

Old neutron stars as a new probe of relic neutrinos and sterile neutrino dark matter

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We study the kinetic cooling (heating) of old neutron stars due to coherent scattering with relic neutrinos (sterile neutrino dark matter) via Standard Model neutral-current interactions. We take into account several important physical effects, such as gravitational clustering, coherent enhancement, neutron degeneracy and Pauli blocking. We find that the anomalous cooling of nearby neutron stars due to relic neutrino scattering might actually be observable by current and future telescopes operating in the optical to near-infrared frequency band, such as the James Webb Space Telescope (JWST), provided there is a large local relic overdensity that is still allowed. Similarly, the anomalous heating of neutron stars due to coherent scattering with keV-scale sterile neutrino dark matter, could also be observed by JWST or future telescopes, which would probe hitherto unexplored parameter space in the sterile neutrino mass-mixing plane.

INTRODUCTION

Cosmic Neutrino Background ($C\nu B$) is a robust prediction of the standard cosmological model of the Universe [1]. However, the existence of $C\nu B$ has only been indirectly inferred from big bang nucleosynthesis (BBN) [2], cosmic microwave background (CMB) [3], and large-scale structure (LSS) [4] data; see Ref. [5] for a recent review. In spite of having the largest flux among all natural and artificial sources of neutrinos [6], direct detection of $C\nu B$ remains elusive [7–9]. This is mainly due to its small kinetic energy today which results in extremely small cross-section and energy/momentum transfer in a laboratory setup [10].

One way to potentially overcome this difficulty is by using coherent scattering [11] where the neutrino may collectively interact with all particles within a sphere of radius equal to the de Broglie wavelength corresponding to the momentum transfer. Owing to their small momenta, relic neutrinos can receive a huge enhancement in their scattering cross-section in a dense astrophysical environment like a neutron star (NS), where they can coherently interact with a gigantic number of neutrons even in a small volume. Since $C\nu B$ behaves as a cold, non-relativistic fluid, its interaction with neutrons in a hot NS medium will result in an anomalous cooling of the NS; see Fig. 1. This was studied long ago in Ref. [12], which found a negligible kinetic cooling rate. In this paper, we improve the old calculation by taking into account gravitational clustering, coherent scattering, neutron degeneracy and Pauli blocking effects. Although we get orders of magnitude improvement over the results in Ref. [12], we still find the kinetic cooling rate to be subdominant compared to the standard cooling rate via black-body emission of photons, which makes the kinetic cooling effect unobservable, unless there is a large local $C\nu B$ overdensity.

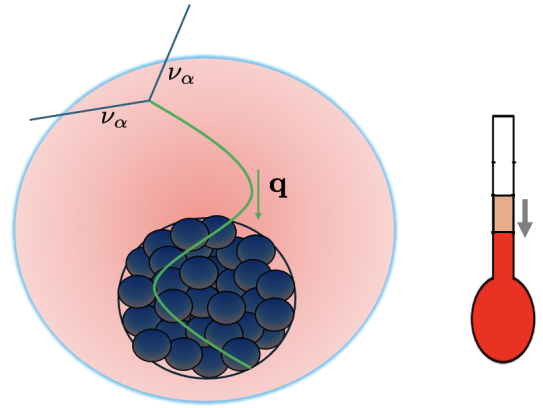


FIG. 1. An artist's rendition of the NS kinetic cooling via $C\nu B$. Gravitationally captured relic neutrinos coherently scatter off neutrons via the neutral-current interaction within a sphere of radius $\lambda_\nu = 2\pi/|\mathbf{q}|$, allowing energy transfer from NS to $C\nu B$, thus anomalously cooling down the NS.

We next consider the possibility of anomalous NS heating due to its interaction with heavier neutrino species with mass much larger than the NS temperature. Due to cosmological [3, 4] and laboratory [13] upper bounds on the absolute neutrino mass scale, the active neutrino mass eigenstates must be at sub-eV scale, and therefore, cannot contribute to NS heating. However, there could be additional sterile neutrino species which are much heavier. In fact, keV-scale sterile neutrinos having tiny mixing with active neutrinos can be an excellent dark matter (DM) candidate [14]. Assuming this to be the case, we find that sterile neutrino accumulation inside NS can lead to the anomalous heating of NSs (see Fig. 2), which is potentially detectable for a local cold old NS by optical and near-infrared telescopes such as the currently operating James Webb Space

Telescope (JWST) [15], near-future European Extremely Large Telescope (ELT) [16], and far-future Thirty Meter Telescope (TMT) [17]. Using the projected sensitivities of these instruments in terms of NS temperatures, we derive new constraints in the sterile neutrino mass-mixing plane, which are comparable to the most stringent astrophysical constraints from X -ray line searches.

NEUTRON STAR TEMPERATURE

NSs are born hot in supernova explosions, with internal temperature $T_{\text{int}} \sim 10^{11}$ K. When relatively young ($t \lesssim 10^5$ yr), they dominantly lose energy via neutrino emission, while at later stages ($t \gtrsim 10^5$ yr) they mainly cool via thermal surface emission of photons [18]. According to the minimal cooling paradigm [19–21], old ($t > 10^9$ yr) isolated NSs exhaust all their thermal and rotational energy, cooling down to temperatures below $\mathcal{O}(100)$ K and making them impossible to observe.

In fact, most of the isolated NSs detected so far are young radio pulsars with $t \lesssim 10^6$ yr and surface temperature observed at infinity T_s^∞ between 10^5 and 10^6 K [22]. Older NSs have only been detected indirectly as companions in binary systems, such as millisecond pulsars [23–25]. The coldest known NS on record, PSR J2144-3933, located at 172 pc from Earth, has its effective surface temperature bounded at $T_s^\infty < 42,000$ K by the Hubble Space Telescope [26]. Its spin-down age is $\sim 3 \times 10^8$ yr, which implies that its temperature would be at most a few hundred K in the minimal cooling models [19–21]. Based on Monte-Carlo orbital simulations of the spatial distribution of galactic NSs, we expect 1-2 (100-200) such cold, old, isolated NSs lurking within 10 (50) pc [27–29]. On the observational front, the currently operating JWST [15] and future ELT [16] and TMT [17] are capable of detecting black-body peak temperatures of 1300-4300 K with their optical to near-infrared imaging instruments. Therefore, it is timely to consider anomalous heating/cooling of old local NSs as a new probe of relic and sterile neutrinos.

COHERENT SCATTERING

The $C\nu B$ temperature today is very small: $T_\nu = (4/11)^{1/3} T_\gamma \simeq 1.9$ K $\simeq 1.7 \times 10^{-4}$ eV, where $T_\gamma \simeq 2.7$ K is the CMB temperature [3]. Assuming that the neutrino rest mass $m_\nu > T_\nu$ which we know is true for at least two of the three active neutrino mass eigenstates in order to fit the observed mass-squared differences [30], we can treat the $C\nu B$ as a cold, non-relativistic fluid. For a relic neutrino scattering against a neutron in old NSs, since the momentum transfer $|\mathbf{q}| \simeq T_{\text{NS}} \lesssim 0.1$ eV (where T_{NS} is the NS surface temperature) is very small, the corresponding de Broglie wavelength $\lambda_\nu = 2\pi/|\mathbf{q}| \simeq$

10^{-3} cm (0.1 eV/ $|\mathbf{q}|$) is much larger than the size of the nucleon, and hence, the neutrino will scatter off the entire neutron, with a scattering cross-section (summed over the spins) given by

$$\sigma_{\nu n} = \frac{G_F^2}{4\pi} E_\nu^2 \simeq (4 \times 10^{-65} \text{ cm}^2) \left(\frac{E_\nu}{10^{-4} \text{ eV}} \right)^2, \quad (1)$$

where G_F is the Fermi constant and E_ν is the neutrino energy. This is an extremely small cross-section, but coherent scattering can give a significant enhancement. The cross-section for coherent elastic neutrino-nucleus scattering (CE ν NS), up to leading order in E_ν/m_N (where m_N is the nucleus mass), and averaging over the spin states, is given by (see e.g., Ref. [31])

$$\sigma_{\nu N} = \frac{G_F^2}{8\pi} (Q_V^2 + 3Q_A^2) E_\nu^2, \quad (2)$$

where $Q_V = A - 2Z(1 - 2\sin^2\theta_w) \simeq A - Z$ and $Q_A = A - 2Z$ are respectively the vector and axial charges of the nucleus, given in terms of its mass and atomic numbers A and Z respectively, and θ_w is the weak mixing angle. Note that the CE ν NS process has already been observed by the COHERENT experiment with CsI [32], Ar [33] and Ge [34] detectors. For an NS with mostly neutrons, we can set $A = 1$ and $Z = 0$. However, since the number density of neutrons inside NS is huge, the coherent cross-section will be enhanced by a factor of

$$N_C = \frac{4}{3}\pi\lambda_\nu^3 n_n \simeq 3 \times 10^{30} \left(\frac{0.1 \text{ eV}}{|\mathbf{q}|} \right)^3 \left(\frac{n_n}{4 \times 10^{38} \text{ cm}^{-3}} \right), \quad (3)$$

which is the number of neutrons within a sphere of radius equal to the de Broglie wavelength $\lambda_\nu = 2\pi/|\mathbf{q}| \simeq 10^{-3}$ cm (0.1 eV/ $|\mathbf{q}|$). Note that although λ_ν is much smaller than the typical NS radius $R_{\text{NS}} \simeq 10$ km, due to the huge neutron number density, N_C turns out to be a large number. A more precise calculation of N_C is given in the Appendix.

