

# Third order QCD predictions for fiducial $W$ -boson production

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## Abstract

Measurements of  $W$ -boson production at the LHC have reached percent-level precision and impose challenging demands on theoretical predictions. Such predictions directly limit the precision of measurements of fundamental quantities such as the  $W$ -boson mass and the weak mixing angle. A dominant source of uncertainty in predictions is from higher-order QCD effects. We present a calculation of  $W$ -boson production at the level of  $\alpha_s^3$  at fixed order and including transverse-momentum resummation. We further show predictions for a direct comparison with low-pileup ATLAS transverse-momentum and fiducial cross-section measurements at  $\sqrt{s} = 5.02$  TeV. We discuss in detail the impact of modern PDFs. Our calculation including the matching to  $W$ +jet production at NNLO will be publicly available the upcoming CuTe-MCFM release and allows for theory-data comparison at the state-of-the-art level.

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## 1. Introduction

The production of  $W$  bosons at hadron colliders has one of the highest cross-sections of all Standard Model (SM) processes. Together with its relative ease of detection, through a large branching ratio into a lepton and neutrino, it has been measured with very high precision at multiple colliders. At the LHC, measurements range from center of mass energies of 2.76 TeV to 13 TeV, as performed by ATLAS [1–4], CMS [5–8] and LHCb in the forward region [9–13]. Since the first measurements at the LHC, luminosity uncertainties have reduced from 2–3% to 1% [14, 15], setting the upper bound on the precision reached in current measurements [16].

The precision of these analyses opens the door to the measurement of fundamental quantities appearing in the weak sector of the SM Lagrangian, such as the  $W$ -boson mass [17–22], the weak mixing angle, as well as stringent constraints on parton distribution functions (PDFs) [4, 23, 24] and charge asymmetries [25–29].

However, the interpretation of these measurements – and their ultimate precision – depends crucially on the sophistication of the theoretical predictions with which they are compared and analyzed. First N<sup>3</sup>LO predictions for  $W$ -boson production have been presented for total inclusive cross-sections in ref. [30], where large cancellation effects between initial-state channels have been observed that lead to significant N<sup>3</sup>LO corrections of about  $-2.5\%$ .

Fixed-order  $\alpha_s^3$   $W$ +jet predictions [31] (NNLOjet) have been matched to the RadISH resummation [32, 33] in ref. [34] at the level of N<sup>3</sup>LL and compared to Pythia results. Higher-order transverse-momentum resummation at the level of N<sup>3</sup>LL' matched to lower-order  $\alpha_s^2$  predictions has also been studied in refs. [35, 36]. Recent studies of threshold resummation in rapidity distributions were presented in refs. [37, 38]. Fixed-order N<sup>3</sup>LO predictions for  $W$ -boson production have been presented in ref. [39] (transverse mass distribution, rapidity, charge asymmetry), which also includes a Tevatron study using fiducial cuts.

Generally the residual QCD truncation uncertainties at the level of  $\alpha_s^3$  are estimated to be at the level of 1–2%. Apart from QCD effects, other Standard Model effects

play a role at the level of 1% precision. Among these, mixed QCDxEW corrections were reported in ref. [40–44] and QED-QCD transverse-momentum resummation in ref. [45].

While all of these recent higher-order corrections are important for percent-level comparison with kinematic measurements, they are crucial to improve the  $W$ -boson mass measurement. In particular, even though few measurements have been performed of the  $W$ -boson transverse momentum directly [16, 46, 47], it is a key part in the  $W$ -boson mass analyses. A comprehensive review of how theoretical contributions and uncertainties impact the  $W$ -boson mass measurement was presented in ref. [48] (2016). An estimate for the impact of mixed QCDxEW corrections has since also been performed [49].

In this paper we present a calculation of fully differential  $W$ -boson production at N<sup>3</sup>LO fixed-order and including transverse-momentum resummation up to a logarithmic order of N<sup>4</sup>LL matched to N<sup>3</sup>LO fixed-order.<sup>1</sup> Together with our calculation of  $Z$  production at  $\alpha_s^3$  [50], it completes the set of two crucial processes entering many Standard Model precision analyses, for example the  $W$ -boson mass determination. It allows for QCD predictions at the highest level with an independent implementation of higher-order ingredients and resummation. The availability of multiple calculations in different resummation formalisms enables reliable uncertainty estimates. In this paper we compute predictions for comparison with the  $\sqrt{s} = 5.02$  TeV ATLAS analysis from ref. [16]. The calculation will be made publicly available, so that its results can be used for future analyses, in the upcoming release of **CuTe-MCFM**.

In section 2 we describe our setup and cross-checks performed on our calculation. In section 3.1 we first discuss predictions of the fiducial cross-section and compare with the measurement. In particular we focus on the impact of PDFs. We move on to differential distribution in section 3, where we discuss the  $W$ -boson transverse-momentum distribution (section 3.2) and transverse mass and charged lepton transverse-momentum distributions (section 3.3). Since the impact of PDFs is significant, we discuss their impact on these distributions in section 3.4. We conclude and present an outlook in

section 4.

## 2. Setup

We implement QCD corrections to  $pp \rightarrow W(\rightarrow e\nu)$  production at fixed-order and including the effect of  $q_T$ -resummation in **CuTe-MCFM** [50–53]. We achieve  $\alpha_s^3$  fixed-order and transverse momentum renormalization-group-improved (RG-improved) logarithmic accuracy, counting  $\log(q_T^2/Q^2) \sim 1/\alpha_s$ . Note that the logarithmic accuracy of N<sup>4</sup>LL ( $\alpha_s^3$ ) relies on the availability of N<sup>3</sup>LO PDFs, in particular on the four-loop DGLAP evolution. PDFs at this order are so far only available by the MSHT group in an approximation [54]. The ingredients of our calculation and the checks we have performed are detailed below.

**Resummation.** The implementation of the resummation formalism follows our study on  $Z$  production [50], it is based on the SCET-derived  $q_T$ -factorization theorem developed in refs. [55–57] and originally implemented to N<sup>3</sup>LL as **CuTe-MCFM** in ref. [51]. Large logarithms  $\log(q_T^2/Q^2)$  are resummed through RG evolution of hard- and beam functions in a small- $q_T$  factorization theorem. Rapidity logarithms are directly exponentiated through the collinear-anomaly formalism.

The transition from the resummation which is valid only at small  $q_T$ , to fixed-order predictions at large  $q_T$ , is achieved through the use of a transition function that smoothly interpolates between those two regions [51]. The overlap between fixed-order and resummation is removed through a fixed-order expansion of the factorization formula. The difference between these two parts is referred to as matching corrections. While for  $Z$  production matching corrections quickly approach zero for  $q_T \rightarrow 0$ , even at the level of  $\alpha_s^3$ , we find that for  $W$  production they are at the level of a few percent even at relatively small  $q_T$  (as described in detail in section 2.1 below).

The higher-order ingredients in the resummation calculation are identical to those in  $Z$  production. We briefly summarize the most important ones here, and refer for more details to our implementation of  $Z$  production [50]. Three loop transverse momentum dependent beam functions that allow for resummation at the level of N<sup>3</sup>LL' have been calculated in refs. [58–60]. We include

<sup>1</sup>The logarithmic accuracy of N<sup>4</sup>LL is only reached within the limitations of current N<sup>3</sup>LO PDF approximations.

the four loop rapidity anomalous dimension [61, 62], which together with N<sup>3</sup>LO PDFs allows for N<sup>4</sup>LL resummation. The five-loop cusp anomalous dimension is numerically negligible, which is true already at the four-loop level.

The formalism of refs. [55–57] further employs an additional power counting that improves the resummation at small  $q_T$  [56] and avoids a Landau pole prescription, as the relevant scales are always set in the perturbative regime. Implementing this requires the inclusion of higher-order terms of the beam functions which were reconstructed from the beam-function RGEs in ref. [50].

