ \text{and}$$
$$B_{d,s}^0 \to \phi\phi,\ K^0K^0,\ K^{*0}\overline{K}^0,\ K^0\overline{K}^{*0},\ K^{*0}\overline{K}^{*0}.$$

Experimentally studying these decays can give direct measurements of the size of the W-loop diagrams.

Once $|V_{ub}|$ is established to be nonzero, we ought to determine its precise value. It is then worthwhile to measure the dynamically simplest $B \to \tau\nu_\tau$ in a dedicated experiment. Another way is to find other b particle decays which have the same Quark-Diagram amplitudes as listed above except with known mixing matrix elements. Then the ratios of such decays can give ratios of $|V_{ub}/V_{cb}|$. The idea here is similar to a previous well known case in charm decays[3,6] $\tilde{\Gamma}(D^+ \to \pi^0\pi^+)/\tilde{\Gamma}(D^+ \to \overline{K}^0\pi^+) = \frac{1}{2}|V_{cd}/V_{cs}|^2$. The ratios of decay widths which give $|V_{ub}/V_{cb}|$ are, e.g.

$$\frac{2\tilde{\Gamma}(B_u^+ \to F^+\pi^0)}{\tilde{\Gamma}(B_d^0 \to D^-K^+)} = \frac{\tilde{\Gamma}(B_d^0 \to D^0K^0)}{\tilde{\Gamma}(B_d^0 \to \overline{D}^0K^0)} = \frac{\tilde{\Gamma}(B_s^0 \to F^+K^-)}{\tilde{\Gamma}(B_s^0 \to F^-K^+)}$$
$$= \frac{2\tilde{\Gamma}(B_s^0 \to D^0\pi^0)}{\tilde{\Gamma}(B_s^0 \to D^-\pi^+)} = |\frac{V_{cs}}{V_{us}}|^2 \cdot |\frac{V_{ub}}{V_{cb}}|^2 .$$

Here we ought to be careful about final-state interactions. It is however very likely that final-state interactions are negligible since the B masses are far beyond known resonances.

The second part of the paper deals with CP noninvariance in partial decay rate differences. Despite many recent dedicated searches, the original observation made in 1964 of the minute CP noninvariance effects of $K_L \to 2\pi$,[7] and in $K_L \to \pi^\pm e^\mp \nu_e$ stays still as the only system and the sole kind of CP noninvariance. Yet CP noninvariance makes up one of the essential ingradiences in explaining the matter dominance of our universe.[8] Further, the origin of CP noninvariance seems to reside at the same reservoir of ignorance of our fundamental understanding of the now triumphant electroweak unified theories, i.e. the mass generating mechanism. It is natural that the searching for new kind of CP noninvariance in reactions other than the K_L system continue to be dedicated efforts at various laboratories, and to be the central theme in the discussions on possible searches in future experiments.

In weak decays basically there are two types of mechanisms making CP noninvariance: the mass-matrix type (also called superweak type),[9] originated in the particle-antiparticle mixing of neutral mesons, and are therefore limited only to neutral mesons; and the decay-amplitude type, originated directly in the weak decay amplitudes. The ϵ from $K_L \to 2\pi$ belongs to the mass-matrix type. $\epsilon' \equiv |\eta_{+-}/\eta_{00}| - 1$ is of the decay amplitude type.[(10)] Unfortunately ϵ' is suppressed relative to ϵ due to the $\Delta I = \frac{1}{2}$ rule, i.e. $\Gamma(K^+ \to \pi^+\pi^0)/\Gamma(K^0 \to \pi^+\pi^-) \approx \frac{1}{670}$. Ironically such suppression was vital for the discovery of parity violation in the 1950's. So far ϵ' has still eluded experimental observation.[11] Dedicated experiments are being carried out to search for its possible presence.[12] And to avoid the $\Delta I = \frac{1}{2}$ rule suppression of decay-amplitude CP noninvariance ϵ', it was pointed out[13] that the time integrated partial decay rate difference in $K^0, \overline{K}^0 \to 2\gamma$ can be as large as ϵ. It is therefore important to do an experiment on this decay. In section II, we shall mainly concentrate on CP noninvariance of partial decay rate differences in heavy quark decays.

A general convenient way to search for such decay-amplitude CP noninvariance is to measure the partial decay rate differences between particle and its antiparticle. It was shown that the Quark Diagram Scheme can provide a model-independent general survey of whether decay-amplitude CP noninvariance can exist in a particular exclusive decays in the KM way of violating CP invariance, and such general survey has been carried out in Ref. (14), (see also Ref. (15)-(17)). Many channels of charm and beauty particle decays were shown to possibly have partial decay rate differences via basic interferences between the quark diagrams, belonging to the following major categories:[(14)] interferences among the tree graphs alone, between the tree graphs and the loop graphs, and between the loop graphs themselves. How big these partial decay rate differences are depends upon specific model calculations. A series of investigations has been carried out based on this Quark-Diagram Scheme,[(18,19)] using the currently available models. A general picture has emerged that the partial decay rate differences can be very large, 10's%, in beauty particle decays. Such large partial decay rate differences are very encouraging.

