Noise reduction in quantum simulations of scalar QED via qudit encodings: Why physics has a qudit bias

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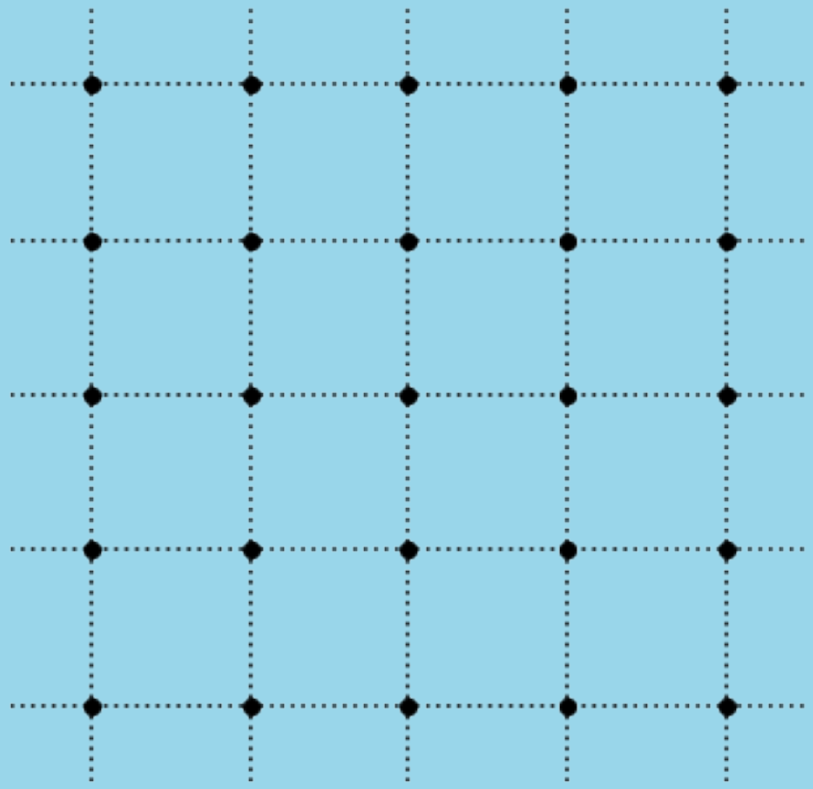
Fermilab

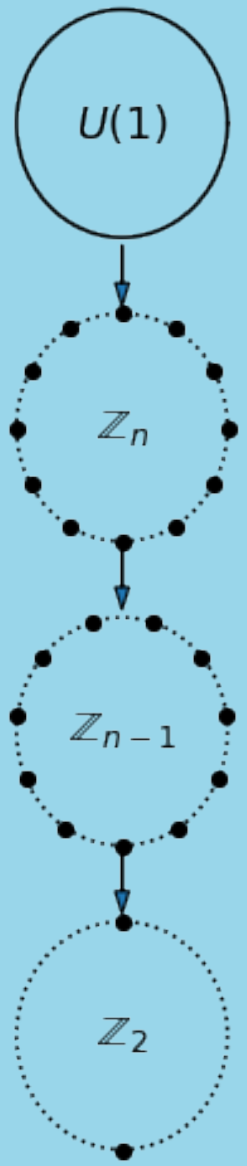
Why does Lattice Gauge Theory need Quantum Computers?

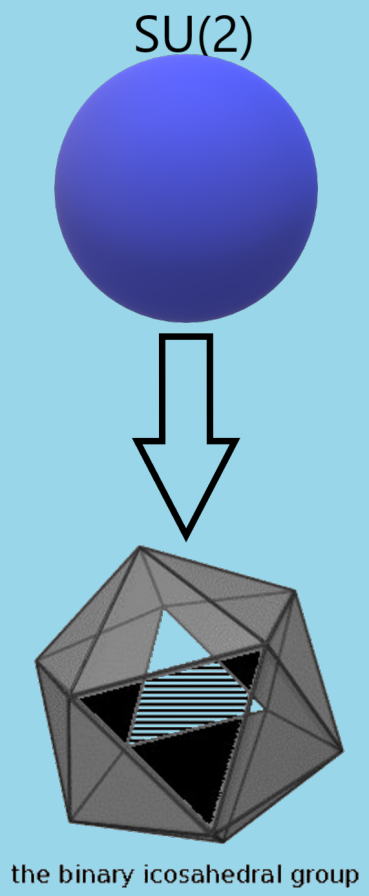
- Some sectors of the standard model are non-perturbative
- Lattice Field Theory is systematically improvable way to compute non-perturbative results
- Some standard model physics, e.g. viscosities, are classically inaccessible



So why do we care about qudits?

- Our theories are infinite dimension $\rightarrow$ regulate to finite resources
- Most discretizations of $U(1)$, $\mathbb{Z}_n$, don't map onto powers of 2.
- Discretizations of $SU(2)$ and $SU(3)$ don't map onto powers of 2. [a]
- Fermions may be more compactly implemented by qudits rather than a collection of qubits.





[a] Y. Ji, Phys. Rev. D(102), 114513

So what are we simulating?[1]

