FERMILAB-PUB-23-0826-SQMS-V

Five-point Superluminality Bounds

Francesco Serra*
Department of Physics and Astronomy, Johns Hopkins University, Baltimore, MD 21218, USA

Leonardo G. Trombetta†
CEICO, Institute of Physics of the Czech Academy of Sciences,
Na Slovance 1999/2, 182 00, Prague 8, Czechia.
(Dated: December 13, 2023)

We investigate how the speed of propagation of physical excitations is encoded in the coefficients of five-point interactions. This leads to a superluminality bound on scalar five-point interactions, which we present here for the first time. To substantiate our result, we also consider the case of four-point interactions for which bounds from S-matrix sum rules exist and show that these are parametrically equivalent to the bounds obtained within our analysis. Finally, we extend the discussion to a class of higher-point interactions.

INTRODUCTION

Fundamental interactions are universally characterized, at low enough energies, by the relevant field degrees of freedom, relevant energy scales and by the symmetries that govern the dynamics. This effective field theory (EFT) characterization allows to describe low energy processes with a given precision by specifying the coefficients of a finite number of interaction operators. Faced with the vast parameter space of yet undetermined coefficients that describe fundamental interactions in nature, an important and ambitious task is to develop a systematic understanding of how key dynamical features manifest at the level of these coefficients. A remarkable example of such manifestation is given by unitary, Lorentz-invariant effective field theories that respect microcausality, i.e. (anti)commutation of operators evaluated on space-like separated regions. These theories do not admit arbitrary coefficients for large classes of low-energy interactions, see e.g. [1–3]. More in detail, in these theories one expects the S-matrix to be an analytic function of the center of mass energy invariant, aside for particle production branch cuts and simple poles, as well as possible anomalous thresholds [4], see e.g. [5] for a pedagogic review. This property, together with crossing symmetry and boundedness of the S-matrix elements implied by unitarity, allows to constrain the coefficients of certain low-energy interactions through sum rules. For two-to-two S-matrix elements, this constraints impose either the positivity or relative boundedness of the coefficients characterizing the low energy 4-point interactions [6–8]. Beyond two-to-two S-matrix elements little is known. The crossing and analytic properties of the S-matrix are less transparent for higher-point processes [9–11], making it more difficult to obtain sum rules [12, 13].

While the principles underpinning the S-matrix sum rules characterize both low-energy (IR) and high-energy (UV) processes, similar constraints to those extracted from the S-matrix elements can be derived without any apparent statements regarding UV physics. This, by requiring that low energy excitations should not propagate faster than light [14–17]. Although the relation between these two different sets of bounds is not rigorously understood, their similarity can be traced back to how microcausality determines both the analyticity of the S-matrix elements and the absence of superluminal propagation at low energies [2, 3].

In light of this, one can sidestep obstacles in the sum rules and make progress in charting the parameter space of interactions by studying superluminality at low energies. Here we follow this idea and investigate how the absence of superluminality constrains 5-point interactions, finding for the first time a two-sided bound on quintic derivative interactions.

For simplicity, we consider the case of a real, shift-symmetric scalar field with derivative self-interactions as a proxy for more phenomenologically relevant systems. Bounds on some of the 4-point interaction coefficients appearing in these effective theories have already been derived through S-matrix sum rules, see e.g. [7, 8, 18], as well as considering superluminality at low energies [15]. The results from S-matrix sum rules in this context can be interpreted in terms of symmetries compatible with microcausality and Lorentz invariance. In particular, at the level of four-point interactions, the S-matrix analysis implies that the Galileon symmetry of the scalar, $\phi \to \phi + a + b_\mu x^\mu$ [19], with $a\,,b_\mu$ constants, is not compatible with the sum rules and that the Galileon invariant 4-point interactions should be subleading with respect to the Galileon breaking ones in the EFT.

In the following we study superluminal propagation on the simplest possible scalar field background. We derive bounds on 4-point interactions that are parametrically equivalent to those found in the literature. We then derive the central result of this work, a bound on 5-point interactions. This bound states that also at 5-point, Galileon symmetry is not consistent with microcausality. We finally generalize the bounds beyond Galileon operators and extend the analysis to a class of higher-point operators.

