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## The DELVE Quadruple Quasar Search I. A Lensed Low Luminosity AGN

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## ABSTRACT

A quadruply lensed source, J125856.3–031944, has been discovered using the DELVE survey and WISE  $W1 - W2$  colors. Followup direct imaging carried out with the MPIA 2.2 m and the Baade 6.5 m telescopes is analyzed, as is spectroscopy from the 2.5 m Nordic Optical Telescope. The lensed image configuration is kite-like, with the faintest image 2 magnitudes fainter than the other three. Redward of  $6000\text{\AA}$  that image is badly blended with the lensing galaxy, which is elongated along the symmetry axis of the kite. Magellan direct imaging carried out in Sloan  $g$  permits better deblending. As the lensed image configuration is nearly circular, simple models give individual magnifications of at least 25 for the 3 brighter images. The source’s narrow emission lines and low intrinsic luminosity qualify it as a type 2 AGN.

*Keywords:* Strong gravitational lensing (1643), Seyfert Galaxies (1447), Quasars (1319)

## 1. INTRODUCTION

Quadruply lensed quasar systems are routinely used to study the lensing galaxies, the quasars themselves, and the geometry of the universe through which their light propagates. But they are rare, both because quasars are themselves rare and because they have only an  $\mathcal{O}(10^{-4})$  chance of being quadruply lensed (Oguri & Marshall 2010). Worse yet, several observational considerations make them difficult to identify, the most important of which is their small angular extent on the sky,  $\sim 1''$ .

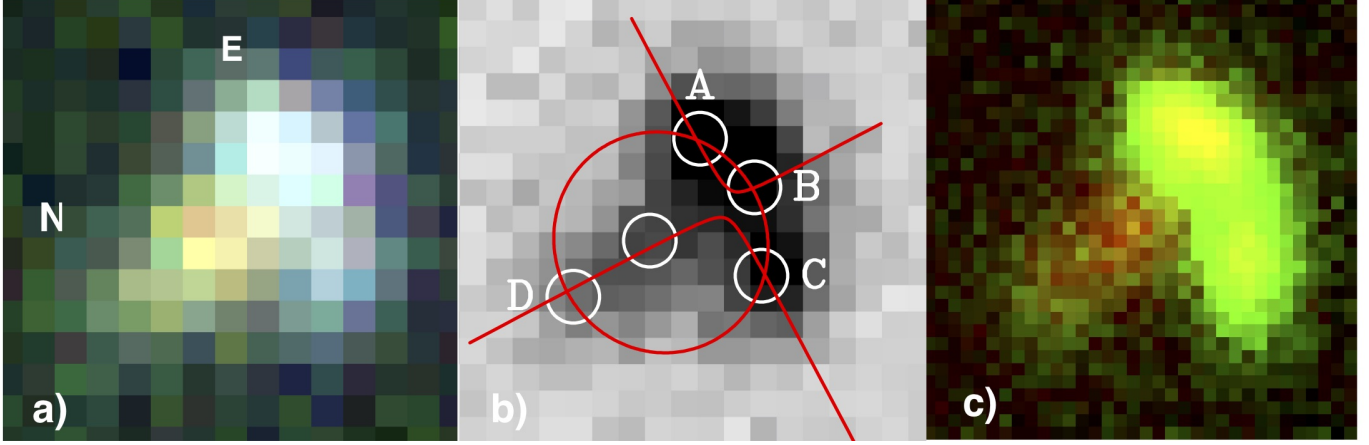
Here we report the discovery of the first confirmed quadruply lensed AGN found in the DELVE Quadruply Lensed Quasar (henceforth DELQQ) survey, carried out in the DELVE footprint. (Drlica-Wagner et al. 2021) using DECam at the 4m Victor M. Blanco telescope. J125856.3–031944 (henceforth DELQQ 1258–0319) has a small Einstein radius,  $\approx 0''.98$ , and the faintest quasar image is both close to and blended with the light from a non-negligible lensing galaxy. These circumstances presented challenges in identifying and subsequently confirming this system as a quadruply lensed AGN. As deeper surveys are carried out, quadruple quasars with increasingly faint images relative to the galaxies that lens them will become more prevalent and their identification will grow still more difficult.