Hamiltonian

Meson Creation

Gauge Strength

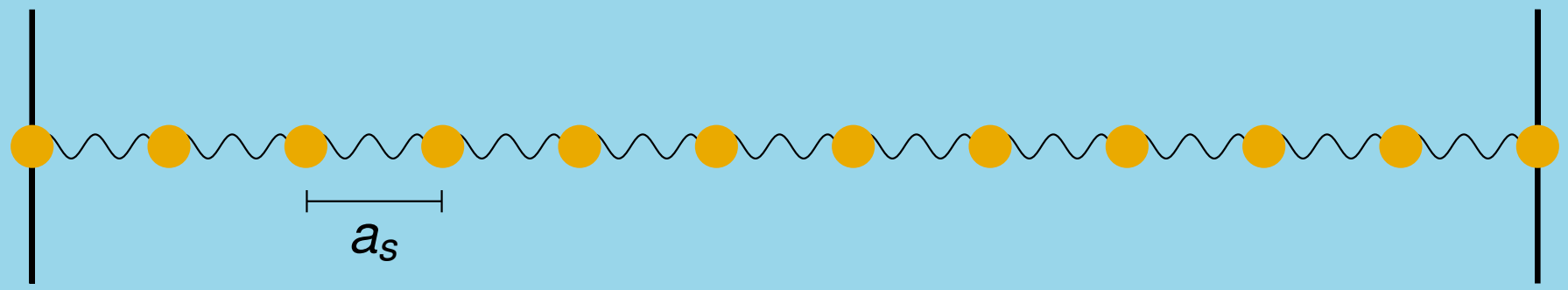
$$\hat{H} = \frac{g^2 a_s}{2} \sum_{i=1}^{N_s} (\hat{L}_i^z)^2 - \frac{2}{a_s} \sum_{i=1}^{N_s} \hat{U}_i^x + \frac{1}{4a_s} \sum_{i=1}^{N_s-1} (\hat{L}_i^z - \hat{L}_{i+1}^z)^2 + \frac{1}{4a_s}\left((\hat{L}_1^z)^2 + (\hat{L}_{N_s}^z)^2\right)$$

Particle Number

g^2 is the coupling strength, a_s is the spacing between lattice sites, U^x is not unitary

[1] Bazavov et al. Phys. Rev. D 92, 076003 (2015); Bazavov et al. arXiv:1512.01737; Zhang et al. Phys. Rev. Lett. 121, 223201 (2018); Unmuth-Yockey et al. Phys Rev D (98) 2018

compact Scalar QED



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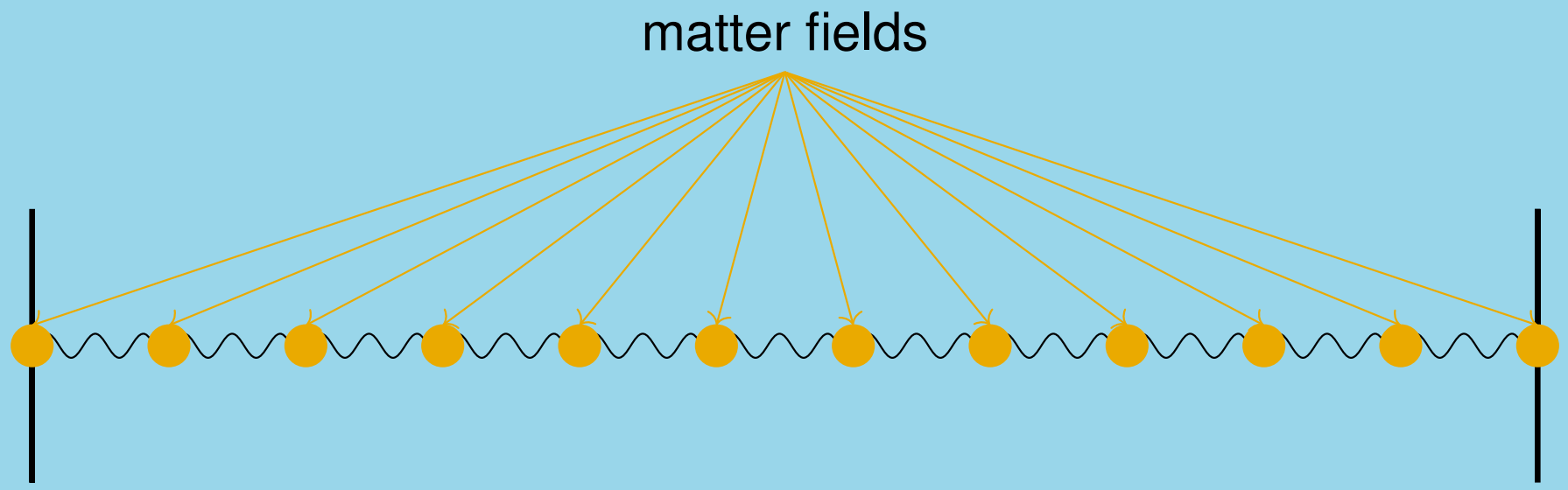