The decay of $B_d^0 \to K^+\pi^-$ calculated in Ref. (19) has been singled out in the physics studies for SSC due to its simplicity in detecting its final particles K^- and π^+, which are all charged. Here some clarifying discussions are in order to compare the advantages of searching partial decay rate differences in the charged $B^\pm$ decays over the neutral B° decays. It was pointed out in Ref. (19), the partial decay rate differences in neutral B^0 decays like $B_d^0 \to K^+\pi^-$, D^-F^+, etc., i.e. $B_d^0 \to$ total $s = 0$, $c = 0$ states, which are not CP self-conjugate state and only B^0 (not $\overline{B}^0$) can decay into it, are purely decay-amplitude type of CP noninvariance. There is no $B^0 \leftrightarrows \overline{B}^0$ mixing effects nor mass-matrix CP noninvariance involved. Therefore the interpretations of any observations of such partial decay rate differences are straight forward. There are neutral channels both B^0 and $\overline{B}^0$ can decay into, e.q. $B_d^0 \to \pi^+\rho^-$, $\pi^-\rho^+$, $\pi^+\pi^-$, D^+D^-, etc. Their time integrated partial decay rate differences are results of all three distinct effects: decay-amplitude CP noninvariance, mass-matrix CP noninvariance, and $B^0 \leftrightarrows \overline{B}^0$ mixing.[(19,20)] Therefore the interpretation of observed results of partial decay rate differences in these decays are more complicated. To untangle these three effects, two more other types of experiments must be supplemented. Here I shall confine my discussions on the decays in which partial decay rate differences are purely from the decay-amplitude CP noninvariance. The charged $B^\pm$ decays for studying partial decay rate differences have the further advantage over the neutral B^0 decays, for which we must tag experimentally in order to avoid the contamination from $B^0 \leftrightarrows \overline{B}^0$ mixing in the measurement of integrated partial decay rate differences.

In Section II, I elaborate these points and then point out a few more such beauty particle decays, which have large partial decay rate differences, yet simple for experimental observation, e.g.

$$B_u^\pm \to K^\pm\rho^0,\ \overline{K}^{*0}\pi^\pm,\ K^\pm\pi^+\pi^-,\ \text{etc.}$$

These decays are from similar kinds of quark diagrams as in $B_d^0 \to K^-\pi^+$, so we expect similar partial decay rate differences ($\sim 10's\%$) and branching ratios ($\sim 10^{-4}$ to 10^{-5}).[(21)]

I shall also give a simple explanation why the partial decay rate differences are naturally large in beauty particle decays, but rather small in charm decays. The partial decay rate differences in t particle decays can also be large, as in the beauty particle decays. Unfortunately the branching ratios of two or three particle decays of t particles are very much suppressed due to the weightiness of the t particles. To enhance the branching ratio, high-multiplicity final states must be studied. This certainly complicates the experimental search. In charm particle decays, the difficulty in the search for CP noninvariance of partial decay rate differences is quite different. In general, the branching ratios of two-body exclusive decays of charm particles are bigger than in beauty particle decays, but the partial decay rate differences are in general much smaller than in beauty particle decays.

In Section III, some highlights will be pointed out in the quark mixing matrix, and CP noninvariance in the events that higher than three generations of quarks exist.

I. Quark Mixing Matrix and the Search for $|V_{ub}|$ and $|V_{ub}/V_{cb}|$ Using the Quark - Diagram Scheme

First I give a brief account of the status of the KM matrix. Recent results show that the quark mixing matrix can be parameterized most conveniently in the following form[22]

$$V = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_y & s_y \\ 0 & -s_y & c_y \end{bmatrix} \begin{bmatrix} c_z & 0 & s_z e^{-i\phi} \\ 0 & 0 & 0 \\ -s_z e^{i\phi} & 0 & c_z \end{bmatrix} \begin{bmatrix} c_x & s_x & 0 \\ -s_x & c_x & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{array}{ccc} d & s & b \end{array}$$

$$= \begin{bmatrix} c_x c_z & s_x c_z & s_z e^{-i\phi} \\ -s_x c_y - c_x s_y s_z e^{i\phi} & c_x c_y - s_x s_y s_z e^{i\phi} & s_y c_z \\ s_x s_y - c_x c_y s_z e^{i\phi} & -c_x s_y - s_x c_y s_z e^{i\phi} & c_y c_z \end{bmatrix} \begin{array}{c} u \\ c \\ t \end{array}$$

$$\underset{s_i \approx 0}{\approx} \begin{bmatrix} 1 & s_x & s_z e^{-i\phi} \\ -s_x - s_y s_z e^{i\phi} & 1 - s_x s_y s_z e^{i\phi} & s_y \\ s_x s_y - s_z e^{i\phi} & -s_y - s_x s_z e^{i\phi} & 1 \end{bmatrix}. \quad (1.1)$$