The hard function entering the factorization formula consists of  $\overline{\text{MS}}$ -renormalized virtual corrections. For  $Z$  production these are more complicated due to separate singlet and axial-singlet contributions, which are not present for  $W$  production. The hard function is therefore solely given by the three-loop (vector) form-factor [63–65].

**Fixed order.** To obtain fixed-order N<sup>3</sup>LO results we use  $q_T$  slicing, which was implemented in ref. [50] using the same factorization theorem and ingredients as laid out above for the resummation.

The  $W$ +jet NNLO calculation, which is necessary above the  $q_T$  slicing cutoff, is based on ref. [66] using 1-jettiness subtractions [66–68] and the 1-jettiness soft-function of refs. [69, 70]. We have thoroughly cross-checked all elements of this calculation. For example, we find agreement between all amplitude expressions and RecoLa [71]. Further checks were performed as part of the validation of  $Z$ +jet production, for example a re-implementation of the non-singlet hard function using refs. [72–74] that was originally taken from the code PeTeR [75, 76]. We have identified and corrected small inconsistencies in the original implementation of NLO subtraction terms in the  $W$ +2 jet process [77], a component in our above-cut calculation. We have further checked our final NNLO  $W$ +jet results against a fully independent calculation presented in ref. [78].

## 2.1. Cutoff effects.

We study fixed-order and resummed results, which are both affected by cutoff effects in different ways. The

resummed calculation requires a cutoff of the matching corrections, while our N<sup>3</sup>LO fixed-order calculation is based on a nested slicing approach, regularizing N<sup>3</sup>LO singularities using  $q_T$  subtractions. In both cases the NNLO  $W$ +jet calculation is performed using 1-jettiness slicing, and therefore, unlike for the local antenna subtractions used in the NNLOjet calculation [31], we have to particularly pay attention to residual slicing cutoff effects.

For the plots here and throughout the results section, we use the cuts of a recent ATLAS study [16] shown in eq. 1 in section 3. The choice of symmetric  $q_T$  cuts on the  $W$  decay leptons makes the calculation numerically challenging [79], even when including linear power corrections in the  $q_T$  slicing method.

Unlike for  $Z$ -boson production [50] the cutoff effects are not negligible at the order of  $\alpha_s^3$  for cutoff values of 3 GeV to 5 GeV that we achieve here. We therefore take care to display the limitations of this throughout our results.

**Cutoff effects in the resummation.** We first discuss cutoff effects in the matched resummed result. In fig. 1 we show the matching corrections relative to the purely resummed  $\alpha_s^3$  result at different orders in  $\alpha_s$ . While they are small and quickly approach negligible levels at lower orders, the  $\alpha_s^3$  corrections are substantial.

At lower orders we use a matching corrections cutoff of 1 GeV, with negligible impact on the results, while at  $\alpha_s^3$  we use a cutoff of 3.16 GeV and a  $q_T$ -dependent dynamic jettiness cutoff of the NNLO  $W$ +jet calculation of  $\frac{0.03}{1050} \sqrt{q_T^2 + m_U^2}$ , so about 0.002 GeV at small  $q_T$ . Lower values of  $q_T$  would require smaller values of a jettiness cutoff, significantly increasing computational resources.

From fig. 1, the matching corrections are still about 3% around our cutoff of 3.16 GeV. We estimate the uncertainty due to missing matching corrections by multiplying the purely resummed result integrated up to 3.16 GeV by three percent. The impact of this is different in various kinematical distributions and also depends on the binning. For example in the  $W$ -boson  $q_T$  distribution in ATLAS binning [16] the first bin ranges from 0 GeV to 7 GeV. The effect of neglected matching corrections in this bin is up to about 1.5%, while

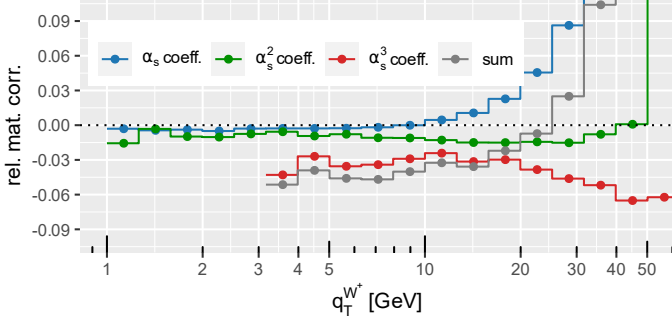


Figure 1: Matching corrections for  $W^+$  production at  $\sqrt{s} = 5.02$  TeV with fiducial cuts as in section 3.

there is no such error in the other bins. Overall, the effect of neglecting matching corrections is therefore not negligible and needs to be included in uncertainty estimates.

In distributions other than the  $W$ -boson  $q_T$  this effect is smeared out and we include it as an additional error bar. Since we know that the effect is likely to lead to a reduction of the cross-section, we display the error bar only in the downwards direction.

The size of the matching corrections also indicates where a transition function needs to switch between the resummed and fixed-order calculations. The matching corrections become sizeable beyond 40 GeV, which motivates a transition function as detailed in ref. [51] using  $x_T^{\max} = (q_T^{\max}/M_Z)^2$  with  $q_T^{\max}$  in the range 35 to 60 GeV. With this choice we find that transition uncertainties, obtained by varying the transition function ( $x_T^{\max} = 0.2, 0.4, 0.6$ ), are then comparable to uncertainties in the fixed-order and resummation regions. They are therefore insensitive to the precise range and shape of the transition.

**Cutoff effects at fixed order.** Our NNLO and N<sup>3</sup>LO fiducial fixed-order cross-sections are computed using  $q_T$  slicing. For the resummed calculation linear power corrections are included automatically through a recoil prescription [80, 81]. In the fixed-order case they have to be added separately, although this is straightforward [82].

The size of the power corrections for the  $\alpha_s^k$  coefficient relative to the full  $\alpha_s$  result is shown in fig. 2

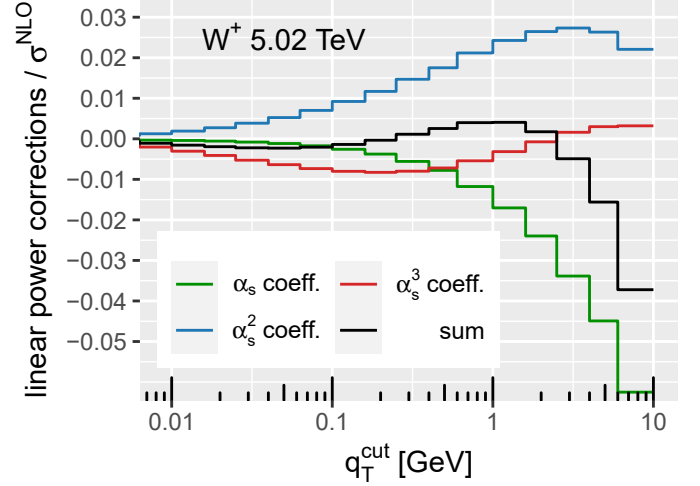


Figure 2: Linear power corrections for  $W^\pm$  production at  $\sqrt{s} = 5.02$  TeV relative to the NLO cross-section, with input parameters as in section 3.

for  $W^+$  production, as a function of the  $q_T$  slicing cutoff,  $q_T^{\text{cut}}$ .<sup>2</sup> At NNLO we use a  $q_T$  slicing cutoff of 1 GeV with linear power corrections on the cross-section at the level of 2%. For the N<sup>3</sup>LO coefficient we use a cutoff of 3.16 GeV and the power corrections are a few per-mille. The large size of the power corrections is an effect of the symmetric lepton cuts [79].