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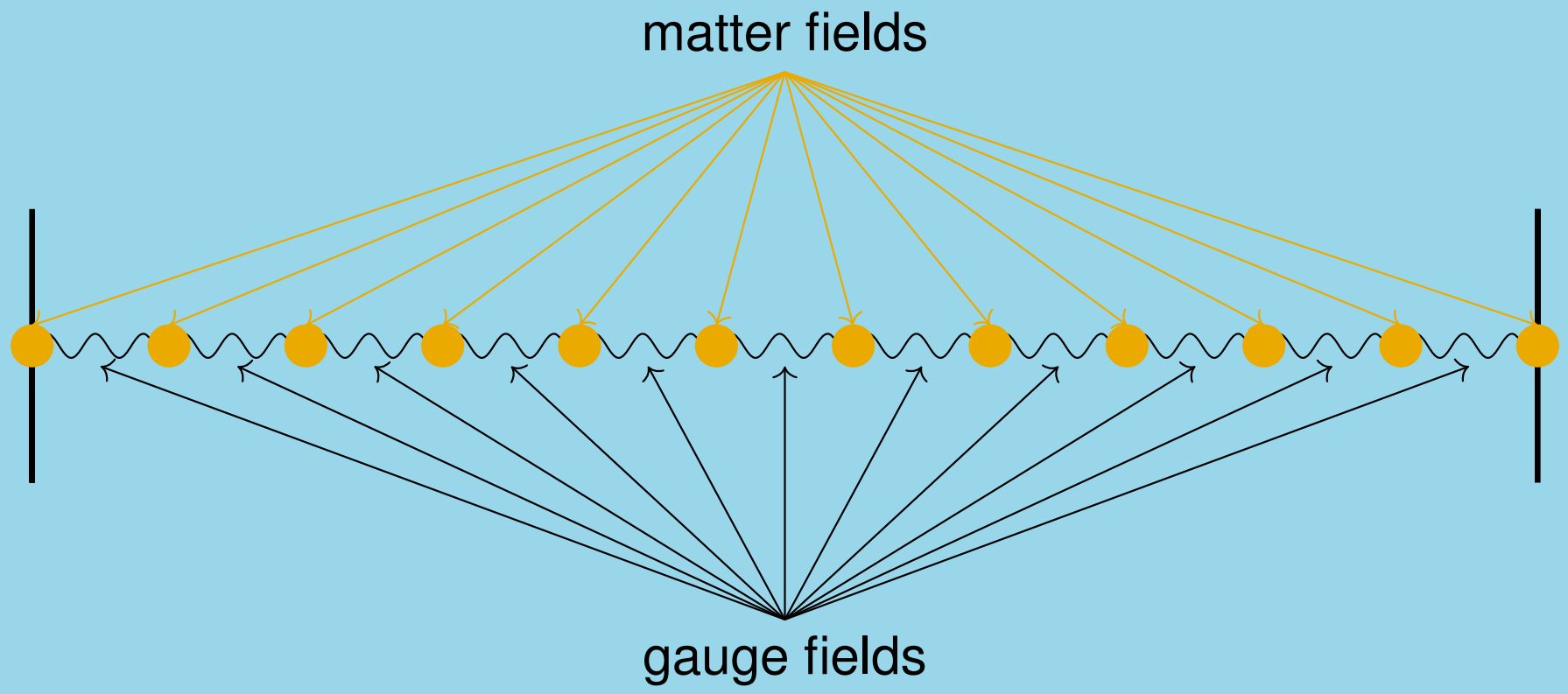
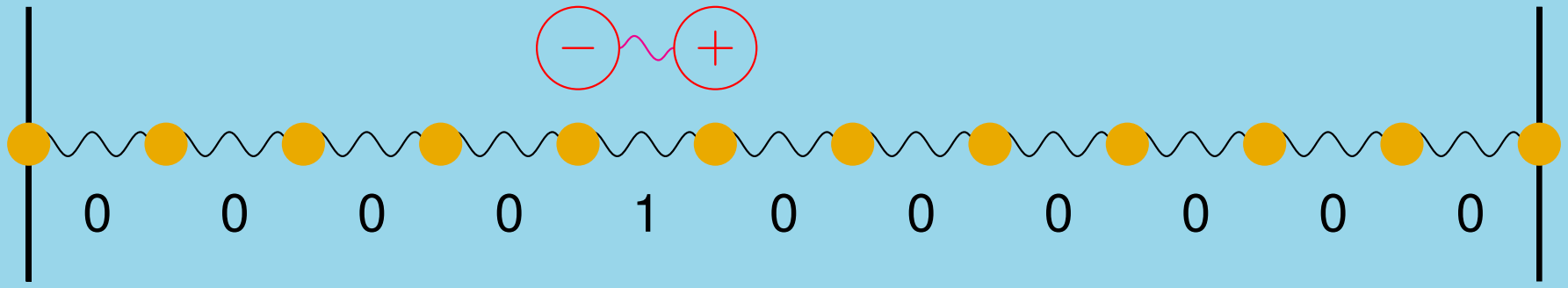


compact Scalar QED

$$(\hat{L}^z_i - \hat{L}^z_{i+1}) \rightarrow \text{scalar particle number}$$



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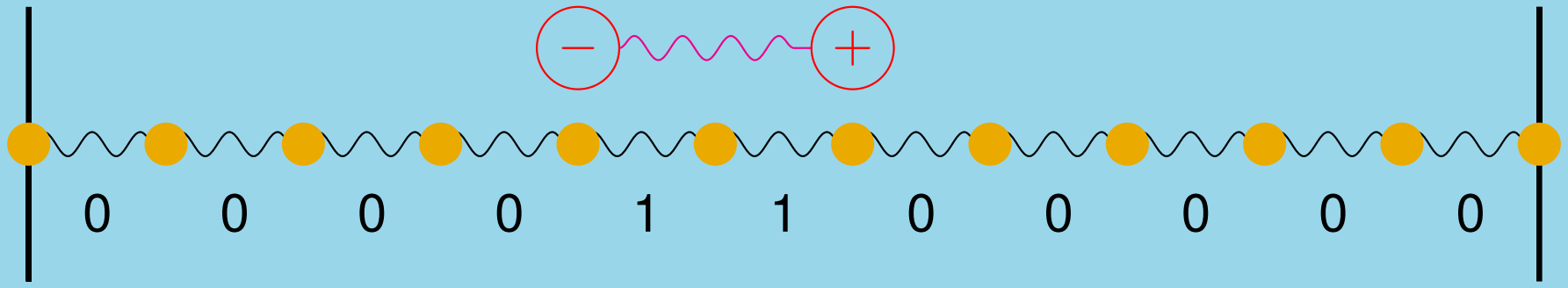
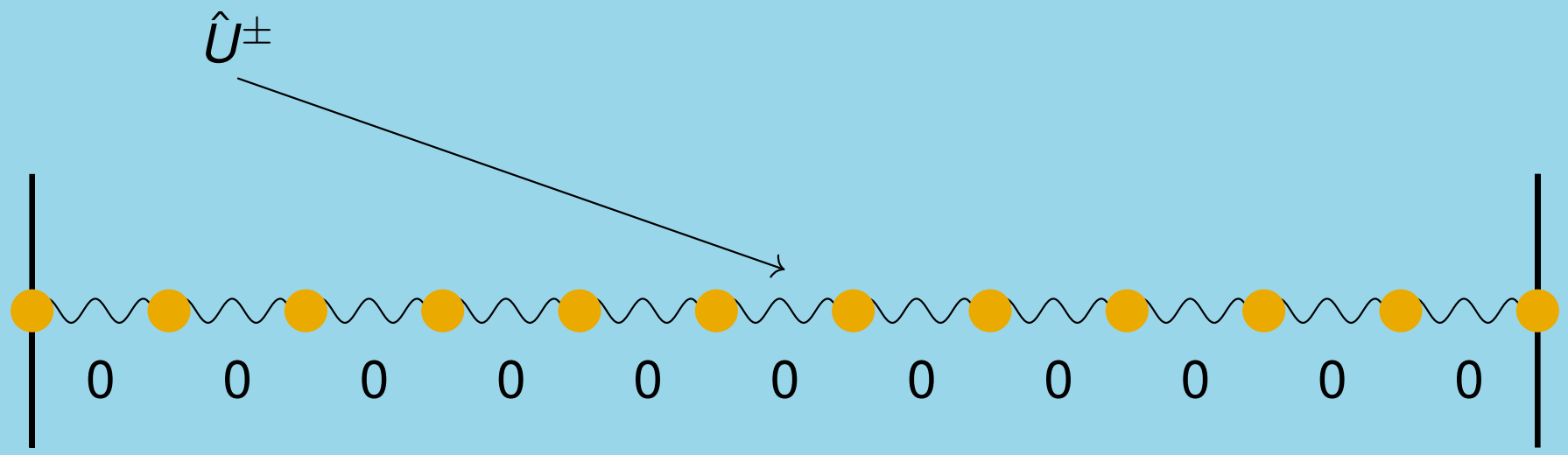
$(\hat{L}^z_i - \hat{L}^z_{i+1}) \rightarrow$ scalar particle number