An important feature of the three-generation quark mixing matrix as first shown in Ref. (22) is that all CP noninvariance in decays of those three-generation quark particles are described by one universal phase-convention-independent parameter X_{CP}

$$\begin{aligned} X_{CP} &= V_{us}V_{ud}^*(V_{cs}V_{cd}^*)^*; \text{ for strange particle decays;} \\ &= V_{cs}V_{us}^*(V_{cd}V_{ud}^*)^*; \text{ for charm particle decays;} \\ &= V_{ub}V_{ud}^*(V_{cb}V_{cd}^*)^*, \\ &= V_{ub}V_{us}^*(V_{cb}V_{cs}^*)^*; \text{ for beauty particle decays;} \\ &= V_{tb}V_{cb}^*(V_{ts}V_{cs}^*)^*, \\ &= V_{tb}V_{ub}^*(V_{ts}V_{us}^*)^*; \text{ for t particle decays;} \\ &= s_x s_y s_z s_\phi c_x c_y c_z^2, \\ &\approx 10^{-5}, \quad \text{if } m_t \approx 50 \text{ GeV is used in fitting } \epsilon. \quad (1.2) \end{aligned}$$

Because of this property we can put the phase factor at the most convenient place, i.e. at where the matrix element is the smallest.[23] This has the advantage that for all practical purposes without involving CP noninvariance, the matrix can be considered to be real. Further in Section II I shall demonstrate that this parametrization is also the simplest in considering CP noninvariance of partial decay rate differences.

Another important feature of this parametrization is that it takes advantage of all the experimental information: each sine is directly related to one type of experiment[24]

$$s_x \approx 0.22, \text{ determined from strange particle decays,} \quad (1.3a)$$

$$s_y \approx 0.05, \text{ from b particle life time,} \quad (1.3b)$$

$$s_z \lesssim 0.01, \text{ from bounds on } (b \to u)/(b \to c). \quad (1.3c)$$

This is the inherited convenience from the original Maiani parametrization,[25] though the rotation order was found originally in Ref. (22) and with final form in the real angles agreeing with Maiani's. We pick the subscript x, y, z because that was the order of experimental measurements and easiest to remember. Such parametrization via sequences of rotations and put phases at the furthest corners can be generalized to cases with higher generating of quarks.[26,27] See Section III.

In this parametrization we can see easily that if $V_{ub} = 0$, there is no phase factor for CP noninvariance, and the value of $|V_{ub}|$ needed for ϵ is not too far below the upper bound[28] on $|V_{ub}|$ from $b \to \ell^- X$. It is very important to settle the issue whether $|V_{ub}|$ is nonzero or not.

Theoretically the clearest way to look for V_{ub} is from $B \to \tau\overline{\nu}_\tau$, since it has the least complication dynamically. (Here we pick the heaviest of the leptons to avoid the helicity suppression in the light-lepton case, as $\pi \to e\nu_e$ is very suppressed compared to $\pi \to \mu\nu_\mu$.)[29] Unfortunately, the branching ratio is very small:

$$\Gamma(B_u \to \tau\nu_\tau) = |V_{ub}|^2 (G_F^2 f_B^2/8\pi) m_B m_t^2 (1 - m_t^2/m_B^2), \quad (1.4a)$$

using

$$\Gamma(b \to cX) \approx |V_{cb}|^2 G_F^2 m_b^5/(192\pi^3), \quad (1.4b),$$

we obtain

$$\begin{aligned} Br(B \to \tau\nu_\tau) &= |V_{ub}/V_{cb}|^2 \times 2.1 \times 10^{-3}, \\ &\lesssim (0.01/0.05)^2 \times 2.1 \times 10^{-3} \approx 8.4 \times 10^{-5}. (1.4c) \end{aligned}$$

Therefore we need to find other ways to establish the nonzeroness of $|V_{ub}|$. The observation of charmless $B \to \ell^+ X_{c=0}$ inclusively or exclusively will establish $|V_{ub}| = 0$. Another way is to study those hadronic decay channels which is nonzero only when $|V_{ub}|$ is not zero. Here the model-independent description of Quark-Diagram Scheme can be very useful. From the Quark-Diagram Scheme, we find the following exclusive decays which are nonzero only if $|V_{ub}| \neq 0$:

$B_u^+ \to F^+ X_{c=0,s=0}$, including the following decay modes:

