EXPERIMENTAL MEASUREMENTS FOR EXTRACTING NONLINEAR INVARIANTS *

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Abstract

Nonlinear integrable optics (NIO) are a promising alternative approach to lattice design. The integrable optics test accelerator (IOTA) at Fermilab has been constructed for dedicated studies of magnetostatic elliptical elements as described by Danilov and Nagaitsev. The most compelling verification of correct implementation of the NIO lattice is direct observation of the analytically expected invariants. This report outlines the experimental and analytical methods for extracting the nonlinear invariants of motion from data gathered in the last IOTA run.

INTRODUCTION

Nonlinear integrable optics are an attractive way to improve the range of amplitude-dependant detuning without negatively affecting the dynamic aperture. NIO has a long history [5, 9], but has been typically limited to exotic focusing elements. Danilov and Nagaitsev were the first to describe a two dimensional NIO system which could be realized as traditional magnetostatic focusing elements [4]. The base of the DN system is a "bare" linear lattice composed of a matching section with linear optics and a drift region for placement of nonlinear inserts. The matching section of the lattice must satisfy three conditions

1. Matched horizontal and vertical beta functions in the nonlinear insert
2. The minimum beta function in the drift is located at the center of the drift
3. An integer multiple of π phase advance across the matching section outside the insertion

To complete the DN system a nonlinear potential with elliptical geometry is inserted into the drift space. The two invariants preserved by the system are presented in a complex parameterization [10] in Eq.(1) and Eq.(2). The author recommends using this parameterization over the original, it is more numerically stable about the $y = 0$ line for simulation purposes and avoids a number of pitfalls in interpretation.

$$H = \frac{1}{2}\left[p_x^2 + p_y^2 + x^2 + y^2\right] - t\mathrm{Re}\left(\frac{z}{\sqrt{1-z^2}}\arcsin(z)\right) \quad (1)$$

$$I = (xp_y - yp_x)^2 + p_x^2 + x^2 - t\mathrm{Re}\left(\frac{x}{\sqrt{1-z^2}}\arcsin(z)\right) \quad (2)$$

Here the phase space variables are normalized to the "bare" lattice Courant-Snyder lattice functions, with $\tilde{q}, \tilde{p_q}$ as the unormalized phase space coordinates.

$$q = \frac{\tilde{q}}{c\sqrt{\beta}}, \qquad p_q = \tilde{p}_q\frac{\sqrt{\beta}}{c} + \tilde{q}\frac{\alpha}{c\sqrt{\beta}}, \qquad z = x + iy \quad (3)$$

The t and c parameters are characteristic of the DN system and describe the scaling of nonlinear field and the geometry of singularities in the potential.

The nonlinear potential must be longitudinally scaled with respect to the bare lattice beta function in the drift. In practice, this means slicing up the nonlinear insert into a number of individual elements which are individually scaled to approximate the ideal smooth scaling of the potential. To evaluate practical implementation of the DN system, the IOTA storage ring was constructed at Fermilab [1]. IOTA is a small (40 m), easily re-configurable ring for beam dynamics studies and general accelerator R&D. The centerpiece of IOTA for NIO studies is a 1.8 m long 18 element DN insert constructed by Radiabeam [11]. The NIO program at IOTA consists of two stages. The first stage is experiments with a 150 MeV electron pencil beam to probe the potential. The second stage is experiments with a 2.5 MeV proton beam to study the interaction of NIO with intense space charge.

IOTA Run 4 was conducted in 2024 as a part of the NIO electron program. Experimental measurement of conservation of the DN nonlinear invariants is a straightforward indication of implementation of the DN system.

EXPERIMENTAL METHODS

Due to the strict requirements on the linear lattice set by the DN system, a simulation study was performed to evaluate the range of bare lattice parameters that still exhibited good conservation of the nonlinear invariants. These values were used as targets for the linear optimization of the lattice. Optimization was performed using the implementation of LOCO in SixDSimulation [12]. The target for the LOCO optimization was the IOTA design lattice, a simulation model also developed in SixDSimulation. The target parameters and experimentally realized values are found in Table 1.

In addition to the linear lattice optimization, sextupoles in IOTA were adjusted for operations. Simulation studies have indicated that successful operation of the DN system requires matched chromaticity [13]. For turn-by-turn (TBT) analysis, it is desirable to maximize the decoherence time by minimizing chromaticity. Based on these requirements the chromaticity was minimized and matched to between -1

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Table 1: IOTA Run 4 Bare Lattice Parameters

Parameter	Target	Result
Phase Advance Accuracy	0.001	0.001(5)
Dispersion in Insert	<1 cm	0.5(2) cm
Closed Orbit Deviation	<50 µm	40(5) µm
Beta Function Deviation	1%	2%
Beta Beating	1%	1%