## 2. DISCOVERY

2.1. *The DELQQ survey: selection*

Our survey, which we call DELQQ, covered the 6000 square degrees of the first DELVE data release (Drlica-Wagner et al. 2021). Candidate quasars with color  $W1 - W2 > 0.7$  and  $W1 < 15.5$  as measured by the WISE (Wright et al. 2010) satellite were chosen following the color criterion proposed by Stern et al. (2012), for a total of 150,000 sources. Square FITS cutouts, 45 pixels on a side, were then generated for those candidates from the DELVE survey  $g, r, i$  and  $z$  images.

A still evolving computer program, `trifurcator`, first described in Schechter et al. (2018), was used to split sources into three components, which were classified as pointlike,  $\mathcal{P}$ , galaxy-like,  $\mathcal{G}$ , or ambiguous  $\mathcal{Q}$ . Systems for which none of the components were pointlike were eliminated.



**Figure 1.** Square cutouts,  $3''.7$  on side, of exposures of DELQQ 1258–0319 taken with a) DECam, b) WFI and c) IMACS. DES  $r$ ,  $i$ , and  $z$  were used to construct the color composite in a). Sloan  $g$  and  $i$  were used to construct c). The white circles are plotted on the WFI  $R_c$  exposure b), at the positions of the four quasar images and of the lens as derived from the IMACS  $g$  exposure. The red conic sections in b) represent a singular isothermal elliptical model for the lens potential, which locates the quasar image positions where the ellipse and hyperbola cross. The lensing galaxy is predicted to lie on the more nearly centered branch of the hyperbola. The ellipse is aligned with the asymptotes of the hyperbola, and has axis ratio  $q = 0.95$  elongated  $63^\circ$  E of N, perpendicular to the elongation of the potential. DECam, WFI and IMACS have scales of  $0''.26$ ,  $0''.21$  and  $0''.11$  per pixel, respectively.

Cutouts in all four filters were visually inspected for systems in which two components (one of which was pointlike) had near-identical  $g - r$ ,  $r - i$  and  $i - z$  colors except for those whose colors were galaxy-like. Roughly 1500 systems were visually inspected.

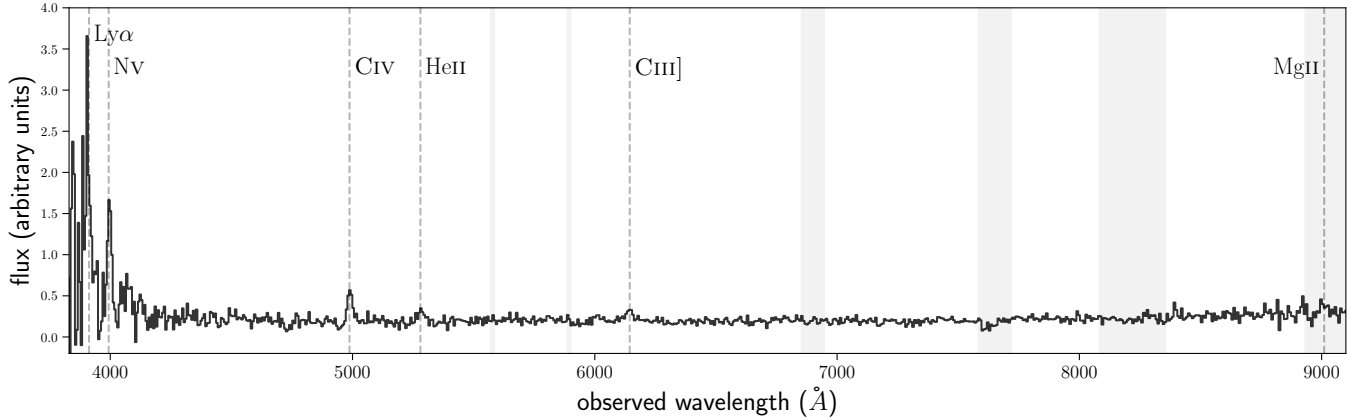
Five quadruply lensed quasars with  $W1 - W2 > 0.7$  had previously been identified in our DELQQ sample (J1310–1714 and J1537–3010 were too blue), of which `trifurcator` recovered three (J1131–4419, J1134–2103 and J1131–1231). The largest image separations for the two systems not recovered were  $0''.66$  (B1113–0641) and  $10''$  (J1651–0417) – either too close to be resolved by DECam or too widely separated. One known quad (J1606–2333) lacked an  $r$  exposure. We are unaware of any quadruply lensed systems discovered within the DELVE footprint in the time since our analysis.