$F^+\pi^0 \times \sqrt{2}$,
$F^+\eta_8 \times \sqrt{6}$,
$F^+\eta_0 \times \sqrt{3}$,
$F^+\rho^0$, $F^+\pi^+\pi^-$, etc.: $\quad V_{cs}V_{ub}^*\ (A)$; (1.5a)

$F^+\phi$, $F^+K^+K^-$, etc.: $\quad V_{cs}V^*_{ub}(\mathcal{D})$; (1.5b)

$\to DX_{c=0,s=+1}$:
D^0K^+, D^0K^{*+},
$D^{*0}K^+$, $D^{*0}K^{*+}$, etc.: $\quad V_{cs}V^*_{ub}(\mathcal{B}+\mathcal{D})$; (1.5c)

$B^0_d \to F^+X_{c=0,s=0}$:
$F^+\pi^-$, $F^+\rho^-$, $F^+\pi^-\pi^0$, etc.: $V_{cs}V^*_{ub}\ (\mathcal{A})$; (1.5d)

$\to DX_{c=0,s=+1}$:
D^0K^0, D^0K^{0*}, $D^0K^+\pi^-$
$D^{*0}K^0$, $D^{*0}K^{0*}$, etc.: $\quad V_{cs}V^*_{ub}\ (\mathcal{B})$; (1.5e)

$B^0_s \to F^+X_{c=0,s=-1}$:
F^+K^-, F^+K^{*-},
$F^+\overline{K}^0\pi^-$, etc.: $\quad V_{cs}V^*_{ub}(\mathcal{A}+\mathcal{C})$; (1.5f)

$\to DX_{c=0,s=0}$:
$D^0\pi^0\times\sqrt{2}$,
$D^0\eta_8\times\sqrt{6}$,
$D^0\eta_0\times\sqrt{3}$,
$D^+\pi^-$, $D^+\rho^-$,
$D^+\pi^-\pi^0$,
D^0J/ψ, etc.: $\quad V_{cs}V^*_{ub}\ (\mathcal{C})$. (1.5g)

(For simplicity, here we use the same amplitude symbols for final states from different multiplets. But do remember that they are in general different.) Notice that all these decays have a net charm quantum number equal to one in the final state. Essentially they are all from graphs with $\bar{b}\to\bar{u}W^+$ and $W^+\to c\bar{s}$, therefore it is always the combination $V_{cs}V_{ub}{}^*$ appearing in these decays. There are other decays involving $V_{cd}V_{ub}{}^*$, e.g. $D^+\pi^-$, $D^0\pi^0$ which are nonzero only if $|V_{ub}|\neq 0$, but their branching ratios are suppressed by a factor $|V_{cd}/V_{cs}|^2\approx 0.05$ compared to the previous case.

Decays with $V_{ud}V_{ub}{}^*$ factors always can have other diagrams involving $V_{cd}V^*_{cb}$. For example the charmless decay $B^0_d\to\pi^+\pi^-$ are given by the following quark amplitudes and quark mixing factors,

$$B^0_d\to\pi^+\pi^-:\ V_{ud}V^*_{ub}(\mathcal{A}+\mathcal{C}+\mathcal{E}_{u-t})+V_{cd}V^*_{cb}(\mathcal{E}_{c-t}). \tag{1.6a}$$

We see that even when $|V_{ub}|=0$, $B^0_d\to\pi^+\pi^-$ may not be zero due to the W-loop diagrams.

So the search for $|V_{ub}|$ is not best in the charmless decays of beauty particles but in those specific beauty particle decays into one charm particle states Eqs. (1.5).

However if we can establish experimentally that the W-loop diagram contributions even with their large mixing matrix coefficient are negligible comparing to the tree-diagram contributions (in many cases with very small mixing matrix coefficients), then the charmless decays like $B^0_d\to\pi^+\pi^-$ can give estimates on $|V_{ub}|$. To do such an analysis we need to measure the W-loop diagrams independently. From the Quark-Diagram Scheme, we find the following decays can have only W-loop contributions (even with the dominant coefficient $V_{cs}V^*_{cb}$),

$$B^0_d\to K^0\overline{K}^0,\ K^{*0}\overline{K}^0,\ K^0\overline{K}^{*0},\ K^{*0}\overline{K}^{*0}:$$
$$V_{ud}V^*_{ub}(\mathcal{E}_{u-t}+\mathcal{F}_{u-t})+V_{cd}V_{cb}(\mathcal{E}_{c-t}+\mathcal{F}_{c-t});$$

