

Twin Sterile Neutrino Dark Matter

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(Dated: May 12, 2023)

We propose that the dark matter of our universe could be sterile neutrinos which reside within the twin sector of a mirror twin Higgs model. In our scenario, these particles are produced through a version of the Dodelson-Widrow mechanism that takes place entirely within the twin sector, yielding a dark matter candidate that is consistent with X-ray and gamma-ray line constraints. Furthermore, this scenario can naturally avoid the cosmological problems that are typically encountered in mirror twin Higgs models. In particular, if the sterile neutrinos in the Standard Model sector decay out of equilibrium, they can heat the Standard Model bath and reduce the contributions of the twin particles to N_{eff} . Such decays also reduce the effective temperature of the dark matter, thereby relaxing constraints from large-scale structure. The sterile neutrinos included in this model are compatible with the seesaw mechanism for generating Standard Model neutrino masses.

INTRODUCTION

Despite its considerable empirical successes, the Standard Model (SM) does not explain a number of observed phenomena, including the origin and identity of dark matter (DM), the existence of neutrino masses, and the large hierarchy between the electroweak and Planck scales. In the SM, radiative corrections are naively expected to increase the mass of the Higgs boson to near the Planck scale, leading to what is known as the electroweak hierarchy problem.

Supersymmetry, if broken near the TeV-scale, could facilitate the cancellation of the largest of these corrections, but this class of solutions to the hierarchy problem appears increasingly fine-tuned in light of null results from the Large Hadron Collider [1–6]. An alternative approach, known as “neutral naturalness,” stabilizes the electroweak hierarchy by introducing new symmetries and new particles without color or other SM gauge charges, making it possible to cancel the quadratic contributions to the Higgs mass while simultaneously satisfying current constraints from colliders [7–16].

The prototypical example of neutral naturalness is the twin Higgs framework [7–9, 13, 17–20], which introduces “twin” copies of SM particles that are charged under twin versions of the SM gauge group. A discrete \mathbb{Z}_2 twin symmetry between the SM and twin sectors helps to stabilize the Higgs mass up to one-loop, allowing the hierarchy problem to be addressed at a higher energy scale, such as

through a supersymmetric UV completion [20–22]. For such a model to be phenomenologically viable, the \mathbb{Z}_2 symmetry must be softly broken, allowing the Higgs vacuum expectation value (VEV) to be different in each sector. For a ratio of twin-to-SM VEVs of $v_B/v_A \sim 3 - 5$, such a scenario can address the hierarchy problem while allowing the 125 GeV Higgs boson to behave approximately as predicted by the SM, containing only a $\sim v_A^2/v_B^2 \sim \mathcal{O}(10\%)$ admixture of the twin sector Higgs boson.

Even with asymmetric VEVs, cosmological problems can arise in “mirror twin Higgs” (MTH) models, in which the SM and twin sectors contain the same particle content. Because the twin and SM sectors are initially in equilibrium, large energy densities of twin photons and twin neutrinos will be present in the early universe in MTH models, increasing the Hubble expansion rate in conflict with measurements of the effective number of neutrino species, N_{eff} . Proposed solutions to this problem include introducing asymmetric and out-of-equilibrium decays of new particles to dilute the energy density in the twin sector [23, 24], reducing the number of twin species as in “fraternal twin Higgs” models [13, 25], or delaying the thermal decoupling of the two sectors through SM-twin neutrino mixing [26].

A variety of DM candidates can arise within the context of twin Higgs models [25, 27–43], including the twin tau, which in fraternal models can be stable, functioning as a prototypical WIMP [25, 27, 28]. However, in light of increasingly stringent constraints from LZ and other direct detection experiments [44], it is well-motivated to consider other dark matter candidates that might arise in the twin Higgs framework.

