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Model-independent description of $B \rightarrow D\pi\ell\nu$ decays

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Abstract

In this contribution we present a novel, model-independent description of semileptonic $B \rightarrow D\pi\ell\nu$ decays. In addition, we discuss recent developments in the understanding of coupled-channel $D\pi$ - $D\eta$ - $D_s K$ S-wave scattering and, for the first time, apply them to semileptonic decays. We not only obtain model-independent predictions for kinematic distributions in $B \rightarrow D\pi\ell\nu$ decays, but also rule out the hypothesis that the gap between the inclusive $B \rightarrow X\ell\nu$ branching fraction and the sum over exclusive channels is made up predominantly by $B \rightarrow D^{(*)}\eta\ell\nu$ decays.

1 Introduction

Semileptonic $B \rightarrow D\pi\ell\nu$ decays, not including on-shell $B \rightarrow D^*(\rightarrow D\pi)\ell\nu$ decays, make up approximately 5% of all semileptonic B meson decays. Not only are they a signal component in inclusive $B \rightarrow X_c\ell\nu$ and $B \rightarrow X\tau\nu$ decays, but they also constitute an important background for studies of $B \rightarrow D^{(*)}\ell\nu$ decays, as well as measurements of $R(D^{(*)})$. Consequently, they contribute to both sides of the $|V_{cb}|$ inclusive-exclusive discrepancy and are relevant to determine if there are effects beyond the Standard Model

in $b \rightarrow c\tau\nu$ transitions. Yet, experimental studies and the theoretical understanding of $B \rightarrow D\pi\ell\nu$ decays are not as mature as of $B \rightarrow D^{(*)}\ell\nu$ decays.

Quark models predict the existence of two low-lying doublets of excited D -meson states decaying to $D^{(*)}\pi$. The first one contains a scalar, the D_0^* , decaying to $D\pi$ and an axial-vector, the D_1' , decaying to $D^*\pi$ through the S-wave. Both are expected to have a large width due to their S-wave nature. The second doublet contains two narrow states: one axial-vector, the D_1 , and a tensor, the D_2^* , which is the only of the four states decaying to both final states. The semileptonic decays of B mesons into these four states are most commonly described by the HQET-based Leibovich-Ligeti-Stewart-Wise (LLSW) parametrization [1, 2], connecting transitions of B mesons into the respective doublet partners.

On the experimental side, the masses and widths of the narrow states have been measured at the sub-MeV level by the LHCb collaboration in nonleptonic $B \rightarrow D^{(*)}\pi\pi$ decays. Yet, the masses and widths of the two broad states have large uncertainties since they do not appear as clear peaks in invariant mass spectra. Furthermore, the only available background-subtracted differential spectra in $B \rightarrow D\pi\ell\nu$ decays have been measured by the Belle experiment more than 15 years ago [3].

These spectra, together with the nonleptonic $B \rightarrow D^{**}(\rightarrow D^{(*)}\pi)\pi$ branching ratios are the experimental input entering the two most detailed studies of $B \rightarrow D^{**}\ell\nu$ decays [4, 5, 6]. Inspired by Dalitz-plot analyses in nonleptonic decays, the more recent study includes, in addition to the D^{**} modes, a possible virtual D^* component, i.e. does account for the fact, that a very narrow Breit-Wigner distribution has a tail that drops like $1/(p^2 - M^2)^2$.

This treatment of the D^* is supported by the most recent study of $B \rightarrow D\pi\ell\nu$ decays by Belle [7], where a falling component is required to fit the data and a smaller than expected D_0^* signal is observed.

2 A model-independent parameterization

In Ref. [8] we introduce a form-factor decomposition inspired by the treatment of $B \rightarrow D\ell\nu$ and $B \rightarrow D^*\ell\nu$ decays by Boyd, Grinstein and Lebed (BGL) [9, 10, 11], but extended, for the first time, to allow for two hadrons in the final state and arbitrary angular momenta of the intermediate states. The BGL parameterization itself is model-independent, but implements unitarity constraints on the q^2 -dependence of form factors in a rigorous way. Consequently, it has proven to be very successful in experimental studies and Lattice QCD calculations of $B \rightarrow D^*\ell\nu$ decays.