$$B^0_d\to K^0\phi,\ K^{*0}\phi:$$
$$V_{ud}V^*_{ub}(\mathcal{E}_{u-t})+V_{cd}V^*_{cb}(\mathcal{E}_{c-t}); \tag{1.6b}$$
$$B_d\to\phi\phi:\ V_{ud}V^*_{ub}(\mathcal{F}_{u-t})+V_{cd}V^*_{cb}(\mathcal{F}_{c-t});$$
$$B^0_s\to\overline{K}^0\phi:\ V_{ud}V^*_{ub}(\mathcal{E}_{u-t})+V_{ud}V^*_{ub}(\mathcal{E}_{c-t});$$
$$B^0_s\to\phi\phi,\ K^0\overline{K}^0,\ K^{*0}\overline{K}^0,\ K^0\overline{K}^{*0},\ K^{*0}\overline{K}^{*0}:$$
$$V_{us}V^*_{ub}(\mathcal{E}_{u-t}+\mathcal{F}_{u-t})+V_{cs}V^*_{cb}(\mathcal{E}_{c-t}+\mathcal{F}_{c-t}).$$

(Here $\mathcal{E}_{i-j}\equiv\mathcal{E}_i-\mathcal{E}_j$). Combined analysis of decays of Eq. (1.6b) and charmless beauty decays like Eq. (1.6a) can help to shed light on $|V_{ub}|$. It is interesting to note that $B_d\to\phi\phi$ comes only from the vertical W-loop diagram $\mathcal{F}$; and $B_d\to K^0\phi$, $K^{*0}\phi$, $B^0_s\to\overline{K}^0\phi$, $\overline{K}^{*0}\phi$ comes only from the horizontal loop diagram $\mathcal{E}$.

We can also use ratios of nonleptonic decays to measure $|V_{ub}/V_{cb}|$. In the following we list some of the nonleptonic decay amplitudes which are proportional to V_{cb}, e.g.

$$B^0_d\to D^-K^+:\quad V_{us}V^*_{cb}(\mathcal{A}); \tag{1.7a}$$
$$\to\overline{D}^0K^0:\quad V_{us}V^*_{cb}(\mathcal{B}); \tag{1.7b}$$
$$B^0_s\to F^-K^+:\quad V_{us}V^*_{cb}(\mathcal{A}+\mathcal{C}); \tag{1.7c}$$
$$\to D^-\pi^+:\quad V_{us}V^*_{cb}(\mathcal{C}). \tag{1.7d}$$

Assuming that the final state interaction effects are small,[5,21] by taking the proper ratios between the decays in Eqs. (1.5) and in Eqs. (1.7), we obtain

$$\frac{2\tilde{\Gamma}(B^+_u\to F^+\pi^0)}{\tilde{\Gamma}(B^0_d\to D^-K^+)}=\frac{\tilde{\Gamma}(B^0_d\to D^0K^0)}{\tilde{\Gamma}(B^0_d\to\overline{D}^0K^0)}=\frac{\tilde{\Gamma}(B^0_s\to F^+K^-)}{\tilde{\Gamma}(B^0_s\to F^-K^+)}$$
$$=\frac{2\tilde{\Gamma}(B^0_s\to D^0\pi^0)}{\tilde{\Gamma}(B^0_s\to D^-\pi^+)}=|\frac{V_{cs}}{V_{us}}|^2\cdot|\frac{V_{ub}}{V_{cb}}|^2. \tag{1.8}$$

Here $\tilde{\Gamma}$ is the reduced width with unequal phase factors factored out. Thus measuring such ratios we can measure $|V_{ub}/V_{cb}|^2$.

II. Partial Decay Rate Differences form the Quark-Diagram Scheme

The partial decay rate differences are defined as

$$\Delta\equiv\frac{|A|^2-|\overline{A}|^2}{|A|^2+|\overline{A}|^2}, \tag{2.1}$$

where A, $\overline{A}$ stand for decay amplitudes of particle and antiparticle into a specific exclusive final state and their corresponding antiparticle state. To map out which exclusive decay channels can possibly have CP noninvariance of partial decay rate differences, the general Quark Diagram Scheme has been indispensible.[3,14,18,19] It provides a model-independent way to find which channels can not have CP noninvariance and which channels can.

The best way to explain the scheme is to give an example: $B_u^+ \to K^+\rho^0$ is given in terms of the quark diagrams as follows,

$$\begin{aligned} A &= V_{ub}V_{us}^*(\mathcal{A}+\mathcal{B}+\mathcal{D}+\mathcal{E}_{u-t}) + V_{cb}V_{cs}^*(\mathcal{E}_{c-t}), \\ &\equiv V_{ub}V_{us}^*A_1 + V_{cb}V_{cs}^*A_2, \quad (2.2a) \\ \overline{A} &= V_{ub}^*V_{us}(\mathcal{A}+\mathcal{B}+\mathcal{D}+\mathcal{E}_{u-t}) + V_{cb}^*V_{cs}(\mathcal{E}_{c-t}), \\ &\equiv V_{ub}^*V_{us}A_1 + V_{cb}^*V_{cs}A_2. \quad (2.2b) \end{aligned}$$