The problem of non-zero neutrino masses can be simply explained by supplementing the SM with right-handed neutrinos. Through the seesaw mechanism [45–47], this

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can lead to light “active” neutrinos which are mostly left-handed, and heavier “sterile” states which are mostly right-handed. Being sufficiently long-lived, cold, and feebly interacting, sterile neutrinos have long been seen as an attractive candidate for DM [48–52]. They can be generated in the early universe through the Dodelson-Widrow mechanism, in which active neutrinos oscillate, interact, and collapse into a sterile state [48].

The simplest Dodelson-Widrow scenario has been entirely ruled out in recent years by searches for DM decaying to X-ray [53–60] or gamma-ray lines [61], as well as by constraints on warm DM from observations of large-scale structure [62–65]. For sterile neutrinos to be a viable DM candidate, they must be produced through resonant [49] or non-thermal [66–71] mechanisms, or involve other exotic elements [72–74].

In this *Letter*, we propose incorporating sterile neutrinos into a mirror twin Higgs model, identifying the twin sterile neutrino as a new DM candidate. We show that by combining these two frameworks, we can address the hierarchy problem, explain the origin of neutrinos masses, and provide a viable DM candidate, while resolving each of these frameworks’ respective tensions with cosmological and astrophysical observations. In this scenario, the twin sterile neutrino DM is produced through a Dodelson-Widrow mechanism taking place entirely within the twin sector. When the DM decays, it produces few observable SM photons, evading X-ray and gamma-ray constraints. Meanwhile, a heavier SM sector sterile neutrino decays and heats the SM bath [23], reducing the relative energy density in the twin sector and thus solving the N_{eff} problem of the MTH model.

MODEL

We consider a minimal MTH model whose \mathbb{Z}_2 -symmetric Lagrangian is supplemented with the terms

$$\mathcal{L} \supset - \sum_{i=A,B} \left(y_\nu^i H^i l_L^i \nu_R^i + \frac{M_i}{2} \nu_R^i \nu_R^i \right) - M_{AB} \nu_R^A \nu_R^B + \text{h.c.},$$

where A and B represent the SM and twin sectors, respectively. For each sector, H^i is the Higgs doublet, l_L^i is the lepton doublet that contains a left handed neutrino ν_L^i , ν_R^i is the right handed neutrino, and y_ν^i is the Yukawa coupling. We allow for explicit \mathbb{Z}_2 breaking in the neutrino sector, such that $M_A \neq M_B$ and $y_\nu^A \neq y_\nu^B$. The last term allows the right-handed neutrinos, ν_R^A and ν_R^B , to mix between sectors through the coefficient M_{AB} . For simplicity, we consider only one generation of neutrinos, although this scenario can be extended to the case of three generations.

After electroweak symmetry breaking, the Yukawa terms yield Dirac masses of $m_i = y_\nu^i v_i / \sqrt{2}$. We adopt $v_B/v_A = 3$ throughout, where the SM Higgs VEV

$v_A = 246$ GeV. We also assume a mass hierarchy of $M_A, M_B \gg M_{AB} \gg m_A, m_B$, so that the 4×4 neutrino mass matrix is diagonalized via an active-sterile seesaw mechanism [45–47] within each sector, as well as a SM-twin seesaw which mixes the sterile states between the two sectors. To leading order in the small mixing angles, this yields two active neutrino mass eigenstates,

$$\begin{aligned} \nu_A &= \nu_L^A - \theta_A \nu_R^A \\ \nu_B &= \nu_L^B - \theta_B \nu_R^B, \end{aligned} \quad (1)$$

and two sterile neutrino mass eigenstates,

$$\begin{aligned} N_A &= \nu_R^A + \theta_A \nu_L^A + \theta_{AB} \nu_R^B \\ N_B &= \nu_R^B + \theta_B \nu_L^B - \theta_{AB} \nu_R^A. \end{aligned} \quad (2)$$