## 2.2. A candidate

Only one candidate emerged as a possible quadruply lensed quasar system, DELQQ J1258–0319. A color image generated from the DELVE  $r$ ,  $i$ , and  $z$  images is shown in Figure 1a.

`Trifurcator` split the  $r$  image into three components: a pointlike  $\mathcal{P}$  component to the SW,  $B$  in Figure 1b, and two galaxy-like  $\mathcal{G}$  components, one centrally located (a blend of image  $D$  and the lensing galaxy), and one toward the East (a blend of images  $A$  and  $B$ ). The colors of pointlike image  $C$  agree with the galaxy-like blend of  $A$  and  $B$  with an rms of 0.14 magnitudes, after allowing for a modest slope due to micro-lensing, extinction and PSF mismatch. They were flagged as possible components of a quadruply lensed quasar. The redder galaxy-like component would then be the lensing galaxy.

The program assigned a DELVE  $g$  magnitude of 22.68 to image  $C$  and  $g - r$ ,  $r - i$  and  $i - z$  colors of 0.373, 0.279, and 0.291 respectively. The latter are useful for establishing achromaticity but less so for deriving quasar properties, as the exposures spanned five observing seasons.



**Figure 2.** NOT spectrum of the brightest quasar component *A*, extending from 3800 to 9000 Å. The high ionization lines of N V, C IV and He II seen at  $z = 2.225$  are narrow, with widths ranging from 1300-2000 km/s, suggesting a type 2 AGN rather than a quasar.

While it was possible to split the central component into two point sources, one of which would be a fourth quasar image, we could not persuade ourselves that the data justified this. A plausible alternative interpretation would have been a “naked cusp” configuration like that of J0457–7820 (Lemon et al. 2022).

Using a program called `clumpfit`, assembled from pieces of DoPHOT (Schechter et al. 1993), it was possible to fit five point sources simultaneously. Though the decomposition of the two  $\mathcal{G}$  images into point sources gave positions that were in qualitative agreement with expectations for a quadruply lensed source, alternate choices of possible PSFs produced large deviations in the positions.

### 2.3. Prior detections

Prior to its detection in DELVE, DELQQ J1258–0319 had been catalogued as a source both in the Pan-STARRS catalogue (Chambers et al. 2016) and the VLASS (Lacy et al. 2020). Its non-detection in Gaia DR2 indicates that it is substantially fainter than the large majority of known quadruply lensed quasars.

## 3. FOLLOWUP MPIA-WFI IMAGING

Four Cousins  $R_c$  followup exposures of 320 s each were obtained with the Wide Field Imager (WFI) of the MPIA 2.2 m telescope on La Silla in seeing of roughly  $0''.64$ . A co-addition of these is shown in Figure 1b. Notwithstanding the good seeing, there is no clear separation of the lensing galaxy and the faintest of the quasar images. Attempts by two of the authors to determine the flattening of the lensing galaxy using differing techniques yielded substantially different results.

## 4. FOLLOWUP NOT SPECTROSCOPY

A 40 minute spectrum was obtained with the Alhambra Faint Object Spectrograph and Camera (ALFOSC) on the Nordic Optical Telescope (NOT) on 2021 April 16 and is shown in Figure 2. The slit was oriented so as to pass through the brightest of the four images and the lensing galaxy. Narrow emission lines are clearly visible, corresponding to a redshift  $z = 2.225$ .