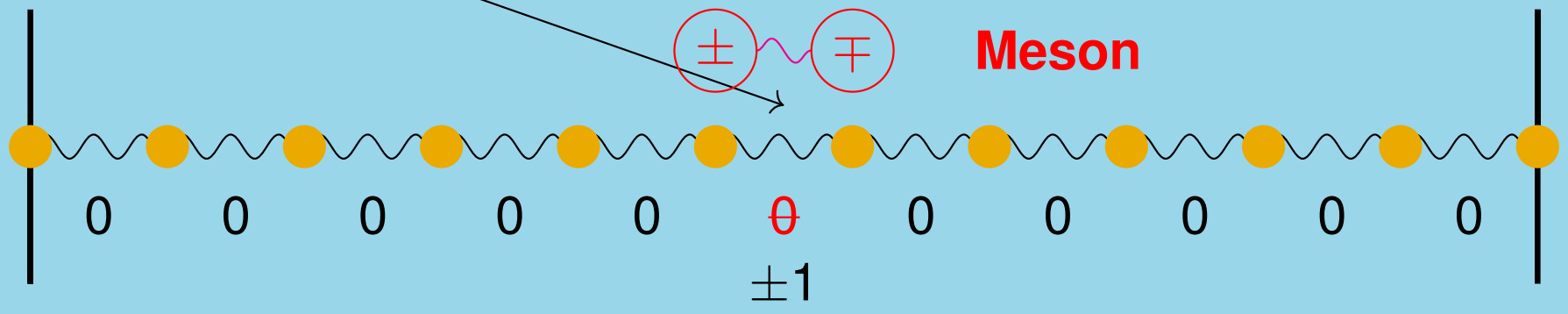
$(\hat{L}^z)^2 \rightarrow$ photons (ish)

