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Ultimate Colliders [†]

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Summary:

Our understanding of the Universe critically depends on the fundamental knowledge of particles and fields, which represents a central endeavor of modern high-energy physics. Energy frontier particle colliders – arguably, among the largest, most complex and advanced scientific instruments of modern times – for many decades have been at the forefront of scientific discoveries in high-energy physics. Due to technology advances and beam physics breakthroughs, the colliding beam facilities have progressed immensely and now operate at energies and luminosities many orders of magnitude greater than the pioneering instruments of the early 1960s.

While the Large Hadron Collider and the Super-KEKB factory represent the frontier hadron and lepton colliders of today, respectively, future colliders are an essential component of a strategic vision for particle physics. Conceptual studies and technical developments for several exciting near- and medium-term future collider options are underway internationally. Analysis of numerous proposals and studies for far-future colliders indicate the limits of the collider beam technology due to machine size, cost, and power consumption, and call for a paradigm shift of the particle physics research at ultra-high energy but low luminosity colliders approaching or exceeding 1 PeV center-of-mass energy scale.

Keywords: Particle physics, accelerators, colliders, protons, ions, electrons, muons, positrons.

Subjects: High Energy Physics, Particle Accelerators

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Contents

1 Introduction

Particle accelerators are unique scientific instruments which offer access to unprecedented energy per constituent, using well-focused, high-density beams of electrons (e^-), positrons (e^+), protons (p), antiprotons ($\bar{p}$), ions, muons (μ^+, μ^-), mesons, photons, and gamma quanta (γ), among others [Shiltsev, 2020]. Three Nobel prizes were awarded for seminal advancements in accelerator science and technology: to Ernest O. Lawrence in 1939 for invention of the first modern accelerator, the cyclotron [Lawrence and Livingston, 1932], to John Cockcroft and Ernest Walton in 1951 for their invention of the eponymous linear accelerator [Cockcroft and Walton, 1932], and to Simon van der Meer in 1984 for conceiving and developing the novel method of stochastic cooling [Van Der Meer, 1985]. Of course, highly notable are applications of accelerators - for example, they were of critical importance for about a quarter of the most acclaimed physics discoveries since 1939, resulting on average in a Nobel Prize for Physics every three years [Haussecker and Chao, 2011]. Electron microscopes and accelerator-based synchrotron radiation and spallation neutron sources were instrumental for numerous Nobel Prize-winning research achievements in chemistry, physiology and medicine, such as those recognized in 1997, 2003, 2006, 2009, 2012, 2017, 2019, and 2021.

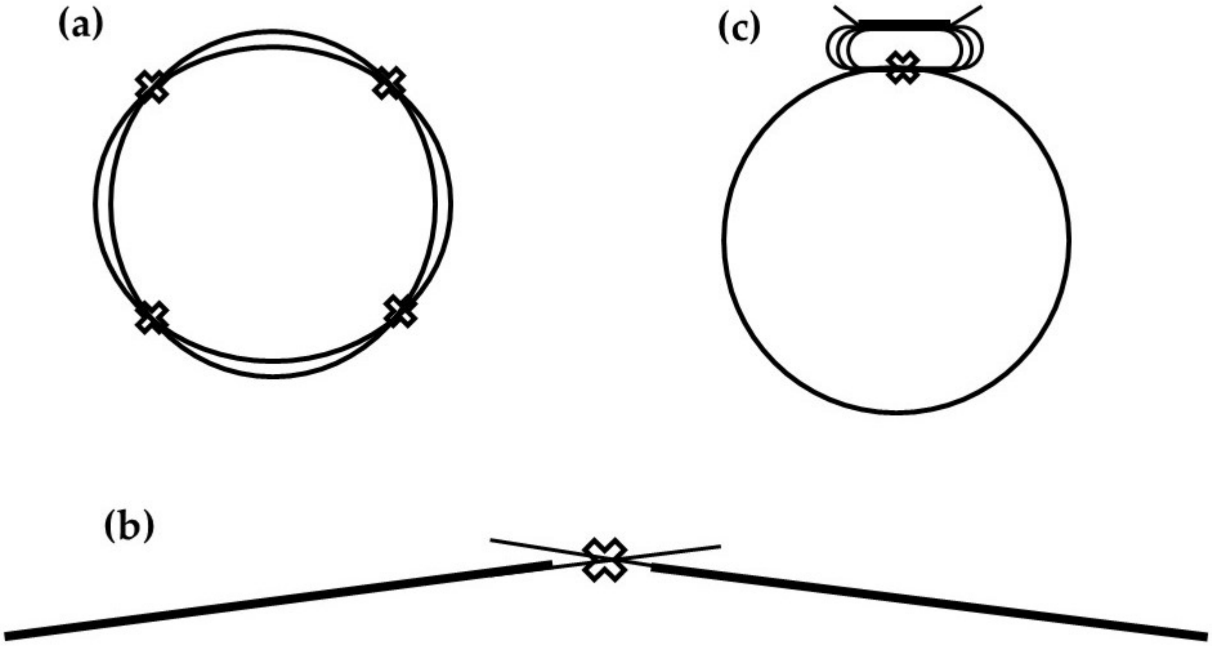

Figure 1: Schematics of most common particle collider types: a) circular, b) linear, c) ring-ERL(energy recovery linac). Beam collision points are marked by crosses.

At present, there about 140 accelerators of all types worldwide devoted to fundamental research [Faus-Golfe and Edgecock, 2017]. Among them, the most complex and technologically advanced are higher-energy accelerators and, especially, colliders for nuclear and particle physics. While they are of different sizes and shapes, based on different technologies and employing different types of particles, they have common functional elements and basic stages – charged particles are produced in dedicated sources, often go through a preparatory stage to arrange the particles in suitable beams of bunches, and then get accelerated to very high kinetic energies. (Here we will generally assume that all the particles are ultra-relativistic and their kinetic energy and full energy are the same $E = \gamma mc^2$, where m is the particle's mass, c is the speed of light, and relativistic Lorentz factor $\gamma \gg 1$.) In order to be most effective in getting insights into the interesting physics of nuclei and/or elementary particles, the beams usually get compressed in a sequence of dedicated elements, like focusing magnets, before being sent to strike other particles, causing reactions that transform the particles into new particles. Sophisticated detectors are needed to identify and analyse products of the reactions of interest.

What makes colliders distinct is the use of two similar but counter-propagating beams directed onto each other in one or several interaction points (IPs) – see Fig.1. While such arrangement makes the machines significantly more complex [Shiltsev and Zimmermann, 2021], it is fully justified by enormous kinematic advantage in so called center-of-mass energy, resulting in much larger momentum transfers and, therefore, opportunity to generate new particles of much higher masses. Indeed, for the head-on collision of two ultra-relativistic particles with same energy E, the center of mass energy (c.m.e.) is:

$$E_{cm} \approx 2E. \tag{1}$$

(The equation for unequal particle energies $E_1 \neq E_2$ is $E_{cm} \approx 2\sqrt{E_1 E_2}$). In the absence of the opposite beam, high-energy particles can be sent of a stationary target resulting in $E_{cm} \approx \sqrt{2Emc^2}$, where m is the mass of the target material particles. Take, for example, the highest energy cosmic rays observed on Earth reaching $E \sim 10^{21}$ eV, or a million PeV (1 PeV=1000 TeV=1000,000 GeV=10^{15} eV). Their collisions with stationary protons ($mc^2 \approx 1$ GeV) result in the c.m.e. of 1.4 PeV. In comparison, the same c.m.e. would be possible in a particle collider with only E=0.7 PeV=700 TeV energy per beam, i.e., with a million(!) times smaller particle energies. The highest beam and center-of-mass energies achieved to date are, of course, much lower - E=0.007 PeV and E_{cm}=0.014 PeV in the Large Hadron Collider (LHC), see Table 1 – and below we discuss the ultimate limits of particle colliders.

2 Colliders: Energy, Luminosity, History

As noted above, colliders essentially shaped modern particle physics, and 31 of them have so far reached the operational stage (some in several successive configurations), with seven operational now (2023) – see Table 1. Two colliders are under construction and almost three dozen proposals for future colliders are under discussion, some of which are also listed in Table 1. The idea of using colliding beams to gain the above mentioned kinematic advantage was first given serious consideration by the Norwegian engineer and inventor Rolf Wideröe, who in 1943 had filed a patent for the collider concept (and received the patent in 1953) [Wideroe, 1953, Waloschek, 2013], and then further developed by Donald Kerst [Kerst et al., 1956] and Gerry O'Neill [O'Neill, 1956]. In the early 1960s, almost concurrently, three earely colliders went into operation in the Soviet Union (e^-e^- collider VEP-1), France (to where the e^+e^- AdA had been moved), and the USA (e^-e^- CBX).

The first colliders, as well as all but one follow up machine, were built in a storage ring (circular) configuration – see Fig. 1a – where particles of each beam circulate in the same or two different rings and repeatedly collide. In linear colliders, first proposed in Ref. [Tigner, 1965] and realized in the 1990s in the SLAC Linear Collider (SLC), the two colliding beams are accelerated in linear accelerators (linacs) and transported to a collision point, either in a simple two-linac configuration as depicted in Fig. 1c or with use of the same linac and two arcs, as in the SLC. Other configurations

Colliders	Species	E_{cm}, GeV	C, m	$\mathcal{L}$, 10^{32}	Years	Host lab, country
AdA	e^+e^-	0.5	4.1	10^{-7}	1964	Frascati/Orsay
VEP-1	e^-e^-	0.32	2.7	5×10^{-5}	1964-68	Novosbirsk, USSR
CBX	e^-e^-	1.0	11.8	2×10^{-4}	1965-68	Stanford, USA
VEPP-2	e^+e^-	1.34	11.5	4×10^{-4}	1966-70	Novosibirsk, USSR
ACO	e^+e^-	1.08	22	0.001	1967-72	Orsay, France
ADONE	e^+e^-	3.0	105	0.006	1969-93	Frascati, Italy
CEA	e^+e^-	6.0	226	0.8×10^{-4}	1971-73	Cambridge, USA
ISR	pp	62.8	943	1.4	1971-80	CERN
SPEAR	e^+e^-	8.4	234	0.12	1972-90	SLAC, USA
DORIS	e^+e^-	11.2	289	0.33	1973-93	DESY, Germany
VEPP-2M	e^+e^-	1.4	18	0.05	1974-2000	Novosibirsk, USSR
VEPP-3	e^+e^-	3.1	74	2×10^{-5}	1974-75	Novosibirsk, USSR
DCI	e^+e^-	3.6	94.6	0.02	1977-84	Orsay, France
PETRA	e^+e^-	46.8	2304	0.24	1978-86	DESY, Germany
CESR	e^+e^-	12	768	13	1979-2008	Cornell, USA
PEP	e^+e^-	30	2200	0.6	1980-90	SLAC, USA
S$p\bar{p}$S	$p\bar{p}$	910	6911	0.06	1981-90	CERN
TRISTAN	e^+e^-	64	3018	0.4	1987-95	KEK, Japan
Tevatron	$p\bar{p}$	1960	6283	4.3	1987-2011	Fermilab, USA
SLC	e^+e^-	100	2920	0.025	1989-98	SLAC, USA
LEP	e^+e^-	209.2	26659	1	1989-2000	CERN
HERA	ep	30+920	6336	0.75	1992-2007	DESY, Germany
PEP-II	e^+e^-	3.1+9	2200	120	1999-2008	SLAC, USA
KEKB	e^+e^-	3.5+8.0	3016	210	1999-2010	KEK, Japan
VEPP-4M	e^+e^-	12	366	0.22	1979-	Novosibirsk, Russia
BEPC-I/II	e^+e^-	4.6	238	10	1989-	IHEP, China
DAΦNE	e^+e^-	1.02	98	4.5	1997-	Frascati, Italy
RHIC	p, i	510	3834	2.5	2000-	BNL, USA
LHC	p, i	13600	26659	210	2009-	CERN
VEPP2000	e^+e^-	2.0	24	0.4	2010-	Novosibirsk, Russia
S-KEKB	e^+e^-	7+4	3016	6000^*	2018-	KEK, Japan
NICA	p, i	13	503	1^*	2024(tbd)	JINR, Russia
EIC	ep	10+275	3834	105^*	2032(tbd)	BNL, USA