NS COOLING BY $C\nu B$

To calculate the energy transfer in coherent neutrino-neutron scattering inside an NS, we model the NS as a homogeneous sphere of radius $R_{\text{NS}} = 10$ km and mass $M_{\text{NS}} = 1.5M_\odot$, where $M_\odot = 2 \times 10^{33}$ g is the solar mass. Since very small momentum transfers are involved, we can neglect the form factor dependence. We also assume a homogeneous energy density distribution with $\rho_n \simeq 7 \times 10^{14}$ g/cm³. Neutrons form a highly degenerate Fermi sea inside the NS with a Fermi momentum of $p_{f,n} \simeq 450$ MeV. Note that although there is some uncertainty in these numbers depending on the exact equation of state used [35], for concreteness, we have just used one

benchmark equation of state for cold non-accreting NS with Brussels–Montreal functionals [36].

A neutron star moving in the sea of relic neutrinos gravitationally captures them within an impact parameter [37]:

$$b_{\max} = \left(\frac{2GM_{\text{NS}}R_{\text{NS}}}{v_{\text{rel}}^2} \right)^{1/2} \left(1 - \frac{2GM_{\text{NS}}}{R_{\text{NS}}} \right)^{-1/2}. \quad (4)$$

where G is Newton’s gravitational constant and v_{rel} is the relative velocity of the NS traveling in the comoving frame of the Universe. We set $v_{\text{rel}} = 200$ km/s by following Ref. [38] which studies the velocity distribution of old NSs by performing the numerical simulation. Eq. (4) can be translated into the number of relic neutrinos captured by a NS per unit time as

$$\dot{N} = \pi b_{\max}^2 v_{\text{rel}} n_{\nu}, \quad (5)$$

where $n_{\nu} \simeq 336 \text{ cm}^{-3}$ is the total number density of relic neutrinos today, including 3 flavors and both neutrinos and antineutrinos. Gravitational clustering could enhance the local $C\nu\text{B}$ number density around the NS, but only by a factor of $\mathcal{O}(1)$ [39–42]. A gravitationally captured relic neutrino gains energy while rolling down the gravitational potential of the NS. The energy of the relic neutrino on the NS surface is $E_{\nu, \text{surface}} = m_{\nu}(1 - 2GM_{\text{NS}}/R_{\text{NS}})^{-1/2} + K_{\nu}^{\infty}$ where $K_{\nu}^{\infty} \simeq 3T_{\nu}$ is the kinetic energy of the relic neutrino at infinite distance from the NS. In the case of light relic neutrinos (with $m_{\nu} \lesssim 0.1$ eV) where their kinetic energy at the NS surface, $K_{\nu} = E_{\nu, \text{surface}} - m_{\nu} \simeq 0.3m_{\nu} \ll T_{\text{NS}}$, energy is transferred from the NS to relic neutrinos in the coherent scattering process, i.e., the NS undergoes anomalous cooling. Although the amount of energy transferred in each scattering is small, $|\Delta E| \simeq T_{\text{NS}} \simeq 0.1$ eV, each neutrino can take away a maximum energy $\simeq 3T_{\text{NS}}$ after a large number of coherent scatterings with the neutrons. Note that since the neutrons are highly degenerate inside the NS, energy transfer to neutrinos is Pauli-blocked, and the maximum energy transfer will be of order $3T_{\text{NS}}$, much smaller than the typical Fermi energy $E_{f,n}$ of the neutrons.

Thus, the energy loss rate of a NS due to $C\nu\text{B}$ coherent scattering is given by

$$L_{C\nu\text{B}} = \dot{N} \times 3T_{\text{NS}} \min \left(1, \frac{\langle \sigma \rangle}{\sigma_{\text{th}}} \right), \quad (6)$$

where $\langle \sigma \rangle$ is the thermally averaged neutrino-NS scattering cross-section that includes the phase-space integration, coherent enhancement factor N_C , and Pauli-suppression factor T_{NS}/p_f ; see Appendix for details. Here σ_{th} is the threshold cross-section (see e.g., Ref. [43])

$$\sigma_{\text{th}} \simeq \frac{\pi R_{\text{NS}}^2 m_n}{M_{\text{NS}}} \left(\frac{3T_{\text{NS}} - K_{\nu}}{\langle \Delta E \rangle} \right)$$

$$\simeq 1.8 \times 10^{-45} \text{ cm}^2 \times \left(\frac{R_{\text{NS}}}{10 \text{ km}} \right)^2 \left(\frac{1.5M_{\odot}}{M_{\text{NS}}} \right) \left(\frac{3T_{\text{NS}} - K_{\nu}}{\langle \Delta E \rangle} \right), \quad (7)$$

which corresponds to the cross-section that is enough for neutrinos to obtain kinetic energy $\simeq 3T_{\text{NS}}$ after a series of collisions, with the average energy transfer given by $\langle \Delta E \rangle$; see Appendix for details. Note that the energy loss rate is independent of the neutrino mass, as long as its kinetic energy is much smaller than the NS temperature.

To see whether the anomalous cooling rate given by Eq. (6) is observable or not, we have to compare it with the standard photon cooling rate (for a fixed NS radius):

$$L_{\gamma} = 4\pi R_{\text{NS}}^2 \sigma_{\text{SB}} T_{\text{NS}}^4, \quad (8)$$

where σ_{SB} is the Stefan-Boltzmann constant. We thus find that the ratio of the kinetic cooling rate from $C\nu\text{B}$ to the black-body cooling rate from photon emission is

$$\frac{L_{C\nu\text{B}}}{L_{\gamma}} = \frac{3GM_{\text{NS}}n_{\nu}}{2\sigma_{\text{SB}}R_{\text{NS}}T_{\text{NS}}^3v_{\text{rel}}} \left(1 - \frac{2GM_{\text{NS}}}{R_{\text{NS}}} \right)^{-1} \min \left(1, \frac{\sigma}{\sigma_{\text{th}}} \right) \simeq 3.7 \times 10^{-6} \left(\frac{1000 \text{ K}}{T_{\text{NS}}} \right)^3. \quad (9)$$

Thus, the energy loss induced by $C\nu\text{B}$ scattering is subdominant compared to that due to photon emission.

The anomalous cooling effect can however be observed if there exists a local $C\nu\text{B}$ overdensity $\gtrsim 10^6$. Such large overdensity is still allowed by KATRIN [44], but has to be reconciled with other observational constraints and projections [31, 45–50]. Theoretically, it is possible to achieve such large overdensities, e.g. due to the formation of neutrino bound states in the presence of nonstandard neutrino interactions [51]. In any case, this will result in a higher $C\nu\text{B}$ temperature $T_{\nu} \simeq \eta_{\nu}^{1/3} T_{\nu,0}$, but as long as it remains much smaller than the NS temperature, the above cooling argument applies.

NS HEATING BY STERILE NEUTRINOS

For coherent scattering of heavier particle species with the neutrons in the NS, the situation is very different. In particular, if the energy of the incoming particle is larger than T_{NS} , it will deposit almost all of its energy in the NS and get captured, thus heating the NS. However, given the stringent upper bound on the active neutrino mass $m_{\nu} \lesssim 0.1$ eV (which corresponds to ~ 1000 K) from cosmological [3, 4] as well as laboratory data [13], it is unlikely for $C\nu\text{B}$ to heat up the NS to an observable extent. We should clarify here that even if the cosmological upper limit on the sum of neutrino masses could in principle be evaded in the present Universe by invoking some non-standard cosmology or time-varying neutrino masses [52–56], the KATRIN bound [13] is unavoidable.

It is true that KATRIN only constrains the effective electron neutrino mass [13], while the corresponding bounds on muon and tau neutrino masses [57] are much weaker:

$$m_{\nu_e}^{\text{eff}} \equiv \sqrt{\sum_i |U_{ei}|^2 m_i^2} < 0.45 \text{ eV}, \quad (10)$$

$$m_{\nu_\mu}^{\text{eff}} \equiv \sqrt{\sum_i |U_{\mu i}|^2 m_i^2} < 0.19 \text{ MeV}, \quad (11)$$

$$m_{\nu_\tau}^{\text{eff}} \equiv \sqrt{\sum_i |U_{\tau i}|^2 m_i^2} < 18.2 \text{ MeV}, \quad (12)$$

where U stands for the PMNS mixing matrix and m_i are the three light neutrino mass eigenvalues. However, after taking into account the observed neutrino oscillation data [30], there is no way one could make any of the mass eigenvalues m_i larger than the KATRIN upper limit given in Eq. (10), thus rendering the weaker upper limits in Eqs. (11) and (12) practically irrelevant.

On the other hand, the anomalous heating of NS could be induced by some heavier beyond the Standard Model (BSM) particles, such as DM. The dark kinetic heating of NS has been extensively studied before for \gtrsim GeV-scale DM [35, 43, 58–67]; see Ref. [68] for a review. Here we examine for the first time the NS kinetic heating effect for a keV-scale sterile neutrino DM; see Fig. 2. Sterile neutrinos are well-motivated BSM particles, originally introduced to explain the non-zero neutrino mass by the seesaw mechanism [69–72]. In the minimal scenario, the sterile neutrinos can only interact with the SM sector via their mixing with the active neutrinos. Depending on the smallness of this mixing angle θ , sterile neutrinos can be cosmologically stable and can serve as a warm/cold DM [14, 73]. We assume this to be 100% of the DM, with the energy density given by the local DM density in the galactic halo, $\rho_{\nu_s} \simeq 0.3 \text{ GeV/cm}^3$ [74]. The DM density can be much larger closer to the galactic center; however, since the kinetic heating effect we are interested in can be detectable only for nearby NSs (within a few pc from Earth), the larger DM density at the galactic center does not help us.