A sharp discontinuity in the galaxy spectrum is visible at  $6000\text{\AA}$ , suggestive of the *H&K* break in early type galaxies. In section 5 below we use astrometry obtained with Magellan’s Walter Baade Telescope to obtain colors for the lensing galaxy consistent with this interpretation.

## 5. FOLLOWUP MAGELLAN IMAGING

Three 300s exposures in each of the Sloan  $g$  and  $i$  filters were obtained with the IMACS f/4 camera (Dressler et al. 2011) on the Baade 6.5 m telescope of the Magellan Observatory on 2022 May 24. A color composite constructed from the coadditions of the two filters is shown in Figure 1c. The lensing galaxy is an orange red and the quasar images are green. No attempt has been made to convolve to the same FWHM, making images  $A$ ,  $B$ , and  $C$ , slightly yellow at their centers. The faintest of the four quasar images is appreciably displaced (to the lower left) from the lensing galaxy.

`Clumpfit` was used to fit five point sources simultaneously to the four quasar images and the center of the lensing galaxy in the coadded  $g$  exposure. An analytic pseudo-Gaussian, as described in DoPHOT (Schechter et al. 1993), was used as the common PSF, as attempts to use stars  $\sim 2'$  from the system as templates produced worse residuals.

The image positions from the Magellan  $g$  image were then fixed relative to the most isolated of the images,  $C$ , and were forced onto the Magellan  $i$  exposure, allowing the shape of the lensing galaxy to vary. These proved unsatisfactory. We suspect that imperfect PSF subtraction in the wings of the brighter quasar images distorts the lensing galaxy's shape.

The Magellan images were taken in non-photometric conditions, so we forced the Magellan  $g$  positions (relative to  $C$ ) onto the original DELVE images to give photometry for the lensing galaxy. `Clumpfit` was used to fit the tabulated DELVE point source PSF to the four quasar images and to the galaxy image. The program assigned DELVE colors to the lensing galaxy of  $g - r = 1.373$ ,  $r - i = 0.639$ ,  $i - z = 0.299$ .

Given that the  $g$  magnitude for the galaxy had a formal uncertainty of 0.27 mag, we set aside the DELVE  $g - r$  color and instead adopt the Magellan  $g - i$  color on the DELVE system. The apparent magnitudes are anchored by the `trifurcator` magnitude for the point-like  $C$  component,  $g = 22.68$ .

The Magellan  $g$  positions (again relative to  $C$ ) were likewise forced to the WFI  $R_c$  positions to permit registration of the two images. The Magellan  $g$  positions required shifts of  $0''.11$  and  $0''.03$  to the East and North, respectively. Table 1 reports these shifted Magellan  $g$  positions relative to the lensing galaxy as measured on the WFI image. These are shown as white circles in Figure 1b.

We obtained a photometric redshift using `Bayesian Photometric Redshift` (hereafter BPZ) described by Benítez (2000) and Coe et al. (2006). Prior distributions were derived from Hubble Deep Field North galaxies. We ran BPZ with the point-source fit apparent magnitudes given in Table 1. We took the photometric uncertainties to be  $\pm 0.27$  mag in DELVE  $g$  and  $\pm 0.06$  in the other filters. Further runs were done with apparent magnitudes 0.5 brighter and uncertainties two and four times as large. The best-fit redshift was found to be 0.75. The redshift posterior probability density functions had a HWHM of 0.10 when the errors were taken to be 4 times the uncertainty.

## 6. MODELS

Keeton's `lensmodel` program (Keeton 2010) was used to model the positions of the quasar images and the center of the galaxy as a singular isothermal elliptical potential (SIEP)

$$\psi(b, \eta, \gamma) = b \sqrt{q_{pot} x^2 + \frac{y^2}{q_{pot}}} \quad (1)$$



**Table 1.** DELQQ 1258–0319: Astrometric, Photometric and Modeled Quantities

image	$\Delta$ R.A.	$\Delta$ Dec.	$g$	$g - i^*$	$F_g$	$\mu$	$\kappa^\dagger$	$\Delta t$
	"	"			nJy			days
A	1.03	-0.42	22.30	0.46	1202	25.7	0.4804	0.2
B	0.59	-0.92	22.80	0.81	759	-39.9	0.5128	0.6
C	-0.21	-0.97	22.68	0.67	847	21.8	0.4770	0.0
D	-0.40	0.73	23.89	0.51	278	-5.4	0.5930	20.1
G	0.11	0.03	24.71	2.52				

\*Colors assume a pointlike lens. Extended flux from the much redder lens contaminates the colors for the quasar images.  $^\dagger$ For an SIEP, shear  $\gamma = \kappa$ .

where  $\psi$  is the dimensionless projected potential and  $q_{pot}$  is its axis ratio. All of the positions were given the same uncertainty.