Proposals	Species	E_{cm}, TeV	C, km	$\mathcal{L}^*$, 10^{35}	Years	Host lab, country
FCCee	e^+e^-	0.24	91	0.77	n/a	CERN
ILC-0.25	e^+e^-	0.25	20	0.27	n/a	Japan
CLIC-0.38	e^+e^-	0.38	11	0.23	n/a	CERN
ILC-1	e^+e^-	1	38	0.5	n/a	Japan
LHeC	ep	0.06+7	9+26.7	0.08	n/a	CERN
CLIC-3	e^+e^-	3	50	0.59	n/a	CERN
MC-3	$\mu^+\mu^-$	3	4.5	0.18	n/a	n/a
MC-14	$\mu^+\mu^-$	14	14	4	n/a	n/a
WFA-15	e^+e^-	15	12	5	n/a	n/a
WFA-30	e^+e^-	30	20	32	n/a	n/a
FCChh	pp	100	91	3	n/a	CERN
SPPC	pp	125	100	1.3	n/a	IHEP, China

Table 1: Past, present and several proposed future particle colliders: their particle species, center of mass energy E_{cm}, circumference or length C, maximum peak luminosity $\mathcal{L}$ per interaction point, years of luminosity operation, and host labs. (i is for ions; luminosity is in units of cm^{-2}s^{-1}, * design; see also text.)

are possible and were considered: e.g., collision of beams circulating in a ring and a few-pass energy recovery linac (ERL) (Fig. 1b) or linac-ring schemes.

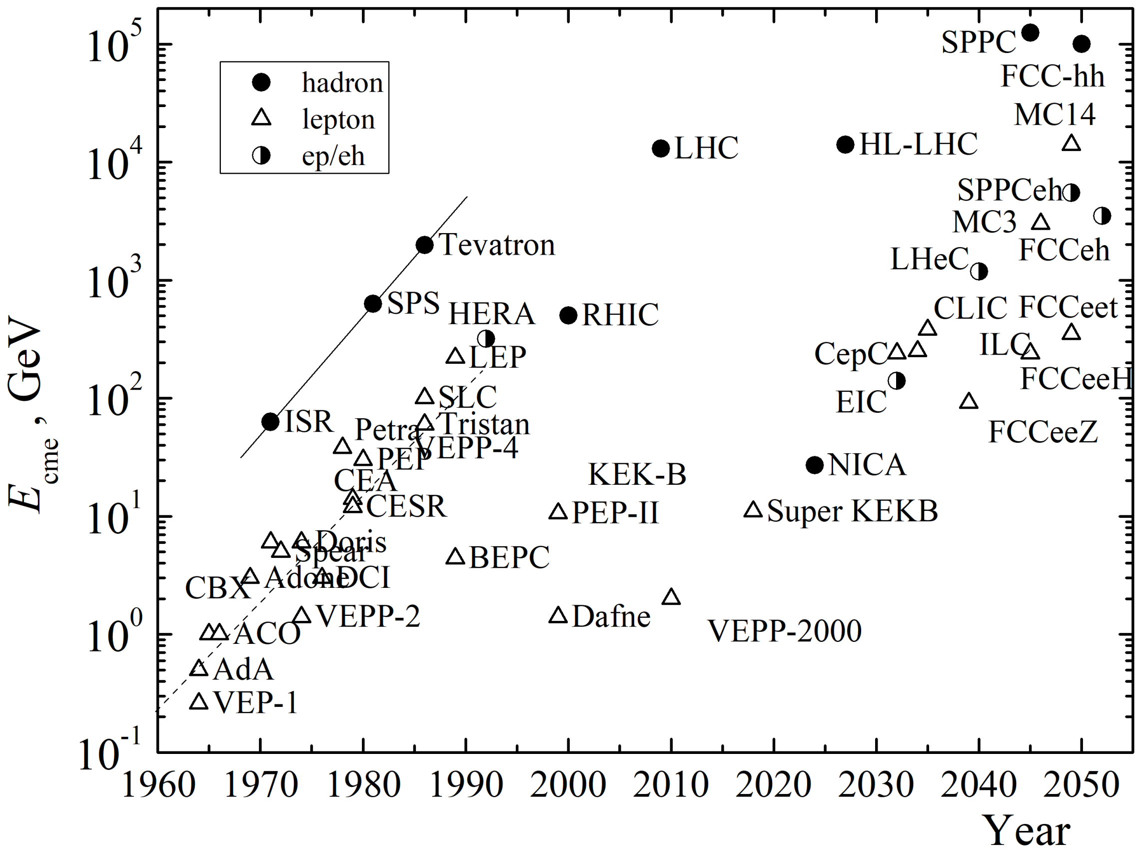

Figure 2: Center of mass energy reach of particle colliders vs their start of operation. Solid and dashed lines indicate a ten-fold increase per decade for hadron (circles), lepton (triangles) and lepton-hadron (half filled circles) colliders (adapted from [Shiltsev and Zimmermann, 2021]).

The ever-growing demands of particle physics research drove a five order of magnitude increase in the beam energy and c.m.e. of colliders, as is demonstrated in Fig. 2. Charged particles gain energy from an electric field. The accelerating field gradients in fast time-varying structures, such as radio-frequency (RF) cavities, are usually orders of magnitude higher than in direct-current (DC) systems, and, therefore, commonly used in modern colliders (with the RF frequencies ranging from 10s of MHz to 10s of GHz). At present, the highest beam accelerating gradients ever achieved in operational machines or beam test facilities are about 31.5 MV/m in 1.3 GHz superconducting RF (SRF) cavities and some $G \approx 100$ MV/m in 12 GHz normal-conducting (NC) ones. In a linear-collider arrangement, illustrated in Fig. 1c, the beam energy E is the product of the average accelerating gradient G and the length of the linac L:

$$E = eG \cdot L \,, \tag{2}$$

where e denotes the elementary (electron) charge, assuming the acceleration of singly charged particles like electrons or protons. For example, reaching just 0.001 PeV=1 TeV energy requires either ∼30 km of SRF linac or 10 km of NC RF accelerator , if the RF cavities occupied all available space – which they usually do not.

Cost considerations (see below) urge for minimization of RF acceleration, e.g., through repeated use of the same RF system, which in that case would boost the energy in small portions $\Delta E = eV_{RF}$ per turn every time a particle passes through the total cavity voltage V_{RF}. Such an arrangement can be realized both in the form of circular colliders (Fig. 1a), which have proven extremely successful, and also through the schemes based on ERLs (Fig. 1b). Circular colliders are most common; here, the momentum and energy of ultra-relativistic particles are determined by the bending radius inside the dipole magnets, ρ, and by the average magnetic field B of these magnets:

$$E = ecB \cdot \rho \quad \text{or} \quad E\,[\text{GeV}] = 0.3(B\rho)\,[\text{Tm}]\,. \tag{3}$$

As the particles are accelerated in a *synchrotron*, the strength of the magnetic field is increased to keep the radius of the orbit approximately constant. Such a condition allows the beam orbit to remain inside the rather limited space provided by the accelerator beam pipe passing through the magnet apertures.

The maximum field of NC magnets is about 2 Tesla (T) due to the saturation of ferromagnetic materials, and while this is sufficient for lower-energy colliders, such as most e^+e^- storage rings, it is not adequate for very high-energy hadron or muon beams because of the implied need for excessively long accelerator tunnels and prohibitively high total magnet power consumption. The development of superconducting (SC) magnets that employ high electric current carrying Nb-Ti wires cooled by liquid helium below 5 K opened up the way towards higher fields and to hadron colliders at record energies [Tollestrup and Todesco, 2008]. For example, the 14 TeV c.m.e. LHC at CERN uses double bore magnets with a maximum field of 8.3 T at a temperature of 1.9 K in a tunnel of $C = 26.7$ km circumference (dipole-magnet bending radius $\rho = 2,800$ m).

The exploration of rare particle physics phenomena at the energy frontier requires not only an appropriately high energy, but also a sufficiently large number of detectable reactions. The number of generated reactions $N_{reaction}$ is given by the product of the cross section of the reaction under study, σ_{reaction}, and the time integral over the instantaneous *collider luminosity*, $\mathcal{L}$:

$$N_{\text{reaction}} = \sigma_{\text{reaction}} \cdot \int \mathcal{L}(t)dt. \tag{4}$$

The luminosity dimension is $[\text{length}]^{-2}[\text{time}]^{-1}$. The integral on the right is referred to as *integrated luminosity* $\mathcal{L}_{\text{int}}$, and, reflecting the smallness of typical particle-interaction cross-sections, is often reported in units of inverse pico-, femto- or attobarns. By definition, 1 barn is equal to 10^{-24} cm^2, and, correspondingly, 1 ab^{-1}=10^{42} cm^2. Figure 3 presents impressive progress in luminosity of colliders – by more than six orders of magnitude. Note that the luminosity progress goes hand in hand with increase of the energy because the cross-sections of many reactions of interest get smaller with energy and often drop as $\sigma_{reaction} \propto 1/E^2_{cm}$. To get reasonably high numbers of events, one needs to raise the luminosity correspondingly – as can be seen from Eq.4). Today's record luminosities are about $2 \cdot 10^{34}$ cm^{-2}s^{-1} and, for example, for the WZ production in the LHC, with the reaction cross-section of about 6 femtobarn or $6 \cdot 10^{-39}$ cm^2, one can expect to see some 1200 of such events over one year of operation (effectively, about 10^7s).

Luminosity of colliders is critically dependent on beam intensities and sizes at the IPs. Colliders usually employ bunched beams of particles with approximately Gaussian distributions, and for n_b bunches containing equal numbers of particles $N_1 = N_2 = N$ colliding head-on with frequency f_{coll}, a basic expression for the luminosity is

$$\mathcal{L} = f_{\text{coll}} n_b \frac{N^2}{4\pi\sigma^*_x\sigma^*_y}\ , \tag{5}$$

where σ_x^* and σ_y^* characterize the rms transverse beam sizes in the horizontal and vertical directions at the IPs. To achieve high luminosity, one therefore needs to maximize the population and number of bunches, and either produce them as narrow as possible or focus them tightly at dedicated colliding locations. Sophisticated detectors usually surround the interaction points in order to collect full information about the reactions that originate from collisions of particles.