The mixing angles can be written

$$\theta_A = \frac{m_A}{M_A}, \quad \theta_B = \frac{m_B}{M_B}, \quad \theta_{AB} = \frac{M_{AB}}{|M_A - M_B|}, \quad (3)$$

and the mass eigenvalues are

$$\begin{aligned} m_{\nu_A} &= \frac{m_A^2}{M_A}, & M_{N_A} &= M_A, \\ m_{\nu_B} &= \frac{m_B^2}{M_B}, & M_{N_B} &= M_B, \end{aligned} \quad (4)$$

demonstrating the familiar seesaw relation in each sector. We identify the twin sterile neutrino, N_B , as the dark matter candidate and the SM sterile neutrino, N_A , as a heavier state that decays in the early universe.

Note that our model contains the same field content as the model in Ref. [23], which preserved a full \mathbb{Z}_2 symmetry between the two sectors and did not consider DM production. However, for sterile neutrino DM in a twin Higgs context, an exact \mathbb{Z}_2 symmetry in the neutrino parameters combined with M_{AB} mixing is problematic because it maximally mixes the sterile states between the sectors ($M_A = M_B \implies \theta_{AB} = \pi/4$). Since the Fermi constant, $G_{F,i} = v_i^{-2}/\sqrt{2}$, is larger in the SM sector, the maximally mixed DM decays predominantly to SM particles, including monoenergetic photons. Such a model is sufficiently similar to the canonical Dodelson-Widrow scenario that it yields comparable X-ray and gamma-ray line signals, which are already excluded by observations.

In contrast, our model overcomes this limitation by explicitly breaking the \mathbb{Z}_2 symmetry between the neutrino parameters of the SM and twin sectors. In the $M_{AB} \ll M_A, M_B$ limit, having $M_A \gg M_B$ not only ensures that θ_{AB} is small, but also that the lighter sterile neutrino, N_B (our DM candidate), couples mostly to the twin sector and is long-lived, while the heavier sterile state, N_A , couples mostly to the SM and decays in the early universe.

COSMOLOGY

The cosmic history in our scenario is schematically illustrated in Fig. 1 and proceeds as follows:

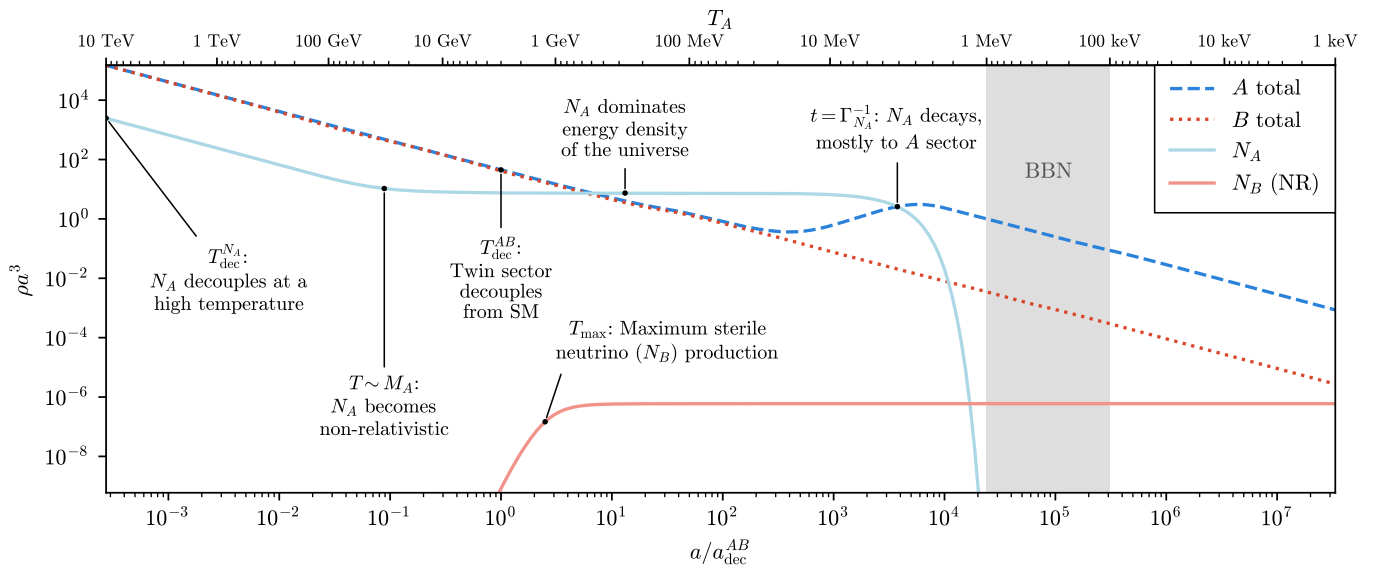


FIG. 1. Timeline of events in one realization of our scenario. The curves indicate the comoving energy densities of radiation in the SM (A) and twin (B) sectors, and of the sterile neutrinos, N_A and N_B , as a function of scale factor, a , or the temperature of the SM bath, T_A . The y -axis is normalized by the comoving energy density of the SM sector at $T_A = 1$ MeV. Only the non-relativistic portion of $\rho_{N_B} = M_{N_B} n_{N_B}$ is shown. This example takes the parameters $M_{N_A} = 100$ GeV, $m_{\nu_A} = 10^{-9}$ eV, $M_{N_B} = 100$ keV, and $m_{\nu_B} = 4 \times 10^{-4}$ eV, for which we find $\Delta N_{\text{eff}} \approx 0.02$ and $\Omega_{N_B} \approx 0.26$, consistent with observations.

1. The N_A sterile neutrino population is initially in equilibrium with the thermal bath of SM and twin particles, but it eventually decouples at temperature $T_{\text{dec}}^{N_A}$.
2. At $T \sim M_A$, the N_A population becomes nonrelativistic and its fraction of the total energy density begins to grow as the universe cools and expands.
3. Higgs portal interactions, due to h^A - h^B mixing, keep the two sectors in equilibrium until $T_{\text{dec}}^{AB} \approx 3$ GeV [23], when the SM and twin sectors decouple.
4. After AB decoupling, the N_A population eventually dominates the energy density of the universe.
5. The N_A population decays out of equilibrium to heat the SM radiation bath. This entropy injection increases the SM temperature and effectively dilutes the relative energy density of the twin sector. Note that steps 1–5 must occur prior to Big Bang nucleosynthesis (BBN) to ensure that any contributions to N_{eff} are minimal.
6. At some time (around T_{max} in Fig. 1) before the epoch of matter-radiation equality, twin sterile neutrino dark matter, N_B , is produced through active-sterile neutrino oscillations (*i.e.* a Dodelson-Widrow mechanism) within the twin sector.

The temperature evolution in the two sectors and the energy density in N_A is described by the following system

of differential equations (see the Supplemental Material for a derivation):

$$\frac{d \ln T_A}{d \ln a} = - \frac{1 - \frac{\langle \Gamma_{N_A \rightarrow A} \rangle \rho_{N_A}}{3 H s_A T_A}}{1 + \frac{1}{3} \frac{d \ln g_{*,s}^A}{d \ln T_A}} \quad (5)$$

$$\frac{d \ln T_B}{d \ln a} = - \frac{1 - \frac{\langle \Gamma_{N_A \rightarrow B} \rangle \rho_{N_A}}{3 H s_B T_B}}{1 + \frac{1}{3} \frac{d \ln g_{*,s}^B}{d \ln T_B}} \quad (6)$$

$$\frac{d \rho_{N_A}}{dt} = -3H(\rho_{N_A} + P_{N_A}) - \langle \Gamma_{N_A} \rangle \rho_{N_A}, \quad (7)$$