Note that the difference between particle and antiparticle decays are mainly in $V_{ij} \to V_{ij}^*$. Substituting Eqs. (2.2) in Eq. (2.1),

$$\begin{aligned} \Delta &= \frac{-4Im[(V_{ub}V_{us}^*(V_{cb}V_{cs}^*)^*]ImA_1A_2^*}{|V_{ub}V_{us}^*A_1 + V_{cb}V_{cs}^*A_2|^2 + |V_{ub}^*V_{us}A_1 + V_{cb}^*V_{cs}A_2|^2}, \\ &= \frac{-4X_{cp}\ ImA_1A_2^*}{2|V_{ub}V_{us}^*A_1 + V_{cb}V_{cs}^*A_2|^2 + 4X_{cp}Im(A_1A_2^*)}. \quad (2.3) \end{aligned}$$

Here $V_{ij}V_{k\ell}^*$ are not phase invariant quantities and are parametrization dependent. They become simplified when we use the parametrization of Eq. (1.1), which always put the imaginary part at the smallest place so that quite often the imaginary part can be dropped. For example $V_{cb}V_{cs}^* = s_y(1 - s_x s_y s_z e^{-i\phi}) \approx s_y$ to a very good approximation.

Also the expression $V_{ub}V_{us}^*A_1 + V_{cb}V_{cs}^*A_2$ holds for partial decay rate differences of many b particle decays, like $B_d^0 \to K^+\pi^-$, which we shall denote in general as $b \to s$. After some rearrangement and using Eqs. (1.1),(1.2) Eq. (2.3) becomes,

$$\Delta_{b\to s} \approx \frac{-4(X_{CP}/s_x s_y s_z)Im(B_1B_2^*)}{2|\eta_{bs}^u B_1 + \eta_{bs}^c B_2|^2 - \text{numerator}}, \quad (2.4)$$

where $B_1 \equiv s_x s_z A_1$, $B_2 \equiv s_y A_2$, $\eta_{bs}^u = e^{-i\phi}$, $\eta_{bs}^c = 1 - s_x s_y s_z e^{-i\phi}$. The important point here is that $X_{CP}/s_x s_y s_z \approx s_\phi$, which can be of order one. In cases when calculations give $|B_1| \sim |B_2|$ and $|\eta_{bs}^u B_1 + \eta_{bs}^c B_2|^2 \sim Im(B_1B_2^*)$, $\Delta_{b\to S}$ become fractional. This does happen quite often.

Similarly for another class of b decays, like $B_s^0 \to K^-\pi^+$, which we denote by $b \to d$

$$\Delta_{b\to d} \simeq \frac{-4(X_{CP}/s_x s_y s_z)Im(B_1'B_2'^*)}{2|\eta_{bd}^u B_1' + \eta_{bd}^c B_2'|^2 - \text{numerator}}, \quad (2.5)$$

where $B_1' \equiv s_z A_1'$, $B_2' \equiv -s_x s_y A_2'$, $\eta_{bd}^u = e^{-i\phi}$, $\eta_{bd}^c = 1 + (s_y s_z/s_x)e^{-i\phi}$. Again the same factor $X_{CP}/s_x s_y s_z \approx s_\phi$ appears, and model calculations can give $\Delta_{b\to d}$ close to one.

For the t quark decays we have the following two cases for the partial decay rate differences:

$$\Delta_{t\to c} = \frac{-4(X_{CP}/s_x^2 s_y^2)I_m(T_1T_2^*)}{2|\eta_{tc}^d T_1 + \eta_{tc}^s T_2|^2 - \text{numerator}}, \quad (2.6a)$$

where $T_1 \equiv s_x^2 s_y A_1$, $T_2 \equiv -s_y A_2$, $\eta_{tc}^d = 1 - (s_z/s_x s_y)e^{i\phi}$, $\eta_{tc}^s = 1 + (s_x s_z/s_y)e^{i\phi}$. Again here $X_{CP}/s_x^2 s_y^2 \approx (s_y/s_x s_y)e^{i\phi} \approx s_\phi$, like in the b particle decays. Another case of the t particle decay is

$$\Delta_{t\to u} = \frac{-4(X_{CP}/s_x^2 s_y^2)I_m(T_1'T_2'^*)}{2|\eta_{tu}^d T_1' + \eta_{tc}^s T_2'|^2 - \text{numerator}}, \quad (2.6b)$$

where $T_1' \equiv s_x s_y A_1'$, $T_2 \equiv -s_x s_y A_2'$, $\eta_{tu}^d = 1 - (s_z/s_x s_y)e^{i\phi}$, $\eta_{tu}^s = 1 + (s_x s_z/s_y)e^{i\phi}$. Here $X_{CP}/s_x^2 s_y^2 \approx s_\phi$ again like in the b decays. Here we see that the partial decay rates of the t particles can be appreciable like in the b particle decays.