The sterile neutrinos can undergo coherent scattering off neutrons in the NS in the same way as the active neutrinos, but now with cross-sections suppressed by $\sin^2 \theta$. For keV-scale sterile neutrinos, their kinetic energy is larger than T_{NS} , and therefore, in each collision, they will lose a significant fraction of their energy to the NS, and the remaining energy will be transferred to the converted active neutrino. We checked that the mean free path of the produced active neutrino is short enough to transfer almost all of the kinetic energy to the NS, provided the sterile neutrino mass is below MeV-scale. Therefore, we can calculate the total energy gain rate of the NS just as a product of the number of captured sterile neutrinos in unit time and the energy transferred to a NS by

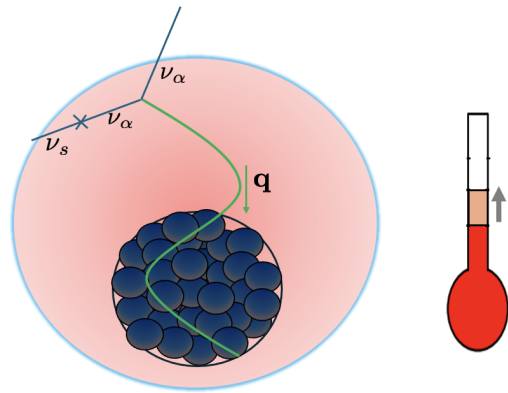


FIG. 2. An artist's rendition of the NS kinetic heating via sterile neutrino DM.

each sterile neutrino. Discussion of energy gain without this approximation can be found, e.g., in Ref. [62]. We find the energy gain of a neutron star per unit time is as follows:

$$L_{\nu_s} = \dot{N}(E_{\nu_s, \text{surface}} - 3T_{\text{NS}}) \min\left(1, \frac{\langle \sigma \rangle}{\sigma_{\text{th}}}\right), \quad (13)$$

where \dot{N} is given by Eq. (4) with n_ν replaced by $n_{\nu_s} = \rho_{\nu_s}/m_{\nu_s}$, $\langle \sigma \rangle$ is the thermally averaged scattering cross-section given in the Appendix scaled by $\sin^2 \theta$, and $E_{\nu_s, \text{surface}} = m_{\nu_s} (1 - 2GM_{\text{NS}}/R_{\text{NS}})^{-1/2}$ is the energy of the sterile neutrino on the NS surface, assuming that its kinetic energy at infinity is negligible. The threshold cross-section in this case will be slightly modified from Eq. (7):

$$\sigma_{\text{th}} = \frac{\pi R_{\text{NS}}^2 m_n}{M_{\text{NS}}} \max\left(\frac{E_{\nu_s, \text{surface}} - 3T_{\text{NS}}}{\langle \Delta E \rangle}, 1\right). \quad (14)$$

The active neutrinos being produced from the sterile neutrino interactions (see Fig. 2) will deposit most their kinetic energy to the NS, but as long as m_{ν_i} is smaller than $0.1T_{\text{NS}}$, they can still escape the gravitational potential of the NS and do not form a Fermi sphere inside.

Including the effect of gravitational redshift, the effective NS surface temperature induced by the kinetic heating is given by

$$4\pi R_{\text{NS}}^2 \sigma_{\text{SB}} T_{\text{heating}}^4 = L_{\nu_s} \left(1 - \frac{2GM_{\text{NS}}}{R_{\text{NS}}}\right).$$

This gives the modified surface temperature observed at infinity: $T_\infty = (T_{\text{NS}}^4 + T_{\text{heating}}^4)^{1/4}$. In Fig. 3, we show different contours of T_∞ in the sterile neutrino mass-mixing plane. The benchmark of 1750 K is motivated by the fact that the corresponding black-body spectrum from an NS located 10 pc from Earth yields a spectral flux density of $f_\lambda \simeq 0.5 \text{ Jy}$ at wavelength $\lambda \sim 2 \mu\text{m}$, which should be detectable by JWST [15]. The 1000 K benchmark is

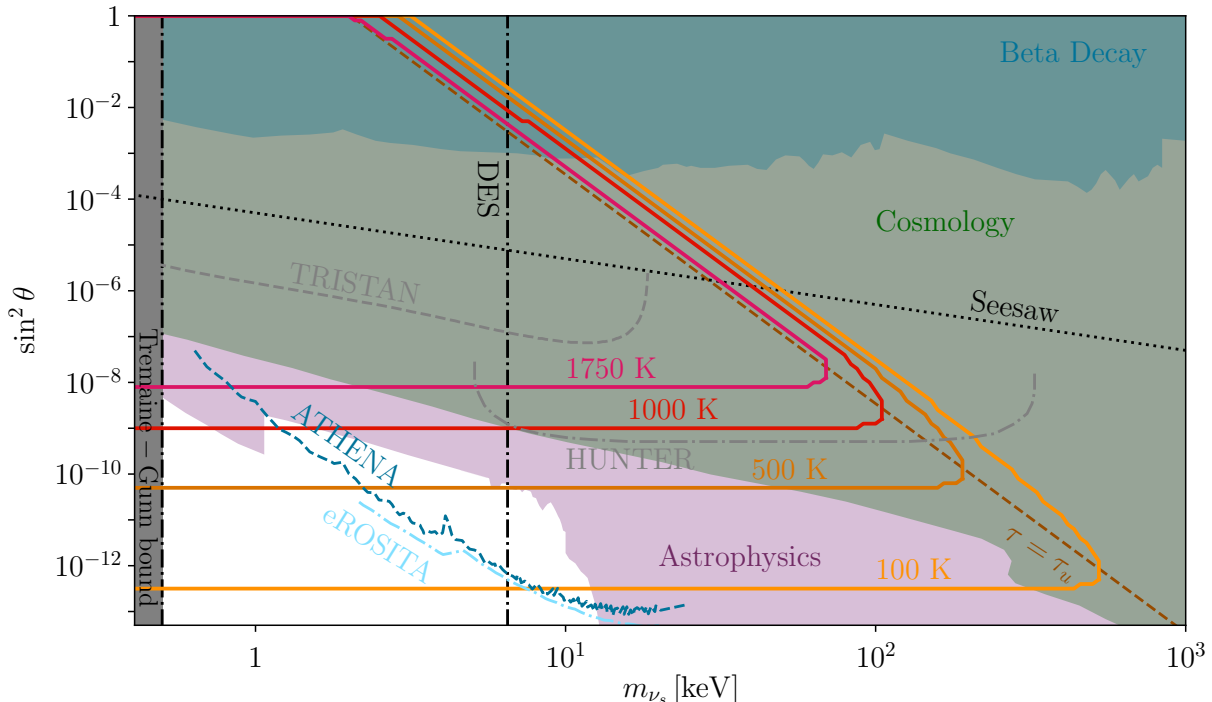


FIG. 3. Projected limits (solid curves) on sterile neutrino mass and mixing from NS kinetic heating observations at different temperatures. The shaded regions are excluded, while the dashed lines show future sensitivities of complementary probes. See text for details.

at the very edge of the detection threshold for JWST, but within reach of future telescopes like ELT and TMT. The 500 K benchmark may lie at the detection threshold for future telescopes, while the 100 K benchmark is very difficult to achieve and is shown here for illustration purpose only.

The dominant decay channel of sterile neutrinos with mass smaller than twice of electron mass is $\nu_s \rightarrow \nu_\alpha \nu_\beta \bar{\nu}_\beta$ whose decay width is given by [75, 76]

$$\Gamma_{\nu_s \rightarrow 3\nu} = \frac{G_F^2 m_{\nu_s}^5}{96\pi^2} \sin^2 \theta \simeq \frac{1}{\tau_U} \left(\frac{m_{\nu_s}}{50 \text{ keV}} \right)^5 \left(\frac{\sin^2 \theta}{1.1 \times 10^{-7}} \right), \quad (15)$$

where $\tau_U = 13.8$ Gyr [3] is the age of the Universe. This is shown in Fig. 3 by the diagonal brown dashed line labeled ‘ $\tau = \tau_U$ ’. The NS heating curves run close to the $\tau = \tau_U$ line on the upper part of Fig. 3, because to the right of this line, the sterile neutrino decays depletes its abundance rapidly and there are not enough of them to interact with the NS. In the lower part of the Figure, the sensitivities are independent of mass, because we have fixed the DM energy density ρ_{ν_s} , which implies that the number density n_{ν_s} goes inversely as m_{ν_s} . The total energy transfer is proportional to $n_{\nu_s} m_{\nu_s}$, which is therefore independent of m_{ν_s} .

There is another radiative decay mode $\nu_s \rightarrow \nu_\alpha \gamma$, which is suppressed by a factor of $27\alpha/8\pi$ with respect to

the 3ν decay mode [75]; however, since this is a 2-body decay with a single photon in the final state having energy at half the sterile neutrino mass, monochromatic X -ray [77, 78]/ γ -ray [79] line searches put a stringent constraint using this channel, as shown by the purple-shaded region labeled ‘Astrophysics’. Future X -ray missions like ATHENA [80] and eROSITA [81] will further improve these bounds, as shown by the dashed blue curves. The ‘Astrophysics’ constraint at lower masses also includes the supernova cooling [82]. Similarly, cosmological observations including ΔN_{eff} , BAO and H_0 exclude the green-shaded region labeled ‘Cosmology’ [83, 84]. There also exist constraints from Lyman- α [85, 86] and Milky Way subhalos [87, 88] and satellite galaxies [89], all of which rule out sterile neutrino DM below a few keV. We show here the latest DES bound of 6.5 keV [89], but although stronger than the theoretical Tremaine-Gunn bound obtained from phase-space considerations [90], it depends on the sterile neutrino DM production mechanism [91, 92], and can in principle be modified in presence of additional interactions [93, 94]; therefore, we do not shade the region labeled ‘DES’. For the same reason, we do not show the DM over/underproduction bounds, which very much depend on the specific production mechanism considered [94]. Finally, for sterile neutrino mixing with electron flavor, there exist constraints from beta-decay spectrum in different nuclei [95, 96], as shown by the cyan-shaded region labeled ‘Beta Decay’.

Future beta-decay experiments like TRISTAN [97] and HUNTER [98] can improve these bounds significantly, as shown by the corresponding dashed curves. From Fig. 3, it is clear that NS kinetic heating observations could put new meaningful constraints on the sterile neutrino DM scenario, which would be complementary to the existing astrophysical, cosmological, and laboratory probes.