where a is the scale factor, T_i is the temperature of the radiation bath in sector i , $H = (8\pi G\rho/3)^{1/2}$ is the Hubble rate, $\rho = \rho_A + \rho_B$ is the total energy density, ρ_i is the energy density in either sector, s_i is the entropy density in either sector, and P_{N_A} is the pressure of the N_A population. $\langle \Gamma_{N_A \rightarrow i} \rangle$ is the thermally-averaged decay width of N_A to sector i (including the effects of time dilation). At rest, the N_A decay width to SM particles is

$$\Gamma_{N_A \rightarrow A} = C_A \frac{G_{F,A}^2}{192\pi^3} \theta_A^2 M_{N_A}^5, \quad (8)$$

where C_A is a factor that is obtained by summing over the decay channels (see Appendix A of Ref. [23]). This depends on the full multi-generation neutrino mass matrix, but for one generation, we find $C_A \approx 12.4$ for $M_{N_A} = 100$ GeV. For all cases we consider here, $\Gamma_{N_A \rightarrow B} \ll \Gamma_{N_A \rightarrow A}$,

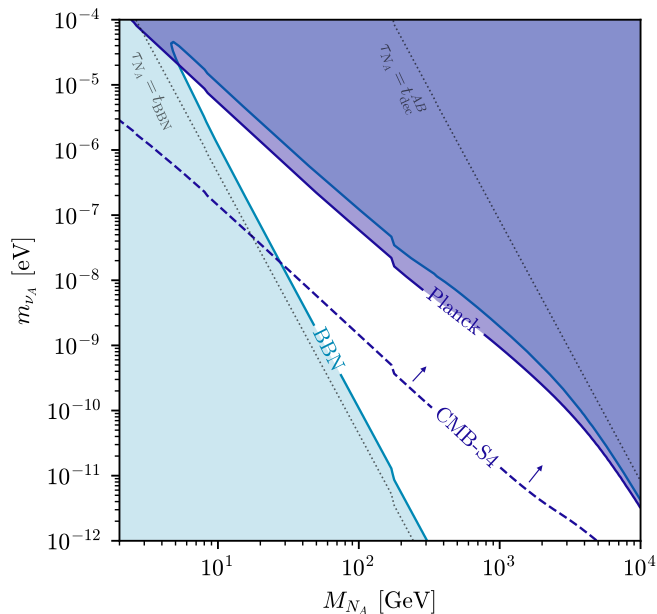


FIG. 2. Constraints on the masses of the decaying sterile neutrino, N_A , and the corresponding active neutrino, ν_A , from measurements of N_{eff} . The shaded regions represent the parameter space that is ruled out by measurements of the light element abundances [76] (light blue) and the cosmic microwave background (dark blue). Current $\Delta N_{\text{eff}}^{\text{CMB}}$ constraints are shown from Planck [77] as well as the projected sensitivity of Stage 4 CMB experiments [78]. In the unshaded regions, the decays of the N_A dilute the energy density of the twin sector to acceptable levels. Dotted gray lines represent curves of constant τ_{N_A} .

so we neglect entropy transfer from N_A to the twin sector. Note that the fermionic contributions to the effective relativistic degrees of freedom in energy and entropy, g_* and $g_{*,s}$, are different in each sector due to the larger masses of the twin fermions, while the QCD phase transition takes place at approximately the same temperature in both sectors [75]. We numerically solve Eqs. 5–7 to find $T_A(a)$, $T_B(a)$, and $\rho_{N_A}(a)$.

The main constraints on this scenario arise from measurements of the energy density in dark radiation, parameterized in terms of the contribution to the effective number of relativistic neutrino species, ΔN_{eff} , which we define as

$$\Delta N_{\text{eff}} = \frac{\rho_B + \rho_{N_A}}{\rho_\nu}, \quad (9)$$

where ρ_ν is the energy density of a single SM active neutrino species. Note that this contribution to N_{eff} is not from the energy injected by the decays of N_A , but rather from the energy density of the twin sector particles that remain after the N_A population has decayed (as well as any N_A particles that have not yet decayed).