Our final fiducial N<sup>3</sup>LO corrections are obtained from a  $q_T^{\text{cut}}$  extrapolation, taking into account that subsequently smaller  $\tau_{\text{cut}}$  values are necessary for small  $q_T^{\text{cut}}$  in the inner  $W$ +jet NNLO calculation. This is shown in fig. 3 for  $W^+$  production. The dependence for  $W^-$  is qualitatively the same. Note that the  $\tau_{\text{cut}}$  dependence is modified by the dynamic choice, our default is  $\tau \propto \sqrt{q_T^2 + m_l^2}$ . Other choices for the functional dependence of  $\tau$ , such as  $\tau \propto q_T$ , may lead to improved performance. We leave a detailed investigation of such choices to a future publication.

The solid red line is from one possible fit of the expected asymptotic behavior. Smaller values of  $q_T^{\text{cut}}$  would be desirable, and there is some uncertainty of the fit that can be exposed for example by varying the number of terms that are included. The overall pattern is that for too large  $\tau_{\text{cut}}$  the cross-section diverges towards more negative values. We find that at the smallest  $q_T^{\text{cut}}$  value

<sup>2</sup>The relative power corrections for  $W^-$  production are virtually identical.

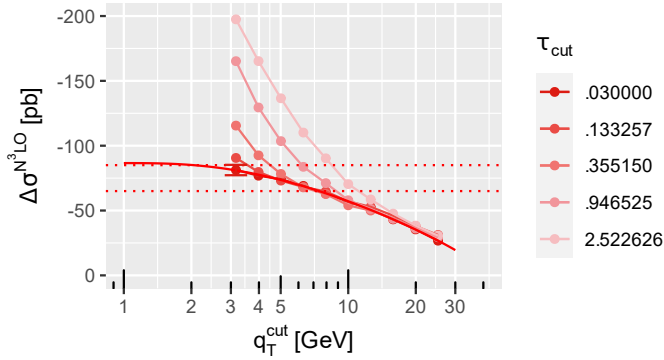


Figure 3:  $q_T^{\text{cut}}$  and  $\tau_{\text{cut}}$  extrapolation of the fiducial N<sup>3</sup>LO  $W^+$  cross-section coefficient at 5.02 TeV (not including the small linear power corrections). The error bar denotes the estimated numerical uncertainty. The solid red line is the result of a fit. The dotted red lines are an estimate of our overall uncertainty.

the two smallest  $\tau_{\text{cut}}$  values are not fully overlapping and therefore we might miss a further slight flattening of the curve that would impact the fit. In addition to the fit uncertainty, the red error bar denotes our numerical uncertainty that affects all points.

Overall we assign a 0.5% technical uncertainty on the fiducial cross-section, which is visualized by the dotted red bars in fig. 3. Our central value is obtained at  $q_T^{\text{cut}} = 4 \text{ GeV}$ . This technical uncertainty includes our numerical statistical uncertainty and the uncertainty in  $q_T^{\text{cut}}$  and  $\tau_{\text{cut}}$  extrapolation. It is large enough to cover the data points from  $3.16 \text{ GeV} < q_T^{\text{cut}} < 7 \text{ GeV}$ .

As an additional check of our setup we worked to reproduce the total inclusive cross-section that can be calculated conveniently with the code `n3lox`s [30, 83]. We find agreement, but unfortunately this calculation is numerically particularly challenging in our nested slicing approach. In the fiducial case the power corrections are more manageable, which allowed us to obtain results with a numerical and slicing error of less than 0.5%. In the total inclusive case the power corrections are larger, but also the numerical integration turns out to be more difficult. We find agreement within a combined uncertainty of about 1%. Given that the total N<sup>3</sup>LO corrections are about  $-1.7\%$  for the choice of invariant mass range and input parameters that we took, this is not as strong of a cross-check as we would have liked. We

also computed the total inclusive cross-section including the effect of  $q_T$  resummation and find an inconclusive difference of  $-1.8\% \pm 1.2\%$  (here also an uncertainty from the transition function enters).

We used about 50000 NERSC Perlmutter core hours (64 nodes for 12 hours) to compute the total inclusive N<sup>3</sup>LO cross-section to the precision of 1%. While these are not huge resources in the context of current calculations [84] which can go into the millions of core hours, reducing the numerical error by a factor of one third would already ten-fold these resource requirements, since Monte-Carlo integration uncertainties decrease like the square root asymptotically. The large power corrections further demand smaller cutoffs to reduce the slicing uncertainty, which also contributes sizeably in the total. Accounting for power corrections in the  $W + \text{jet}$  NNLO calculation provides a possible path for reducing the overall slicing uncertainty [85, 86]. However, ultimately a more efficient approach such as a local subtraction procedure, and ideally local N<sup>3</sup>LO subtractions, will be necessary to substantially reduce these uncertainties.

### 3. Results

In this section we show fiducial cross-sections and distributions for  $W^+$  production at  $\sqrt{s} = 5.02 \text{ TeV}$  corresponding to the ATLAS analysis in ref. [16]. The fiducial cuts for this analysis are,

$$q_T^\ell > 25 \text{ GeV}, \quad q_T^\nu > 25 \text{ GeV}, \quad \left| \eta^\ell \right| < 2.5, \\ m_T = \sqrt{2q_T^\ell q_T^\nu (1 - \cos \Delta\phi_{\ell\nu})} > 50 \text{ GeV}. \quad (1)$$

Note that the theory predictions used in that analysis are at a lower order and do not include any uncertainties. We include the corresponding plots for  $W^-$  production in appendix A since overall the relative perturbative corrections are very similar.

For all our predictions we use the  $G_\mu$  scheme with  $m_W = 80.385 \text{ GeV}$ ,  $m_Z = 91.1876 \text{ GeV}$  and  $G_F = 1.6639 \times 10^5 \text{ GeV}^{-2}$ , as well as  $\Gamma_W = 2.091 \text{ GeV}$ . Different scheme choices can be used to estimate the effect of higher-order electroweak corrections [87] which we do not consider here. We examine the impact of various PDF sets, extracted at different perturbative orders, in our study and will make clear which PDF set is used in each prediction.

In the following, the label  $\alpha_s^k$  at fixed-order denotes  $N^k\text{LO}$ , while for the resummed cross-section it denotes  $N^{k+1}\text{LL}+N^k\text{LO}$ , that is  $\alpha_s^k$  in an RG-improved power counting where large logarithms  $\log(q_T^2/Q^2)$  are counted as  $1/\alpha_s$ . Note that only the prediction using the `MSHT20an3lo` PDF set [54] reaches approximately  $N^4\text{LL}$  logarithmic accuracy, within the limitations of its approximations. For the NNLO PDF sets the effect from missing four-loop splitting functions degrades the formal accuracy to  $N^3\text{LL}$ , see ref. [50].

As well as the fiducial cross section, we choose to discuss the  $W$ -boson transverse momentum, charged-lepton transverse momentum and  $W$ -boson transverse mass distributions, which are of particular interest for the  $W$ -boson mass analyses.

### 3.1. Fiducial cross-sections

We start with a discussion of the total fiducial cross-section that we compare with the recent 5.02 TeV ATLAS measurement [16]. We compare predictions of different perturbative orders in  $\alpha_s$  at fixed order as well as including the effect of  $q_T$  resummation at the respective logarithmic order. Even at the level of the fiducial cross section this is interesting because, in the presence of symmetric lepton  $q_T$  cuts such as those in this analysis (c.f. equation (1)), one expects some difference between resummed and fixed-order predictions due to a strong sensitivity to unphysically low momentum scales [88].

For our theory predictions we match the PDF order with the perturbative cross-section order for consistency, using `MSHT20` PDF fits with  $\alpha_s(m_Z) = 0.118$  [54, 89]. This is particularly important for the logarithmic accuracy in the resummation. Uncertainties associated with missing higher order effects are estimated by performing scale variations following the procedure of ref. [50] for the Drell-Yan process. These are symmetrized based on the maximum excursion for simplicity. We also take into account uncertainties from the PDF determination, which for the case of the `MSHT20` approximate  $N^3\text{LO}$  PDF set [54] accounts for missing higher-order effects within the PDF in addition to uncertainty arising from the fitting procedure. Furthermore, as discussed in section 2.1, our  $\alpha_s^3$  results have an additional uncertainty of 0.5% that covers our remaining uncertainty due to cutoff effects in the slicing procedure and matching corrections.