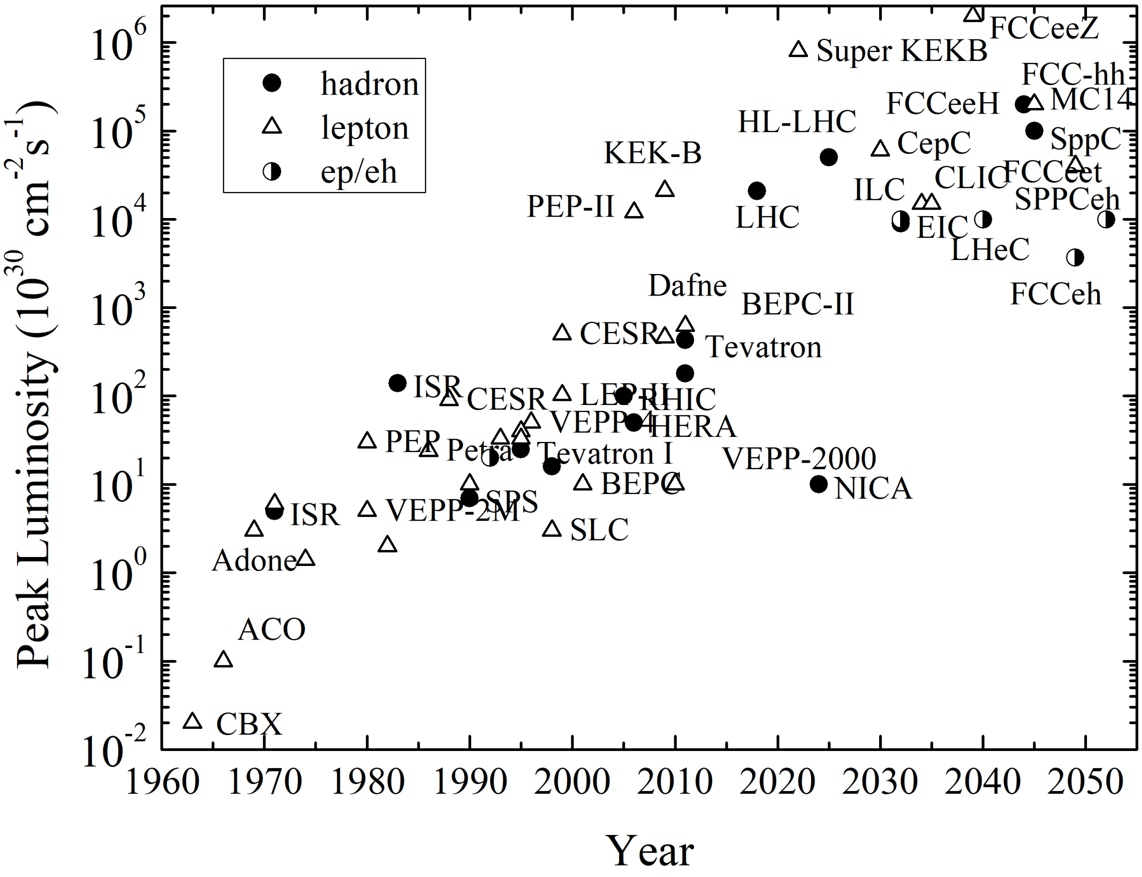

Figure 3: Luminosities of particle colliders: triangles are lepton colliders, full circles are hadron colliders, and half-filled circles for electron-hadron colliders. Values are per collision point (adapted from [Shiltsev and Zimmermann, 2021]).

In our attempt to understand the ultimate limits of colliders, we should note that the great progress of the colliders shown in Figs. 2 and 3 was accompanied by simultaneous increase of their size, power consumption, complexity and cost. Modern colliders employ a number of diverse technologies for power converters and power supplies, ultra-high vacuum systems, particle sources, injection and extraction systems, tunneling, geodesy and alignment, cooling water and cryogenic cooling, beam diagnostics, accelerator control, personnel safety and machine protection, among other equipment. Still, when it comes to the facility size, cost and power consumption, the most important factors are the "core technologies" required for accelerating particles to high energies – normal- and/or superconducting radio-frequency (RF) acceleration systems, and normal- and/or superconducting accelerator magnets – and "beam physics techniques" used to attain the necessary beam qualities such as intensity, brightness, and sometimes polarization, including beam cooling, manipulation and collimation, the production of exotic particles like antiprotons or muons, mitigation of beam instabilities, and countermeasures against beam-size blow up caused by space-charge (SC) and beam-beam effects or intra-beam scattering (IBS), among other phenomena. The energy reach of a collider is mostly defined by its core accelerator technologies, while its luminosity is very much dependent on the sophistication of beam physics techniques [Shiltsev and Zimmermann, 2021].

The energy frontier colliders were and remain costly, often at the brink of financial and political affordability. That poses serious risks, and in the past several projects have been terminated, even after the start of construction. For example, the construction of the 400 GeV c.m.e. ISABELLE *pp* collider (briefly renamed CBA) at the Brookhaven National Laboratory in the USA was stopped in 1983 [Month, 2003, Crease, 2005a, Crease, 2005b]; in the early 1990s two other flagship projects were terminated: the 6 TeV c.m.e. proton-proton complex UNK [Yarba, 1990, Kuiper, 1994] in Protvino, Russia, and the 40 TeV c.m.e. proton-proton Superconducting Super Collider (SSC) in Texas, USA, in 1993 [Wojcicki, 2009, Riordan et al., 2015]. Notwithstanding the above, advances in core accelerator technologies – including the developments of superconducting magnets for ISABELLE/CBA, UNK and SSC – have led to substantial reductions in collider cost per TeV [Shiltsev, 2014]. This progress, together with the growing strength of the high-energy particle physics community, enabled development of frontier machines, such as the currently operational multi-billion dollar LHC. Even larger $10B-scale future collider projects are generally considered feasible (see Section 3) because no other instrument can replace high-energy colliders in the search for the fundamental laws governing the universe.

3 Next Few Decades

The prevailing view of the global HEP community is that the next large collider facility should be an e^+e^- collider that functions as a Higgs/ElectroWeak factory. The physics case for such a collider with c.m.e. range (0.25-0.5) TeV and very high luminosity (0.1-1) ab^{-1}/yr (hence the name "factory") is quite compelling because it would enable detailed exploration of subtle reactions involving the Higgs/ElectroWeak fields (H, W, Z particles and photons) and shed light on possible deviations from the predictions of the *Standard Model* theory of particle physics. Several options for each of these types of colliders are under consideration globally, with variable technical readiness. The leading candidates for a Higgs/EW factory are (1) the e^+e^- Future Circular Collider (FCC-ee) at CERN and the quite similar Circular Electron-Positron Collider (CEPC) in China, (2) the International Linear Collider (ILC) in Japan, and (3) the Compact LInear Collider (CLIC) at CERN – see Table 1. Additional novel options for compact e^+e^- colliders, such as Cool Copper Collider (C^3), high gradient ($\sim$ 70 MV/m) superconducting RF linear collider HELEN (High Energy LEptoN collider), ERL-based circular and linear collider schemes, and a Fermilab Site Filler circular e^+e^- collider, have emerged and are under investigation. For the purpose of our analysis, all Higgs factories can be considered as low-energy machines that can be built based on generally existing technologies and within a reasonable timescale O(10-20 years) from the decision to proceed [Roser et al., 2022]. Many beam physics methods and accelerator technologies developed for the Higgs factories can be employed in much higher energy machines.

At the "energy frontier," the aspirations of the international particle physics community are after a collider with an energy reach of $\sim$ 10 TeV scale to enable New Physics discoveries (i.e., particles and reactions beyond those described by the Standard Model). Energy of such a collider should significantly exceed the 14 TeV c.m.e. of the LHC, which can be provided either by a $\sim$ 100 TeV hadron (*pp*) collider or a $\geq$ 10 TeV lepton (e^+e^- or muon) collider. Here we should note that in very high energy collisions hadrons manifest themselves as compositions of quarks and gluons, and

Proposal		Type	E_{cm} [TeV]	$\mathcal{L}_{int}$/IP [ab^{-1}/yr]	Yrs. of R&D	Yrs. to 1st physics	Constr. cost [2021 B$]	El. power [GW]
ILC-3	e^+e^-	L	3	0.61	5-10	19-24	18-30	$\sim$0.4
CLIC-3	e^+e^-	L	3	0.59	3-5	19-24	18-30	$\sim$0.55
CCC-3	e^+e^-	L	3	0.6	3-5	19-24	12-18	$\sim$0.7
ReLiC-3	e^+e^-	ERL	3	4.7(9.4)	5-10	>25	30-50	$\sim$0.78
$\mu\mu$Collider[1]-3	$\mu^+\mu^-$	C	3	0.23(0.46)	>10	19-24	7-12	$\sim$0.23
LWFA-LC-3	e^+e^-	L	3	1	>10	>25	12-80	$\sim$0.34
PWFA-LC-3	e^+e^-	L	3	1	>10	19-24	12-30	$\sim$0.23
SWFA-LC-3	e^+e^-	L	3	1	5-10	>25	12-30	$\sim$0.17
Muon Collider[1]	$\mu^+\mu^-$	C	10	2(4)	>10	>25	12-18	$\sim$0.3
LWFA-LC-15	e^+e^-	L	15	5	>10	>25	18-80	$\sim$1
PWFA-LC-15	e^+e^-	L	15	5	>10	>25	18-50	$\sim$0.62
SWFA-LC-15	e^+e^-	L	15	5	>10	>25	18-50	$\sim$0.45
FNAL *pp* circ.	*pp*	C	24	0.35(0.7)	>10	>25	18-30	$\sim$0.4
FCC-hh	*pp*	C	100	3(6)	>10	>25	30-50	$\sim$0.56
SPPS	*pp*	C	125	1.3(2.6)	>10	>25	30-50	$\sim$0.4
Collider in Sea	*pp*	C	500	5	>10	>25	>80	>1

Table 2: Main parameters of the multi-TeV lepton collider proposals (3 TeV c.m.e. options) and colliders with 10 TeV or higher parton c.m.e: colliding particles; type of the collider (L for linear, C for circular, ERL for energy recovery linacs); center-of-mass energy (the relevant energies for the hadron colliders are the parton c.m. energy, which is ~ 7 times less than hadron c.m. energy E_{cm} quoted here - see Eq.6); annual integrated luminosity per interaction point (assuming 10^7s per year effective operating time; for colliders with multiple IPs, the total peak luminosity is given in parenthesis); years of the pre-project R&D indicate an estimate of the required effort to get to sufficient technical readiness; estimated years to first physics are for technically limited timeline starting at the time of the decision to proceed; total construction cost range in 2021$ (based on a parametric estimator, including explicit labor, but without escalation and contingency); facility electric power consumption (adapted from the Implementation Task Force report [Roser et al., 2022]).

their total energy is distributed among the constituents. Therefore, the highest accessible c.m.e. E^*_{cm} of individual parton-to-parton collisions is significantly lower than the nominal (proton-proton) $E_{cm} = 2E$, and, e.g., for many reactions one can assume [Al Ali et al., 2022]:

$$E^*_{cm} \simeq (1/7 - 1/10) \times E_{cm} = (1/7 - 1/10) \times 2E. \tag{6}$$

Several ~ 10 TeV c.m.e. scale collider options are under active discussion at present – see Table 2 – including two *pp* colliders, the FCC-hh at CERN and SPPC in China; 3 TeV to 14 TeV muon colliders; as well as novel e^+e^- collider schemes based on plasma wakefield acceleration (WFA).

In the course of the recent *Snowmass'21* US community strategic planning exercise, the Implementation Task Force (ITF) [Roser et al., 2022] of a dozen internationally renowned accelerator experts has been convened and charged with developing metrics and processes to facilitate comparisons between projects. Essentially all (> 30) collider concepts presently considered viable have been evaluated by the ITF using parametric estimators to compare physics reach (impact), beam parameters, size, complexity, power, environmental concerns, technical risk, technical readiness, validation and R&D required, cost and schedule – see Table 2. The significant uncertainty in these values was addressed by giving a range where appropriate. Notably, the ITF choose to use the proponent-provided luminosity and power consumption values. The years of required pre-project R&D is just one aspect of the technical risk, but it provides a relevant and comparable measure of the maturity of a proposal and an estimate of how much R&D time is required before a proposal could be considered for a project start (so called "Critical Decision 0" in the US scientific infrastructure project approval system). The time to first physics in a technically limited schedule includes the pre-project R&D, design, construction and commissioning of the facility, and is most useful to compare the scientific relevance of the proposals over the timeline of interest.

The total project cost follows the US project accounting methods, but without taking into account the inflation escalation and (usually required) contingency. The ITF used Various parametric cost models, also taking into account the estimates provided by the proponents, and – for reference – known costs of existing installations, as well as reasonably expected costs for novel equipment. For future technologies, the pre-project cost reduction R&D may further lower the ITF cost estimate ranges.

As for any large scientific research facility, it is not only the cost that is of importance, but also the number of experts needed for the design, construction and commissioning of the future colliders and the environmental impact, e.g., the electric energy consumption. Therefore, it is of very practical interest for the particle physics community to assess the limits of the ultimate colliders in a quantitative manner.