This manuscript has been authored by Fermi Research Alliance, LLC under Contract No. DE-AC02-07CH11359 with the U.S. Department of Energy, Office of Science, Office of High Energy Physics.

EXTRACTING TIME-SHIFTS FROM THE ACOUSTIC METRIC

We start by considering the following Lagrangian:

$$\mathcal{L} = -\frac{1}{2}\Big\{(\partial\phi)^2 - \frac{1}{2}c_2(\partial\phi)^4 + c_3(\partial\phi)^2\Box\phi + c_4(\partial\phi)^2 E_4 + c_5(\partial\phi)^2 E_5\Big\} + \dots\ , \quad (1)$$

where the coefficients c_i are dimensionful, e.g. $c_2 \sim \Lambda_2^{-4}$, $c_3 \sim \Lambda_3^{-3}$, $c_4 \sim \Lambda_3^{-6}$, $c_5 \sim \Lambda_3^{-9}$ (with $\Lambda_{2,3}$ some energy scales), while $E_4 = (\Box\phi)^2 - (\partial^2\phi)^2$ and $E_5 = (\Box\phi)^3 - 3\Box\phi(\partial^2\phi)^2 + 2(\partial^2\phi)^3$. Dots indicate other operators which we assume to be suppressed by smaller coefficients and therefore negligible. In the following, we will adopt the notation $\Pi_{\mu\nu} = \partial_\mu\partial_\nu\phi$. In this Lagrangian the Galileon symmetry is only broken by the c_2 contribution. The shift-symmetry of the field, $\phi \to \phi + a$, is instead unbroken, leaving the scalar massless. In this theory, we consider a perturbation φ propagating on a background ϕ. By linearizing the equation of motion with respect to φ, and considering a plane-wave ansatz $\varphi = A\exp(-ik_\mu x^\mu)$, we obtain the following [20]:

$$0 = Z^{\mu\nu}k_\mu k_\nu - iD^\mu k_\mu\ , \quad (2)$$

where the acoustic metric is:

$$\begin{aligned} Z_{\mu\nu} = &\left(1 - c_2(\partial\phi)^2 + 2c_3\Box\phi + 3c_4E_4 + 4c_5E_5\right)\eta_{\mu\nu} \\ &-2c_2\,\partial_\mu\phi\,\partial_\nu\phi - 2\left(c_3 + 3c_4\Box\phi + 6c_5E_4\right)\Pi_{\mu\nu} \\ &+6\left(c_4 + 4c_5\Box\phi\right)\Pi_{\mu\alpha}\Pi^\alpha{}_\nu - 24c_5\,\Pi_{\mu\alpha}\Pi^{\alpha\beta}\Pi_{\beta\nu}, \end{aligned} \quad (3)$$

while $D^\nu = -c_2\left(4\partial_\mu\phi\Pi^{\mu\nu} + 2\Box\phi\partial^\nu\phi\right)$ brings in general dissipation or enhancement of the perturbations but can be neglected if propagation takes place over short enough distances, as we will discuss below. If this is the case, one can solve $Z^{\mu\nu}k_\mu k_\nu = 0$, and for a diagonizable $Z^{\mu\nu}$ get:

$$\omega^2 = c_{si}^2 k_i^2\ . \quad (4)$$

Focusing on a single direction, we will assume $c_s = 1 + \delta c$, with $|\delta c| \ll 1$. Superluminality will then correspond to time-like k_μ and wavefronts that propagate outside the lightcone: $\eta^{\mu\nu}k_\mu k_\nu = -\omega^2 + \vec{k}^2 \simeq -2\delta c\,\vec{k}^2$, were we are neglecting $O(\delta c^2)$ corrections.

With this, when a signal propagates through a region of size L in the background ϕ, it will accumulate a phase-shift from which one can extract the following time delay or advance [21]:

$$\Delta T = \int_0^L dx\left(\frac{1}{c_s} - 1\right) \simeq -\int_0^L dx\,\delta c + O\left(\frac{1}{\Lambda_{UV}}\right)\ , \quad (5)$$

where Λ_{UV} is the cutoff of the EFT, which limits the accuracy of the computation of the phase-shift, and cannot be larger than the strong-coupling scale, $\Lambda_{UV} \lesssim \Lambda_3$. It follows that we have a robust prediction of superluminality whenever we find

$$\Delta T \lesssim -\frac{1}{\Lambda_{UV}}\ . \quad (6)$$

This uncertainty is due to lack of precision in the EFT, rather than to the time-localization of the probe. Indeed, one could imagine interferometric measurements of the phase-shift whose precision would not be limited by the probe's frequency.