DISCUSSION

The future detectability of the kinetic cooling (heating) effect studied here relies on the crucial assumption that there is no other active cooling/heating mechanism in play so that we can identify the anomalous cooling (heating) as solely due to the coherent scattering of relic neutrinos (sterile neutrino dark matter). In practice, NSs cool passively since their birth may as well be affected by some other late-time reheating mechanism, either of astrophysical origin [99], such as magnetic field decay, rotochemical heating, vortex creep heating, and crust-cracking, or of BSM physics origin [68], such as accretion of heavier DM and baryon-number violating (di)nucleon decays. These additional heating mechanisms could obfuscate our results; however, dedicated future observational campaigns to measure thermal luminosities of old NSs might resolve some of these ambiguities [100].

In addition, our results could be affected by theoretical uncertainty in the composition of and dynamics in the NS core. Many theoretical models expect the emergence of exotic phases of matter such as hyperon [101] and color superconducting phases [102]. Strong interaction in the matter with a density above the saturation density is abstruse. Moreover, an improved calculation of the coherence effect using dynamic structure functions is desirable, since we are dealing with an interacting quantum many-body system [103]. We leave these directions for future work.

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- [1] S. Weinberg, “Universal Neutrino Degeneracy,” *Phys. Rev.* **128** (1962) 1457–1473.
- [2] C. Pitrou, A. Coc, J.-P. Uzan, and E. Vangioni, “Precision big bang nucleosynthesis with improved Helium-4 predictions,” *Phys. Rept.* **754** (2018) 1–66, [1801.08023].
- [3] **Planck** Collaboration, N. Aghanim *et al.*, “Planck 2018 results. VI. Cosmological parameters,” *Astron. Astrophys.* **641** (2020) A6, [1807.06209]. [Erratum: *Astron. Astrophys.* 652, C4 (2021)].
- [4] **DESI** Collaboration, A. G. Adame *et al.*, “DESI 2024 VI: Cosmological Constraints from the Measurements of Baryon Acoustic Oscillations,” [2404.03002].
- [5] E. Di Valentino, S. Gariazzo, and O. Mena, “Neutrinos in Cosmology,” [2404.19322].
- [6] E. Vitagliano, I. Tamborra, and G. Raffelt, “Grand Unified Neutrino Spectrum at Earth: Sources and Spectral Components,” *Rev. Mod. Phys.* **92** (2020) 45006, [1910.11878].
- [7] G. B. Gelmini, “Prospect for relic neutrino searches,” *Phys. Scripta T* **121** (2005) 131–136, [hep-ph/0412305].
- [8] C. Yanagisawa, “Looking for Cosmic Neutrino Background,” *Frontiers in Physics* **2** (2014) 30.
- [9] P. Vogel, “How difficult it would be to detect cosmic neutrino background?,” *AIP Conf. Proc.* **1666** no. 1, (2015) 140003.
- [10] **PTOLEMY** Collaboration, E. Baracchini *et al.*, “PTOLEMY: A Proposal for Thermal Relic Detection of Massive Neutrinos and Directional Detection of MeV Dark Matter,” [1808.01892].
- [11] D. Z. Freedman, “Coherent Neutrino Nucleus Scattering as a Probe of the Weak Neutral Current,” *Phys. Rev. D* **9** (1974) 1389–1392.
- [12] V. V. Dixit and J. Lodenquai, “ON NEUTRON STAR COOLING BY RELIC NEUTRINOS,” *Lett. Nuovo Cim.* **38** (1983) 174.
- [13] **Katrin** Collaboration, M. Aker *et al.*, “Direct neutrino-mass measurement based on 259 days of KATRIN data,” [2406.13516].
- [14] M. Drewes *et al.*, “A White Paper on keV Sterile Neutrino Dark Matter,” *JCAP* **01** (2017) 025, [1602.04816].
- [15] J. P. Gardner *et al.*, “The James Webb Space Telescope,” *Space Sci. Rev.* **123** (2006) 485, [astro-ph/0606175].
- [16] B. Neichel, D. Mouillet, E. Gendron, C. Correia, J. F. Sauvage, and T. Fusco, “Overview of the European Extremely Large Telescope and its instrument suite,” in *SF2A-2018: Proceedings of the Annual meeting of the French Society of Astronomy and Astrophysics*, P. Di Matteo, F. Billebaud, F. Herpin, N. Lagarde, J. B. Marquette, A. Robin, and O. Venot, eds., p. Di.

- Dec., 2018. [[1812.06639](#)].
- [17] **TMT International Science Development Teams & TMT Science Advisory Committee** Collaboration, W. Skidmore *et al.*, “Thirty Meter Telescope Detailed Science Case: 2015,” *Res. Astron. Astrophys.* **15** no. 12, (2015) 1945–2140, [[1505.01195](#)].
- [18] D. G. Yakovlev, O. Y. Gnedin, M. E. Gusakov, A. D. Kaminker, K. P. Levenfish, and A. Y. Potekhin, “Neutron star cooling,” *Nucl. Phys. A* **752** (2005) 590–599, [[astro-ph/0409751](#)].
- [19] D. Page, J. M. Lattimer, M. Prakash, and A. W. Steiner, “Minimal cooling of neutron stars: A New paradigm,” *Astrophys. J. Suppl.* **155** (2004) 623–650, [[astro-ph/0403657](#)].
- [20] D. Page, U. Geppert, and F. Weber, “The Cooling of compact stars,” *Nucl. Phys. A* **777** (2006) 497–530, [[astro-ph/0508056](#)].
- [21] D. D. Ofengeim and D. G. Yakovlev, “Analytic description of neutron star cooling,” *Mon. Not. Roy. Astron. Soc.* **467** no. 3, (2017) 3598–3603.
- [22] D. G. Yakovlev, W. C. G. Ho, P. S. Shternin, C. O. Heinke, and A. Y. Potekhin, “Cooling rates of neutron stars and the young neutron star in the Cassiopeia A supernova remnant,” *Mon. Not. Roy. Astron. Soc.* **411** (2011) 1977–1988, [[1010.1154](#)].
- [23] A. Wolszczan, “Confirmation of Earth-Mass Planets Orbiting the Millisecond Pulsar PSR B1257+12,” *Science* **264** no. 5158, (Apr., 1994) 538–542.
- [24] D. R. Lorimer, “Binary and Millisecond Pulsars,” *Living Rev. Rel.* **11** (2008) 8, [[0811.0762](#)].
- [25] J. M. Lattimer and M. Prakash, “The Equation of State of Hot, Dense Matter and Neutron Stars,” *Phys. Rept.* **621** (2016) 127–164, [[1512.07820](#)].
- [26] S. Guillot, G. G. Pavlov, C. Reyes, A. Reisenegger, L. Rodriguez, B. Rangelov, and O. Kargaltsev, “Hubble Space Telescope Nondetection of PSR J2144–3933: The Coldest Known Neutron Star,” *Astrophys. J.* **874** no. 2, (2019) 175, [[1901.07998](#)].
- [27] E. O. Ofek, “Space and velocity distributions of Galactic isolated old Neutron stars,” *Publ. Astron. Soc. Pac.* **121** (2009) 814, [[0910.3684](#)].
- [28] N. Sartore, E. Ripamonti, A. Treves, and R. Turolla, “Galactic neutron stars I. Space and velocity distributions in the disk and in the halo,” *Astron. Astrophys.* **510** (2010) A23, [[0908.3182](#)].
- [29] A. Taani, “Systematic comparison of initial velocities for neutron stars in different models,” *Research in Astronomy and Astrophysics* **16** no. 7, (July, 2016) 101. <http://www.nu-fit.org>.
- [30] <http://www.nu-fit.org>.
- [31] M. Bauer and J. D. Shergold, “Limits on the cosmic neutrino background,” *JCAP* **01** (2023) 003, [[2207.12413](#)].
- [32] **COHERENT** Collaboration, D. Akimov *et al.*, “Observation of Coherent Elastic Neutrino-Nucleus Scattering,” *Science* **357** no. 6356, (2017) 1123–1126, [[1708.01294](#)].
- [33] **COHERENT** Collaboration, D. Akimov *et al.*, “First Measurement of Coherent Elastic Neutrino-Nucleus Scattering on Argon,” *Phys. Rev. Lett.* **126** no. 1, (2021) 012002, [[2003.10630](#)].
- [34] S. Adamski *et al.*, “First detection of coherent elastic neutrino-nucleus scattering on germanium,” [[2406.13806](#)].
- [35] N. F. Bell, G. Busoni, and S. Robles, “Capture of Leptophilic Dark Matter in Neutron Stars,” *JCAP* **06** (2019) 054, [[1904.09803](#)].
- [36] J. M. Pearson, N. Chamel, A. Y. Potekhin, A. F. Fantina, C. Ducoin, A. K. Dutta, and S. Gorieli, “Unified equations of state for cold non-accreting neutron stars with Brussels–Montreal functionals – I. Role of symmetry energy,” *Mon. Not. Roy. Astron. Soc.* **481** no. 3, (2018) 2994–3026, [[1903.04981](#)]. [Erratum: *Mon. Not. Roy. Astron. Soc.* 486, 768 (2019)].
- [37] I. Goldman and S. Nussinov, “Weakly Interacting Massive Particles and Neutron Stars,” *Phys. Rev. D* **40** (1989) 3221–3230.
- [38] E. O. Ofek, “Space and Velocity Distributions of Galactic Isolated Old Neutron Stars,” *Publ. of the ASP* **121** no. 882, (Aug., 2009) 814, [[0910.3684](#)].
- [39] A. Ringwald and Y. Y. Y. Wong, “Gravitational clustering of relic neutrinos and implications for their detection,” *JCAP* **12** (2004) 005, [[hep-ph/0408241](#)].
- [40] P. Mertsch, G. Parimbelli, P. F. de Salas, S. Gariazzo, J. Lesgourgues, and S. Pastor, “Neutrino clustering in the Milky Way and beyond,” *JCAP* **01** (2020) 015, [[1910.13388](#)].
- [41] F. Zimmer, C. A. Correa, and S. Ando, “Influence of local structure on relic neutrino abundances and anisotropies,” *JCAP* **11** (2023) 038, [[2306.16444](#)].
- [42] E. B. Holm, S. Zentarra, and I. M. Oldengott, “Local clustering of relic neutrinos: comparison of kinetic field theory and the Vlasov equation,” *JCAP* **07** (2024) 050, [[2404.11295](#)].
- [43] M. Baryakhtar, J. Bramante, S. W. Li, T. Linden, and N. Raj, “Dark Kinetic Heating of Neutron Stars and An Infrared Window On WIMPs, SIMPs, and Pure Higgsinos,” *Phys. Rev. Lett.* **119** no. 13, (2017) 131801, [[1704.01577](#)].
- [44] **KATRIN** Collaboration, M. Aker *et al.*, “New Constraint on the Local Relic Neutrino Background Overdensity with the First KATRIN Data Runs,” *Phys. Rev. Lett.* **129** no. 1, (2022) 011806, [[2202.04587](#)].
- [45] V. Brdar, P. S. B. Dev, R. Plestid, and A. Soni, “A new probe of relic neutrino clustering using cosmogenic neutrinos,” *Phys. Lett. B* **833** (2022) 137358, [[2207.02860](#)].
- [46] Y.-D. Tsai, J. Eby, J. Arakawa, D. Farnocchia, and M. S. Safronova, “OSIRIS-REx constraints on local dark matter and cosmic neutrino profiles,” *JCAP* **02** (2024) 029, [[2210.03749](#)].
- [47] M. Císcar-Monsalvatje, G. Herrera, and I. M. Shoemaker, “Upper Limits on the Cosmic Neutrino Background from Cosmic Rays,” [[2402.00985](#)].
- [48] J. Franklin, I. Martinez-Soler, Y. F. Perez-Gonzalez, and J. Turner, “Constraints on the Cosmic Neutrino Background from NGC 1068,” [[2404.02202](#)].
- [49] A. G. De Marchi, A. Granelli, J. Nava, and F. Sala, “Relic Neutrino Background from Cosmic-Ray Reservoirs,” [[2405.04568](#)].
- [50] G. Herrera, S. Horiuchi, and X. Qi, “Diffuse Boosted Cosmic Neutrino Background,” [[2405.14946](#)].
- [51] A. Y. Smirnov and X.-J. Xu, “Neutrino bound states and bound systems,” *JHEP* **08** (2022) 170, [[2201.00939](#)].
- [52] R. Fardon, A. E. Nelson, and N. Weiner, “Dark energy from mass varying neutrinos,” *JCAP* **10** (2004) 005,