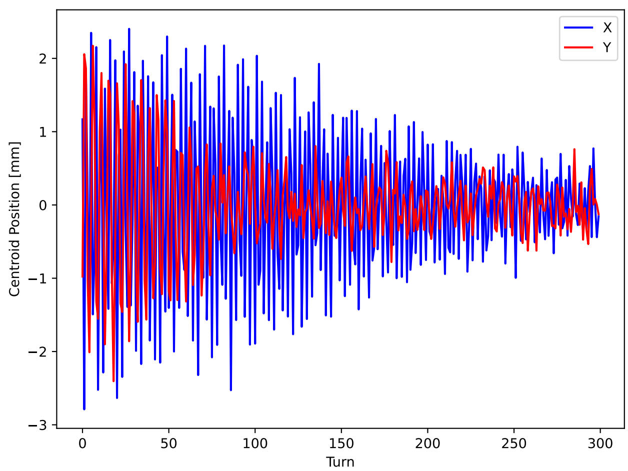

Figure 1: Single BPM response to a kick.

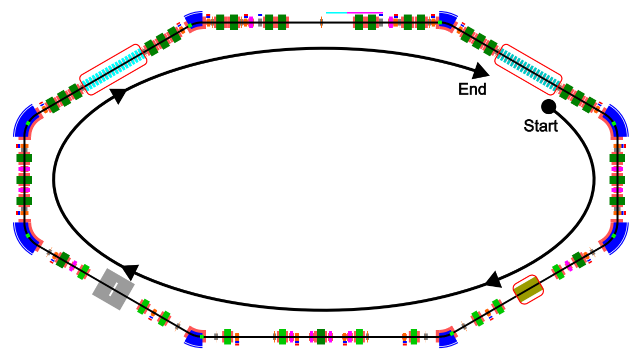

Figure 2: Diagram of IOTA with matching section indicated. All IOTA BPMs lie on the path indicated.

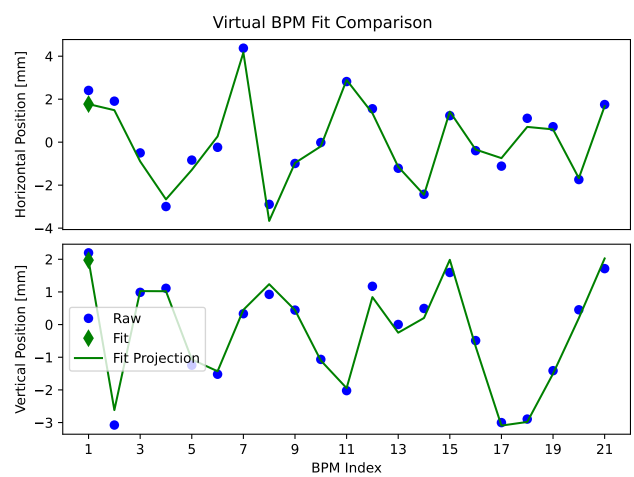

Figure 3: Illustration of fitting method. Blue points are preprocessed tbt centroid positions in each BPM along the matching section indicated in Figure 2. Orange cross indicates the fitted position. Green line is the fitted position propagated through the same transfer matrices used for fitting.

and 0 using sextupole magnets in the matching section of the IOTA lattice.

The TBT measurements in IOTA utilized stripline kickers [2] and the 21 button BPMs [6] in IOTA. For a given measurement, once the electron beam was at its radiative equilibrium and in a sufficient current range for the BPMs to be sensitive and unsaturated, a dipole kick was applied. The TBT centroid position was then measured by the BPM system for several thousand turns. To probe the full physical aperture, both the horizontal and vertical kickers were fired in a single turn to allow for mixed-plane excitations. The strong nonlinearity of the system means that the centroid oscillations decohere in a short time (50-250 turns), with different decoherence times in different planes depending on the amplitude (Figure 1). To effectively probe the nonlinear dynamics of the system, a grid of different horizontal and vertical amplitudes were measured for various values of the DN t-parameter. The pyIOTA library [7] was used for automated lattice control and data acquisition for all TBT data sets.

POSITION RECONSTRUCTION

For calculation of the invariant expressions, the four dimensional (x, p_x, y, p_y) transverse coordinates of the beam need to be reconstructed. This was accomplished through the use of a least squares fitting of the turn by turn BPM data to generate a virtual BPM.

Before fitting the TBT data a few preprocessing steps were taken. BPM calibrations for scaling and roll as fit from the LOCO optimization were applied. A principle component analysis was applied to all of the BPMs. Based on the singular value, the first 8 components of the PCA were used to reduce the uncorrelated noise in the TBT signals.

Since the matching section of the DN system is most of the lattice footprint, all IOTA BPMs are located in this section. For the fitting, he matching section was essentially treated in a channel mode from the end of the nonlinear insert to the beginning of it (Figure 2).

As the linear elements are not adjusted from the bare lattice parameters for different the linear transfer matrices from the design lattice were used to fit the virtual BPM position from up to all 21 BPMs.