4 Limits of Colliders

Our approach to the limits of future colliders starts with an introduction to the issue: definitions of the units and general considerations regarding energy, luminosity, and social cost of the ultimate machines. It is followed by a more detailed look into specific limitations of the circular pp, ee and $\mu\mu$ colliders; linear and plasma-based $ee, \gamma\gamma, \mu\mu$ ones; and some exotic schemes, such as the crystal muon colliders. The social cost considerations (power consumption, financial costs, carbon footprint, availability of experts and time to construct) are most defined for the machines based on extensions of the existing core accelerator technologies (RF and magnets) and less so for the emerging or exotic technologies (ERLs, plasma WFA, crystals, etc).

Three of the most important aspects of the evaluation are feasibility of the c.m. energy E_{cm}, feasibility of the collider luminosity $\mathcal{L}$, and feasibility of the facility cost C. For each machine type (technology), one can start with the current state-of-the-art machines – see, e.g., Ref.[Shiltsev and Zimmermann, 2021] for more details – and attempt to make several (1,2,...) orders of magnitude steps in the energy to see how that would affect the luminosity and the cost.

Below we use 1 PeV = 1000 TeV as the unit for E_{cm}. The units of $\mathcal{L}$ are ab^{-1}/yr that is equal to, e.g., 10^{35} cm^{-2}s^{-1} over 10^7 sec/yr. For reference, the LHC will deliver 0.3 ab^{-1}/yr after its high luminosity upgrade. Due to the spread of expectations for the machine availability and annual operation time, there might be a factor of $\sim$2 uncertainty in peak luminosity demands for any ab^{-1}/yr value. The units of total facility electric power consumption are TWh/yr. For reference, the CERN power consumption is about P_s=200MW of the average power and 1.1-1.3 TWh/yr while operating the LHC (note that the total electric power includes not only the collider and its injectors, but also detectors, infrastructure, lighting, etc). The cost is estimated in "LHC-Units" (LHCU) – the cost of the LHC construction (in present day prices LHCU$\simeq$\$10B). There is no better and more reliable cost estimation method than a detailed analysis, similar to that of the ITF (see above Sec.3). With proper reservations and understanding the caveats, one could rely here on an approximate phenomenological $\alpha\beta\gamma$ collider cost model [Shiltsev, 2014]:

$$C_c \approx \alpha \cdot L_t^{p_1} + \beta \cdot E_{cm}^{p_2} + \gamma \cdot P_s^{p_3} \tag{7}$$

where the cost is understood as a total project cost (TPC is of an all new facility without previous investments, taking into account labour cost, escalation due to inflation, contingency, R&D, management, etc) that scales with just three facility-specific parameters — the length of the tunnels

L_t, the center-of-mass or beam energy E_{cm}, and the total required site power P_s. The second part reflects the cost of accelerator components (magnets, RF, etc. and associated auxiliary subsystems); it is very much technology dependent and often dominates the total cost. Comparison with the cost of recently built large accelerators and the ITF cost estimates indicates that the model estimates are good within a factor of 2 if the exponents are set equal $p_1 = p_2 = p_3 = 1/2$, and the coefficients are $\alpha \approx 0.1\text{LHCU}/\sqrt{10\text{km}}$, $\gamma \approx 0.3\text{LHCU}/\sqrt{\text{TWh/yr}}$ and the accelerator technology dependent coefficient $\beta_{\text{MAG}} \approx 6\text{LHCU}/\sqrt{\text{PeV}}$ for high-field magnets and $\beta_{\text{RF}} \approx 30\text{LHCU}/\sqrt{\text{PeV}}$ for RF accelerating structures [Shiltsev, 2014, Roser et al., 2022]. The $\alpha\beta\gamma$-model should be used with caution as it still needs to be properly extended to the advanced technologies (plasma WFA, lasers, crystals, etc).

4.1 General Limitations

The most obvious limit to consider is the size of the collider. Indeed, as Eqs.2 and 3 indicate, the larger the length of a linac or circumference (radius) of a ring, the higher beam energies E can be envisioned. For example, if the available site length is limited to $L_t \simeq 100$ km, then two linacs of 50 km each could allow the energy to reach up to $E_{cm} \simeq 0.01$ PeV with the current state of the art normal-conducting RF cavities with $G = 0.1$ GeV/m and up to $E_{cm} \simeq 0.2$-0.5 PeV with the potentially achievable average accelerating gradient of $G = 2 - 5$ GeV/m in plasma wakefield structures. In comparison, a 100 km long circular tunnel (ρ=16 km radius) can fit a ~0.1 PeV collider based on the 16T Nb_3Sn SC bending magnets or a 0.25 PeV collider with ~40T high-temperature superconducting (HTS) magnets. Of course, larger circumference tunnels could fit proportionally higher c.m. energy machines.

Note that not all kinds of particles can be accelerated in high-energy circular colliders due to imminent synchrotron radiation (SR) that results in the energy loss per turn of [Sands, 1970]:

$$\Delta E_{SR} = \frac{1}{3\epsilon_0} \frac{e^2 \beta^3 \gamma^4}{\rho} \, , \tag{8}$$

which increases with the fourth power of energy $E = \gamma mc^2$ and scales with the inverse of the bending radius (here, ϵ_0 is the permittivity of vacuum and $\beta = \sqrt{1 - 1/\gamma^2}$). At the limit of practicality, one should demand the SR loss per turn to be at least less than the total beam energy $\Delta E_{SR} \leq E$, and that defines the c.m. energy limit for circular colliders as:

$$E_{cm}[\text{PeV}] \leq 0.001 \cdot (m/m_e)^{4/3} (\rho/10[\text{km}])^{1/3} \ \ , \tag{9}$$

that is ~1 TeV for electrons, ~1.2 PeV for muons ($m_\mu \approx 210 m_e$) and ~25 PeV for protons ($m_p \approx 2000 m_e$). Beyond these energies, a sheer energy economy will demand that the colliders be linear (thus, needing no bending magnets).

Survival of the particles in very long accelerators may set another energy limit. Indeed, if, for example, a 0.5 PeV linear collider will be based on individual 5 GeV plasma wakefield accelerating stages, then $M = 10^5$ of them will be required. For the beam of particles to propagate through such a chain without losing too much intensity (and power), the stage-to-stage transfer efficiency must be much better than $\eta_{stage} \geq 1 - 1/M = 0.99999$ – an extremely hard challenge. Also, if the particles are unstable, they may decay before the end of the acceleration process. To guarantee delivery to the collision point, the minimum accelerator gradient must significantly exceed $G \gg mc/\tau_0$ – where with the lifetime at rest τ_0 - that is, e.g., 0.3 MeV/m for muons (relatively easy to achieve even with present day technologies) and 0.3 GeV/m for tau-leptons (quite a challenge even for the most optimistic currently envisioned advanced acceleration schemes) [**?**].

Performance (luminosity) reach of the ultimate colliders can be limited by many factors and effects – particle production, beamstrahlung, synchrotron radiation power per meter, IR radiation damage, neutrino-radiation dose, beam instabilities, jitter/emittance growth, etc – which are machine specific and will be considered below. But the most fundamental is the limit on the total beam power $P_b = 2 \times f_0 n_b N \gamma mc^2$ (the factor of 2 accounts for two colliding beams). Indeed, the luminosity equation (5) can be re-written as:

$$\mathcal{L} = \frac{1}{16\pi f_{coll} n_b \varepsilon \beta^* mc^2} \cdot \frac{P_b^2}{E} \propto \frac{P_b^2}{E}\,, \tag{10}$$

here we have substituted $\sigma_x^* \sigma_y^* = \varepsilon_n \beta^* / \gamma$ with so called "normalized beam emittance" ε_n and the so called "beta-function at IP" β^* generally not explicitly dependent on energy - see [Shiltsev and Zimmermann, 2021]. Particle accelerators in their essence are transformers of electric plug site power P_s into the high-energy beam power $P_b = \eta P_s$ with much less than 100% efficiency (in the best case scenario $\eta \sim 0.1 - 0.3$). It is hard to know precisely where the ever-changing societal limits on the power consumption of large accelerators will be in the future, but they will surely include "carbon footprint" considerations and the environmental impact of future accelerators' construction and operation. For reference, with the present world-average power consumption rates, 1,000,000 people get ~3TWh/yr, which is three time larger than the CERN annual site energy usage. Wherever the limit is, Eq.(10) points out that the luminosity will decrease with energy at least as $L \propto 1/E$. Such dependence on energy is markedly different from the traditional HEP demand for the luminosity to follow the cross-section scaling $L \propto E^2$, and from what we know now, other factors f_{coll}, n_b, ε_n, β^*, η could be of only limited help in avoiding performance degradation in our quest for two to three orders of magnitude higher energies.

Of course, there will be societal limits on the collider's total cost C_c, too. While the total cost is dependent on the technology (core accelerator technology, civil construction technology, electric power production, delivery and distribution technology, etc.), the probability of approval and realization for a technically feasible future collider facility typically decreases with cost increase beyond "reasonably acceptable", perhaps as $\propto 1/C_c^{\kappa}$. As a guide, such a decrease with the exponent $\kappa \approx 2 - 3$ is characteristic for real estate sales price distributions. Also, to note: i) the costs of civil construction and power systems are mostly driven by the larger economy and are not that dependent on the collider type and accelerator R&D advances, ii) having an injector complex available (sometimes up to 1/3 of the total cost) results in potential increase of a factor of 2 in the energy reach - see Eq.(7); iii) the collider cost is usually a relatively weak function of luminosity (the latest example is the HL-LHC \$1B project that will increase luminosity of the O(\$10B) LHC by a factor of 5); iv) so, one can consider starting future machines with high E and relatively low $\mathcal{L}$ in anticipation of eventual performance upgrades (for example, in the past, CESR and Tevatron witnessed $\mathcal{L}$ increase $O(100)$, LHC by a factor ≥10, etc); v) the total cost C_c is moderately weakly dependent on the tunnel length/circumference L_t, but it is critically dependent on E_{cm} and the choice of the acceleration technology.

Construction time of large accelerator projects to date is usually between 5 and 11 years and approximately scales as $T \propto \sqrt{C_c}$. It is often limited by the peak annual spending rate, typically in the range \$0.2 to \$0.5 B/yr (compare to the world's global HEP budget ~\$4B), and the number of available technical experts. So far the period of technical commissioning of colliders ("one particle reaches design energy") was $O(1)$ yr – and it is shorter for known technologies and longer for new ones and for larger numbers of accelerator elements. Progress towards the design (or ultimate) luminosity is dependent on the machine's "complexity" [Shiltsev, 2011], and for the luminosity risk of 100 (ratio of the initial $\mathcal{L}$ to the ultimate one), it can take as long as ~9 yrs.