In Fig. 2, we plot the parameter space in this scenario. Contours are shown for values of ΔN_{eff} at the

time of BBN ($T_A \approx 1$ MeV), and at the time of recombination ($T_A \approx 0.3$ eV). Current constraints require $\Delta N_{\text{eff}}^{\text{CMB}} \lesssim 0.3$ [77] and $\Delta N_{\text{eff}}^{\text{BBN}} \lesssim 0.4$ [76], while Stage 4 CMB experiments are projected to be sensitive down to $\Delta N_{\text{eff}}^{\text{CMB}} \lesssim 0.03$ [78]. The constraints can be understood in terms of two criteria: In the upper right region of this figure, the N_A population decays before the SM and twin sectors decouple, so that both sectors quickly thermalize again, leading to unacceptably large contributions to N_{eff} . In the lower left portion of the figure, N_A decays during or after BBN, again leading to large $\Delta N_{\text{eff}}^{\text{BBN}}$ (but not necessarily large $\Delta N_{\text{eff}}^{\text{CMB}}$).

Because the rates for sterile neutrino production and decay are set by the same model parameters, there is an inherent tension between establishing a thermal population of N_A particles (required for step 1), and having these states decay at the right time (required for step 5). Satisfying both of these requirements within the viable parameter space of Fig. 2 requires an additional \mathbb{Z}_2 breaking interaction that enables the N_A population to reach equilibrium at early times, without altering the subsequent cosmological timeline (for a discussion, see the Supplemental Material). The lower right boundary of the viable parameter space shown in Fig. 2 depends on the choice of these additional interactions, so we leave it unconstrained. Given a concrete realization of such interactions, a lower limit will appear, but its precise location is model-dependent.

TWIN STERILE NEUTRINO DARK MATTER

The production of twin sterile neutrino DM through the Dodelson-Widrow mechanism in the twin sector is described by the following Boltzmann equation:

$$\frac{dn_{N_B}}{dt} + 3Hn_{N_B} = 2 \int \frac{d^3p}{(2\pi)^3} \frac{\Gamma_{\nu_B}^{\text{int}}}{4} \sin^2 2\theta_B^m (f_{\nu_B} - f_{N_B}), \quad (10)$$

where n is the number density of a given species and f is its phase space distribution. The quantity $\Gamma_{\nu_B}^{\text{int}} \propto G_{F,B}^2 T_B^5$ is the scattering rate of the twin active neutrinos, which we calculate using the values tabulated in Ref. [79] (for the case of neutrinos of muon flavor), rescaled by a factor of $(v_B/v_A)^{-4}$. The quantity θ_B^m is the effective active-sterile mixing angle in the twin sector in the presence of matter. This is related to the corresponding mixing angle in vacuum according to the following expression [48, 51]:

$$\sin^2 2\theta_B^m = \frac{\Delta^2 \sin^2 2\theta_B}{\Delta^2 \sin^2 2\theta_B + (\Delta \cos 2\theta_B - V_T)^2}, \quad (11)$$

where $\Delta = (M_{N_B}^2 - m_{\nu_B}^2)/2p$ is the vacuum oscillation factor for a neutrino of momentum p , and the thermal

potential is given by [51, 80]

$$V_T(p) = -\frac{8\sqrt{2}G_{F,B}p}{3} \left(\frac{\rho_{\nu_B} + \rho_{\bar{\nu}_B}}{m_{Z_B}^2} + \frac{\rho_{l_B} + \rho_{\bar{l}_B}}{m_{W_B}^2} \right), \quad (12)$$

where ρ_ν and $\rho_{\bar{\nu}}$ denote the energy densities of the twin active neutrinos and antineutrinos, and ρ_l and $\rho_{\bar{l}}$ are the energy densities of the twin charged leptons and antileptons of the same flavor.