Our results, and comparison to the ATLAS measurement, are shown in fig. 4.<sup>3</sup> We find that perturbative corrections are small for both the fixed-order and resummed results. Ultimately this is due to the effect of the (approximate)  $N^3\text{LO}$  PDFs, which we discuss in the following. Scale uncertainties at  $N^3\text{LO}$  (<1%) are at the level of NNLO uncertainties, which is a feature already observed in the literature in both inclusive and differential cases [30, 39]. On the other hand, scale uncertainties for our resummed results consistently decrease, but are still about 2% at order  $\alpha_s^3$ .

The  $\alpha_s^3$  fixed-order and resummed cross-sections are marginally compatible, and overall uncertainties from both predictions are still too large to indicate a significant difference that would indicate the need for resummation [88]. In addition, the experimental precision is limited by about a 1% luminosity uncertainty and is compatibly with both predictions. Of course a direct window on this issue comes from a comparison of predictions with the measurement of the  $q_T^W$  distribution since the bulk of the cross-section comes from the region of small  $q_T$  where resummation is required. We defer this discussion to section 3.2.

**PDF dependence.** We now extend our discussion by considering PDFs from different groups, where it is not possible to consistently match the order in  $\alpha_s$  of the PDF fit to that of the perturbative calculation. In this section we therefore use the same PDF set at each order of the perturbative calculation. We will compare results using the `MSHT20nnlo_as118` PDF set with those from the NNLO determinations `MSHT20nnlo_as118`, `CT18NNLO` [90] and `NNPDF40nnlo_as_0118` [91].

Our results are shown in fig. 5. We first focus on the fixed-order and resummed  $\alpha_s^3$  predictions using `MSHT20` at NNLO and  $aN^3\text{LO}$ . Since the same data was used in both PDF fits, the difference between these predictions therefore solely results from higher-order corrections in the PDF and the inclusion of  $N^3\text{LO}$  K-factors in the predictions of some cross-sections. The  $aN^3\text{LO}$  PDF increases the  $\alpha_s^3$  results by about 3% compared to using the NNLO PDF. This is a significant deviation also in terms of the PDF uncertainties. Without taking into account the  $aN^3\text{LO}$  PDFs we would conclude a cross-

<sup>3</sup>We have added the ATLAS systematic and statistical experimental uncertainties in quadrature.

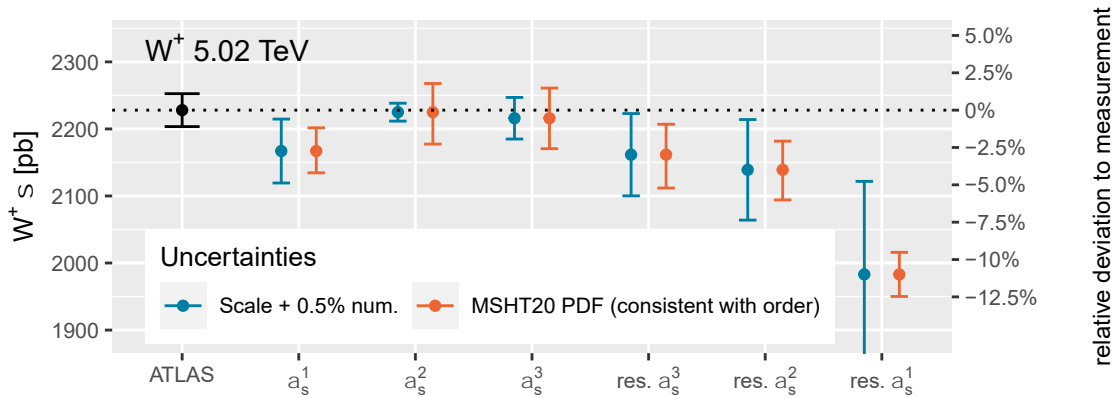


Figure 4:  $W^+$  cross-sections in comparison with the ATLAS 5.02 TeV measurement [16]. Error bars show uncertainties from scale variation and from the MSHT20 PDF sets corresponding to the perturbative order. The  $\alpha_s^3$  results have an additional numerical and cutoff uncertainty of 0.5% that we added linearly to the scale uncertainties for display.

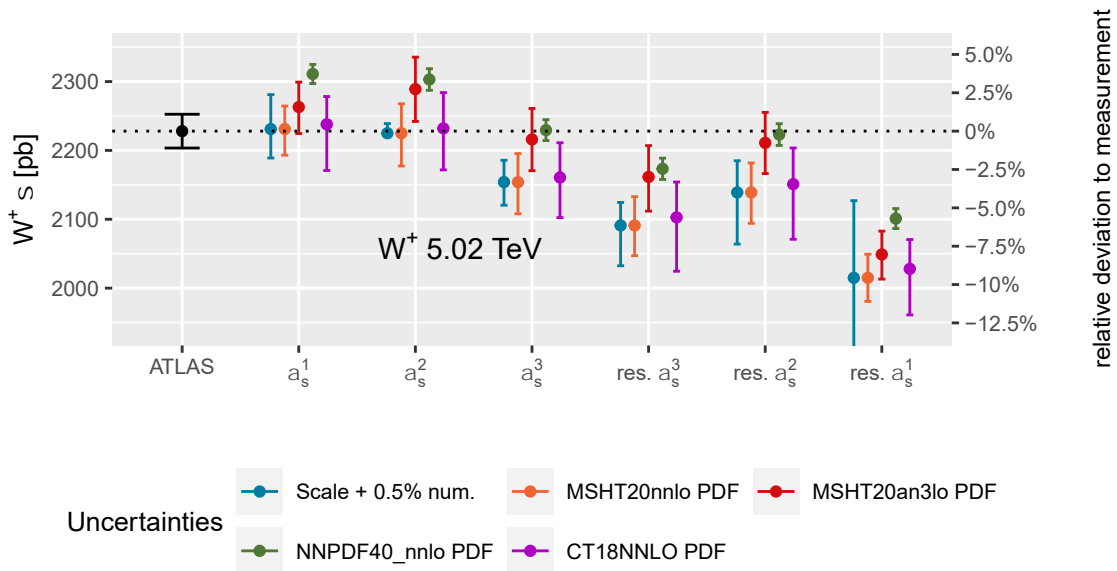


Figure 5:  $W^+$  cross-sections in comparison with the ATLAS 5.02 TeV measurement [16]. Error bars show uncertainties from scale variation and from different PDF sets with their respective uncertainties. The  $\alpha_s^3$  results have an additional numerical and cutoff uncertainty of 0.5% that we have added linearly to the scale uncertainties for display.

section decrease of about 3.2% at fixed-order and about 2.3% using the resummed result.<sup>4</sup> This is similar in size to the effects observed in previous calculations of this process more inclusively, where the same PDF set has been employed across all orders of the calculation [30, 39].

The size of the aN<sup>3</sup>LO PDF effects, and the delicate cancellation between different partonic channels, indicates that a consistent order is important, see also fig. 4. On the other hand the NNPDF4.0 NNLO PDF set is consistent with the larger cross-sections of the MSHT20 aN<sup>3</sup>LO PDF. This leads to the question of what impact N<sup>3</sup>LO effects will have in a NNPDF4.0 fit. Judging by the pattern in MSHT20 one would expect a sizeable positive shift, which would lead the fixed-order prediction to overshoot the measurement. From this it is clear that the improvement of PDFs is a priority for precision predictions and measurements of this process.

To conclude the discussion of the fiducial cross-section, we find that theory uncertainties are overall at the level of 3-5% for the  $\alpha_s^3$   $W$ -boson cross-section. This includes scale uncertainties, the envelope of PDF uncertainties and the difference between fixed-order and resummed results. All uncertainties are at the same level and it will require effort in all directions to significantly reduce the overall theoretical uncertainty. In particular it will require careful investigation of PDFs in terms of higher-order effects and systematics as well as the estimation of statistical uncertainties, which vary significantly between modern PDF sets. The comparison of differential cross-sections is likely to shed further light on these issues, which we discuss further when comparing differential PDF uncertainties in section 3.4.