The key behind the extension to multi-hadron final states is a partial-wave decomposition of the $D\pi$ system. This approach is natural and widely used in the study of nonleptonic three-body decays, as all hadronic resonances have definite angular momentum, e.g. the D_2^* only appears in the $D\pi$ D-wave, but not in the P- or S-wave. Thus, each partial wave is described by four (two for the S-wave) q^2 - and $M_{D\pi}^2$ -dependent form factors. Formally, the unitarity bounds are derived by considering the three-hadron contributions to two-point functions of the weak current. While each partial

wave contributes to a given bound, due to the partial-wave expansion, there are no cross-terms, resulting in diagonal bounds.

To obtain a practically useful parameterization taking into account the unitarity bounds, we observe that the weak $b \rightarrow c$ transition takes place at much smaller length scales than the residual strong interactions between the two final-state hadrons. Thus, we write each form factor as

$$f^{(l)}(q^2, M_{D\pi}^2) = \hat{f}^{(l)}(q^2, M_{D\pi}^2) g^{(l)}(M_{D\pi}^2), \quad (2.1)$$

where the function $g^{(l)}$ is the same for all form factors of a given partial wave and encodes the effect of final state interactions in the $D\pi$ system, such as the appearance of resonances. The remainder of the form factor only mildly depends on $M_{D\pi}^2$ and thus can be approximated. For the case of a partial wave with a single Breit-Wigner resonance of mass M_R , we could write:

$$\hat{f}^{(l)}(q^2, M_{D\pi}^2) \approx \tilde{f}^{(l)}(q^2) + (M_R^2 - M_{D\pi}^2) \bar{f}^{(l)}(q^2) + \mathcal{O}((M_R^2 - M_{D\pi}^2)^2). \quad (2.2)$$

Neglecting all higher order terms, the function $\tilde{f}^{(l)}(q^2)$ can be treated just as a regular form factor in the BGL parameterization with modified outer functions encoding the effect of $g^{(l)}(M_{D\pi}^2)$:

$$\tilde{f}_l(q^2) = \frac{1}{\phi_l^{(f)}(q^2) B_f(q^2)} \sum_{i=0}^{\infty} a_{li}^{(f)} z^i, \quad (2.3)$$

where B_f is a Blaschke factor including subthreshold B_c resonances,

$$z(q^2, q_0^2) = \frac{q_0^2 - q^2}{(\sqrt{q_+^2 - q^2} + \sqrt{q_+^2 - q_0^2})^2}. \quad (2.4)$$

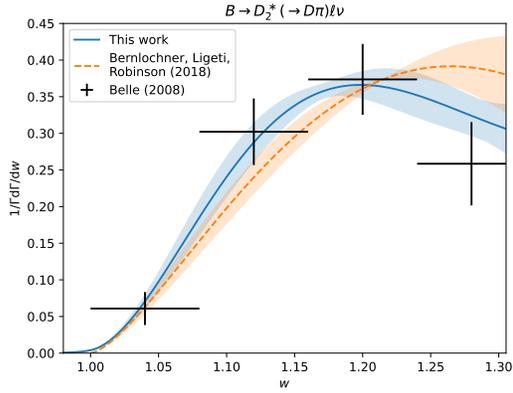
and the unitarity bound

$$\sum_{i,l} |a_{li}^{(f)}|^2 < 1. \quad (2.5)$$

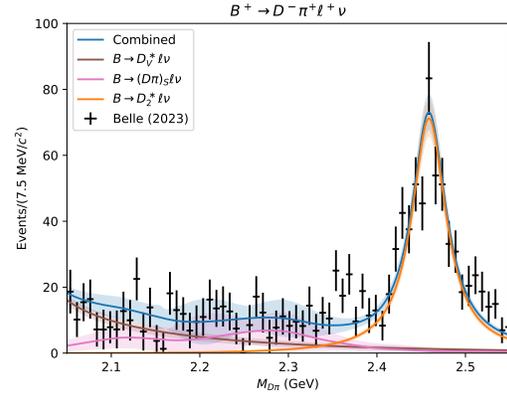
Including the suppressed term $\bar{f}^{(l)}(q^2)$ would lead to terms mixing the expansion coefficients of $\tilde{f}^{(l)}(q^2)$ and $\bar{f}^{(l)}(q^2)$ and consequently to a non-diagonal unitarity bound.