- [astro-ph/0309800].
- [53] G. Krnjaic, P. A. N. Machado, and L. Necib, “Distorted neutrino oscillations from time varying cosmic fields,” *Phys. Rev. D* **97** no. 7, (2018) 075017, [1705.06740].
- [54] C. S. Lorenz, L. Funcke, E. Calabrese, and S. Hannestad, “Time-varying neutrino mass from a supercooled phase transition: current cosmological constraints and impact on the Ω_m - σ_8 plane,” *Phys. Rev. D* **99** no. 2, (2019) 023501, [1811.01991].
- [55] G.-y. Huang, M. Lindner, P. Martínez-Miravé, and M. Sen, “Cosmology-friendly time-varying neutrino masses via the sterile neutrino portal,” *Phys. Rev. D* **106** no. 3, (2022) 033004, [2205.08431].
- [56] F. Goertz, M. Hager, G. Laverda, and J. Rubio, “Phasing out of Darkness: From Sterile Neutrino Dark Matter to Neutrino Masses via Time-Dependent Mixing,” [2407.04778].
- [57] **Particle Data Group** Collaboration, R. L. Workman *et al.*, “Review of Particle Physics,” *PTEP* **2022** (2022) 083C01.
- [58] C. Kouvaris, “WIMP Annihilation and Cooling of Neutron Stars,” *Phys. Rev. D* **77** (2008) 023006, [0708.2362].
- [59] C. Kouvaris and P. Tinyakov, “Can Neutron stars constrain Dark Matter?,” *Phys. Rev. D* **82** (2010) 063531, [1004.0586].
- [60] A. de Lavallaz and M. Fairbairn, “Neutron Stars as Dark Matter Probes,” *Phys. Rev. D* **81** (2010) 123521, [1004.0629].
- [61] J. Bramante, A. Delgado, and A. Martin, “Multiscatter stellar capture of dark matter,” *Phys. Rev. D* **96** no. 6, (2017) 063002, [1703.04043].
- [62] N. F. Bell, G. Busoni, and S. Robles, “Heating up Neutron Stars with Inelastic Dark Matter,” *JCAP* **09** (2018) 018, [1807.02840].
- [63] N. F. Bell, G. Busoni, S. Robles, and M. Virgato, “Improved Treatment of Dark Matter Capture in Neutron Stars,” *JCAP* **09** (2020) 028, [2004.14888].
- [64] N. F. Bell, G. Busoni, S. Robles, and M. Virgato, “Improved Treatment of Dark Matter Capture in Neutron Stars II: Leptonic Targets,” *JCAP* **03** (2021) 086, [2010.13257].
- [65] S. Chatterjee, R. Garani, R. K. Jain, B. Kanodia, M. S. N. Kumar, and S. K. Vempati, “Faint light of old neutron stars and detectability at the James Webb Space Telescope,” *Phys. Rev. D* **108** no. 2, (2023) L021301, [2205.05048].
- [66] A. Ávila, E. Giangrandi, V. Sagun, O. Ivanytskyi, and C. Providência, “Rapid neutron star cooling triggered by dark matter,” *Mon. Not. Roy. Astron. Soc.* **528** no. 4, (2024) 6319–6328, [2309.03894].
- [67] N. F. Bell, G. Busoni, S. Robles, and M. Virgato, “Thermalization and Annihilation of Dark Matter in Neutron Stars,” [2312.11892].
- [68] J. Bramante and N. Raj, “Dark matter in compact stars,” *Phys. Rept.* **1052** (2024) 1–48, [2307.14435].
- [69] P. Minkowski, “ $\mu \rightarrow e\gamma$ at a Rate of One Out of 10^9 Muon Decays?,” *Phys. Lett. B* **67** (1977) 421–428.
- [70] R. N. Mohapatra and G. Senjanovic, “Neutrino Mass and Spontaneous Parity Nonconservation,” *Phys. Rev. Lett.* **44** (1980) 912.
- [71] T. Yanagida, “Horizontal gauge symmetry and masses of neutrinos,” *Conf. Proc. C* **7902131** (1979) 95–99.
- [72] M. Gell-Mann, P. Ramond, and R. Slansky, “Complex Spinors and Unified Theories,” *Conf. Proc. C* **790927** (1979) 315–321, [1306.4669].
- [73] T. Asaka, S. Blanchet, and M. Shaposhnikov, “The nuMSM, dark matter and neutrino masses,” *Phys. Lett. B* **631** (2005) 151–156, [hep-ph/0503065].
- [74] T. Callingham, M. Cautun, A. J. Deason, C. S. Frenk, W. Wang, F. A. Gómez, R. J. J. Grand, F. Marinacci, and R. Pakmor, “The mass of the Milky Way from satellite dynamics,” [1808.10456].
- [75] P. B. Pal and L. Wolfenstein, “Radiative Decays of Massive Neutrinos,” *Phys. Rev. D* **25** (1982) 766.
- [76] V. D. Barger, R. J. N. Phillips, and S. Sarkar, “Remarks on the KARMEN anomaly,” *Phys. Lett. B* **352** (1995) 365–371, [hep-ph/9503295]. [Erratum: Phys.Lett.B 356, 617–617 (1995)].
- [77] K. C. Y. Ng, B. M. Roach, K. Perez, J. F. Beacom, S. Horiuchi, R. Krivonos, and D. R. Wik, “New Constraints on Sterile Neutrino Dark Matter from NuSTAR M31 Observations,” *Phys. Rev. D* **99** (2019) 083005, [1901.01262].
- [78] B. M. Roach, K. C. Ng, K. Perez, J. F. Beacom, S. Horiuchi, R. Krivonos, and D. R. Wik, “NuSTAR Tests of Sterile-Neutrino Dark Matter: New Galactic Bulge Observations and Combined Impact,” *Phys. Rev. D* **101** no. 10, (2020) 103011, [1908.09037].
- [79] R. Laha, J. B. Muñoz, and T. R. Slatyer, “INTEGRAL constraints on primordial black holes and particle dark matter,” *Phys. Rev. D* **101** no. 12, (2020) 123514, [2004.00627].
- [80] A. Neronov and D. Malyshev, “Toward a full test of the ν MSM sterile neutrino dark matter model with Athena,” *Phys. Rev. D* **93** no. 6, (2016) 063518, [1509.02758].
- [81] A. Dekker, E. Peerbooms, F. Zimmer, K. C. Y. Ng, and S. Ando, “Searches for sterile neutrinos and axionlike particles from the Galactic halo with eROSITA,” *Phys. Rev. D* **104** no. 2, (2021) 023021, [2103.13241].
- [82] X. Shi and G. Sigl, “A Type II supernovae constraint on electron-neutrino - sterile-neutrino mixing,” *Phys. Lett. B* **323** (1994) 360–366, [hep-ph/9312247]. [Erratum: Phys. Lett.B324,516(1994)].
- [83] A. C. Vincent, E. F. Martinez, P. Hernandez, M. Lattanzi, and O. Mena, “Revisiting cosmological bounds on sterile neutrinos,” *JCAP* **1504** no. 04, (2015) 006, [1408.1956].
- [84] S. Bridle, J. Elvin-Poole, J. Evans, S. Fernandez, P. Guzowski, and S. Soldner-Rembold, “A Combined View of Sterile-Neutrino Constraints from CMB and Neutrino Oscillation Measurements,” *Phys. Lett. B* **764** (2017) 322–327, [1607.00032].
- [85] A. Garzilli, A. Magalich, O. Ruchayskiy, and A. Boyarsky, “How to constrain warm dark matter with the Lyman- α forest,” *Mon. Not. Roy. Astron. Soc.* **502** no. 2, (2021) 2356–2363, [1912.09397].
- [86] V. Iršič *et al.*, “Unveiling dark matter free streaming at the smallest scales with the high redshift Lyman-alpha forest,” *Phys. Rev. D* **109** no. 4, (2024) 043511, [2309.04533].