compact Scalar QED

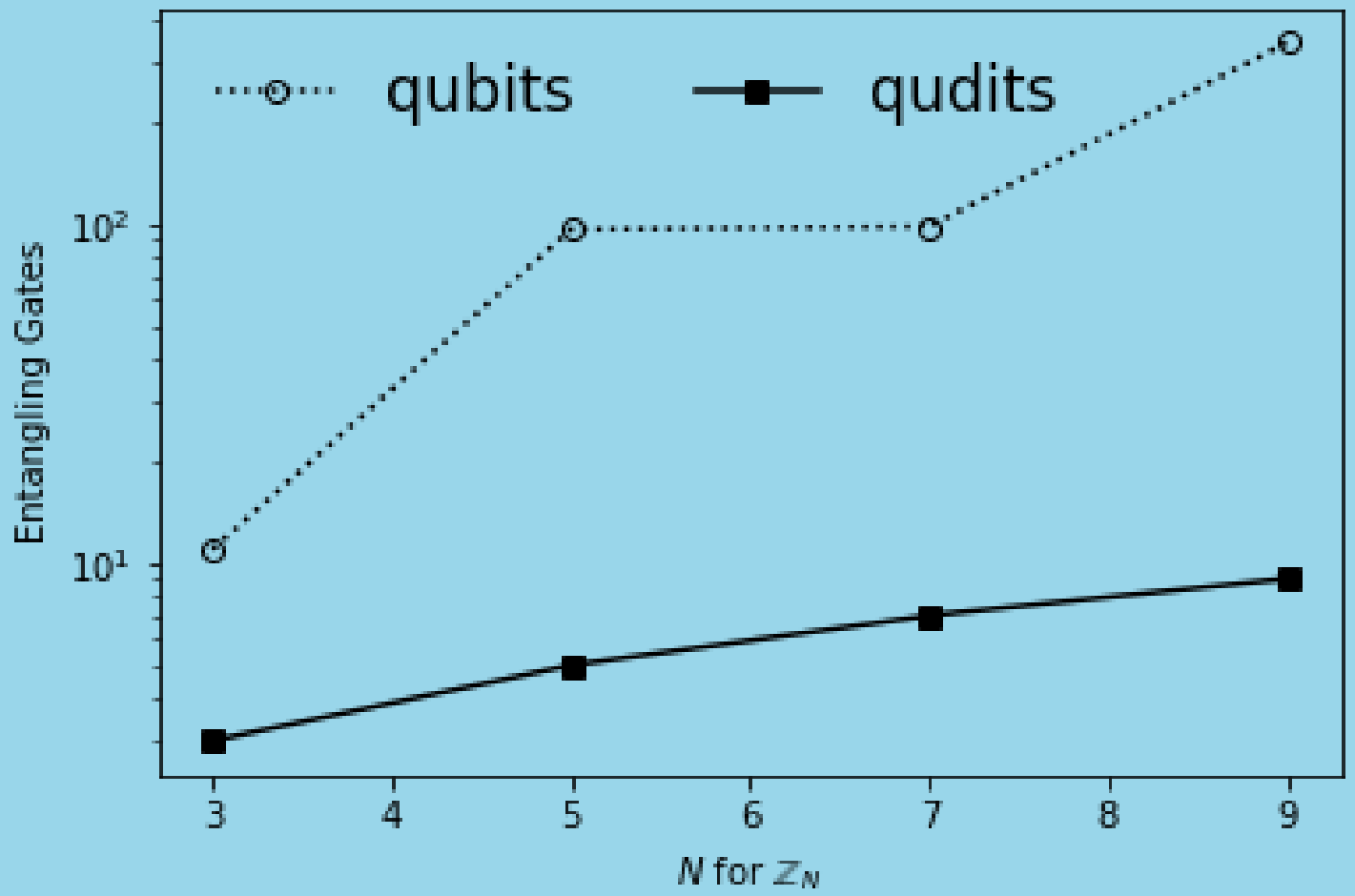


compact Scalar QED

$\hat{U}^{\pm} \quad \rightarrow \quad$ Meson creation / annihilation operator



Circuit Cost: qubits v. qudits[2]

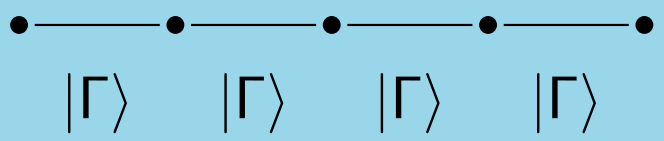


- Qubit Gates: Arbitrary SU(2) + CNOT
- Qudit Gates: Arbitrary SU(d) + Controlled Sum

[1] Phys. Rev. D 103, 114505 (2021), arXiv:2201.04546

Desired Observable: Two point correlator / mass gap

$|\Gamma\rangle$ local ground state, $|\Omega\rangle = (|\Gamma\rangle^{\otimes})^4$



Correlator: $\langle\Omega| \sum_i \hat{U}^- e^{-itH} \hat{U}^+ |\Omega\rangle$

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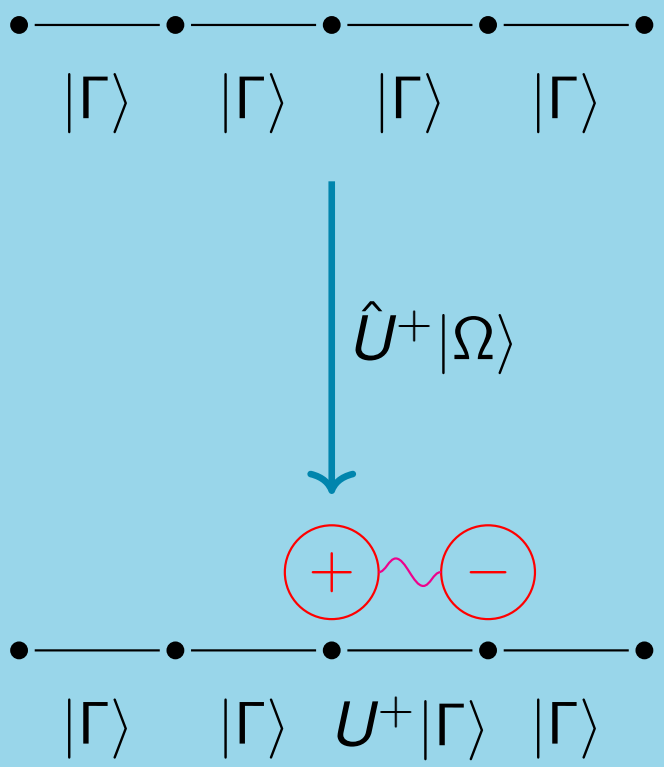

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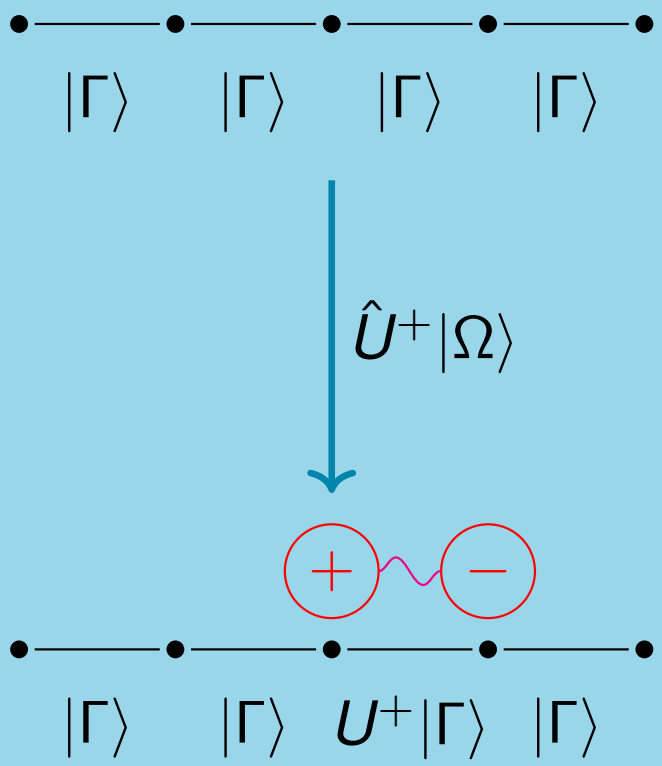