### 3.2. Transverse momentum distributions

Moving towards differential quantities, we show the  $W^+$  transverse momentum distribution at  $\sqrt{s} = 5.02$  TeV in fig. 6. Results for  $W^-$  production can be found in fig. 12 in the appendix. To highlight the effect of short-distance corrections, we use the MSHT20nn1o\_as118 PDF set for all orders of our predictions in this and in section 3.3.

We neglect negative matching corrections of up to about 3% below 3.16 GeV from  $\alpha_s^3$  contributions. This can

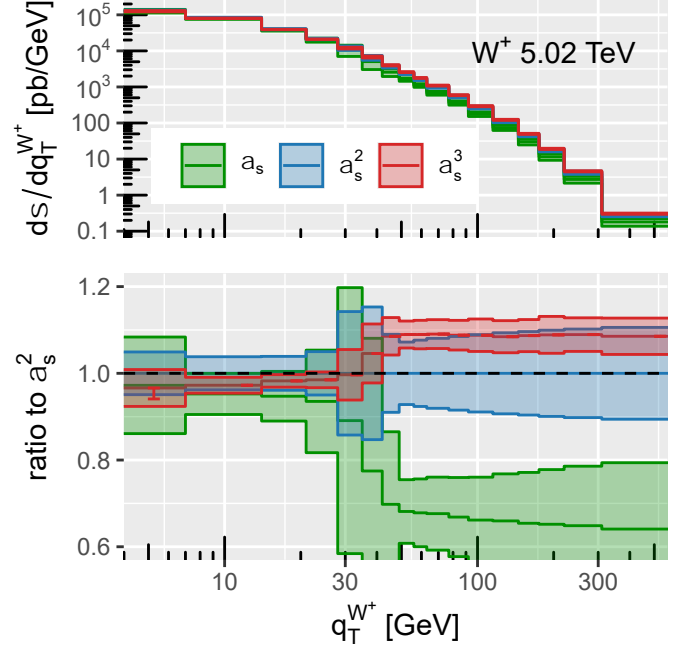


Figure 6:  $W^+$  transverse momentum distribution at 5.02 TeV using the PDF set MSHT20nn1o\_as118 throughout.

amount to an effect of up to about 1.5% in the first bin in the experimental distribution which ranges up to 7 GeV. We show the effect of neglected matching corrections with an additional error bar. Since it is expected that the effect of the neglected matching corrections is negative we plot an error bar of 1.5% only in the negative direction, the amount by which the predictions could shift downwards. Note that our numerical precision in the first bin is similar, at the level of 1%.

Furthermore, towards very small  $q_T$  scale uncertainty estimates become potentially unreliable, since a downward variation requires a cutoff to not reach into the non-perturbative regime. This further motivates a symmetrization of uncertainty bars, in addition to the fact that distinguishing between and up- and downwards scale variation is unphysical.

Overall the relative corrections are, as expected, very similar to our Drell-Yan results [50] with about 10% corrections in the tail from fixed-order, and smaller corrections at small  $q_T$  through higher-order resummation. Uncertainties consistently decrease at higher orders, including the uncertainty associated with transi-

<sup>4</sup>Note that our numerical and cutoff uncertainty is about 0.5%.



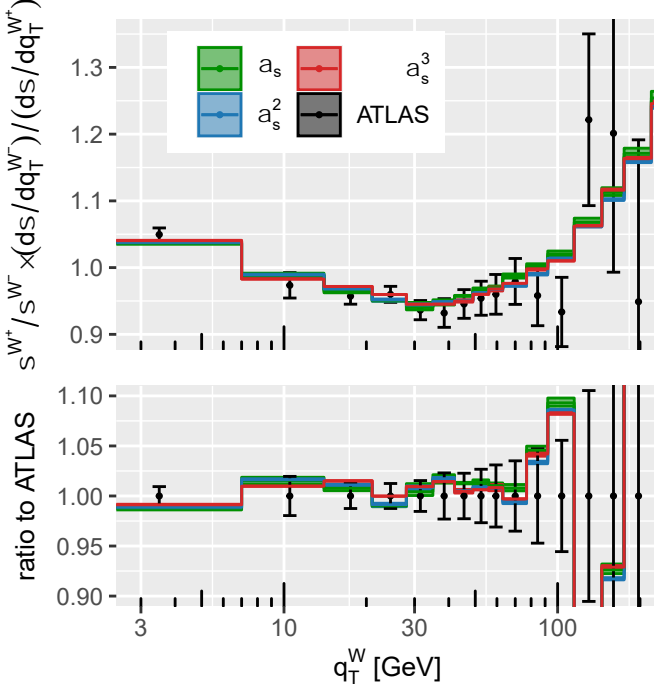


Figure 7: Ratio of  $W^+/W^-$  transverse momentum distributions in comparison with the 5.02 TeV ATLAS measurement [16] using the PDF set MSHT20nnlo\_as118 throughout.

tioning between fixed-order and resummation around 30 GeV to 50 GeV which shrinks considerably at higher orders. The uncertainty bands almost completely overlap between  $\alpha_s^3$  and  $\alpha_s^2$ , indicating a stabilization of the perturbative series over the whole range. N<sup>3</sup>LO effects in the PDF mostly cause a positive shift at small  $q_T$  in the first two bins, see section 3.4.

In fig. 7 we show the ratio of normalized  $W^+$  to  $W^-$  transverse momentum distributions. For this distribution we were able to digitize the plotted ATLAS measurements in ref. [16]. We find that the predictions at all orders are compatible within 1% of numerical noise. We also find excellent agreement with the measurement, as already observed in ref. [16].

The agreement of the three perturbative orders within numerical uncertainties indicates that to estimate uncertainties one should compute the ratios in a correlated way, as we have done. This leads to uncertainties smaller than 1%, negligible in comparison with measurement uncertainties and their difference to the predictions.

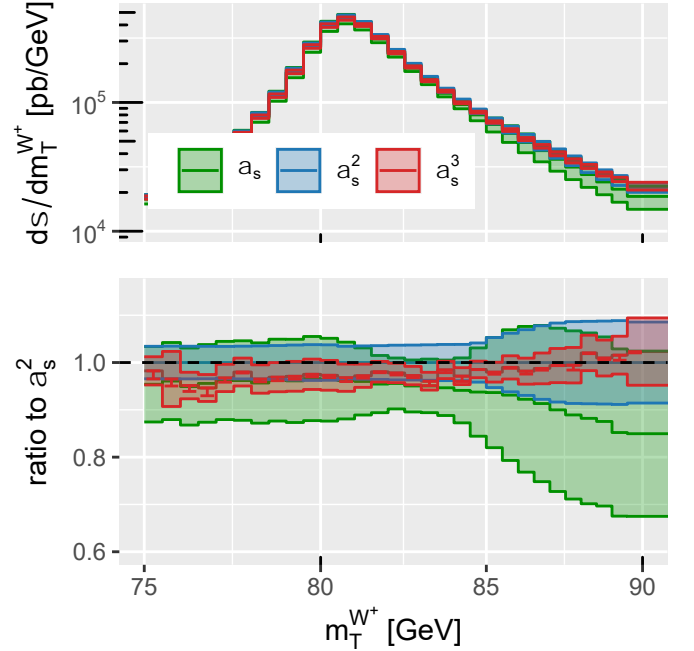


Figure 8:  $W^+$  transverse mass distribution using the PDF set MSHT20nnlo\_as118 throughout.

### 3.3. Transverse mass and charged lepton transverse momentum distributions

We present the transverse mass distribution for  $W^+$  in fig. 8 and for  $W^-$  in fig. 13 in the appendix. While this distribution could comfortably be computed at fixed order, we only show predictions including the effect of  $q_T$  resummation here.