To keep our analysis as simple as possible, in the following we will consider backgrounds in which $\Pi^\mu{}_\nu = \partial^\mu\partial_\nu\phi$ is a constant, diagonal matrix. This means that, aside for the c_2 contribution, the acoustic metric will also be in a diagonal form: $Z^\mu{}_\nu = (1+A)\delta^\mu{}_\nu + B_1\,\Pi^\mu{}_\nu + B_2\,(\Pi^2)^\mu{}_\nu + B_3\,(\Pi^3)^\mu{}_\nu - 2c_2\,\partial^\mu\phi\,\partial_\nu\phi$, with $\partial_\mu\phi = \Pi_{\mu\nu}x^\nu$ given a suitable choice of coordinates. Note that instead, choosing a background with constant gradient $\partial_\mu\phi = b_\mu$ straightforwardly leads to the positivity bound $c_2 > 0$ [2].

Before closing the section, we remark that one could further study whether the Null Dominant Energy Condition (NDEC) [22] is satisfied by the backgrounds considered. NDEC violation would diagnose superluminal transport of energy in the background, further reducing the space of coefficients that are compatible with microcausality. However, by inspection of the Stress-Energy tensor, we see that the NDEC violation can only lead to bounds parametrically similar to the ones we derive in this work.

SUPERLUMINALITY IN QUARTIC INTERACTIONS

We can start analyzing how the interplay between the different quartic interactions can give raise to superluminality when higher derivative Galileon-like interactions become larger than lower derivative shift-symmetric interactions. In practice, let us start by assuming that only c_2 and c_4 are non zero. We will discuss in a later section how c_3^2 contributions enter this picture, obtaining bounds on the same combination of c_4 and c_3^2 as the one bounded by S-matrix sum rules [23]. We will work in the perturbative regime in which $(c_4\Pi^2 = \epsilon_4\ ,\ c_2(\partial\phi)^2 = \epsilon_2) \lesssim \epsilon \ll 1$. These conditions, together with limiting the frequency of perturbation to low enough values, grant that the background will satisfy the Null Energy Condition (NEC) and that no ghost instabilities will appear. With this setup, Eq. (3) leads to:

$$\begin{aligned} Z^{\mu\nu}k_\mu k_\nu = &(1 + 2c_2(\partial\phi)^2 + 3c_4E_4)k^2 - 2c_2(x\cdot\Pi\cdot k)^2 \\ &+6c_4 k\cdot\Pi\cdot\Pi\cdot k + O(\epsilon^2)\ , \end{aligned} \quad (7)$$

where we are using the approximate equation of motion for the background $\eta^{\mu\nu}\Pi_{\mu\nu} = O(\epsilon)$. The linear-in-$k$ term instead takes the form:

$$iD^\mu k_\mu = -4ic_2 x\cdot\Pi\cdot\Pi\cdot k\ . \quad (8)$$

On the constant Π background, at leading order in ϵ this term produces an imaginary part in the frequency of perturbations, schematically $\text{Im}(\omega) \sim c_2\Pi^2 x$. The consequent instability leads to enhancement or depletion of perturbations which however will always be negligible when considering propagation over small enough regions. We will verify below that superluminality ensues within regions that are small enough for this to be the case. Therefore, we can solve $Z_{\mu\nu}k^\mu k^\nu = 0$, and from this dispersion relation we can derive the soundspeed deviation from lightspeed. For instance, for a perturbation with momentum $k_\mu = (-c_s k_1, k_1, 0, 0)$ traveling along $x^\mu = (x_1/c_s, x_1, 0, 0) + O(\epsilon)$, we find:

$$\delta c = -c_2 x_1^2(\Pi_{00}+\Pi_{11})^2 + 3c_4(-\Pi_{00}^2+\Pi_{11}^2) + O(\epsilon^2)\ . \quad (9)$$