- [87] S. Horiuchi, P. J. Humphrey, J. Onorbe, K. N. Abazajian, M. Kaplinghat, and S. Garrison-Kimmel, “Sterile neutrino dark matter bounds from galaxies of the Local Group,” *Phys. Rev. D* **89** no. 2, (2014) 025017, [1311.0282].
- [88] A. Dekker, S. Ando, C. A. Correa, and K. C. Y. Ng, “Warm dark matter constraints using Milky Way satellite observations and subhalo evolution modeling,” *Phys. Rev. D* **106** no. 12, (2022) 123026, [2111.13137].
- [89] DES Collaboration, E. O. Nadler *et al.*, “Milky Way Satellite Census. III. Constraints on Dark Matter Properties from Observations of Milky Way Satellite Galaxies,” *Phys. Rev. Lett.* **126** (2021) 091101, [2008.00022].
- [90] S. Tremaine and J. E. Gunn, “Dynamical Role of Light Neutral Leptons in Cosmology,” *Phys. Rev. Lett.* **42** (1979) 407–410.
- [91] S. Dodelson and L. M. Widrow, “Sterile-neutrinos as dark matter,” *Phys. Rev. Lett.* **72** (1994) 17–20, [hep-ph/9303287].
- [92] X.-D. Shi and G. M. Fuller, “A New dark matter candidate: Nonthermal sterile neutrinos,” *Phys. Rev. Lett.* **82** (1999) 2832–2835, [astro-ph/9810076].
- [93] R. An, V. Gluscevic, E. O. Nadler, and Y. Zhang, “Can Neutrino Self-interactions Save Sterile Neutrino Dark Matter?,” *Astrophys. J. Lett.* **954** no. 1, (2023) L18, [2301.08299].
- [94] M. D. Astros and S. Vogl, “Boosting the production of sterile neutrino dark matter with self-interactions,” *JHEP* **03** (2024) 032, [2307.15565].
- [95] A. Atre, T. Han, S. Pascoli, and B. Zhang, “The Search for Heavy Majorana Neutrinos,” *JHEP* **0905** (2009) 030, [0901.3589].
- [96] P. D. Bolton, F. F. Deppisch, and P. S. Bhupal Dev, “Neutrinoless double beta decay versus other probes of heavy sterile neutrinos,” *JHEP* **03** (2020) 170, [1912.03058].
- [97] KATRIN Collaboration, S. Mertens *et al.*, “A novel detector system for KATRIN to search for keV-scale sterile neutrinos,” *J. Phys. G* **46** no. 6, (2019) 065203, [1810.06711].
- [98] C. J. Martoff *et al.*, “HUNTER: precision massive-neutrino search based on a laser cooled atomic source,” *Quantum Sci. Technol.* **6** no. 2, (2021) 024008.
- [99] D. Gonzalez and A. Reisenegger, “Internal heating of old neutron stars: contrasting different mechanisms,” *Astron. Astrophys.* **522** (Nov., 2010) A16, [1005.5699].
- [100] N. Raj, P. Shivanna, and G. N. Rachh, “Reheated Sub-40000 Kelvin Neutron Stars at the JWST, ELT, and TMT,” [2403.07496].
- [101] V. A. Ambartsumyan and G. S. Saakyan, “The Degenerate Superdense Gas of Elementary Particles,” *Sov. Astron.* **4** (Oct., 1960) 187.
- [102] M. Alford and S. Reddy, “Compact stars with color superconducting quark matter,” *Phys. Rev. D* **67** (2003) 074024, [nucl-th/0211046].
- [103] L. Van Hove, “Correlations in space and time and born approximation scattering in systems of interacting particles,” *Phys. Rev.* **95** (Jul, 1954) 249–262.
- [104] G. Chauhan, “Neutron stars as probe of cosmic neutrino background,” to appear.
- [105] J. A. Formaggio and G. P. Zeller, “From eV to EeV: Neutrino Cross Sections Across Energy Scales,” *Rev. Mod. Phys.* **84** (2012) 1307–1341, [1305.7513].
- [106] V. Domcke and M. Spinrath, “Detection prospects for the Cosmic Neutrino Background using laser interferometers,” *JCAP* **06** (2017) 055, [1703.08629].
- [107] V. Shtabovenko, R. Mertig, and F. Orellana, “FeynCalc 9.3: New features and improvements,” *Comput. Phys. Commun.* **256** (2020) 107478, [2001.04407].
- [108] S. Reddy, M. Prakash, and J. M. Lattimer, “Neutrino interactions in hot and dense matter,” *Phys. Rev. D* **58** (1998) 013009, [astro-ph/9710115].

Appendix: Coherent scattering of neutrinos inside a neutron star

In this section, we give a detailed analytic evaluation of the neutrino coherent cross-section inside the NS via neutral-current interactions. The mass basis of neutrinos is used throughout this section and we assume Dirac neutrinos.

In general, for a neutrino-nucleus coherent scattering, assuming that the coherence length is larger than the size of the nucleus, the squared amplitude of a coherent scattering between the (anti)neutrino (with energy E_ν much lower than the rest mass m_N) and a single nucleus (with the mass number A and the atomic number Z), averaging over target nucleus spins, and summing over the final state helicities, is given by [31, 105, 106]

$$\begin{aligned} \langle |\mathcal{M}_N(\nu_{i,s}^D)|^2 \rangle &= \frac{G_F^2}{16} \mathcal{T}_{\alpha\beta} \mathcal{T}_X^{\alpha\beta}, \\ \langle |\mathcal{M}_N(\bar{\nu}_{i,s}^D)|^2 \rangle &= \frac{G_F^2}{16} \bar{\mathcal{T}}_{\alpha\beta} \mathcal{T}_X^{\alpha\beta}, \end{aligned} \quad (16)$$

where s is the helicity of the incoming (anti)neutrino mass eigenstate i , and the traces $\mathcal{T}_{\alpha\beta}$, $\bar{\mathcal{T}}_{\alpha\beta}$, and $\mathcal{T}_X^{\alpha\beta}$ are expressed as follows:

$$\begin{aligned} \mathcal{T}^{\alpha\beta} &= \text{Tr} \left[\gamma^\alpha (\not{k}' + m_{\nu_i}) \gamma^\beta (1 - \gamma^5) u(k, r) \bar{u}(k, r) (1 + \gamma^5) \right] \\ &= 4 \left[-g^{\alpha\beta} (k - m_{\nu_i} S) \cdot k' + k'^\alpha (k^\beta - m_{\nu_i} S^\beta) + k'^\beta (k^\alpha - m_{\nu_i} S^\alpha) - i(k_\rho - m_{\nu_i} S_\rho) k'_\sigma \epsilon^{\alpha\beta\rho\sigma} \right], \\ \bar{\mathcal{T}}^{\alpha\beta} &= \text{Tr} \left[\gamma^\alpha \not{v}(k, r) \bar{v}(k, r) \gamma^\beta (1 - \gamma^5) (\not{k}' - m_{\nu_i}) (1 + \gamma^5) \right] \end{aligned} \quad (17)$$

$$= 4 [-g^{\alpha\beta}(k + m_{\nu_i}S) \cdot k' + k'^{\alpha}(k^{\beta} + m_{\nu_i}S^{\beta}) + k'^{\beta}(k^{\alpha} + m_{\nu_i}S^{\alpha}) + i(k_{\rho} + m_{\nu_i}S_{\rho})k'_{\sigma}\epsilon^{\alpha\beta\rho\sigma}], \quad (18)$$

$$\begin{aligned} \mathcal{T}_X^{\alpha\beta} &= \text{Tr} [\gamma^{\alpha} (\not{p}' + m_X) \gamma^{\beta} (Q_V - Q_A \gamma^5) (\not{k} + m_X) (Q_V + Q_A \gamma^5)] \\ &= 4(Q_V^2 + Q_A^2)(-g^{\alpha\beta}(p \cdot p') + p^{\alpha}p'^{\beta} + p^{\beta}p'^{\alpha}) - 8iQ_V Q_A p_{\rho}p'_{\sigma}\epsilon^{\alpha\beta\rho\sigma} + 4m_X^2(Q_V^2 - Q_A^2)g^{\alpha\beta}. \end{aligned} \quad (19)$$

where k , k' , p , and p' are the momenta of incoming neutrino, outgoing neutrino, incoming nucleus X , and outgoing nucleus X , respectively. Here r specifies the spin state of the incoming neutrino. The outer products of spinors are given by

$$u(k, r) \bar{u}(k, r) = \frac{1}{2}(\not{k} + m_{\nu_i})(1 + \gamma^5 \not{S}), \quad (20)$$

$$v(k, r) \bar{v}(k, r) = \frac{1}{2}(\not{k} - m_{\nu_i})(1 + \gamma^5 \not{S}), \quad (21)$$

where S can be chosen as

$$S^{\mu} = r \left(\frac{|\mathbf{k}|}{m_{\nu_i}}, \frac{E_{\nu}}{m_{\nu_i}} \frac{\mathbf{k}}{|\mathbf{k}|} \right). \quad (22)$$

With this choice, r can be interpreted as the helicity of the incoming neutrino. Relic neutrinos are ultra-relativistic at decoupling, and the helicity r of all relic neutrinos coincides with their chirality, which is -1 in the SM, at decoupling time. Since the relic neutrinos propagate through the Universe nearly freely, we can safely assume that their helicity has not been flipped since the time of decoupling.