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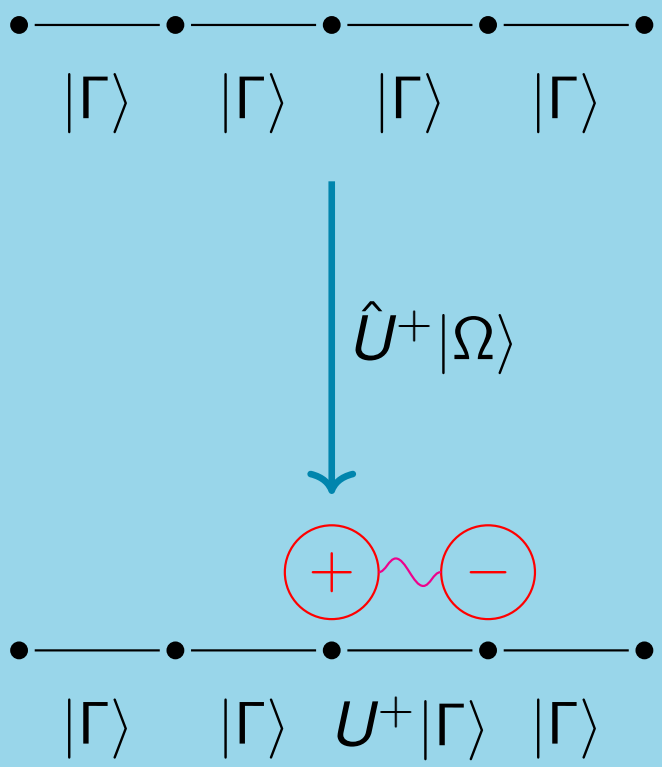
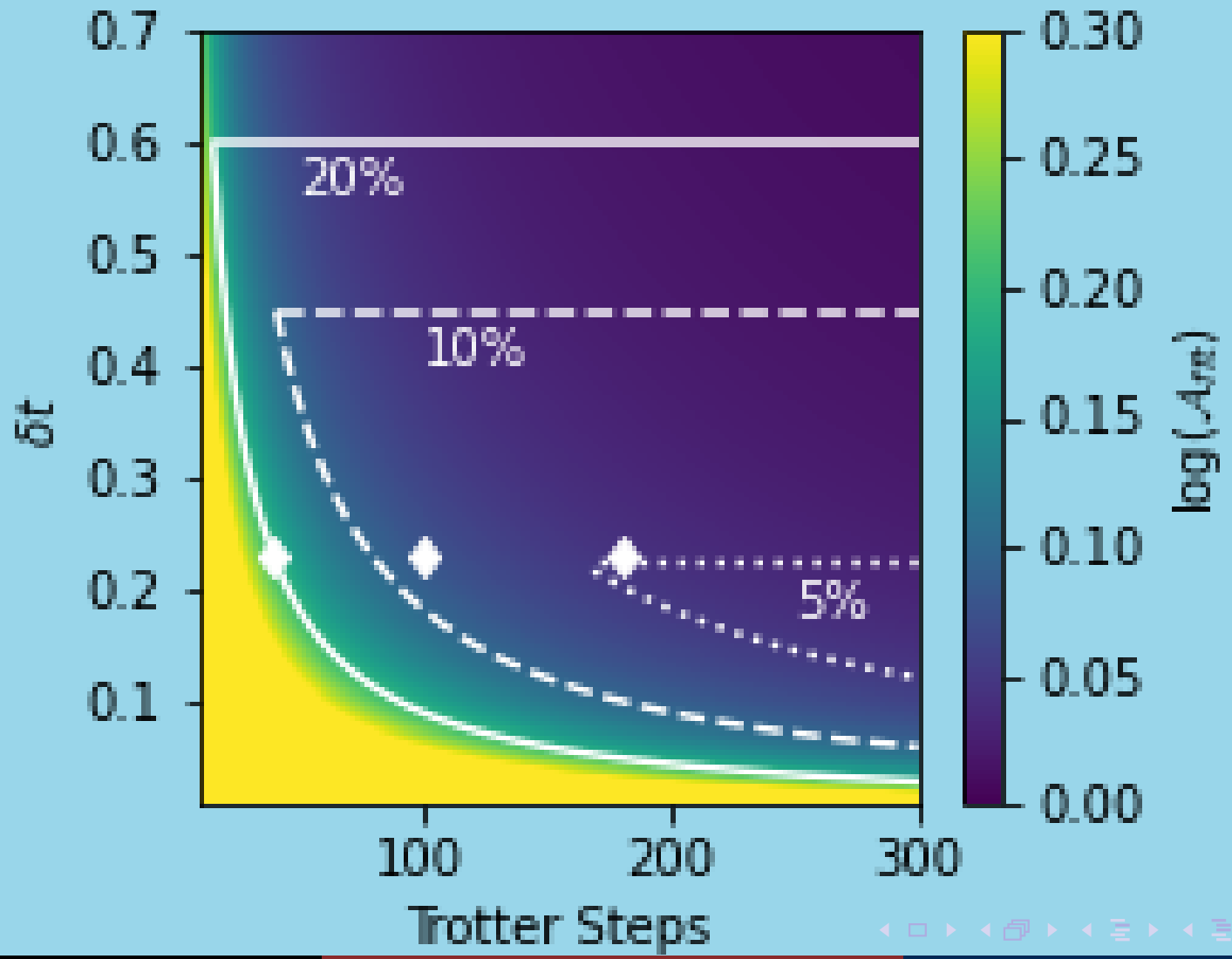