Along this trajectory, our perturbative approximation remains valid for a finite length L_{max} such that $c_2(\partial\phi)^2 = \epsilon_2 \ll 1$:

$$L_{max} = \frac{\epsilon_2^{1/2}}{(c_2|\Pi_{00}^2 - \Pi_{11}^2|)^{1/2}}\ . \quad (10)$$

Since $\text{Im}(\omega)x \simeq c_2(\partial\phi)^2$, restricting our solution to a region of size L_{max} also grants that the instability can be neglected. Outside of this region we can imagine the background to change in such a way to quench the instability, e.g. turning into $\Pi = 0$. With this, using Eq. (5), we find the time-shift accumulated with respect to a lightray:

$$\begin{aligned}\Delta T &= \frac{c_2}{3}L_{max}^3(\Pi_{00}+\Pi_{11})^2 - 3c_4 L_{max}(-\Pi_{00}^2+\Pi_{11}^2) \\ &\quad + O(L_{max}\epsilon^2)\ . \end{aligned} \quad (11)$$

From this expression we can readily derive the positivity bound $c_2 > 0$, by considering the case in which $\Pi_{00} \sim \Pi_{11}$. To inspect other cases, it is useful to parameterize the background as:

$$|-\Pi_{00}^2 + \Pi_{11}^2| = \frac{\Lambda^2}{c_2} \quad , \quad \text{with} \quad \Lambda \sim \frac{\epsilon_2^{1/2}}{L_{max}}\ , \quad (12)$$

and $(\Pi_{00} - \Pi_{11})^2 = \alpha\frac{\Lambda^2}{c_2}$. Choosing also $\epsilon_4 = c_4(-\Pi_{00}^2 + \Pi_{11}^2)$, we find:

$$\Delta T = L_{max}\left(\frac{\alpha}{3}\epsilon_2 - 3\epsilon_4\right)\ . \quad (13)$$

From this we find that regardless of the sign of c_4, there are backgrounds for which one has a robust estimate of superluminality when

$$|c_4|\Lambda^2 \gtrsim c_2\left(\frac{\Lambda}{\Lambda_{UV}}\frac{1}{3\epsilon_2^{1/2}} + \frac{\alpha\epsilon_2}{9}\right)\ , \quad (14)$$

which is well compatible with the condition of perturbativity $\epsilon_4 < 1$. Neglecting the second term and choosing $\epsilon_2 \sim 0.1$, the bound to avoid superluminality becomes:

$$|c_4|\Lambda\Lambda_{UV} \lesssim c_2 \quad , \quad \Lambda \sim \frac{1}{3L_{max}}\ . \quad (15)$$

Since Λ can be pushed towards Λ_{UV} at the expense of perturbative ease, this bound is parametrically equivalent to the one found in [8, 18].

SUPERLUMINALITY IN QUINTIC INTERACTIONS

We now consider the main case of interest, in which the quintic interaction has a possibly large coefficient c_5. For simplicity, we neglect c_3 and c_4, keeping only track of the positive c_2, and we work in the perturbative regime in which $(c_5\Pi^3 = \epsilon_5\ ,\ c_2(\partial\phi)^2 = \epsilon_2) \lesssim \epsilon \ll 1$. In this case, evaluating the acoustic metric on the background ϕ and contracting it with the 4-vector of a generic perturbation we find:

$$\begin{aligned} Z^{\mu\nu}k_\mu k_\nu = &\ (1 + 2c_2(\partial\phi)^2 + 4c_5 E_5)k^2 - 2c_2(x\cdot\Pi\cdot k)^2 \\ & -12c_5(E_4 k\cdot\Pi\cdot k + 2k\cdot\Pi\cdot\Pi\cdot\Pi\cdot k) + O(\epsilon^2), \end{aligned} \quad (16)$$

where again we are using the approximate equation of motion for the background. With the same notation as in the previous section, for a perturbation with momentum $k_\mu = (-c_s k_1, k_1, 0, 0)$ traveling along $x^\mu = (x_1/c_s, x_1, 0, 0) + O(\epsilon)$ we obtain:

$$\begin{aligned}\delta c &= -c_2 x_1^2(\Pi_{00}+\Pi_{11})^2 + 6c_5(\text{tr}[\Pi^2](\Pi_{00}+\Pi_{11}) \\ &\quad -2(\Pi_{00}^3 + \Pi_{11}^3)) + O(\epsilon^2)\ , \end{aligned} \quad (17)$$

where $\text{tr}[\Pi^2] = \Pi_{00}^2 + \Pi_{11}^2 + \Pi_{22}^2 + \Pi_{33}^2$. From this, we see that as in the previous case the Galileon contribution can lead to superluminal speed regardless of the sign of c_5. Similarly to before, L_{max} is given by Eq. (10). Interestingly, in contrast with the case of the quartic interaction, the c_5 contribution is not proportional to $\Pi_{00}^2 - \Pi_{11}^2$, meaning that one could make the c_2 contribution small without suppressing the overall time advance. Rewriting the Π^3 coefficient of c_5 as $\frac{\Lambda^3}{c_2^{3/2}}$, with again $\Lambda \sim \frac{\epsilon_2^{1/2}}{L_{max}}$, we have:

$$\Delta T = \frac{c_2}{3}L_{max}^3(\Pi_{00}+\Pi_{11})^2 - 6c_5 L_{max}\frac{\Lambda^3}{c_2^{3/2}} + O(\epsilon^2)\ . \quad (18)$$

Since we are working in perturbation theory, we can make $\Pi_{00} + \Pi_{11}$ as small as $O(\epsilon)$. In this case the c_2 contribution scales as $\epsilon^3 L_{max}$, with $L_{max} \sim \frac{\epsilon_2^{1/2}}{\Lambda}$. Therefore we find superluminality as soon as:

$$|c_5|\Lambda^3 \gtrsim \frac{c_2^{3/2}}{6}\left(\frac{\Lambda}{\Lambda_{UV}\epsilon_2^{1/2}} + O(\epsilon^2)\right)\ . \quad (19)$$

Similarly to the quartic case, this prediction of superluminality is obtained well in the perturbative regime $\epsilon_5 < 1$. Therefore, the requirement of absence of superluminal propagation can be read as the following parametric bound:

$$|c_5|\Lambda^2\Lambda_{UV} \lesssim c_2^{3/2} \quad , \quad \Lambda \sim \frac{\epsilon_2^{1/2}}{L_{max}}\ . \quad (20)$$

This is a novel two-sided bound on 5-point interactions. Similarly to the case of 4-point interactions, this bound parametrically states that 5-point interactions cannot display weakly broken Galileon symmetry unless micro-causality is broken.

FIELD REDEFINITIONS AND DEGENERACY OF THE COEFFICIENTS

In the previous sections we have studied cases in which c_3 was set to zero. However, we can discuss the cases in which c_3 is present by means of a field redefinition. We find that the specific choice:

$$\phi \;=\; \hat{\phi} + \frac{c_3}{2}(\partial\hat{\phi})^2 + \frac{c_3^2}{2}\,\partial\hat{\phi}\cdot\partial^2\hat{\phi}\cdot\partial\hat{\phi} + \frac{c_3^3}{2}\left(\partial\hat{\phi}\cdot\partial^2\hat{\phi}\cdot\partial^2\hat{\phi}\cdot\partial\hat{\phi} + \frac{1}{3}\partial^3\hat{\phi}\cdot(\partial\hat{\phi})^3\right), \quad (21)$$

where $\partial^3\hat{\phi}(\partial\hat{\phi})^3 = \partial^{\mu\nu\rho}\hat{\phi}\,\partial_\mu\hat{\phi}\,\partial_\nu\hat{\phi}\,\partial_\rho\hat{\phi}$, maps the Galileon invariant Lagrangian with coefficients $\{c_3, c_4, c_5\}$ to another Galileon invariant Lagrangian without cubic Galileon term:

$$\{\, c_3\,,\, c_4\,,\, c_5 \,\} \;\to\; \left\{\, 0\,,\, c_4 - \frac{c_3^2}{2}\,,\, c_5 - c_3 c_4 + \frac{c_3^3}{3} \,\right\}. \quad (22)$$