Perturbative corrections at  $\alpha_s^3$  are flat in the peak region and therefore, as expected from the total fiducial results presented earlier, negative and about 2%. Note that this statement is based on using NNLO PDFs throughout. As we will show in section 3.4, moving towards the aN<sup>3</sup>LO set, one observes a considerable shift of about 5% below the peak.

In fig. 9 we show the  $W^+$  lepton transverse momentum distribution at different orders in  $\alpha_s$  (c.f. fig. 14 in the appendix for  $W^-$ ). This distribution is particularly important for the  $W$ -boson mass determination, since it is sensitive to  $m_W$  and does not depend on missing-energy estimations. While typically used in template fits, a new asymmetry observable based on this distribution has recently been proposed in ref. [92].

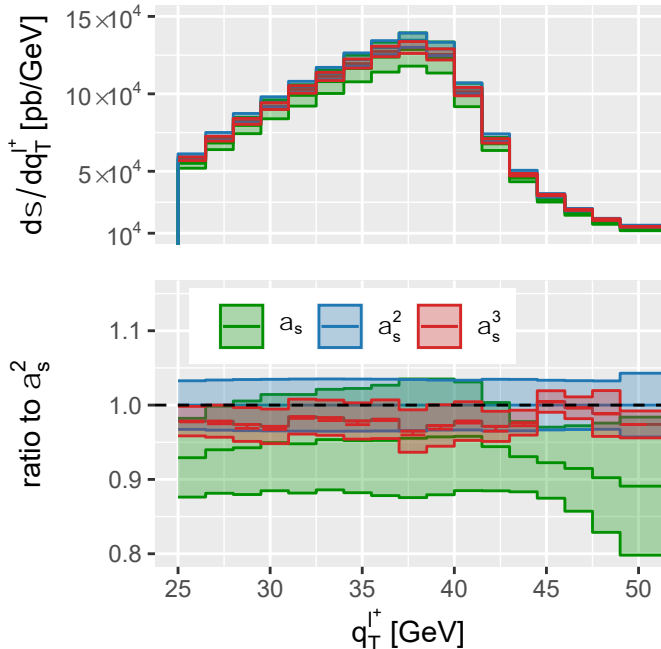


Figure 9:  $W^+$  lepton transverse momentum distribution using the PDF set MSHT20nnlo\_as118 throughout.

Within numerical bin-to-bin fluctuations the  $\alpha_s^3$  corrections are flat with smaller uncertainties. This statement also holds while using the aN<sup>3</sup>LO PDFs and other sets, at least below the peak region.

### 3.4. Impact of PDFs

As already indicated by the fiducial cross-sections in section 3.1, PDFs are among the biggest limitation in precise predictions. Most insight will be gained by studying differential distributions.

In fig. 10 we show the impact of four modern PDF sets for the  $W^+$  transverse momentum, charged lepton transverse momentum, and transverse mass distributions. These relative PDF uncertainties are computed using  $\alpha_s^2$  matrix elements. The differences to  $\alpha_s^3$  are at the per-mille level and insignificant for this discussion. Even using  $\alpha_s$  matrix elements leads to qualitatively the same conclusions [52]. Further, the results for  $W^-$  production are virtually the same, but are included for completeness in the appendix in fig. 15.

Effects from different PDF sets can be significant, de-

pending on the distribution and region up to 10%. The MSHT and CTEQ NNLO PDF sets are broadly similar, which are the sets considered in the ATLAS study in ref. [16], in addition to NNPDF3.1.

The most interesting comparison is between MSHT NNLO and MSHT aN<sup>3</sup>LO as a higher-order effect, and then considering NNPDF4.0 NNLO. We find significant shape changes for several distributions when utilizing these PDF sets.

The effects of using MSHT aN<sup>3</sup>LO are even more important differentially than inclusively, inducing a significant cross-section increase below the peaks for the transverse mass and lepton  $q_T$  distributions, while dropping off beyond. In the  $W$ -boson transverse-momentum distribution the most significant change is a positive shift of about 7% in the first bin containing the Sudakov peak. PDF uncertainties even range up to 10%. Clearly such a range can be constrained within QCD uncertainties in future fits, and even precise Drell-Yan measurements and predictions [50] will constrain this significantly.

Predictions using NNPDF4.0 NNLO for  $m_t^W$  and  $q_T^l$  are much flatter with respect to MSHT20 NNLO, except for  $q_T^W$ , which predicts a similar enhancement of about 5% in the first bin, but drops off slower than MSHT20 aN<sup>3</sup>LO.

## 4. Conclusions & Outlook

In recent years the experimental precision of  $Z$  and  $W$ -boson production has reached new levels at the LHC. In particular this has been achieved through better measurements of the luminosity uncertainty which is now down to 1%. Precise measurements of  $W$ -boson kinematics enter many Standard Model inputs like the weak mixing angle, parton distribution functions, and  $W$ -boson mass. At the same time, theoretical calculations have become more advanced, reaching new levels of precision in fixed-order and resummed predictions, and allowing for more refined PDF determinations. However these calculations have presented new challenges for performing precision measurements. N<sup>3</sup>LO QCD corrections are surprisingly large, at the level of minus 2-3% [30] (disregarding the effect of N<sup>3</sup>LO PDFs), and more statistically precise PDF fits begin to reveal systematic discrepancies that are challenging to reconcile.

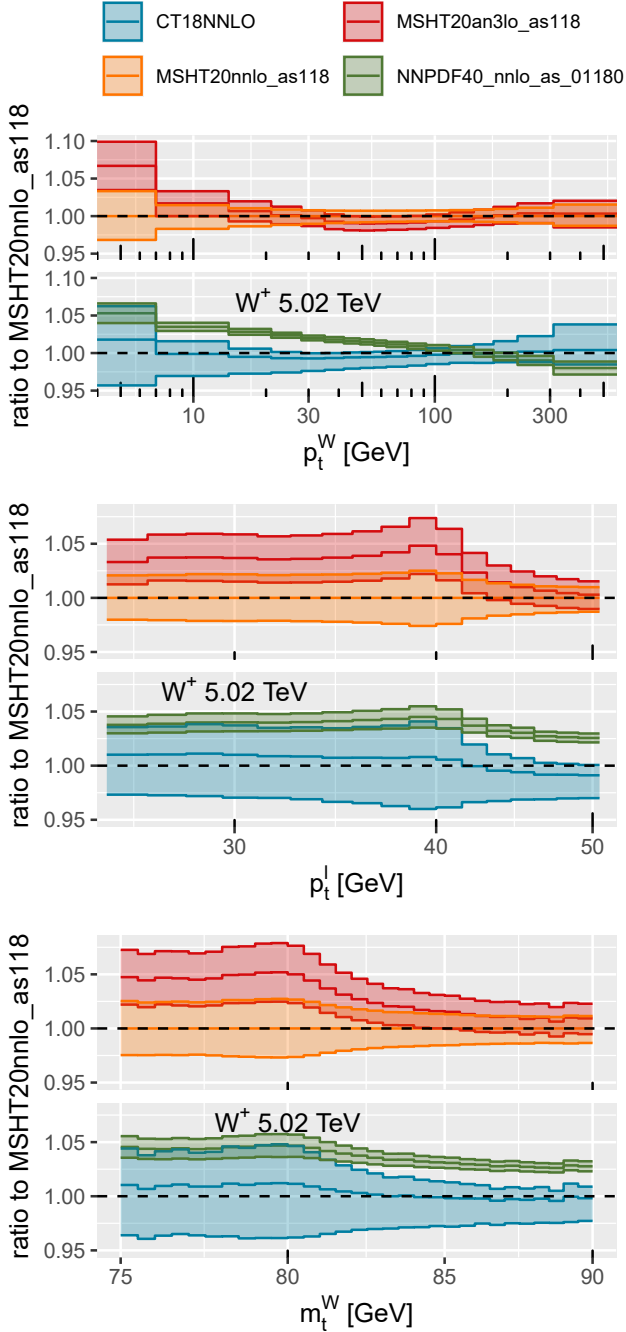


Figure 10: Relative PDF uncertainties of different observables for  $W^+$ . Note that MSHT20an3lo includes uncertainties from missing higher orders, which are not included in the other sets.