For charm decays the partial decay rate difference has one form:

$$\Delta_c \approx \frac{-4(X_{CP}/s_x^2)Im(C_1C_2^*)}{2|\eta_{cu}^d C_1 + \eta_{cu}^s C_2|^2 - \text{numerator}}, \quad (2.7)$$

where $C_1 = -s_x A_1$, $C_2 = s_x A_2$, $\eta_{cu}^d = 1 + (s_y s_z/s_x)e^{i\phi}$, $\eta_{cu}^s = 1 - s_x s_y s_z e^{i\phi}$. Here $X_{CP}/s_x^2 \approx 10^{-2}s_\phi$, which is 10^{-2} of that of the b and t particle decays. Except when there is exact cancellation between the real parts of C_1, C_2 the first term in the denominator dominates over the second term, and $\Delta_c << 1$. In model calculations usually $I_m(C_1C_2^*)/|\eta_{cu}^d C_1 + \eta_{cu}^s C_2|^2 \approx 1$, thus $\Delta_c \lesssim 10^{-2}$.

To state it simply and intuitively, the reason for the possibilities of large partial decay rate differences Δ in the b and t particle decays is that Δ must come from mixing matrix suppressed diagrams. In the three generation case the numerator of Δ is proportional to a universal factor X_{cp}, therefore those decays with the smaller denominator in Δ win. The b and t decays have smaller denominators in Δ than charm decays since the mixing-matrix suppressions are much more severe in the b and t particle decays than in the charm particle decays.

Let's now look at the branching ratios. The widths (or lifetimes) of heavy-quark particle decays are usually calculated using the simple W-emission diagram, e.g.

$$\Gamma_B = |V_{cb}|^2 \quad \frac{G_F^2 m_B^5}{192\pi^3}. \quad (2.8)$$

Let's compare this to the decay width of $B \to P_1P_2$, two pseudoscalar final states, via a W-emission diagram

$$\Gamma_B \overset{\mathcal{A}}{\to} P_1P_2 = \frac{2m_B|\vec{p}_1||V_{ij}V_{k\ell}^*(G_F/\sqrt{2})f_{P_1}F^{BP_2}m_B^2|^2}{16\pi m_B^3}$$

$$\approx |V_{ij}V_{kl}^*|^2 f_{P_1}^2 \frac{G_F^2 m_B^3}{32\pi}, \qquad (2.9)$$

where $|\vec{p}_1|2m_B = [m_B^2 - (m_1+m_2)^2]^{\frac{1}{2}} \times [m_B^2 - (m_1 - m_2)^2]^{\frac{1}{2}}$, and f_{P_1} is the P_1 particle decaying constant and usually we take $f_{P_1} \approx f_B \approx f_\pi \approx 130$ MeV is taken, and F^{BP_2} is the form factor between B and P_2, usually we take $F^{BP_2} \approx 1$. Taking the ratio of Eq. (2.8)/Eq (2.9), we obtain the branching ratio.

$$\begin{aligned} B_r(B \to P_1 P_2) &= \frac{\mathcal{A}}{\Gamma_B} \approx 6\pi^2 \frac{|V_{ij}V_{k\ell}^*|^2 f_{P_1}^2}{|V_{cb}|^2 m_B^2}, \\ &\approx 10^{-2}, \text{ for } V_{ij}V_{k\ell}^* = V_{cb}V_{cs}^*; \\ &\approx 10^{-4} \sim 10^{-5}, \text{ for } V_{ij}V_{k\ell}^* = V_{ub}V_{us}^*. \end{aligned} \qquad (2.10)$$

In partial decay rate differences, the W-loop graphs $\mathcal{E}$, $\mathcal{F}$ have coefficients $V_{cb}V_{cs}^*$, but in the current model calculations, the loop graphs are much smaller such that $V_{cb}V_{cs}^*\mathcal{E} \approx V_{ub}V_{us}^* \mathcal{A}$. Therefore the branching ratio is rather small for the B decay. Actually it is the smallness of the decay width helping to make Δ_b large since $\Gamma_{B\to P_1P_2}$ do appear in the denominator of Δ_b. This actually is a nice trade off since the number of events needed is inversely proportional to the square of Δ and inversely proportional to the branching ratio, Br., i.e.

$$N \gtrsim \Delta^{-2} B_r^{-1}, \qquad (2.11)$$

as was discussed in Ref. (18).

Another point we see from Eq. (2.10) is that the two-body-decays branching ratio is inversely proportional to the square of the mass of the decaying particle. Therefore the charm two-body-decays in general have larger branching ratio then beauty two-body decays, and the two body decays of the t quark particles will have very small branching ratios. All these explain why it is most favorable to look for partial decay rate differences in particle decays.