These combinations are those typically found in 4-pt and 5-pt amplitudes [23, 24]. The field redefinition will also give rise to higher derivative 6 and 7 fields interactions, whose contribution is suppressed in the dispersion relation as long as k is small enough. In this new field frame, we will find results similar to those discussed in the previous sections. When the quintic interaction is negligible with respect to the quartic we will have, parametrically:

$$\left| c_4 - \frac{c_3^2}{2} \right| \Lambda_{UV}^2 \lesssim c_2 \,. \quad (23)$$

If we exploit the non-renormalization properties of Galileons [25] to tune the coefficients c_3 and c_4 so as to make the quartic interaction suppressed and negligible, $c_4 = c_3^2/2$, we will have the following parametric bound:

$$\left| c_5 - \frac{c_3^3}{6} \right| \Lambda_{UV}^3 \lesssim c_2^{3/2} \,. \quad (24)$$

In particular, this bound restricts the coefficients c_3 , c_4 even when their contribution to four-point amplitudes is negligible.

Examining the computation in the original field frame, one finds a contribution linear in c_3 to the dispersion relation. Despite this, c_3 enters the phase-shift starting at quadratic order. Even more, it can be shown that this observable is sensitive only to the coefficients in the combinations of Eq. (22). This originates from the same dynamical redundancy that makes possible to remove the cubic Galileon interaction with a field redefinition, and confirms the intuition that the phase-shift inherits the field reparametrization invariance of the S-matrix.

Another way to understand this is by rephrasing the problem of the propagation of the high frequency fluctuations in geometrical terms, provided the term $D^\mu\partial_\mu\varphi$ can be neglected. In this regime, the quadratic action takes the form of a free massless scalar field in curved space with effective metric $G_{\mu\nu}$, such that $\sqrt{-G}\,G^{-1\,\mu\nu} = Z^{\mu\nu}$. The optical path of a scalar perturbation is given by the G-null geodesics described by $\tilde{k}^\mu \equiv G^{-1\,\mu\nu}k_\mu$:

$$\tilde{k}^\mu\,\nabla^G_\mu \tilde{k}^\nu = 0, \quad (25)$$

with ∇^G_μ the covariant derivative compatible with $G_{\mu\nu}$ [26]. In this description, Shapiro-like time differences between neighboring geodesics are associated with a non-vanishing curvature $\mathcal{R}^\mu{}_{\nu\rho\sigma}(G)$, which contains no linear in c_3 contribution.

GOING BEYOND GALILEONS

It is worth considering how the previous analysis applies to other shift-symmetric, but non-Galileon invariant operators. Consider quartic and quintic operators schematically of the form

$$\tilde{c}_4\partial^6\phi^4 \quad , \quad \tilde{c}_5\partial^8\phi^5 \,, \quad (26)$$

with the same power counting as their Galileon counterparts but with different contractions. While these operators lead to higher-than-second-order equations of motion, in the EFT framework they can be treated consistently at low energies ($E \ll \tilde{c}_4^{-1/6}, \tilde{c}_5^{-1/9}$). For the dispersion relation, this means that any new roots for $\omega(\vec{k})$ lie above the cutoff scale, while the low energy ones get only perturbatively modified.

The above operators can lead to a non-zero Galileon-like contribution, plus higher-order terms which add $\vec{k}$ dependent pieces to the soundspeed that can a priori become dominant. However, most of the contributions beyond the Galileon-like one are either: (a) proportional to $k_\mu k^\mu \simeq 0$, (b) proportional to $\Box\phi \simeq 0$, (c) containing derivatives acting on $\Pi = const$, or (d) odd-in-k_μ and then purely imaginary. In any of these cases, their effect on c_s will be negligible in the perturbative regime.

For the quartic operators, those are all the contributions aside for the Galileon-like term. When this is present, the soundspeed is given just by substituting $c_4 \to \tilde{c}_4$ in Eq. (9). In this case, we conclude that the bounds discussed before on the quartic Galileon operator also apply here with no changes. From the S-matrix perspective, this follows from the fact that there is only one contribution to the on-shell 4-point scattering amplitude at this order in derivatives.