Taking all the above into account, below we will analyse various types of future colliders and assess their potential energy and luminosity reach – maximum E_{cm} and peak $\mathcal{L}$ – under the assumption of the societal limits on the site power consumption and cost:

$$P_s \leq 3\,\mathrm{TWh/yr} \ \text{, and } \ C_c \leq 3\,\mathrm{LHCU}. \tag{11}$$

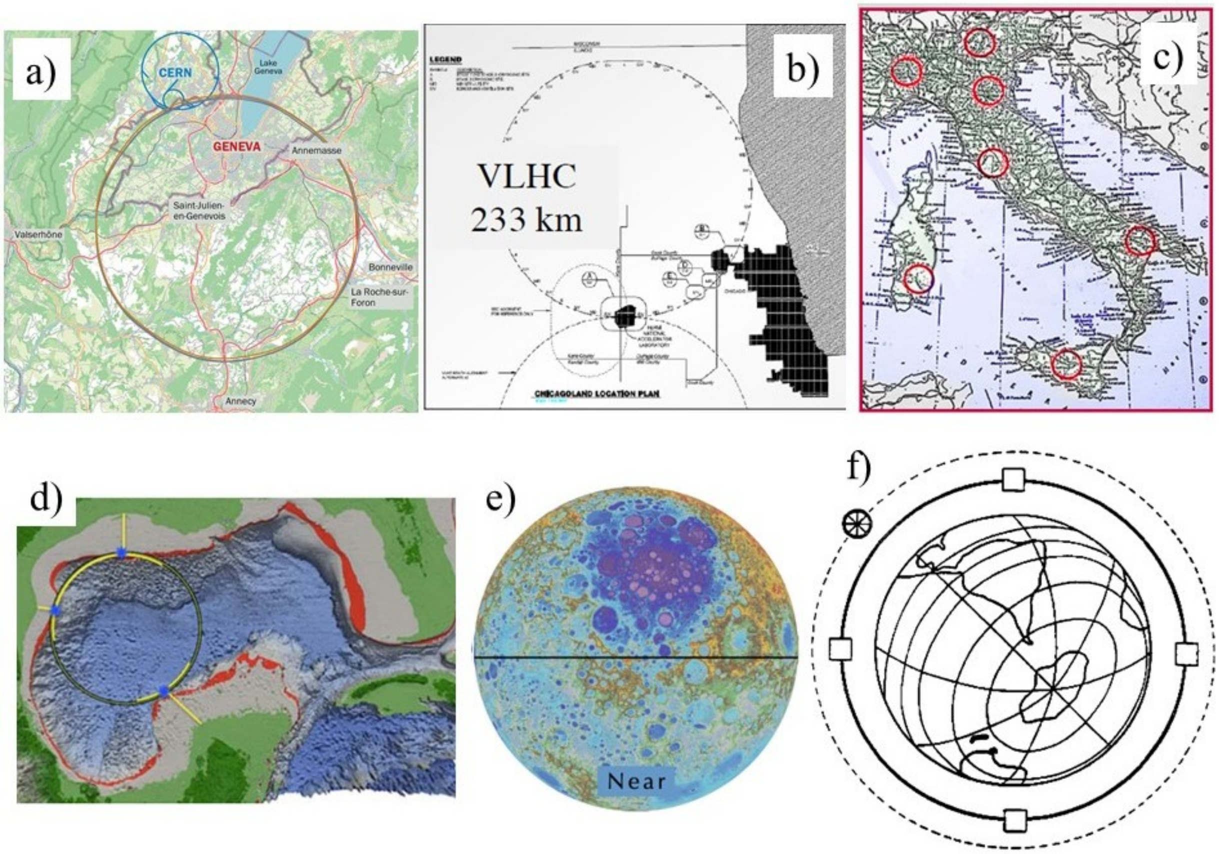

Figure 4: Very large hadron collider proposals (not in scale): a) FCChh (91 km circumference, 100 TeV), b) VLHC (233 km, 175 TeV); c) Eloisatron (300km, 200 TeV); d) "Collider in the Sea" (1,900 km, 500 TeV), e) collider on the Moon (11,000 km, 14 PeV), f) Enrico Fermi's accelerator encircling our Earth ("globe-tron", 40,000km, 2.9 PeV)

4.2 Circular e^+e^- colliders

As mentioned above, the synchrotron radiation of light leptons e^+, e^- limits the energy reach of such colliders to $E_{cm} \leq 1$ TeV, which is far below even the energy reach of the LHC, to say nothing of the aspired to PeV energies. High luminosity could be a potential rationale for an interest in these types of colliders, but it is limited by synchrotron radiation power losses $P_{SR} = 2f_{\text{revolution}}en_bN \cdot \Delta E_{SR}$ and very quickly drops with energy as:

$$\mathcal{L}_{ee\,\mathrm{cir}} = \mathcal{F}_{ee}\frac{P_{\mathrm{SR}}\rho}{\gamma^3}\,, \tag{12}$$

The factor $\mathcal{F}_{ee}$ above accounts for the IP vertical focusing parameters and a dimensionless *beam-beam parameter* that reflect the severity of the electromagnetic disruption of one beam after collision with another - see exact expression elsewhere [Shiltsev and Zimmermann, 2021]. Of importance for our discussion is that $\mathcal{F}_{ee}$ is weakly dependent on the beam energy and all practical limits of the luminosity of e^+e^- circular colliders scales as $1/E^{3-3.5}$. These facilities naturally call for larger radius ρ and circumference $O(100\text{ km})$ – see Eq.(12) – and are considered quite promising tools at low energies, e.g., as high-luminosity Higgs/ElectroWeak factories with typical $E_{cm} \simeq 0.25$ TeV, but even those have an energy demand of some (1.5-2) TWh/yr and cost ∼(1.5-2) LHCU. Significant energy savings are possible with usage of the RF energy recovery (ERLs), but that expands the c.m. energy reach of circular e^+e^- colliders to only $E_{cm} \sim 0.5$ TeV.

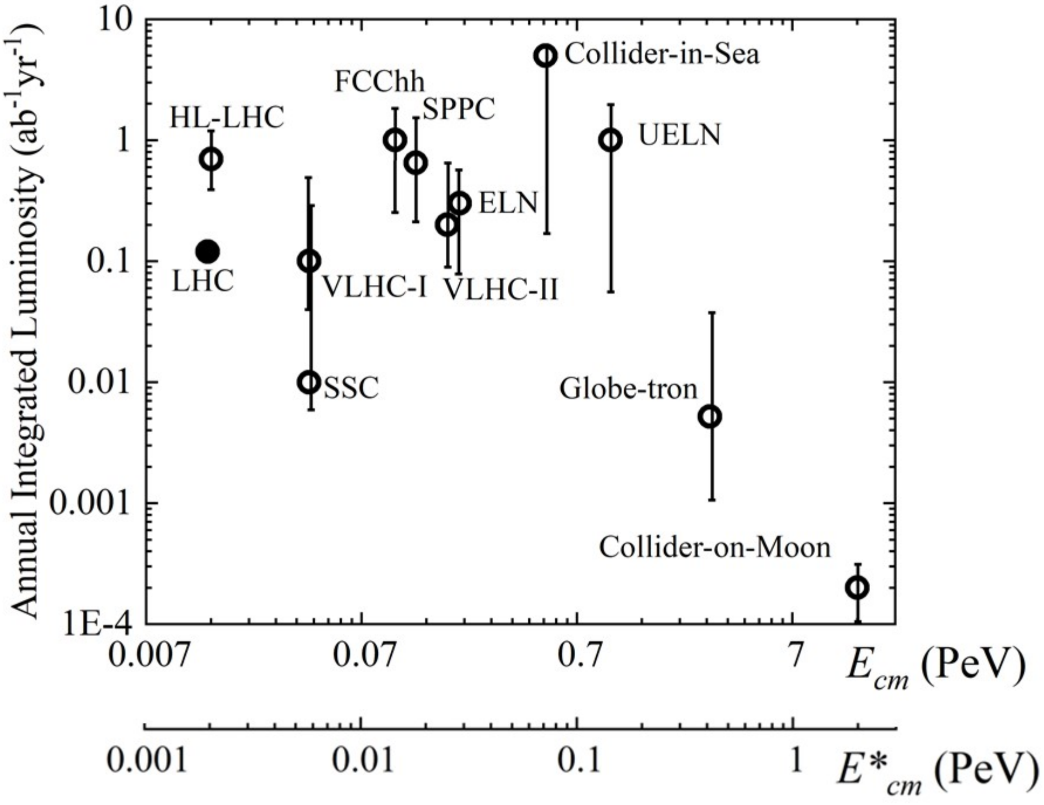

Figure 5: Estimates of the annual integrated luminosity for very high energy circular hadron colliders vs the center-of-mass energy E_{cm}. The second horizontal axis is for the approximate equivalent parton cme $E^*_{cm} \approx E_{cm}/7$.

4.3 Circular *pp* colliders

Being significantly less subjected to the limits of the synchrotron radiation losses – see Eq.(9), protons can be accelerated in circular machines to multi-PeV energies, and, according to Eq.(3), the limit is fully determined by the maximum field B of the bending magnets and the tunnel circumference $L_t \simeq 2\pi\rho$. Fig.4 presents several *pp* collider proposals aimed for higher and higher energies which are based on increase of either B or L_t or both. Most appropriate magnet technologies currently assume limits on the maximum bending field: about 2 T for normal-conducting magnets (usually, room temperature copper conductor and steel yoke), some 8 T for NbTi SC technology, up to 16 T for Nb_3Sn SC technology [Schoerling and Zlobin, 2019], and 20 T to (max) ∼40 T high temperature superconductor (HTS) technologies (e.g, based on rare earth oxides like ReBCO, or iron-based superconductors).

There is significant knowledge in the physics community on how to design, build and operate circular pp colliders – as reference points, one can turn to experience with the Tevatron $p\bar{p}$ collider (E_{cm}=0.002 PeV, B =4.5T, 6.3 km circumference) [Lebedev and Shiltsev, 2014] and 0.014 PeV LHC (8T, 27km) [Evans and Bryant, 2008]. Also, there are designs and/or parameter sets available for the Superconducting Super Collider (SCC, 0.04 PeV, 6.6T, 87km), Super proton-proton Collider (SppC, 0.075-0.125 PeV, 12-24T, 100km) [CEPC Study Group, 2018], Future Circular pp Collider (FCC-hh, 0.1 PeV, 16T, 100km) [Benedikt et al., 2019], Very Large Hadron Collider (VLHC, 0.175 PeV, 12T, 233km) [Ambrosio et al., 2001], the Eloisatron (0.3 PeV, 10T, 300km) [Barletta, 1996], and "Collider-in-the-Sea" (in the Gulf of Mexico, 0.5 PeV, 3.2T, 1900km) [McIntyre et al., 2016]. Going to the extreme, Enrico Fermi had thought of an accelerator encircling the Earth which could reach about 3 PeV c.m.e. with inexpensive normal-conducting magnets ([Cronin, 2004]), and, more recently, a circular collider on the Moon was discussed (CCM, 14 PeV, 20T, 11,000 km) [Beacham and Zimmermann, 2022].

The first and foremost limitations come from large size (related to the magnetic field B technological limit) and very high power consumption requirements, resulting in high cost. Already the 100-km machines like the FCChh and SPPC could be approaching an energy need of 3 TWh/yr and be over the 3 LHCU cost limits given by Eq.11. Of course, even that is incomparable with the lunar CCM cost (about 20-40 LHCU just for the SC magnets) and the energy needs, which are (2-5)$\cdot 10^4$ TWh/yr (O(30%) of the world's current production).

Even more serious are limitations on the maximum attainable luminosity $\mathcal{L}_{pp}$. With the increase of beam energy, limiting detrimental effects include beams disruption due to opposite bunch EM forces experienced at each IP (beam-beam effects) and coherent beam instabilities induced by the beams' own EM interaction with induced image charges, currents and wakefields (especially dangerous in large circumference high intensity machines). Unavoidable will be fast beam burn-off – destruction due to inelastic interactions of high-energy protons as the result of repetitive collisions – leading to shorter and shorter beam lifetime:

$$\tau_{pp} = \frac{n_b N}{\mathcal{L}_{pp}\sigma_{tot}} \,. \tag{13}$$

The total pp cross-section grows slowly from ∼100 mbarn to ∼300 mbarn with an increase of E_{cm} from 0.001 PeV to 1 PeV. The burn-off at very high energies results in several undesired effects: first, a short beam lifetime τ_{pp} of about an hour or even minutes, which requires the frequent injection and acceleration of new bunches of particles. The injection and acceleration in a chain of SC magnet-based boosters is a lengthy process and, therefore, a smaller fraction of the operation time is left for collisions and the entire accelerator complex efficiency drops. Secondly, the particle detectors get flooded with products of the inelastic interactions – the so-called *pile-up* effect makes it extremely difficult to entangle the huge number of tracks originating from approximately 1000 or more pp reactions per bunch collision with luminosity $O(10^{35}\ \mathrm{cm}^{-2}\mathrm{s}^{-1})$. And, thirdly, growing problems are also anticipated with radiation protection of the detectors and collider elements and collimation of the higher energy density beams.