Poof! $\langle\Omega| \sum_i \hat{U}_i^-$

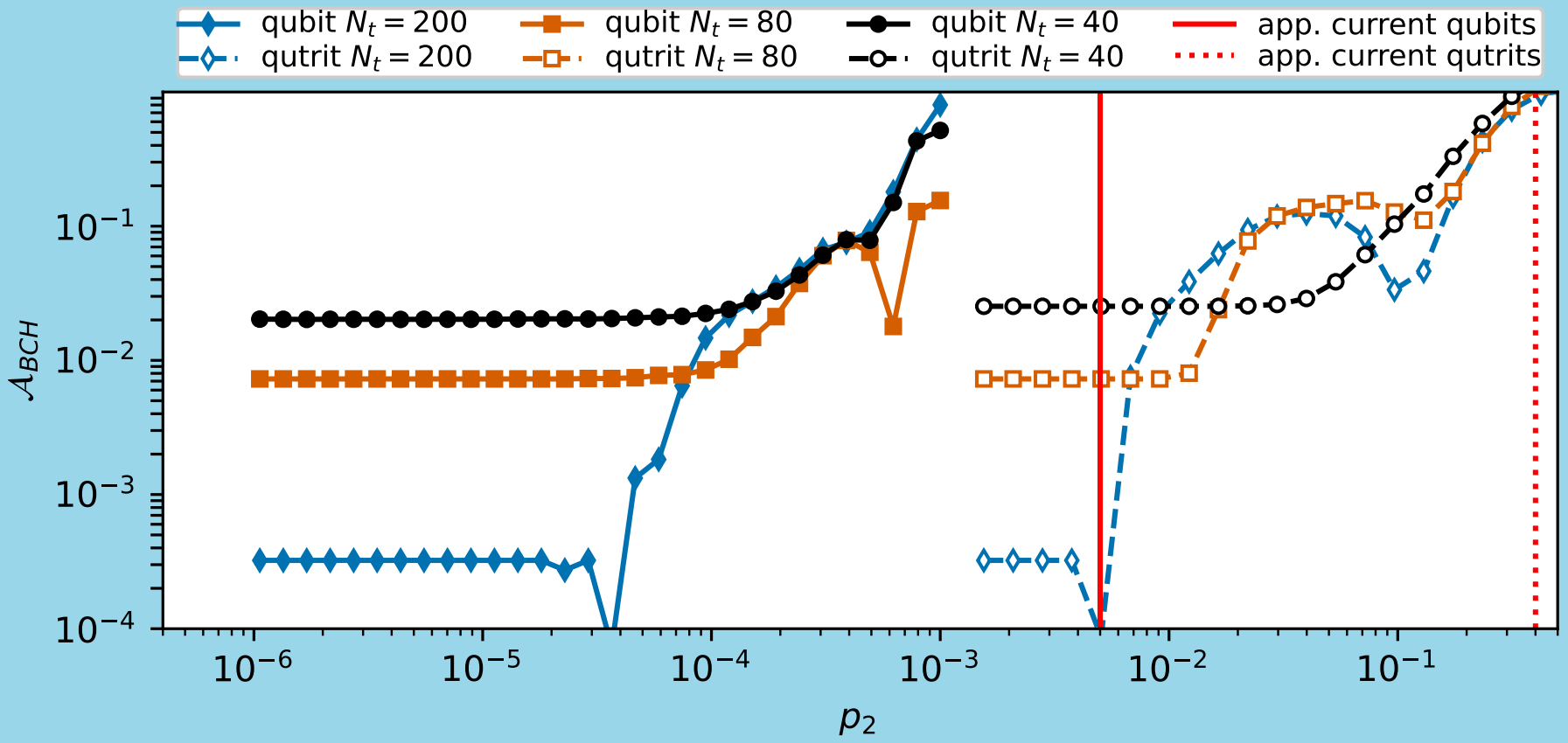
e^{-itH}

Correlator: $\langle\Omega| \sum_i \hat{U}^- e^{-itH} \hat{U}^+ |\Omega\rangle$

Parameter Selection: $g^2 a_s = 5$, 4 sites, $\delta t = 0.235$, truncate to 3 states