The quasar image positions for the model are located where the red ellipse in Figure 1b crosses the red hyperbola (Wynne & Schechter 2018). The agreement is somewhat better for the three brighter quasar images, as might be expected from their smaller measurement errors. The center of the lensing galaxy is predicted to lie on one of the branches of the hyperbola. The observed offset may result either from the larger measurement error in the galaxy’s position or the shortcomings of the model.

The ellipse in Figure 1b is quite round, with an ellipticity  $e = 0.05$ . The elliptical potential has the same axis ratio, but is oriented perpendicular to that ellipse. Rounder systems have larger magnifications, and the model predicts brightenings of  $\sim 3.5$  mag (a factor of 25) for each of the three brightest images. This would make DELQQ 1258–0319 one of the most highly magnified systems known. The quasar host would be similarly magnified. Taking  $z_L \sim 0.75$  gives a longest time delay (relative to the leading image) of 20 days for a  $\Lambda$ CDM cosmology with  $H_0 = 70$  km/sec/Mpc, and two delays of a day or less.

Predicted magnifications  $\mu$  for the individual quasar images are given in Table 1.

## 7. QUASAR, LLAGN OR SEYFERT?

Based on the narrow emission lines seen in Figure 2, Khachikian & Weedman (1971) and Khachikian & Weedman (1974) – the first of these in Russian and the second in English – would have classified the source in DELQQ 1258–0319 as a Seyfert II galaxy rather than a Seyfert I or a quasar. Subsequent work indicated that physics of the “active galactic nuclei” (AGNs) of such systems is somewhat decoupled from the properties of the galaxies that host them. In a review of “unified models” Antonucci (1993) presents a cartoon model in which type 2 AGN have narrow emission lines with FWHM  $\mathcal{O}(1000)$  kms/s, and type 1 AGN have broad emission lines with FWHM  $\mathcal{O}(10000)$  kms/s. Such systems are classified as Seyferts only if the host galaxy is observed, which may eventually happen if the system is observed at sufficiently high spatial resolution and surface brightness sensitivity.

The source in the DELQQ 1258–0319 system is not very red, as would have been the case if dust were obscuring both its broad line region and continuum emission from its accretion disk. The high magnifications derived from our model,  $\sim 25$  (3.5 magnitudes) for each of the three bright quasar

images and the photometry from the original DELVE images point to a source absolute magnitudes  $M_{AB}$ , fainter than those of the low luminosity quasars in the SHELLQs survey (Matsuoka et al. 2016). It would therefore seem that the first quadruply lensed AGN in the DELVE Quadruple Quasar survey does not qualify as quasar. As we do not, as yet, see evidence of the host galaxy, the source is classified a low luminosity AGN, an “LLAGN,” and not a Seyfert.

## 8. CONCLUSIONS

With the benefit of hindsight, none of the challenges described in the preceding sections is entirely surprising. Many of the brighter quadruply imaged quasars with wider separations have already been discovered, particularly since the publication of Gaia DR2 (Delchambre et al. 2019; Lemon et al. 2022). The  $W2 - W1 > 0.7$  criterion will not be satisfied by systems for which the lensing galaxy contributes a substantial fraction of the infrared light. Lensed image configurations often include a faint image relatively close to the lensing galaxy (a saddlepoint of the light travel time), making deblending more difficult. Seyfert galaxies are more common than quasars (Willott et al. 2000), and may account for an increasingly large fraction of quadruply lensed sources in future surveys that push to fainter apparent magnitudes.

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