As a first application, we fit the $D\pi$ D-wave form factors to the differential decay rates measured by Belle [3]. The resulting w -spectrum is shown in Fig. 1a and compared to the results of Ref. [5].

The second novelty of Ref. [8] is the treatment of the $D\pi$ S-wave contribution. Lattice QCD studies of $D\pi$ S-wave scattering [12, 13, 14] point to a lower mass of the D_0^* than obtained from quark models: approximately 2.1 GeV instead of 2.3-2.4 GeV. In the context of unitarized chiral perturbation theory it was found that the calculation of Ref. [12] leads to S-wave scattering matrices that contain two poles near $(2.1 - i0.1)$ and $(2.45 - i0.13)$ GeV [15, 16, 17], with the former coupling predominantly to the $D\pi$ final state and the latter to the $D_s K$ final state. The resulting $D\pi$ lineshape can not



(a) Normalized $B \rightarrow D_2^* \ell \nu$ w -spectrum



(b) Fit of the measured $M_{D\pi}$ -spectrum.

be described in terms of a sum of Breit-Wigner curves and thus we follow a different strategy.

Below the onset of large $D\pi\pi\pi$ inelasticities, analyticity and unitarity dictate that the imaginary part of $f^{(0)}$ is given by the coupled-channel $D\pi$ - $D\eta$ - $D_s K$ scattering T -matrix, which we take from Ref. [12]:

$$\text{Im } \vec{f}(q^2, M_{D\pi}^2 + i\epsilon) = T^*(M_{D\pi}^2 + i\epsilon) \Sigma(M_{D\pi}^2) \vec{f}(q^2, M_{D\pi}^2 + i\epsilon), \quad (2.6)$$

$$\vec{f}(q^2, M_{D\pi}^2) = \Omega(M_{D\pi}^2) \vec{P}(q^2, M_{D\pi}^2), \quad (2.7)$$

$$\text{Im } \Omega(s + i\epsilon) = \frac{1}{\pi} \int_{s_{\text{thr}}}^{\infty} \frac{T^*(s') \Sigma(s') \Omega(s')}{s' - s - i\epsilon} ds'. \quad (2.8)$$

Here the vector \vec{f} is a vector in channel-space, Σ collects phase-space factor and Ω is the Muskhelishvili-Omnès matrix [18, 19]. The function $\vec{P}(q^2, M_{D\pi}^2)$ is a polynomial in $M_{D\pi}^2$ and we truncate it at zeroth order.

Combining our description of the S- and D-waves with the tail of the D^* resonance in the P-wave, we fit to the $M_{D\pi}$ distributions recently measured by the Belle experiment [7]. We obtain a good fit, the result displaced in Fig. 1b, showing that semileptonic data are compatible with a two-pole structure in the S-wave. However, in contrast to nonleptonic decays [20] we can not rule out the quark model picture of a single, broad, S-wave resonance yet.

3 Conclusion & Outlook

We have presented a model-independent parameterization of $B \rightarrow D\pi\ell\nu$ decay, a novel treatment of the $D\pi$ S-wave and compared to available data. The coupled channel treatment of the S-wave allows us to infer the branching ratios of $B \rightarrow D\eta\ell\nu$ and, through heavy quark spin-symmetry, $B \rightarrow D^*\eta\ell\nu$ decays, which are found to be at a level of 10^{-5} . Thus, they can not account for the gap between the inclusive $B \rightarrow X\ell\nu$ branching fraction and the sum over exclusive states.

Our work opens the door to future studies of $1 \rightarrow 2$ -hadron semileptonic decays in a model-independent manner and will be crucial for direct measurements of the $D\pi$ S-wave scattering phase-shift, allowing to obtain the position of the lowest scalar D meson pole from experiment.

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