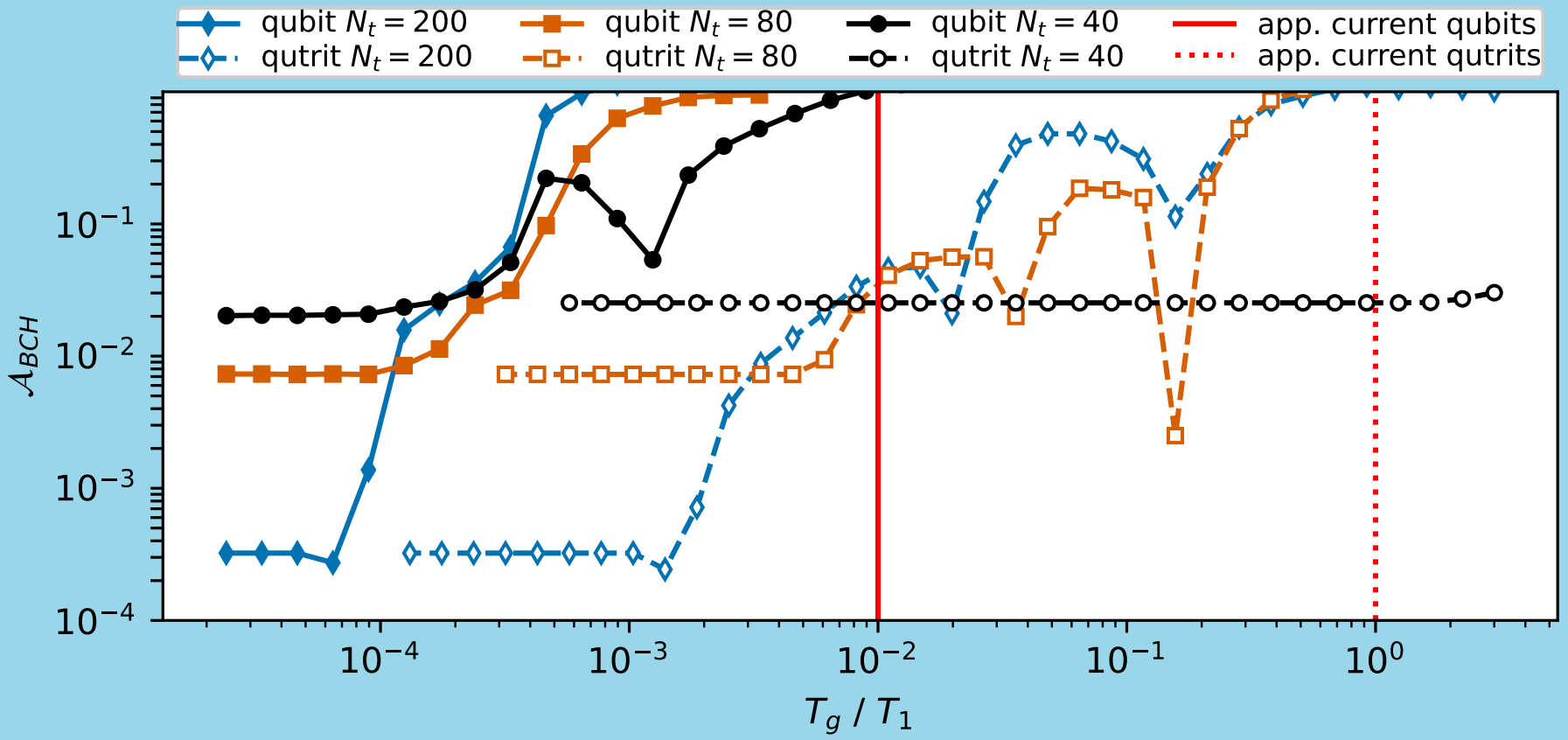


Accuracy v. Two qubit Pauli error p_2



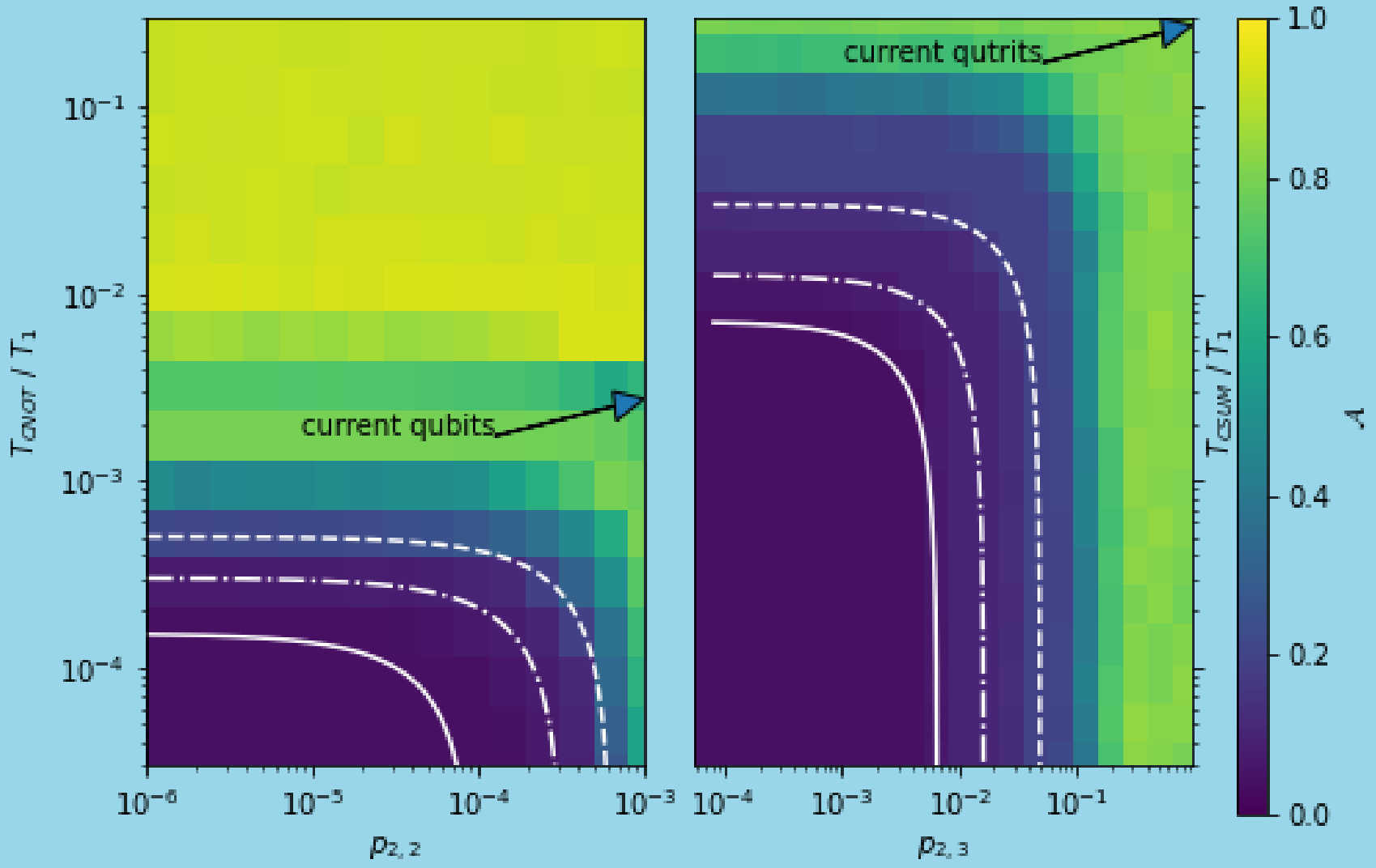
For equivalent error accuracies and precision Qutrits can be 100x noisier than qubits

Accuracy v. Amplitude damping time T_1



For equivalent error accuracies and precision Qutrits can have 10-100x longer T_1

Qubit v. Qutrit Both channels



Qubit v. Qutrit

$\mathcal{A}$	qubit error bound	qutrit error bound
20%	$\frac{T_{CNOT}}{T_1} \lesssim 0.0005 - 0.8013 p_{2,2}$	$\frac{T_{CSUM}}{T_1} \lesssim 0.030 - 0.618 p_{2,3}$
10%	$\frac{T_{CNOT}}{T_1} \lesssim 0.0003 - 0.9365 p_{2,2}$	$\frac{T_{CSUM}}{T_1} \lesssim 0.013 - 0.774 p_{2,3}$
5%	$\frac{T_{CNOT}}{T_1} \lesssim 0.0002 - 1.6456 p_{2,2}$	$\frac{T_{CSUM}}{T_1} \lesssim 0.007 - 1.091 p_{2,3}$

- Qubits need to have a 1 to 2 order of magnitude lower noise rate than qutrits

Take away

- qutrits can reduce number of entangling gates by a factor of 3
- qutrits can be 10-100x noisier than qubits and achieve same physics
- Straight forward encoding on qutrits
- not obvious encoding on qubits when $dim \neq 2^n$
- the tradeoff between decreased fidelity / gate and needing fewer gates with qutrits might be worth it
- on going work: non-Abelian gauge theories, e.g. D_4; multiple sites to 1 qudit; higher qudit states

Acknowledgments

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