Turning to five-point interactions, there is a new relevant k^4 contribution to the dispersion relation:

$$\tilde{c}_5 \left(\Pi^{\mu\nu}\partial^\alpha\phi\,\partial^\beta\phi\right) k_\mu k_\nu k_\alpha k_\beta + O(\epsilon)\ . \tag{27}$$

However, being proportional to $(\partial\phi\cdot k)^2$ this contribution can be made $O(\epsilon)$ suppressed with respect to the Galileon-like contribution by taking $\Pi_{00}\sim -\Pi_{11}$. In this way, we find that our bound on quintic interactions is also applicable beyond the Galileon-symmetric case, provided a $\tilde{c}_5(\Pi_{\mu\nu})^3$ term is present. Furthermore, the contribution in Eq. (27) could make the bounds even stronger, although we can expect them to remain parametrically similar, since no physical scale separates k from Λ.

Before concluding, we consider the case of Galileon-like operators with higher number of fields, such as:

$$c_n(\partial\phi)^2\Pi^{n-2}\ , \tag{28}$$

with $n\geq 6$. In the presence of $c_2(\partial\phi)^4$, c_n will give a contribution to the time-shift of order

$$c_n L_{max}\frac{\Lambda^{n-2}}{c_2^{(n-2)/2}}(1+(kL_{max})^2)+O(\epsilon^2)\ . \tag{29}$$

Then, if the combination of constant background Π^{n-2} can consistently get both signs, we will get the following two-sided bounds:

$$|c_n|\Lambda^{n-3}\Lambda_{UV}\lesssim c_2^{(n-2)/2}\ . \tag{30}$$

DISCUSSION

In this letter we studied how the presence of superluminal propagation at low energies is reflected in the coefficients of 5-point interactions. In the case of a self-interacting scalar field, we derived two-sided bounds on these coefficients by computing the time advance accumulated by a scalar perturbation traveling through a region with a given background. Our analysis is made particularly straightforward by considering propagation on a finite region over which both the background and the dynamics of perturbations have a simple description. This simplicity comes at the expense of the sharpness of our bounds, which despite being robust are only parametrically precise. For concreteness, we derived explicitly the bound on the quintic Galileon interaction, Eq. (20), extending them later to other shift-symmetric 5-point scalar self-interactions having the same number of derivatives, Eq. (26). These bounds state superluminality will appear when these quintic interactions are large compared to the shift symmetric quartic ones. If we write $c_2\sim 1/\Lambda_2^4$, $c_5\sim 1/\Lambda_3^9$, then the bound can be read parametrically as stating that $\Lambda_3^9\gtrsim\Lambda_2^6\Lambda_{UV}^3$. This means that a situation of (weakly broken) Galileon invariance, in which Λ_3 is low compared to the other scales, is not compatible with microcausality at the level of 5-point interactions.

Besides 5-point interactions, we considered 4-point interactions and showcased how our simplified analysis can reproduce the existing constraints arising from S-matrix sum rules in this context.

Moreover, we argued that Galileon-like interactions with higher number of fields should be compatible with microcausality only when Eq. (30) holds. These results go in the direction of deriving a definitive statement about Galileon symmetry. However, that would require deriving bounds on the coefficients of all higher derivative operators as well, a task that we leave for future work.

In conclusion, it will be interesting to employ the simple analytic strategy presented here to investigate other classes of interactions, both at higher-points and in contexts more relevant for phenomenology.

ACKNOWLEDGMENTS

The authors would like to thank B. Bellazzini, D. E. Kaplan, D. Kosmopoulos, S. Rajendran, S. Ramazanov, J. Serra, G. Trenkler, A. Vikman for useful discussions, and especially I. Sawicki and E. Trincherini for helpful comments on the manuscript. The work of F.S. was supported by the U.S. Department of Energy (DOE), Office of Science, National Quantum Information Science Research Centers, Superconducting Quantum Materials and Systems Center (SQMS) under Contract No. DE-AC02-07CH11359 as well as by the Simons Investigator Award No. 827042 (P.I.: Surjeet Rajendran). The work of L.G.T. was supported by European Union (Grant No. 101063210). We thank the hospitality of the Centro de Ciencias de Benasque Pedro Pascual during the initial stages of this work.