**Conclusions.** In this paper we have presented a calculation of  $W$ -boson production at the level of  $\alpha_s^3$  at fixed-order and including the effect of  $q_T$  resummation. We find that the size of the corrections depends crucially on whether higher-order corrections are included consistently in the short-distance calculation and the PDFs. We considered MSHT20 PDFs, which, taken at NNLO, lead to results with about  $-3\%$  corrections at  $\alpha_s^3$ . On the other hand using the aN<sup>3</sup>LO version [54] to consistently match the matrix-element we find that corrections are less than half a percent, which is about our numerical precision. When comparing with data, it seems important to consistently match orders. Other PDF sets like NNPDF4.0 overshoot the experimental measurement at NNLO, but then match it at N<sup>3</sup>LO due to the large corrections. Whether N<sup>3</sup>LO effects in other PDF fits lead to a similar increase as for MSHT20, and how the systematic differences between them are resolved, will have to be studied in detail in the future.

Our fiducial cross-sections with  $q_T$  resummation are overall smaller than at fixed order, but we find larger missing-higher-order uncertainties of about 2.5%. Here in particular the higher-order DGLAP evolution is important to achieve full N<sup>4</sup>LL accuracy and it is therefore important to match the PDF order.

We calculated differential distributions for the  $W^\pm$  transverse momentum, charged lepton transverse momentum and transverse mass distributions. While data for such distributions has not been made public yet, we have digitized the  $W^+/W^-$  transverse momentum ratio and found agreement, confirming earlier results at lower orders.

We have illustrated the impact of different PDF sets and their uncertainties for these three kinematic distributions. We find large shape differences between MSHT20 NNLO and aN<sup>3</sup>LO, but also between NNPDF40\_nnlo and the other sets considered. The differences and shape changes reach 5–10%. It is therefore clear that the precise measurements will further strongly constrain PDFs.

In the future it will be interesting to include the effect of non-perturbative corrections in the context of transverse-momentum-dependent PDFs [93–95]. In the typical formalisms the disentangling of perturbative and non-perturbative effects in a model-independent way is difficult [96], but which is simpler here since no direct

Landau pole regularization is needed [97]. A further phenomenological avenue will be to study the effect of higher-order resummation on angular coefficients which are used in experimental studies to dress parton-shower results with higher-order corrections.

Our calculation including the matching to  $W$ +jet production at NNLO will be publicly available the upcoming CuTe-MCFM release and allows for theory-data comparison at the state-of-the-art level.

**Calculational outlook.** While our calculation at  $N^3$ LO is computationally expensive compared to NLO or NNLO results, which can be computed in a short time on modern multi-core desktops, it is relatively low compared to other state-of-the-art calculations [84]. The precision reached in this paper is about 0.5% for total fiducial cross-sections, and is limited by the double-real  $W$ +jet NNLO calculation at small  $q_T$ . We have used about 1500 NERSC Perlmutter node hours for this, which translates into about 100,000 core hours.

Unfortunately we are roughly limited to a precision of 0.5% for the chosen set of fiducial cuts which have large matching corrections in the case of resummation, and require small  $q_T^{\text{cut}}$  values to reach  $q_T$ -subtraction asymptotics at fixed order. In our nested slicing approach such small  $q_T^{\text{cut}}$  values come at the price of correspondingly smaller  $\tau_{\text{cut}}$  in the  $W$ +jet NNLO slicing calculation. For example, we estimate that decreasing the cutoff to less than 1 GeV will require an order of magnitude smaller  $\tau_{\text{cut}}$ . Since Monte-Carlo integration uncertainties decrease asymptotically only like  $1/\sqrt{N}$ , where  $N$  is the number of calls (or runtime), one quickly reaches the limit of reasonable runtimes. Future improvements based on local subtractions for the  $W$ +jet NNLO calculation, and ultimately for the  $N^3$ LO cross-section will naturally improve this.

In practice the numerical uncertainty of 0.5% does not pose a problem. In the  $W$ -boson  $q_T$  distribution it only affects the first bin, typically at the level of 1% but dependent on the exact extent of the bin. In our other resummed distributions it is smeared out and still small compared to our estimation of missing-higher-order uncertainties.

**Acknowledgments.** We would like to thank NERSC for use of the Perlmutter supercomputer that enabled this calculation. This manuscript has been authored by Fermi Research Alliance, LLC under Contract No. DE-AC02-07CH11359 with the U.S. Department of Energy, Office of Science, Office of High Energy Physics. Tobias Neumann is supported by the United States Department of Energy under Grant Contract DE-SC0012704. This research used resources of the National Energy Research Scientific Computing Center (NERSC), a U.S. Department of Energy Office of Science User Facility located at Lawrence Berkeley National Laboratory, operated under Contract No. DE-AC02-05CH11231 using NERSC award HEP-ERCAP0023824 “Higher-order calculations for precision collider phenomenology”.

## A. Results for $W^-$ production

A comparison of the fiducial cross-section measurement at  $\sqrt{s} = 5.02$  TeV with our theoretical predictions for  $W^-$  production is shown in fig. 11. The relative perturbative corrections to the  $W^-$ -boson transverse momentum (fig. 12), the  $W^-$ -boson transverse mass (fig. 13), and the charged lepton transverse momentum (fig. 14) distributions are very similar to the corresponding  $W^+$  results in the main text, see figs. 6, 8 and 9. The impact of different PDF fits with uncertainties for  $W^-$  distributions is shown in fig. 15. We find that the relative positions of the individual predictions shift only slightly compared to  $W^+$  production.

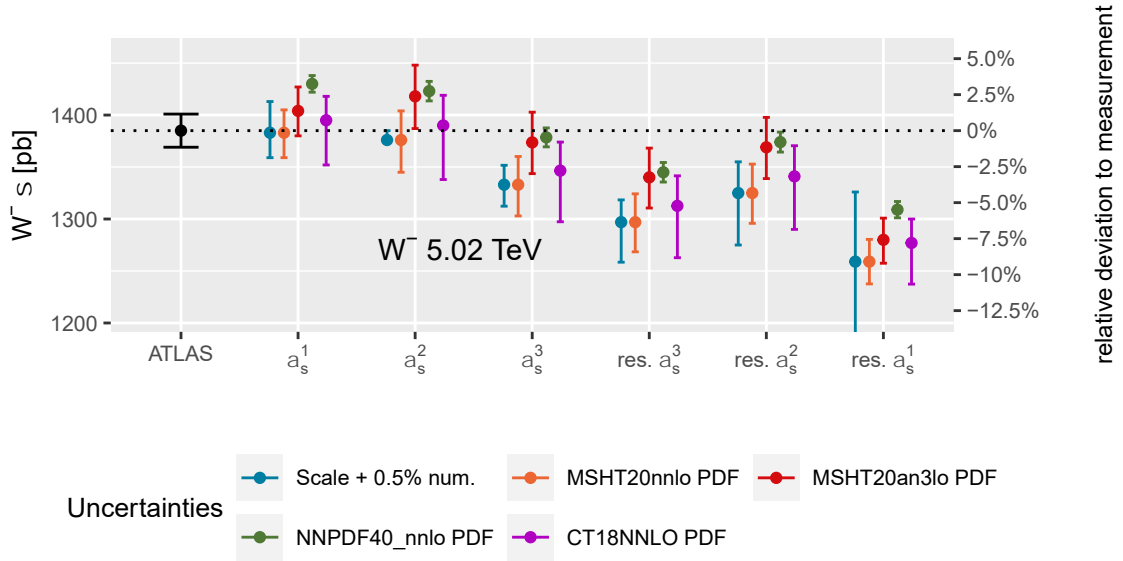


Figure 11:  $W^-$  cross-sections in comparison with the ATLAS 5.02 TeV measurement [16]. Error bars show uncertainties from scale variation and from different PDF sets with their respective uncertainties. The  $\alpha_s^3$  results have an additional numerical and cutoff uncertainty of 0.5% that we added linearly to the scale uncertainties.