In the early universe, these matter effects make $\sin^2 2\theta_B^m$ small, suppressing the oscillation rate of ν_B into N_B . The rate of N_B production increases as the temperature falls, but not necessarily enough for the N_B abundance to reach its equilibrium value. This continues until the expansion of the universe begins to efficiently suppress the rate of N_B production at later times, and their abundance freezes in. Most twin sterile neutrinos are produced near a temperature of $T_{\max} \approx 270 \text{ MeV} (M_{N_B}/\text{keV})^{1/3}$ (for $v_B/v_A = 3$) [48].

For given values of M_{N_B} and m_{ν_B} , we numerically solve Eq. 10, in conjunction with Eqs. 5–7, for a large number of points within the viable region of Fig. 2. The decays of N_A must be accounted for here because they dilute the twin sterile neutrino DM abundance along with the rest of the twin sector particles.

In Fig. 3, we summarize the constraints on twin sterile neutrino DM. Throughout the white region labeled as “ $\Omega_{N_B} = \Omega_{\text{DM}}$ ”, there exists some combination of M_{N_A} and m_{ν_A} that lead to the observed DM abundance ($\Omega_{\text{DM}} \approx 0.26$ [77]) through the twin sector Dodelson-Widrow mechanism while remaining consistent with the constraints shown in Fig. 2. We do not consider resonant production mechanisms that rely on significant lepton number asymmetry [49], although these could further expand this region. In the lower red region, the DM is consistently underproduced for all viable values of M_{N_A} and m_{ν_A} .

Another important constraint on sterile neutrino DM arises from its impact on large-scale structure formation. In the standard Dodelson-Widrow scenario, sterile neutrinos with masses smaller than 25 keV [62] are unacceptably warm, leading to fewer satellite galaxies than are observed [64, 65, 81]. In our scenario, however, the decays of N_A reduce T_B/T_A , effectively cooling the DM and relaxing this constraint to $M_{N_B, \min} = 25 \text{ keV} (T_B/T_A)$. Throughout the orange shaded region of Fig. 3, the DM is too warm for all viable combinations of M_{N_A} and m_{ν_A} , while the light orange region is too warm for only some of those parameter values. In the green region, the DM decays too rapidly, with $\tau_{N_B} < 288 \text{ Gyr}$ [82], in conflict with observations of large-scale structure [82–85].

So far, we have neglected the effects of mixing between the sectors, but if $\theta_{AB} \neq 0$, constraints can arise from the null results of X-ray line searches [53–55, 60, 64, 86, 87], which are sensitive to loop-induced decays of N_B to an active neutrino and a monoenergetic SM photon [52, 88,