In Eq. (19), $Q_V = A - 2Z(1 - 2\sin^2\theta_w)$ and $Q_A = A - 2Z$ are the vectorial and axial charges of the nucleus respectively, and θ_w is the Weinberg angle, with $\sin^2\theta_w \simeq 1/4$. The coherent cross-section thus acquires a large enhancement $\propto (A - Z)^2$, which depends on the number of neutrons in the target nucleus [11].

Evaluation of the traces (17)-(19) is done using `FeynCalc` [107]:

$$\begin{aligned} \langle |\mathcal{M}_N(\nu_{i,s}^D)|^2 \rangle &= 8G_F^2(k' \cdot p')((k - m_{\nu_i}S) \cdot p) \\ &= 2G_F^2(-m_{\nu_i}^2 - m_X^2 + s)(-2m_{\nu_i}(S \cdot p) - m_{\nu_i}^2 - m_X^2 + s), \end{aligned} \quad (23)$$

$$\begin{aligned} \langle |\mathcal{M}_N(\bar{\nu}_{i,s}^D)|^2 \rangle &= 8G_F^2(p \cdot k')((k + m_{\nu_i}S) \cdot p') \\ &= 2G_F^2(-m_{\nu_i}^2 - m_X^2 + s + t)(-2m_{\nu_i}(S \cdot p') - m_{\nu_i}^2 - m_X^2 + s + t), \end{aligned} \quad (24)$$

where s , t , and u are the usual Mandelstam variables defined as

$$s = (k + p)^2 = (k' + p')^2, \quad (25)$$

$$t = (k - k')^2 = (p' - p)^2, \quad (26)$$

$$u = (k - p')^2 = (k' - p)^2. \quad (27)$$

In Eqs. (23-24), one of the Mandelstam variables u is eliminated by using the relation $s + t + u = 2m_{\nu}^2 + 2m_X^2$. Since \mathbf{p} , the three-momentum of neutrons, is isotropic in the momentum space, only the zeroth component of any product of two four-momenta involving p survives. The momentum of neutrons in the final state \mathbf{p}' is almost isotropic, and we can also approximate $A \cdot p' \simeq A^0 p'^0$ involving any dot product of p' .

The thermally averaged cross-section of the interaction $\nu(k^{\mu}, \mathbf{s}) + X(p^{\mu}, r) \rightarrow \nu(k'^{\mu}, q') + X(p'^{\mu}, r')$ of a particle X in the NS with incoming neutrinos with a helicity \mathbf{s} is given by

$$\begin{aligned} \langle \sigma \rangle &= \frac{g_X}{n_X} \int \frac{d^3p}{(2\pi)^3} f_X(E_X) \frac{1}{4\sqrt{(k \cdot p)^2 - m_{\nu}^2 m_X^2}} \int \frac{d^3k'}{2E'_{\nu}(2\pi)^3} \int \frac{d^3p'}{2E'_X(2\pi)^3} \\ &\quad \times N_C \langle |\mathcal{M}|^2 \rangle (1 - f_{\nu}(E'_{\nu}))(1 \pm f_X(E'_X))(2\pi)^4 \delta^{(4)}(k_{\mu} + p_{\mu} - k'_{\mu} - p'_{\mu}). \end{aligned} \quad (28)$$

Here $\langle |\mathcal{M}|^2 \rangle$ is the scattering amplitude averaged over the helicity states of the incoming neutrino and summed over the helicity states of outgoing particles. g_X is the number of degrees of freedom of X , n_X its number density, r, q', r' are indices that specify spin states, $k^{\mu}, p^{\mu}, k'^{\mu}$, and p'^{μ} are four-momenta, and N_C is the coherent enhancement factor [cf. Eq. (3)]. f_{ν} and f_X are the distribution functions for the neutrino and X , respectively. The factor of $1 \pm f_X(E'_X)$ describes the Bose enhancement or the Pauli blocking effect depending if a particle X is a boson or a fermion. Since neutron stars mostly contain neutrons, we will consider X as neutrons in the rest of our calculation, with $g_n = 2$.

We will perform the integral in Eq. (28) analytically as much as possible following Refs. [64, 108]. The integration over \mathbf{p}' removes the three-dimensional delta function:

$$\begin{aligned} \langle \sigma \rangle &= \frac{g_n}{n_n \langle v_{\text{rel}} \rangle} \int \frac{d^3 k'}{(2\pi)^3} \frac{1}{(2E_\nu)(2E'_\nu)} \\ &\times \int \frac{d^3 p}{(2\pi)^3} N_C \frac{\langle |\mathcal{M}|^2 \rangle (s, t)}{(2E_n)(2E'_n)} (E'_n - E_n) f_n(E_n) (1 - f_n(E'_n)) (2\pi) \delta(E_\nu + E_n - E'_\nu - E'_n). \end{aligned} \quad (29)$$

Here we have treated the neutrinos inside the NS as a non-degenerate system and let the factor of $1 - f_\nu(E'_\nu) \rightarrow 1$. This is certainly valid for light relic neutrinos which eventually escape the NS, as well as for the active neutrinos produced from heavy sterile neutrino interactions in the NS. In addition, we substitute the relation $4\sqrt{(k \cdot p)^2 - m_\nu^2 m_n^2} = (2E_\nu)(2E_n)|v_{\text{rel}}|$ and move $|v_{\text{rel}}|$ to the outside the integral by replacing it with its averaged value:

$$\langle v_{\text{rel}} \rangle (E_\nu) = \frac{\int \frac{d^3 p}{(2\pi)^3} f_n(E_n) \frac{4\sqrt{(k \cdot p)^2 - m_\nu^2 m_n^2}}{(2E_\nu)(2E_n)}}{\int \frac{d^3 p}{(2\pi)^3} f_n(E_n)}. \quad (30)$$

To proceed further, we write $d^3 p = p E_n dE_n d(\cos \theta_n) d\phi_n$ by defining θ_n as an angle between \mathbf{p} and $\mathbf{q} = \mathbf{k} - \mathbf{k}' = \mathbf{p}' - \mathbf{p}$ and ϕ_n as that between $\mathbf{k} - (\mathbf{k} \cdot \tilde{\mathbf{q}})\tilde{\mathbf{q}}$ and $\mathbf{p} - (\mathbf{p} \cdot \tilde{\mathbf{q}})\tilde{\mathbf{q}}$ where $\tilde{\mathbf{q}} \equiv \mathbf{q}/|\mathbf{q}|$ is a unit vector parallel to \mathbf{q} . Letting ΔE be the energy transfer $\Delta E \equiv E'_n - E_n = E_\nu - E'_\nu$, we obtain

$$\begin{aligned} \langle \sigma \rangle &= \frac{g_n}{n_n \langle v_{\text{rel}} \rangle} \int \frac{d^3 k'}{(2\pi)^3} \frac{1}{(2E_\nu)(2E'_\nu)} N_C \\ &\times \frac{1}{16\pi^2} \int \frac{p}{E'_n} dE_n f_n(E_n) (1 - f_n(E'_n)) \int d\phi_n \int_{-1}^1 d \cos \theta_n \langle |\mathcal{M}|^2 \rangle (s, t) \Delta E \delta(E_\nu + E_n - E'_\nu - E'_n). \end{aligned} \quad (31)$$

We then next integrate over $\cos \theta_n$. The delta function fixes the value of $\cos \theta_n$:

$$\cos \theta_n = \frac{(\Delta E)^2 + 2E_n \Delta E - q^2}{2q\sqrt{E_n^2 - m_n^2}}. \quad (32)$$

As a function of $q, \Delta E$, and $\cos \theta_n$, the energy of incoming neutron E_n is expressed as:

$$E_n = \frac{-2\Delta E [q^2 - (\Delta E)^2] \pm \sqrt{4(\Delta E)^2 [q^2 - (\Delta E)^2]^2 - 4[(\Delta E)^2 - q^2 \cos^2 \theta_n] ([q^2 - (\Delta E)^2]^2 + 4q^2 m_n^2 \cos^2 \theta_n)}}{4[(\Delta E)^2 - q^2 \cos^2 \theta_n]}, \quad (33)$$

which has a minimum at $\cos \theta_n = \pm 1$:

$$E_{\text{min}} = -\frac{\Delta E}{2} + \sqrt{\left(m_n + \frac{\Delta E}{2}\right)^2 + \left(\frac{\sqrt{q^2 - (\Delta E)^2}}{2} - \frac{m_n \Delta E}{\sqrt{q^2 - (\Delta E)^2}}\right)^2}. \quad (34)$$

and diverges to infinity at $\cos^2 \theta_n = (\Delta E)^2/q^2$. The integration over $\cos \theta_n$ leaves a factor of pq/E'_n because $E'_n = \sqrt{E_n^2 + q^2} + 2pq \cos \theta_n$. Therefore,

$$\langle \sigma \rangle = \frac{g_n}{64\pi^2 n_n \langle v_{\text{rel}} \rangle} \int \frac{d^3 k}{(2\pi)^3} \int \frac{d^3 k'}{(2\pi)^3} N_C \frac{\Delta E}{E_\nu E'_\nu} \frac{1}{q} \int_{E_{\text{min}}}^{\infty} dE_n f_n(E_n) (1 - f_n(E'_n)) \int d\phi_n \langle |\mathcal{M}|^2 \rangle (s, t). \quad (35)$$