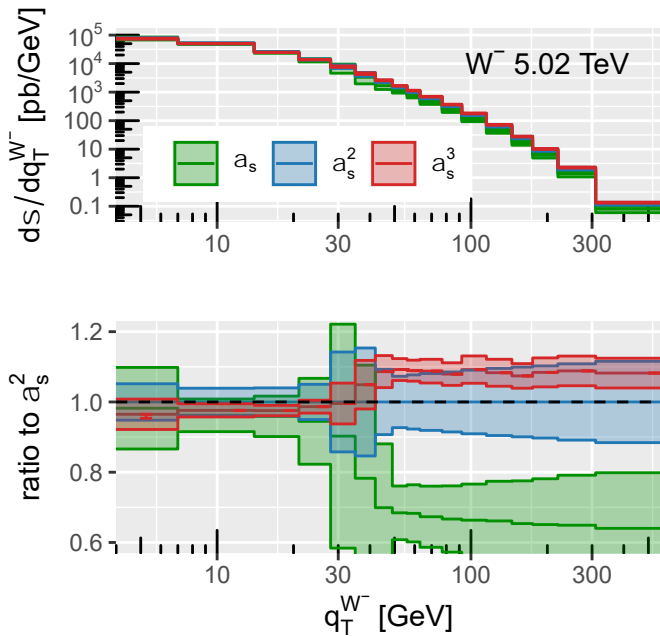


Figure 12:  $W^-$  transverse momentum distribution at 5.02 TeV.

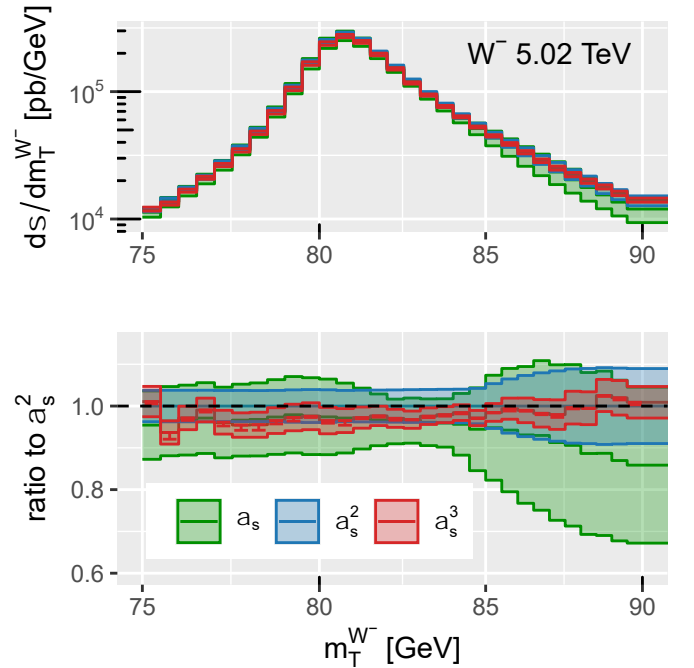


Figure 13:  $W^-$  transverse mass distribution at 5.02 TeV.

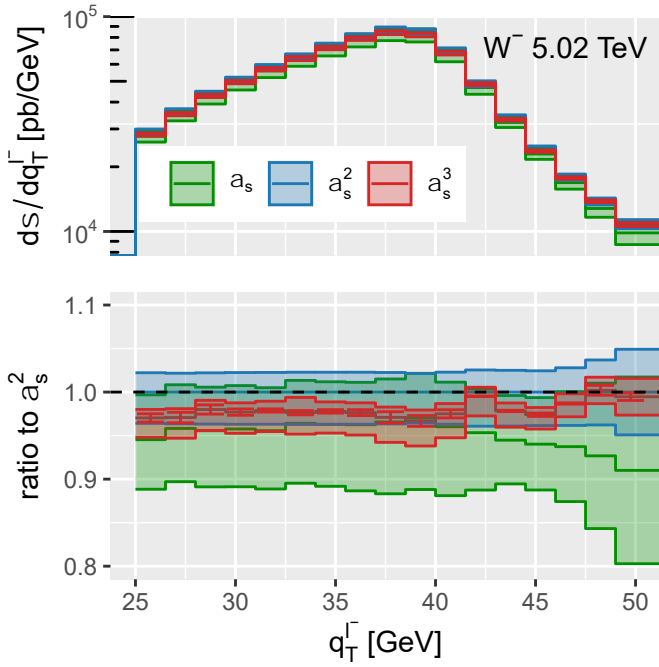


Figure 14:  $W^-$  electron transverse momentum distribution at 5.02 TeV.

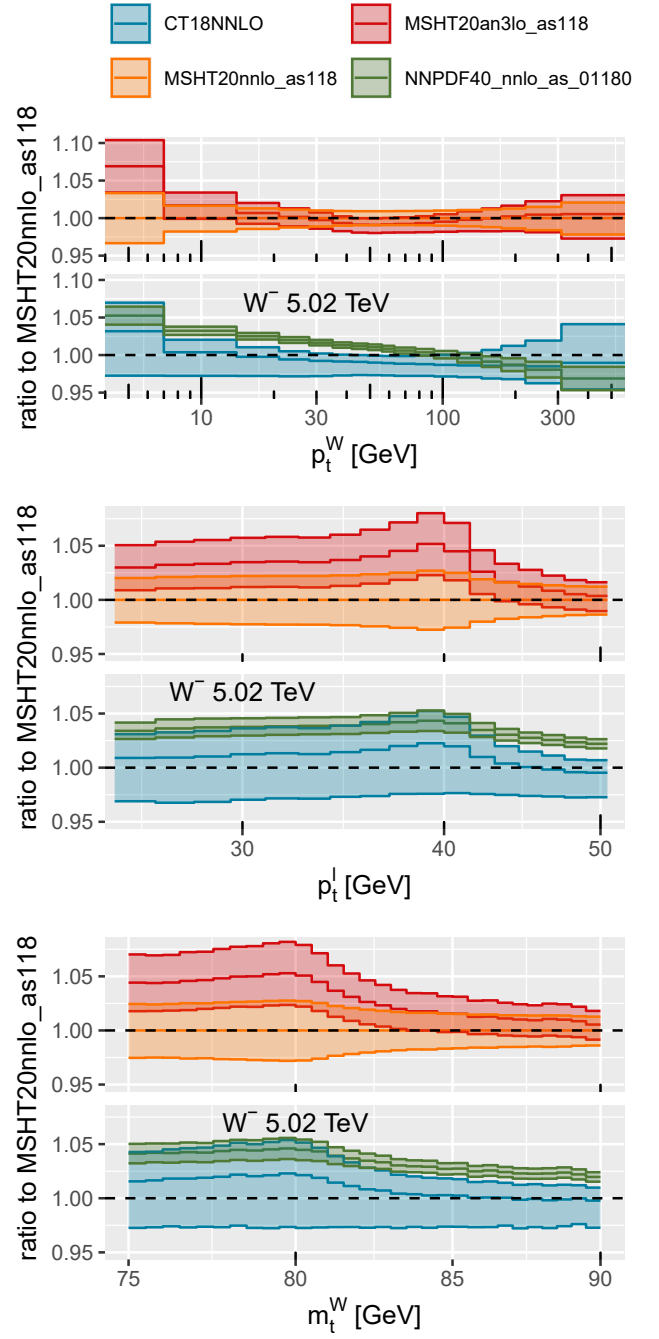


Figure 15: Relative PDF uncertainties of different observables for  $W^-$ . Note that MSHT20an3lo includes uncertainties from missing higher orders, which are not included in the other sets.

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