For the pp colliders with E_{cm} above (0.1-0.2)PeV, synchrotron radiation will essentially limit the maximum attainable luminosity in very much the same fashion as for e^+e^- colliders – see Eq.(12) – because of either the limited RF power available to replenish the SR losses P_{SR} or due to challenges related to the cooling of the SC magnets in which one will need to intercept internally the SR photons and handle at the cryogenic temperatures significant heating caused by these photons.

Individual machine designs may vary in optimization approaches toward the highest luminosity, and Fig.5 presents estimates of performance of circular *pp* colliders vs c.m. energy up to E_{cm} =14 PeV (equivalent to parton c.m. energy $E^*_{cm} \simeq 2$ PeV, according to Eq.(6)). Even with the logarithmically large uncertainties (indicated by the error bars) in the electric power limited scenarios, the very high energy colliders will by necessity have low luminosity.

4.4 Circular $\mu\mu$ colliders

Colliding muons would have two key advantages: i) compared to protons, the same size machine would allow effectively a factor of 7-10 higher energy reach due to the point-like nature of the muons – see Eq.(6); and ii) according to Eq.(8), the synchrotron radiation of muons is $\sim (m_\mu/m_e)^4 = 2$ billion times weaker than that of electrons and positrons, and power- and cost-effective acceleration in rings is possible to about a fraction of a PeV – see Eq.(9). Therefore, the highest energy circular muon colliders are predicted to be more compact, more power-efficient and significantly less expensive than the equivalent energy-frontier hadron or e^+e^- machines [Long et al., 2021].

These advantages come along with difficulties due to the short lifetime of the muon, $\gamma\tau_0$ where τ_0=2.2μs. For example, in one second a 0.1 PeV μ^--meson will decay into an electron (or positron in the case of μ^+ decay) and two neutrinos, each carrying a significant fraction of the initial muon momentum. It is widely believed that this time is more than sufficient to allow fast acceleration to high energy before most, or all, of the muons decay, followed by a storage for some $300B$ turns in a ring with an average bending magnet field of B (in units of Tesla) where μ^- and μ^+ particles will collide with each other [Sessler, 1998].

As schematically shown in Fig.6, the $\mu^+\mu^-$ collider itself will not look much different from the *pp* collider rings – it will consist of accelerating RF cavities and high field (SC) magnets, and the latter will determine the size of the facility for a given E_{cm}. What will be different is a somewhat more complicated system of production of the muons in the reactions resulting from multi-GeV protons hitting stationary targets, collection of these muons, muon beam *cooling* (significant reduction of their sizes and internal velocity spreads), and rapid acceleration to the energy of the collider [Palmer, 2014].

There are parameter sets available for 1.5, 3, 6, 10, and 14 TeV circular $\mu\mu$ colliders which indicate their superior (w.r.t. other collider types) power efficiency in terms of ab^{-1}/TWh [Shiltsev and Zimmermann, 2
Projecting their site power requirements and costs, one can estimate that "the feasibility limits" of 3TWh/yr and 3 LHCU – see Eq.(11) – will take place at E_{cm}=(0.03-0.05) PeV muon colliders.

The average luminosity of a muon collider is equal to:

$$\mathcal{L}_{\mu\mu} = f_{rep}\gamma \frac{c\tau_0}{4\pi\rho} \frac{n_{\rm b} N^2}{4\pi\sigma^*_x\sigma^*_y} = \mathcal{F}_{\mu\mu} B P_b \gamma\,, \tag{14}$$

where f_{rep} is the rate of the facility acceleration cycles, and we emphasize the luminosity scaling with B and with the total beam power $P_b = f_{rep} e n_b N E_{cm}$. Exact expression for the factor $\mathcal{F}_{\mu\mu}$ can be found in, e.g., [Palmer, 2014]. The above Eq.(14) indicates an obvious incentive to have the highest bending magnetic field B and the overall scaling $\mathcal{L}_{\mu\mu} \propto \gamma$ with other limiting parameters fixed.

Unfortunately, above about 0.01 PeV, the intense neutrino flux originating from the muons decaying in the collider poses the challenge of minimizing the environmental impact. The collider complex is usually located underground, and when the produced neutrinos emerge at the surface, a small fraction interacts with the rock (and other material) and produces an ionizing radiation dose

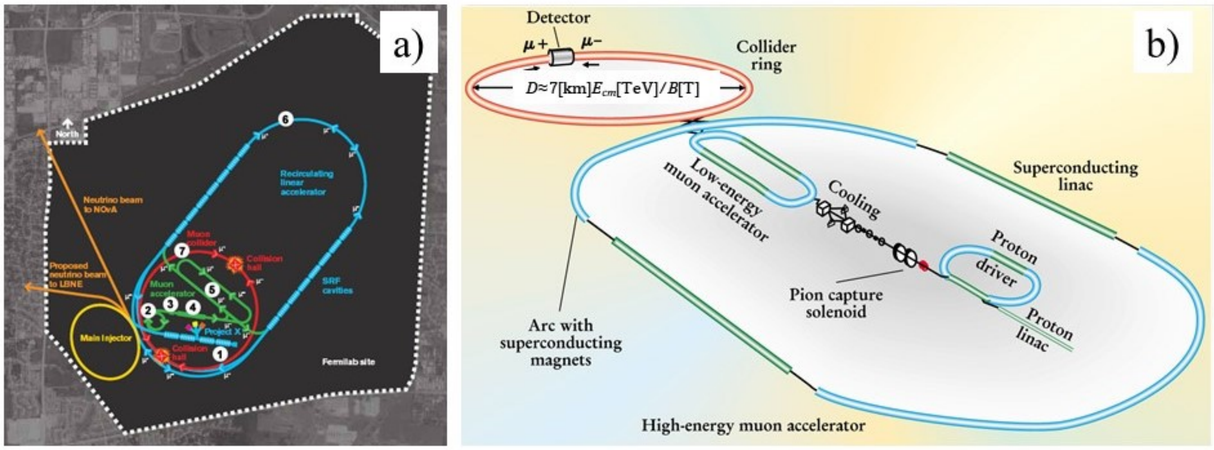

Figure 6: Schematics of high-energy circular muon colliders: a) on the FNAL site, b) a general scheme (adapted from [Sessler, 1998]).

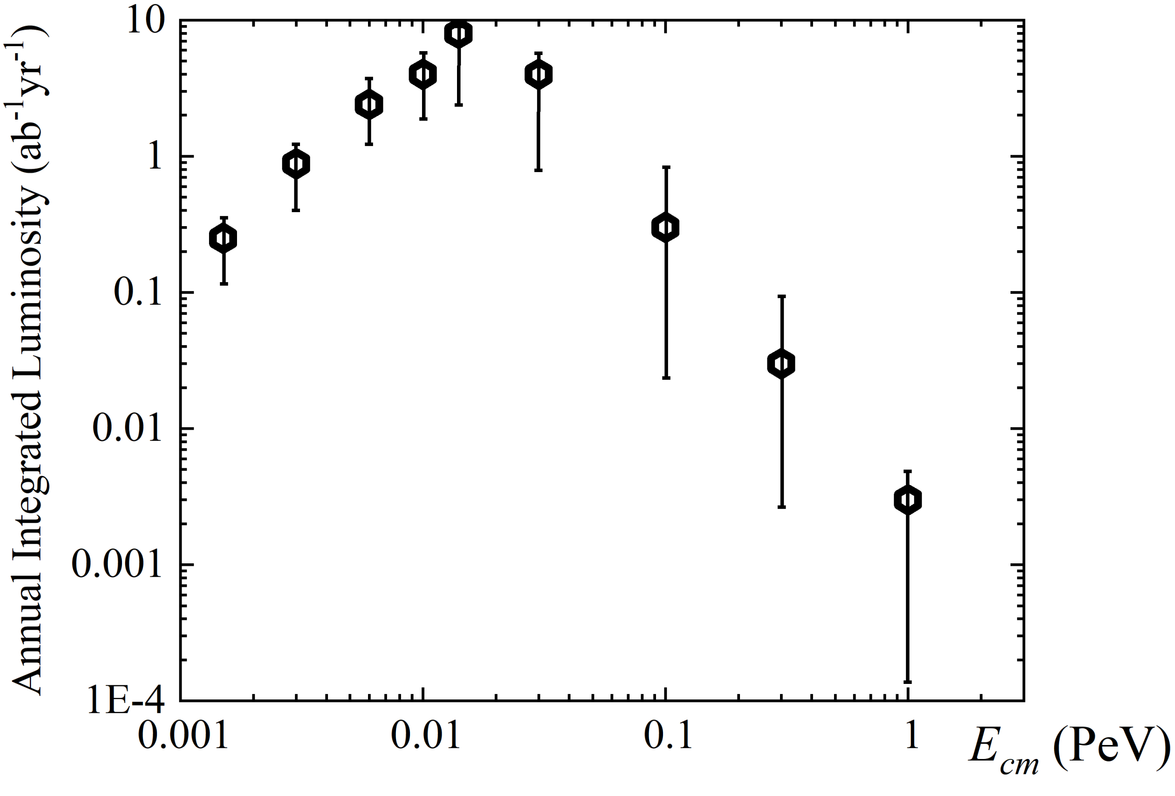

Figure 7: Estimates of the annual integrated luminosity for very high energy circular muon colliders.

that quickly grows with energy $D_\nu \propto f_{rep} n_b N E^3$. The impact of this neutrino-induced radiation can be mitigated, for example, by continually adjusting the orbits of the beams to spread them out on a wider area, by deeper collider tunnels or with a further reduced emittance of the muon beam so that the required luminosity could be obtained using a substantially smaller number of muons. It is believed that the neutrino flux dilution factor Φ could be as high as 10-100 and the ultimate luminosity will depend on it as:

$$\mathcal{L}_{\mu\mu} \propto \frac{D_\nu \Phi}{E_{cm}^2} \,. \tag{15}$$

Additional uncertainty at high energies will be limited capabilities to operate SC magnets with significant deposition of the beam power inside them – muons decay into high-energy electrons, which will be quickly bent by the strong magnetic field into the vacuum chamber/absorber walls, radiating SR on their way.

Therefore, the resulting luminosity projections for muon colliders indicate a promising increase up to E_{cm} ~0.02 PeV followed by fast decline, approximately as shown in Fig.7.

4.5 Traditional, advanced and exotic linear ee or $\mu\mu$ colliders

Acceleration in linear systems (without bending magnets) allows, in principle, the avoidance of the energy limits specified in Eq.(9) due to the SR (the power of a particle's radiation in a longitudinal field is γ^2 times smaller than in an equivalent transverse field). A huge disadvantage of linear colliders (LCs) is that beams are used (collide) only once and then are spent in beam dumps – that leads to intrinsic power inefficiency and, as we will see below, low luminosities. The energy limit will be set by the available length of the tunnel and the average accelerating gradient. Traditional RF accelerating structures are good at best for gradients G_{RF} ~0.2 GV/m before their inner walls become inoperable due to vacuum discharges. As a result, even the most optimized traditional LC designs, like the ILC[Michizono, 2019] and CLIC[Stapnes, 2019], become quite long (30-50 km, see Figs.8a and b) and get to the cost of power limits (3 LHCU and 3 TWh/yr) already at 0.001-0.003 PeV.