To find a formula of the center-of-mass energy $s = m_\nu^2 + m_n^2 + 2E_\nu E_n - 2\mathbf{k} \cdot \mathbf{p}$ as a function of $\phi_n, q, \Delta E$, we choose the z-axis parallel to the vector \mathbf{q} . In this coordinate system,

$$\mathbf{q} = (0, 0, q), \quad (36)$$

$$\mathbf{k} = (k \sin \tilde{\theta}_n, 0, k \cos \tilde{\theta}_n), \quad (37)$$

$$\mathbf{p} = (p \sin \theta_n \cos \phi_n, p \sin \theta_n \sin \phi_n, p \cos \theta_n), \quad (38)$$

where $\tilde{\theta}_n$ is the angle between \mathbf{k} and \mathbf{q} . Since

$$k \cos \tilde{\theta}_n = \frac{\mathbf{k} \cdot \mathbf{q}}{|\mathbf{q}|} = -\frac{(\Delta E)^2 - q^2 - 2E_\nu \Delta E}{2q}, \quad (39)$$

$$p \cos \theta_n = \frac{\mathbf{p} \cdot \mathbf{q}}{|\mathbf{q}|} = -\frac{q^2 - (\Delta E)^2 - 2E_n \Delta E}{2q}, \quad (40)$$

we obtain

$$\begin{aligned} s &= m_\nu^2 + m_n^2 + 2E_\nu E_n - 2\mathbf{k} \cdot \mathbf{p} \\ &= m_\nu^2 + m_n^2 + 2E_\nu E_n - \frac{(q^2 - (\Delta E)^2 - 2E_n \Delta E)((\Delta E)^2 - q^2 - 2E_\nu \Delta E)}{2q^2} \\ &\quad - 2\sqrt{E_n^2 - m_n^2 - \frac{(q^2 - (\Delta E)^2 - 2E_n \Delta E)^2}{4q^2}} \sqrt{E_\nu^2 - m_\nu^2 - \frac{(q^2 - (\Delta E)^2 + 2E_\nu \Delta E)^2}{4q^2}} \cos \phi_n. \end{aligned} \quad (41)$$

The ϕ_n integral can be evaluated analytically and results in a function of t, E_ν, E_n, q , and ΔE :

$$\int d\phi_n \langle |\mathcal{M}|^2 \rangle (s, t) \equiv A(t, E_\nu, E_n, q, \Delta E). \quad (42)$$

We next evaluate the integration over E_n :

$$\begin{aligned} \langle \sigma \rangle &= \frac{g_n}{64\pi^2 n_n \langle v_{\text{rel}} \rangle} \int \frac{d^3 k}{(2\pi)^3} \int \frac{d^3 k'}{(2\pi)^3} N_C \frac{\Delta E}{E_\nu E'_\nu} \frac{1}{q} \int_{E_{\text{min}}}^{\infty} dE_n f_n(E_n) (1 - f_n(E'_n)) A(t, E_\nu, E_n, q, \Delta E), \\ &\equiv \frac{g_n}{64\pi^2 n_n \langle v_{\text{rel}} \rangle} \int \frac{d^3 k}{(2\pi)^3} \int \frac{d^3 k'}{(2\pi)^3} N_C \frac{\Delta E}{E_\nu E'_\nu} \frac{1}{q} B(t, E_\nu, q, \Delta E). \end{aligned} \quad (43)$$

Next, we choose the spherical coordinates for k' -space and choose the angle θ'_ν as the angle between \mathbf{k} and \mathbf{k}' . Using the relations

$$t = -q^2 + (\Delta E)^2, \quad (44)$$

$$\Delta E = E_\nu - \sqrt{k'^2 + m_\nu^2}, \quad (45)$$

$$q^2 = k^2 + k'^2 - 2kk' \cos \theta'_\nu, \quad (46)$$

we perform coordinate transformations from k' and θ'_ν to ΔE and t . We thus obtain

$$\langle \sigma \rangle = \frac{g_n}{1024\pi^4} \frac{1}{n_n \langle v_{\text{rel}} \rangle} \frac{1}{kE_\nu} \int d(\Delta E) \Delta E \int dt \frac{B(t, E_\nu, \Delta E)}{\sqrt{(\Delta E)^2 - t}} N_C. \quad (47)$$

To get the equation above, we used $d(\Delta E) = -k' dk' / E'_\nu$, $2q dq = -2kk' d(\cos \theta'_\nu)$, and $dt = -2q dq$. The region of integration is determined so that $|\cos \theta'_\nu| \leq 1$. From Eq. (44), we find

$$t = 2 \left[E_\nu (\Delta E - E_\nu) + m_\nu^2 + k \sqrt{(E_\nu - \Delta E)^2 - m_\nu^2} \cos \theta'_\nu \right]. \quad (48)$$

Therefore, the lower/upper bound of the integral t_-/t_+ is

$$t_\pm = 2 \left[E_\nu (\Delta E - E_\nu) + m_\nu^2 \pm k \sqrt{(E_\nu - \Delta E)^2 - m_\nu^2} \right]. \quad (49)$$

In summary, the cross-section averaged over the thermal distribution of neutrons is given by

$$\langle \sigma \rangle = \frac{g_n}{1024\pi^4} \frac{1}{n_n \langle v_{\text{rel}} \rangle} \frac{1}{kE_\nu} \int_{-\infty}^{E_\nu - m_\nu} d(\Delta E) \Delta E \int_{t_-}^{t_+} dt \frac{B(t, E_\nu, \Delta E)}{\sqrt{(\Delta E)^2 - t}} N_C. \quad (50)$$

We note that the cross-section $\langle \sigma \rangle$ shows neutrino mass-independent behavior in the regime $T_{\text{NS}} \ll K_{\nu, \text{surface}} \ll m_n$ where the $\nu - n$ cross-section scales as $\sigma \propto E_\nu^2$ while the coherent enhancement factor and a fraction r of neutrons

involving a scattering behave as $N_C \propto p_\nu^{-3}$ and $r \simeq |\mathbf{q}|/p_{f,n} \propto p_\nu$ respectively, canceling out the m_ν -dependence of the cross-section.

Similarly, the average energy transfer is given by

$$\langle \Delta E \rangle = \frac{\sum_{r,q',r'} \frac{1}{2E_\nu} \int N_C |\mathcal{M}|^2 \Delta E f_n(E_n) (1 - f_\nu(E'_\nu)) (1 - f_n(E'_n)) (2\pi)^4 \delta^{(4)}(k_\mu + p_\mu - k'_\mu - p'_\mu)}{\sum_{r,q',r'} \frac{1}{2E_\nu} \int N_C |\mathcal{M}|^2 f_n(E_n) (1 - f_\nu(E'_\nu)) (1 - f_n(E'_n)) (2\pi)^4 \delta^{(4)}(k_\mu + p_\mu - k'_\mu - p'_\mu)}, \quad (51)$$

where the integral is over the three-momenta p, p', k' :

$$\int \equiv \int \frac{d^3 p}{2E_n (2\pi)^3} \int \frac{d^3 k'}{2E'_\nu (2\pi)^3} \int \frac{d^3 p'}{2E'_n (2\pi)^3}. \quad (52)$$

As mentioned in the main text, the incoming neutrino coherently interacts with many neutrons, which lie inside a sphere with a radius equal to the de Broglie wavelength of the momentum transfer, and the corresponding enhancement factor is given by Eq. (3). However, this equation gives inaccurate results when the mean free path λ is shorter than the de Broglie wavelength. Therefore, we estimate the coherent enhancement factor by assuming particles inside a sphere with a radius λ rather than $\lambda_\nu = 2\pi/|\mathbf{q}|$ interact with neutrino at the same time, that is, $N_C = 4\pi\lambda^3 n_n/3$, where λ is determined in a self-consistent way as follows: First, we divide N_C by a factor of 2 as nearly half of the sphere will be vacant in the scattering with neutrons on the surface of NS. The interaction rate, i.e. the number of collisions that each neutrino experiences per unit time, is given by

$$\Gamma = \sum_{r,q',r'} \frac{1}{2E_\nu} \int \frac{d^3 p}{2E_n (2\pi)^3} \int \frac{d^3 k'}{2E'_\nu (2\pi)^3} \int \frac{d^3 p'}{2E'_n (2\pi)^3} \frac{4\pi}{3} r \lambda^3 n_n |\mathcal{M}|^2 f_n(E_n) (1 - f_\nu(E'_\nu)) (1 - f_n(E'_n)) \times (2\pi)^4 \delta^{(4)}(k_\mu + p_\mu - k'_\mu - p'_\mu). \quad (53)$$

One can see the consistency of this relation with the definition of the mean free path $\lambda \equiv 1/n_n \langle \sigma \rangle$ by using the relation $4\sqrt{(k \cdot p)^2 - m_\nu^2 m_n^2} = (2E_\nu)(2E_n)|v_{\text{rel}}|$ and $\Gamma = \langle v_{\text{rel}} \rangle / \lambda$ where v_{rel} is the relative velocity between neutrinos and neutrons. With Eq. (53), we define the self-consistent mean free path λ as:

$$\frac{1}{\lambda^4} = \frac{1}{\langle v_{\text{rel}} \rangle} \sum_{r,q',r'} \frac{1}{2E_\nu} \int \frac{d^3 p}{2E_n (2\pi)^3} \int \frac{d^3 k'}{2E'_\nu (2\pi)^3} \int \frac{d^3 p'}{2E'_n (2\pi)^3} \frac{4\pi}{3} r n_n |\mathcal{M}|^2 f_n(E_n) (1 - f_\nu(E'_\nu)) \times (1 - f_n(E'_n)) (2\pi)^4 \delta^{(4)}(k_\mu + p_\mu - k'_\mu - p'_\mu). \quad (54)$$

We checked that self-consistency of the mean free path needs to be considered only when relic neutrinos are very light $m_\nu \ll 0.1$ eV. For heavier neutrinos, it suffices to compute the enhancement factor by just using the momentum transfer, $N_C = 4\pi n_n/3 \times (2\pi/|\mathbf{q}|)^3 \times 1/2$.

We find that $\langle \Delta E \rangle$ is nearly independent of NS temperature except when neutrons are more energetic than incoming relic neutrinos. $\langle \Delta E \rangle$ increases linearly with neutrino masses, while it falls to negative values once K_ν drops below $3T_{\text{NS}}$. This roughly defines the boundary between kinetic heating versus cooling of the NS.