Here I shall list a few nice charged $B^\pm$ decay modes:

$$B_u^+ \to K^\pm \rho^0,\ \overline{K}^{*0}\pi^\pm,\ K^\pm\pi^+\pi^-, \text{ etc.} \qquad (2.12)$$

which have similar quark-diagram structure as $B_d^0 \to K^+\pi^-$. Therefore it is very natural to expect that the decay modes of Eq. (2.12) have the similar partial decay rate differences of 10's%, and branching ratios of 10^{-4} to 10^{-5}. The following decay modes should also be good interesting,

$$B_u^+ \to K^+\phi,\ K^+K^+K^-;\ K^+w,\ K^+\pi^+\pi^-\pi^0, \text{ etc.} \qquad (2.13)$$

though our current calculations are not as reliable for these channels as the ones given in Eq. (2.12).

III. Beyond the Three Generations of Quarks

When there are more than three generations of quarks, we can generalize the sequential rotations as given in Eq. (1.1), e.g. for four generations of quarks, the quark mixing matrix can be written as follows:

$$V_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & c_u & s_u \\ 0 & 0 & -s_u & c_u \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_v & 0 & s_v e^{-i\phi_3} \\ 0 & 0 & 1 & 0 \\ 0 & -s_v e^{i\phi_3} & 0 & c_v \end{bmatrix} \times$$

$$\begin{bmatrix} c_w & 0 & 0 & s_w e^{-i\phi_2} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s_w e^{i\phi_2} & 0 & 0 & c_w \end{bmatrix} \begin{bmatrix} & V_3 & & 0 \\ & & & 0 \\ & & & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad (3.1)$$

where V_3 is the 3x3 matrix given in Eq. (1.1). The reason we put the additional phase ϕ_2, ϕ_3 where there are in Eq. (3.1) is to anticipate that the widening of the generation gap will keep its pace. Of course physical consequences are independent of ways of parameterization. The striking new phenomena are the proliferation of the number of $X_{CP's}$ they are

$$(X_{CP})_{ij}^{\alpha\beta} \equiv Im T_{ij}^{\alpha\beta} = (V_{\alpha j}^* V_{\alpha i} V_{\beta i}^* V_{\beta j}), \qquad (3.2)$$

where α, β, i, j run from 1 to (N-1) with $\alpha < \beta, i < j$. So there are $[(N-1)(N-2)/2]^2$ number of $(X_{CP})_{ij}^{\alpha\beta}$, however totally there are only $(N-1)^2$ real parameters. Thus the X_{CP}'s saturate the whole parameter space of the quark mixing matrix! We see that the three-generation case is a rather singular case. Therefore the estimates of partial decay rates can be very different in the presence of higher than three generations of quarks. One important effect is that the partial decay rate differences in charm particle decays can be large too. Of special interest to note is that some of these large effects involving only tree graphs which are independent of the masses of the 4th or higher generations of quarks, and are dependent solely on the existence of higher generations of quarks. Partial decay rate difference between

$$F^+ \to \overline{K}^0\pi^+,\ F^- \to K^0\pi^- \qquad (3.3)$$

is such an example.

With this scenario of the possible existence of higher than three generations of quarks it is very worthwhile to be on the lookout for CP noninvariant decays in experiments where charm particles are or will be copiously produced.

Concluding Remarks:

For the urgently needed measurements of $|V_{ub}|$, $|V_{ub}/V_{cb}|$, the Quark Diagram Scheme has provided a useful model-independent framework to select out nonleptonic decays, and decay ratios, as given in Eqs. (1.5) and (1.8), which are directly proportional to $|V_{ub}|$ and $|V_{ub}/V_{cb}|$ respectively. Dedicated experimental measurements of these decays are very important for our understanding of the quark mixing phenomena and whether there are more than three generations of quarks. Charmless decays are useful for search of $|V_{ub}|$ only if the W-loop-diagram contributions with dominant mixing-matrix coefficients are negligible. To directly measure the size of the W-loop-diagram contribution, it is useful to study the decays which are purely from the W-loop diagrams as given in Eq. (1.6b).

In our search for CP noninvariance, the Quark Diagram Scheme has been very useful, providing guidance on what decay channels in heavy quark decays can possibly have nonzero partial decay rate differences. Using the scheme we see easily the qualitative reason why the beauty particle decays provide favorable reactions for the CP noninvariance search. Here a few more nice channels for experimental studies are pointed out, Eqs. (2.12), (2.13).

In the presence of higher then three generations of quarks, the number of CP nonvariance parameters increases from a unique X_{CP} to saturate the whole parameter space: e.g. nine X_{CP}'s for four generations of quarks, etc. This gives the possibility that the CP noinvariance of partial decay rate differences can also be appreciable in charm particle decays. It is therefore very worthwhile to be on the lookout for CP noninvariance decays in experiments where charm particles are or will be copiously produced.

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