Ionized plasmas can sustain electron plasma density waves with accelerating electric field gradients up to:

$$G_p = m_e \omega_p c/e \approx 0.1\,[\mathrm{TV/m}]\,\sqrt{n_0[10^{18}\,\mathrm{cm}^{-3}]}, \tag{16}$$

where n_0 denotes the ambient electron number density and $\omega_\mathrm{p} = \sqrt{e^2 n_0/(m_e \varepsilon_0)}$ is the electron plasma frequency [Tajima and Dawson, 1979]. Such gradients can be effectively excited by either powerful external pulses of laser light or electron bunches if they are shorter than the plasma wavelength $\lambda_\mathrm{p} = c/\omega_\mathrm{p} \approx 1$ mm$\times\sqrt{10^{15}\,\mathrm{cm}^{-3}/n_0}$, or by longer beams of protons if their charge density is modulated with the period of λ_p [Gonsalves et al., 2019, Litos et al., 2016, Adli et al., 2018]. Whether plasma acceleration will be suitable for a very high energy collider application is yet to be seen, given the necessity of very high efficiency staging and phase-locking acceleration in multiple plasma chambers [Leemans and Esarey, 2009, Schroeder et al., 2010]. Also, at the present early stage of this advanced plasma wakefield technology development, the cost of such a collider would be extremely high and a potential for a significant, several orders of magnitude improvement in the cost efficiency still needs to be demonstrated. It is clear, though, that any type of linear collider will be power-hungry. Indeed, its luminosity scales as:

$$\mathcal{L}_{\mathrm{lin}} = \mathcal{F}_{\mathrm{lin}} \frac{N_\gamma}{\sigma_y^*} \frac{P_b}{E_{cm}} \,, \tag{17}$$

and it decreases at higher energies if the total beam power is limited. Other factors in the equation above are limited, too, such as the beam sizes at the IP $\sigma_{x,y}^*$ (strongly dependent on the jitter of the

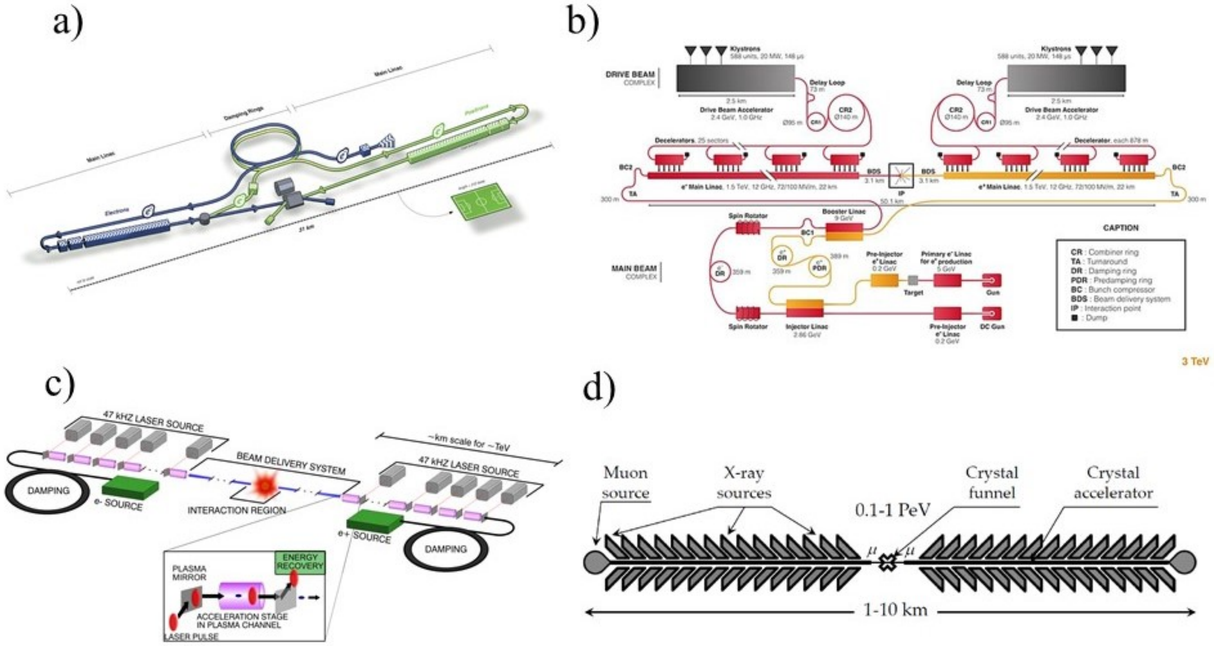

Figure 8: Very high energy linear lepton collider proposals (not in scale): a) 1 TeV cme e^+e^- ILC (31 km long), b) 3 TeV cme e^+e^- CLIC (50 km); c) plasma wakefield linear e^+e^- collider (length depends on energy, e.g., ∼20 km for 30 TeV cme); d) linear crystal wakefield $\mu^+\mu^-$ collider.

collider elements and sophistication of the final focus system) and $N_\gamma \approx 2\alpha r_0 N/\sigma_x^*$ – the number of beamstrahlung photons emitted per $e^\pm$ (α denotes the fine-structure constant). The latter characterizes the energy radiated due to the electromagnetic field of one bunch acting on the particles of the other (*beamstrahlung*) and the corresponding c.m. energy spread that should be controlled to be $\ll E_{cm}$ – see more on that and exact expression for $\mathcal{F}_{\text{lin}}$ in, e.g., [Shiltsev and Zimmermann, 2021].

Most technologically feasible are LCs colliding electrons with electrons, but their effective physics outcome (at the same luminosity) is an order of magnitude worse than that of the e^+e^- colliders. In order to avoid the c.m.e. spread induced by the beamstrahlung, which at high energies $E_{cm} \geq$ 3TeV and luminosities approaches 100%, conversion of electrons into photons – via inverse Compton scattering on the high brightness laser beam right before the IP – was proposed, and the resulting $\gamma\gamma$ collisions would have kinematic advantages for some HEP reactions, though still with significant c.m. energy spread. Proton linear colliders have never been seriously considered because of the factor of 7-10 inefficiency in the c.m. energy reach w.r.t. leptons - see Eq.(6). Until recently, linear muon colliders were not discussed either due to obvious difficulties with muon production and collection. An interesting opportunity of wakefield acceleration of muons in structured solid media, e.g., CNTs or crystals with the charge carrier density n_0 $\sim 10^{20-22}$ cm^{-3}, was proposed in [Tajima and Dawson, 1979]. It promises extreme accelerating gradients 1-10 TV/m, continuous focusing and simultaneous acceleration (no cells, one long channel, particles get strongly cooled via the betatron radiation while channeling between the crystal planes or inside individual CNT channels). A corresponding linear crystal muon collider [Chen and Noble, 1997, Shiltsev, 2012] would be compact in size ($\sim$10 km for 1 PeV) – see, Fig.6 d) - and, therefore, have the promise of low(er) cost. The luminosity of such an exotic LC will still be very low – $O(0.1$ ab^{-1}/yr) at best – for the same reasons as for any linear collider.

Fig.9 presents estimated luminosities of very high energy linear lepton colliders, starting with the 1 TeV ILC and 3 TeV CLIC, and followed by the wakefield acceleration (WFA) 0.01-0.03 PeV LCs based on gaseous plasma, and up to 1 PeV crystal muon LC options.

5 Conclusion

The future of particle physics is critically dependent on the feasibility of future energy frontier colliders. The concept of feasibility is complex and includes at least three factors: feasibility of energy, feasibility of luminosity, and feasibility of cost and construction time. Here we discuss major beam physics limits of ultimate accelerators and take a look into the ultimate energy reach of possible future colliders. We also foresee a looming paradigm change for high-energy particle physics research as the thrust for higher energies by necessity will mean lower luminosity.

The above considerations of ultimate high-energy colliders for particle physics indicate that their major thrust is attainment of the highest possible energy E_{cm}, while the accelerator design challenge is high luminosity $\mathcal{L}$ and the major limit is the cost C_c. The cost is critically dependent on core acceleration technology used to reach the required E_{cm}. While considering the limits on E_{cm}, we assumed the total facility construction cost to be less than three times the cost of the world's most powerful collider to date, the LHC, i.e., $C_c \leq 3$ LHCU. The cost limitations are not well defined, being dependent on such societal factors as the priority and availability of resources to support fundamental research. Consequently, if the affordable collider cost limit can be increased, say, 3-fold to C_c $\sim$10LHCU, that would also push the maximum collider energy E_{cm} by a factor of 3-10, according to Eq.(7). Notably, employment of already existing injectors and infrastructure can greatly help to reduce C_c.

For most collider types, we found that the pursuit of high energy typically results in low(er)

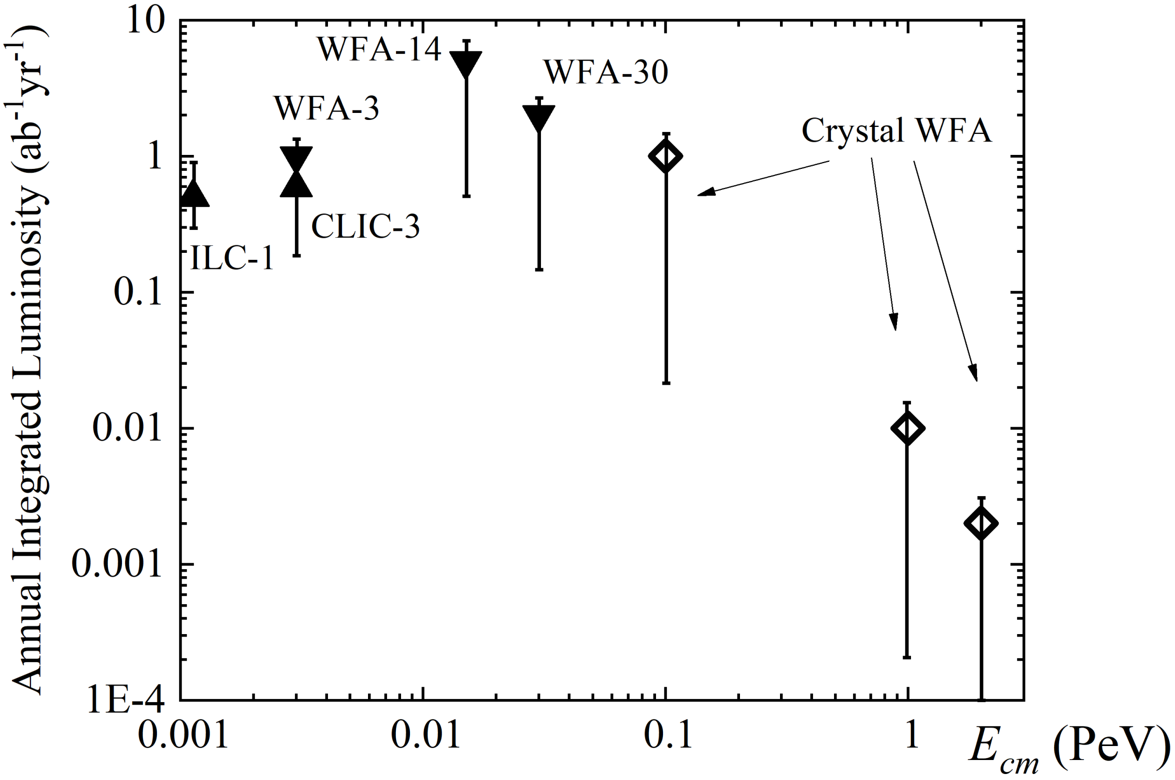

Figure 9: Estimated annual integrated luminosity for very high energy linear lepton colliders: RF-based ILC and CLIC, and plasma wakefield-based e^+e^-, and linear crystal wakefield $\mu^+\mu^-$.

luminosity. So, e.g., one should not expect more than $O(1\ \text{ab}^{-1}/\text{yr})$ at $E_{cm} \geq 30$ TeV to 1 PeV. In the luminosity calculations, one might also assume the total facility (and, therefore, the beam) annual power consumption might by necessity be limited to ∼3 TWh/yr, again, depending on the societal priorities and considerations of the ecological footprint and energy efficiency.

For the considered collider types we found that: i) for circular *pp* colliders the overall feasibility limit is close to or below 100 TeV (∼14 TeV c.m.e. for constituents); ii) for circular *ee* colliders the limit is at ∼0.5 TeV; iii) for circular $\mu\mu$ colliders the limit is about 30 TeV; iv) for linear RF-based lepton colliders and plasma $ee/\gamma\gamma$ colliders the limit is between 3 and 10 TeV; v) there are exotic schemes, such as crystal channeling muon colliders, which potentially offer 100 TeV-1 PeV c.m.e., though at very low luminosity. All in all, muons seem to be the particles of choice for the future ultimate HEP colliders.