Intermittent unreliability and saturation in the BPMs needed to be filtered before applying the fitting algorithm. Individual BPMs could be dropped, up to 10. Studies of reduction in the number of BPMs yielded improvement in the fit quality for one nonstandard BPM, so it was dropped from fitting. All other BPMs were used barring errors. The known nonlinearities in the matching section stemming from the

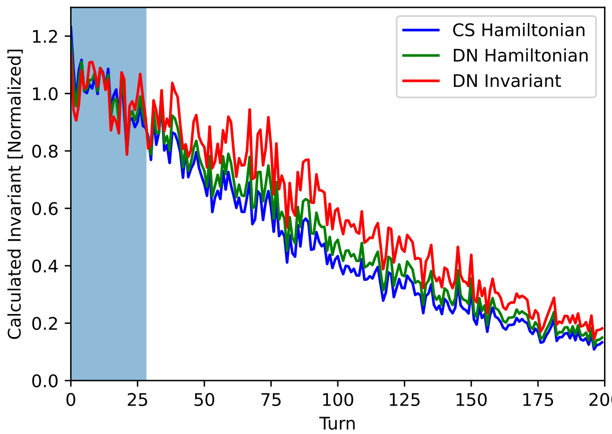

Figure 4: Calculated invariant quantities for t=-0.238 and proportional amplitudes in x and y. All traces are normalized to the mean over their first 28 turns. Note the reduction in value due decoherence in the fitted positions.

sextupole fields were considered, but expanding the method using second order transfer maps yielded poor quality fits of the motion, and strictly linear fits were used for analysis.

INVARIANT CALCULATION

To evaluate the invariant expressions, the raw fitted coordinates needed to be normalized by the bare lattice Courant-Snyder functions at the virtual BPM location. Like the transfer matrices, these quantities were extracted from the design lattice. To benchmark the method, the Courant-Snyder invariants were extracted from the bare lattice.

Due to the TBT noise in the BPM data, any derived parameter will also exhibit TBT noise. Additionally, the fast decoherence of the centroid oscillation will cause the calculated invariants show a correlated fast reduction in value. To determine the quality of conservation the calculated invariants were normalized to their mean value over the first 28 turns.

For reasonable comparison to the equivalent Courant-Snyder invariant, the first order effect on the lattice functions of the DN element was extracted from the design lattice. This effect comes from the quadrupole term of the non-linear field. To avoid apparent invariant changes due to any linear coupling, the sum of the horizontal and vertical invariants (the Hamiltonian) was used for comparison [8]. The first DN invariant, the Hamiltonian of the system, is very similar to the CS Hamiltonian. As a result these two quantities display similar levels of conservation. A better metric is looking at the conservation of the second DN invariant.

Figure 5 is plots the rms of the normalized second invariant against kick amplitude. Primarily horizontal kicks demonstrate superior conservation to primarily vertical kicks.

To evaluate expected conservation levels, single particle simulations were performed for the same amplitude excitation. Using the same conservation metrics, the conservation

Table 2: Invariant Conservation RMS

Invariant	Simulation	Experimental
CS Hamiltonian	1.9%	6.9%
DN Hamiltonain	0.3%	5.6%
DN Invariant	0.3%	6.9%

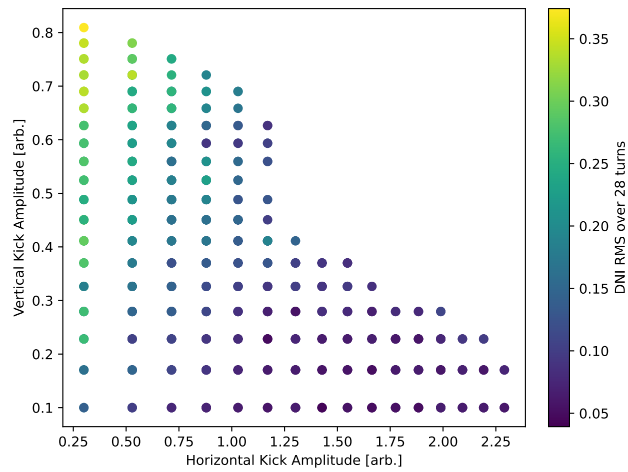

Figure 5: DN second invariant RMS over 28 turns.

levels were found to be below the noise limit of the directly calculated experimental invariants 2

CONCLUSION AND NEXT STEPS

Direct calculation of the analytical invariant quantities from fitted position data has insufficient resolution to demonstrate conservation of these invariants. More sophisticated analysis of the fitted position data is ongoing, including an application of machine learning to search for invariants of motion [3]. Comparison of conservation levels against kick amplitude and t-parameter indicates that signal-to-noise ratio in the current BPM system needs to be better or the available dynamic aperture must be larger to directly observe non-linear contributions to the DN Hamiltonian and invariant using electron beam as a probe. With a larger dynamic aperture, the amplitude of measureable kicks could be increased, at larger amplitudes the nonlinear terms in the DN system become stronger and the conservation ratio between the DN and Courant-Snyder invariants increases. The following contribution [14] describes the measured limitations on the dynamic aperture.

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