* fserra2@jh.edu
† trombetta@fzu.cz

[1] T. N. Pham and T. N. Truong, Phys. Rev. D **31**, 3027 (1985).
[2] A. Adams, N. Arkani-Hamed, S. Dubovsky, A. Nicolis, and R. Rattazzi, JHEP **10**, 014 (2006), arXiv:hep-th/0602178.
[3] N. Arkani-Hamed, T.-C. Huang, and Y.-T. Huang, JHEP **05**, 259 (2021), arXiv:2012.15849 [hep-th].
[4] J. Bros, H. Epstein, and V. J. Glaser, Nuovo Cim. **31**, 1265 (1964).
[5] S. Mizera, Phys. Rept. **1047**, 1 (2024), arXiv:2306.05395 [hep-th].
[6] B. Bellazzini, J. Elias Miró, R. Rattazzi, M. Riembau, and F. Riva, Phys. Rev. D **104**, 036006 (2021), arXiv:2011.00037 [hep-th].
[7] A. J. Tolley, Z.-Y. Wang, and S.-Y. Zhou, JHEP **05**, 255 (2021), arXiv:2011.02400 [hep-th].

[8] S. Caron-Huot and V. Van Duong, JHEP **05**, 280 (2021), arXiv:2011.02957 [hep-th].
[9] J. Bros, V. Glaser, and H. Epstein, Helv. Phys. Acta **45**, 149 (1972).
[10] S. Caron-Huot, M. Giroux, H. S. Hannesdottir, and S. Mizera, (2023), arXiv:2308.02125 [hep-th].
[11] S. Caron-Huot, M. Giroux, H. S. Hannesdottir, and S. Mizera, (2023), arXiv:2310.12199 [hep-th].
[12] J. Bros, Phys. Rept. **134**, 325 (1986).
[13] V. Chandrasekaran, G. N. Remmen, and A. Shahbazi-Moghaddam, JHEP **11**, 015 (2018), arXiv:1804.03153 [hep-th].
[14] X. O. Camanho, J. D. Edelstein, J. Maldacena, and A. Zhiboedov, JHEP **02**, 020 (2016), arXiv:1407.5597 [hep-th].
[15] M. Carrillo Gonzalez, C. de Rham, V. Pozsgay, and A. J. Tolley, Phys. Rev. D **106**, 105018 (2022), arXiv:2207.03491 [hep-th].
[16] F. Serra, J. Serra, E. Trincherini, and L. G. Trombetta, JHEP **08**, 157 (2022), arXiv:2205.08551 [hep-th].
[17] M. Carrillo González, C. de Rham, S. Jaitly, V. Pozsgay, and A. Tokareva, (2023), arXiv:2307.04784 [hep-th].
[18] B. Bellazzini, M. Riembau, and F. Riva, Phys. Rev. D **106**, 105008 (2022), arXiv:2112.12561 [hep-th].
[19] A. Nicolis, R. Rattazzi, and E. Trincherini, Phys. Rev. D **79**, 064036 (2009), arXiv:0811.2197 [hep-th].
[20] An Eikonal ansatz gives the same $Z_{\mu\nu}$ but can modify D^{μ}. However, our discussion is not affected by this.
[21] L. Maiani and M. Testa, Annals Phys. **263**, 353 (1998), arXiv:hep-th/9709110.
[22] E. Curiel, Einstein Stud. **13**, 43 (2017), arXiv:1405.0403 [physics.hist-ph].
[23] B. Bellazzini, F. Riva, J. Serra, and F. Sgarlata, Phys. Rev. Lett. **120**, 161101 (2018), arXiv:1710.02539 [hep-th].
[24] K. Kampf, J. Novotny, F. Preucil, and J. Trnka, JHEP **07**, 153 (2021), arXiv:2104.10693 [hep-th].
[25] M. A. Luty, M. Porrati, and R. Rattazzi, JHEP **09**, 029 (2003), arXiv:hep-th/0303116.
[26] L. Heisenberg and C. F. Steinwachs, JCAP **02**, 031 (2020), arXiv:1909.07111 [hep-th].