6 Acknowledgements and Further Reading

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For those who want to read more deeply on the topics touched in this article, one can recommend the following sources:

- reviews of the modern and future particle colliders – Ref. [Shiltsev and Zimmermann, 2021, Myers and Schopper, 2013, Myers and Brüning, 2016],
- introductory textbooks on high-energy particle physics – Refs. [Perkins, 2000, Barger, 2018],
- on the history of accelerators and colliders – Refs.[Livingston, 1954, Hoddeson et al., 1997, Sessler and Wilson, 2014],
- on accelerator and collider technologies and costs – Refs.[Seryi, 2016, Schoerling and Zlobin, 2019, Shiltsev, 2014, Roser et al., 2022],
- other publications on the topics of limits of particle colliders – Refs.[Shiltsev, 2012, Zimmermann, 2018, Shiltsev, 2019].

References

[Adli et al., 2018] Adli, E. et al. (2018). Acceleration of electrons in the plasma wakefield of a proton bunch. *Nature*, 561(7723):363.

[Al Ali et al., 2022] Al Ali, H., Arkani-Hamed, N., Banta, I., Benevedes, S., Buttazzo, D., Cai, T., Cheng, J., Cohen, T., Craig, N., Ekhterachian, M., et al. (2022). The muon smasher's guide. *Reports on Progress in Physics*.

[Ambrosio et al., 2001] Ambrosio, G. et al. (2001). Design study for a staged very large hadron collider. Technical report, Fermilab.

[Barger, 2018] Barger, V. D. (2018). *Collider physics*. CRC Press.

[Barletta, 1996] Barletta, W. (1996). Maximizing the luminosity of eloisatron, a hadron supercollider at 100 tev per beam. In *AIP Conference Proceedings*, volume 351, pages 56–67. AIP.

[Beacham and Zimmermann, 2022] Beacham, J. and Zimmermann, F. (2022). A very high energy hadron collider on the moon. *New Journal of Physics*, 24(2):023029.

[Benedikt et al., 2019] Benedikt, M. et al. (2019). Fcc-hh: The hadron collider. *The European Physical Journal Special Topics*, 228(4):755–1107.

[CEPC Study Group, 2018] CEPC Study Group (2018). Cepc conceptual design report: Volume 1 - accelerator.

[Chen and Noble, 1997] Chen, P. and Noble, R. J. (1997). Crystal channel collider: Ultra-high energy and luminosity in the next century. In *AIP Conference Proceedings*, volume 398, pages 273–285. AIP.

[Cockcroft and Walton, 1932] Cockcroft, J. D. and Walton, E. T. (1932). Experiments with high velocity positive ions.—(i) further developments in the method of obtaining high velocity positive ions. *Proceedings of the Royal Society of London. Series A, Containing Papers of a Mathematical and Physical Character*, 136(830):619–630.

[Crease, 2005a] Crease, R. P. (2005a). Quenched! the isabelle saga, i. *Physics in Perspective*, 7(3):330–376.

[Crease, 2005b] Crease, R. P. (2005b). Quenched! the isabelle saga, ii. *Physics in Perspective*, 7(4):404–452.

[Cronin, 2004] Cronin, J. W. (2004). *Fermi remembered*, volume 12. University of chicago Press Chicago, IL.

[Evans and Bryant, 2008] Evans, L. and Bryant, P. (2008). Lhc machine. *JINST*, 3:S08001.

[Faus-Golfe and Edgecock, 2017] Faus-Golfe, Á. and Edgecock, R. (2017). Applications of particle accelerators in Europe (EuCARD Report).

[Gonsalves et al., 2019] Gonsalves, A. et al. (2019). Petawatt laser guiding and electron beam acceleration to 8 gev in a laser-heated capillary discharge waveguide. *Physical Review Letters*, 122(8):084801.

[Haussecker and Chao, 2011] Haussecker, E. F. and Chao, A. W. (2011). The influence of accelerator science on physics research. *Physics in Perspective*, 13(2):146. we extended the original analysis of the paper from 1939-2009 to 1939-2018.

[Hoddeson et al., 1997] Hoddeson, L., Brown, L., Dresden, M., and Riordan, M. (1997). *The rise of the standard model: A history of particle physics from 1964 to 1979*. Cambridge University Press.

[Kerst et al., 1956] Kerst, D., Cole, F., Crane, H., Jones, L., Laslett, L. J., Ohkawa, T., Sessler, A., Symon, K., Terwilliger, K., and Nilsen, N. V. (1956). Attainment of very high energy by means of intersecting beams of particles. *Physical review*, 102(2):590.

[Kuiper, 1994] Kuiper, B. (1994). Unk, protvino: A milestone in spite of hardships. *Europhysics News*, 25(3):66–66.

[Lawrence and Livingston, 1932] Lawrence, E. O. and Livingston, M. S. (1932). The production of high speed light ions without the use of high voltages. *Physical Review*, 40(1):19.

[Lebedev and Shiltsev, 2014] Lebedev, V. and Shiltsev, V. (2014). *Accelerator physics at the Tevatron collider*. Springer.

[Leemans and Esarey, 2009] Leemans, W. and Esarey, E. (2009). Laser-driven plasma-wave electron accelerators. *Phys. Today*, 62(3):44–49.

[Litos et al., 2016] Litos, M. et al. (2016). 9 gev energy gain in a beam-driven plasma wakefield accelerator. *Plasma Physics and Controlled Fusion*, 58(3):034017.

[Livingston, 1954] Livingston, M. S. (1954). *High energy accelerators*. Interscience.

[Long et al., 2021] Long, K. R., Lucchesi, D., Palmer, M. A., Pastrone, N., Schulte, D., and Shiltsev, V. (2021). Muon colliders to expand frontiers of particle physics. *Nature Physics*, 17(3):289–292.

[McIntyre et al., 2016] McIntyre, P. M., Assadi, S., Bannert, S., Chavez, D., Gerity, J., Pogue, N., Sattarov, A., Breitschopf, J., and Kellams, J. N. (2016). Collider in the sea: vision for a 500 tev world laboratory. In *2016 North American Particle Accelerator Conf.(NA-PAC'16), Chicago, IL, USA, October 9-14, 2016*, pages 13–16. JACoW Publishing, Geneva, Switzerland.

[Michizono, 2019] Michizono, S. (2019). The international Linear Collider. *Nature Reviews Physics*, 1(4):244–245.

[Month, 2003] Month, M. (2003). *Weep for Isabelle: a rhapsody in a minor key: a historical novel depicting the rise and fall of big physics in the second half of the twentieth century*. AvantGarde Press.

[Myers and Brüning, 2016] Myers, S. and Brüning, O. (2016). *Challenges and Goals for Accelerators in the XXI Century*. World Scientific.

[Myers and Schopper, 2013] Myers, S. and Schopper, H. (2013). Accelerators and colliders. *Landolt Bornstein*, 21.

[O'Neill, 1956] O'Neill, G. K. (1956). Storage-ring synchrotron: device for high-energy physics research. *Physical Review*, 102(5):1418.

[Palmer, 2014] Palmer, R. (2014). Muon colliders. *Reviews of Accelerator Science and Technology*, 7:137–159.

[Perkins, 2000] Perkins, D. H. (2000). *Introduction to high energy physics*. Cambridge University Press.

[Riordan et al., 2015] Riordan, M., Hoddeson, L., and Kolb, A. W. (2015). *Tunnel visions: the rise and fall of the Superconducting Super Collider*. University of Chicago Press.

[Roser et al., 2022] Roser, T., Brinkmann, R., Cousineau, S., Denisov, D., Gessner, S., Gourlay, S., Lebrun, P., Narain, M., Oide, K., Raubenheimer, T., et al. (2022). Report of the snowmass 2021 collider implementation task force. *arXiv preprint arXiv:2208.06030*.

[Sands, 1970] Sands, M. (1970). The physics of electron storage rings: an introduction. Technical Report SLAC-121, Stanford Linear Accelerator Center, Menlo Park, CA (USA).

[Schoerling and Zlobin, 2019] Schoerling, D. and Zlobin, A., editors (2019). *Nb3Sn Accelerator Magnets*. Springer. ISBN 978-3-030-16118-7.

[Schroeder et al., 2010] Schroeder, C., Esarey, E., Geddes, C., Benedetti, C., and Leemans, W. (2010). Physics considerations for laser-plasma linear colliders. *Physical Review Special Topics-Accelerators and Beams*, 13(10):101301.

[Seryi, 2016] Seryi, A. (2016). *Unifying physics of accelerators, lasers and plasma*. Taylor & Francis.

[Sessler, 1998] Sessler, A. M. (1998). Gamma-ray colliders and muon colliders. *Physics Today*, 51(3):48–53.

[Sessler and Wilson, 2014] Sessler, A. M. and Wilson, E. J. (2014). *Engines of discovery: A Century of particle accelerators*. World Scientific.

[Shiltsev, 2011] Shiltsev, V. (2011). On performance of high energy particle colliders and other complex scientific systems. *Modern Physics Letters A*, 26(11):761–772.

[Shiltsev, 2014] Shiltsev, V. (2014). A phenomenological cost model for high energy particle accelerators. *Journal of Instrumentation*, 9(07):T07002.

[Shiltsev, 2019] Shiltsev, V. (2019). Ultimate colliders for particle physics: Limits and possibilities. *International Journal of Modern Physics A*, 34:1943002.

[Shiltsev, 2020] Shiltsev, V. (2020). Particle beams behind physics discoveries. *Physics Today*, 73(4):32–39.

[Shiltsev and Zimmermann, 2021] Shiltsev, V. and Zimmermann, F. (2021). Modern and future colliders. *Reviews of Modern Physics*, 93(1):015006.

[Shiltsev, 2012] Shiltsev, V. D. (2012). High-energy particle colliders: past 20 years, next 20 years, and beyond. *Physics-Uspekhi*, 55(10):965.

[Snwomass'21 Workshop on "Physics Limits of Ultimate Beams",] Snwomass'21 Workshop on "Physics Limits of Ultimate Beams". `https://indico.fnal.gov/event/47217/`. Accessed: February 17, 2023.

[Stapnes, 2019] Stapnes, S. (2019). The compact linear collider. *Nature Reviews Physics*, 1(4):235–237.

[Tajima and Dawson, 1979] Tajima, T. and Dawson, J. M. (1979). Laser electron accelerator. *Physical Review Letters*, 43(4):267.

[Tigner, 1965] Tigner, M. (1965). A possible apparatus for electron clashing-beam experiments. *Il Nuovo Cimento (1955-1965)*, 37(3):1228–1231.

[Tollestrup and Todesco, 2008] Tollestrup, A. and Todesco, E. (2008). The development of superconducting magnets for use in particle accelerators: From the tevatron to the lhc. In *Reviews Of Accelerator Science And Technology: Volume 1*, pages 185–210. World Scientific.

[Van Der Meer, 1985] Van Der Meer, S. (1985). Stochastic cooling and the accumulation of antiprotons. *Reviews of Modern Physics*, 57(3):689.

[Waloschek, 2013] Waloschek, P. (2013). *The infancy of particle accelerators: Life and work of Rolf Wideroe*. Springer Science & Business Media.

[Wideroe, 1953] Wideroe, R. (1953). Patentschrift, nr. 876279, klass 21g, gruppe 36, ausgegeben am 11 mai 1953. Technical report, Deutsches Patentamt.

[Wojcicki, 2009] Wojcicki, S. (2009). The supercollider: the texas days—a personal recollection of its short life and demise. In *Reviews Of Accelerator Science And Technology: Volume 2: Medical Applications of Accelerators*, pages 265–301. World Scientific.

[Yarba, 1990] Yarba, V. (1990). The ihep unk project: Status and development. *Particle Accelerators*, 26:131–139.

[Zimmermann, 2018] Zimmermann, F. (2018). Future colliders for particle physics—“big and small”. *Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment*, 909:33–37.