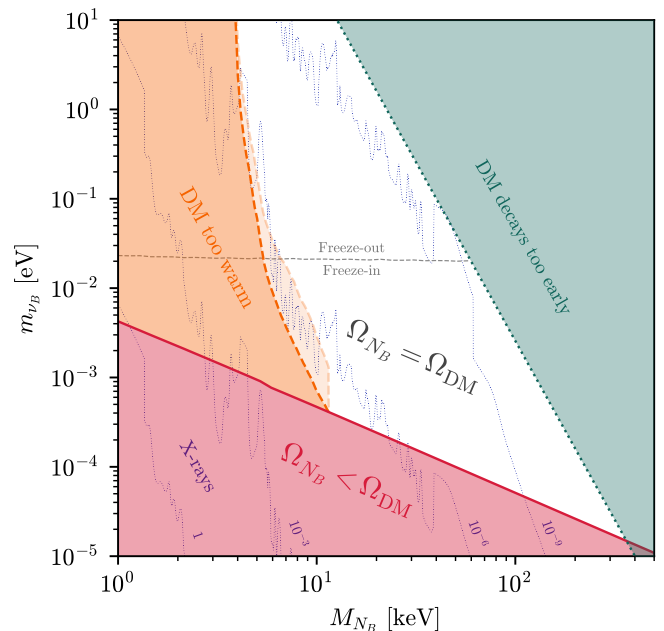


FIG. 3. Constraints on the masses of our dark matter candidate, N_B , and the twin sector active neutrino, ν_B . Shaded regions indicate constraints on the DM abundance (red), temperature (orange), and lifetime (green). Throughout the white region labeled “ $\Omega_{N_B} = \Omega_{\text{DM}}$ ” our model can produce the observed DM abundance while remaining consistent with the constraints on ΔN_{eff} . The dotted blue lines represent constraints on DM decay to X-ray lines for several values of the branching ratio $\Gamma_{N_B \rightarrow \nu_A \gamma_A} / \Gamma_{N_B \rightarrow \nu_B \gamma_B}$; for each choice of this ratio, the parameter space to the right is excluded.

89]. These constraints are shown in Fig. 3 as dotted blue lines. If the branching fraction for $N_B \rightarrow \nu_A \gamma_A$ (which is related to θ_{AB}) is small, this signal can be highly suppressed, well below the reach of existing telescopes.

In some regions of parameter space, $\sin^2 2\theta_B^m$ is large enough that an N_B population can be produced with an equilibrium abundance, so instead of freezing in, it freezes out as a hot relic. The approximate boundary between these two cases is indicated in Fig. 3, showing that the observed DM abundance can be obtained in both cases.

SUMMARY

In this *Letter*, we have proposed a new dark matter candidate in the form of a sterile neutrino in the twin sector of a mirror twin Higgs model. This dark matter candidate can be generated through neutrino oscillations in the twin sector. If the mixing between the Standard Model and twin sectors is sufficiently small, the decays of the dark matter particles will not produce an observable X-ray or gamma-ray line. Furthermore, a sterile neutrino in the Standard Model sector can naturally undergo out-of-equilibrium decays in this model, diluting the energy

density of the twin sector, and thus evading constraints from N_{eff} . Such decays can also relax the constraints on warm dark matter that arise from observations of large-scale structure. This scenario can restore both mirror twin Higgs models and sterile neutrino DM as viable solutions to several long-standing mysteries in particle physics and cosmology, while remaining consistent with all existing constraints.

We would like to thank Patrick Fox and Shunsaku Horiuchi for helpful conversations. IH is supported by generous contributions from Philip Rice and the Brinson Foundation. DH and GK are supported by the Fermi Research Alliance, LLC under Contract No. DE-AC02-07CH11359 with the U.S. Department of Energy, Office of Science, Office of High Energy Physics. DS is supported by the Visiting Scholars Award Program of the Universities Research Association, the U.S. Department of Energy Office of Science under award number DE-SC0020262 and the Grants-in-Aid for Scientific Research (KAKENHI) No. 20H05852.

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Supplemental Material: Derivation of the evolution equations

To determine how the temperature of the SM bath is impacted by a particle decaying out of equilibrium, we begin with the treatment presented in Sec. 5.3 of Ref. [90]. The second law of thermodynamics ($dS = dQ/T$) allows us to relate the change in total entropy, $S = sa^3$, in each sector to the heat, Q , that is injected by decays of N_A . The first law of thermodynamics ($dQ = dE + PdV$) also lets us relate Q to the change in internal energy, $E = \rho_{N_A} a^3$, and pressure, P , of N_A , where $dV = d(a^3)$ is the change in volume due to the expansion of the universe. The time derivative of the total entropy is then

$$\begin{aligned} \frac{dS}{dt} &= \frac{1}{T} \frac{dQ}{dt} = -\frac{1}{T} \left(\frac{d(a^3 \rho_{N_A})}{dt} + P_{N_A} \frac{d(a^3)}{dt} \right) \\ &= -\frac{1}{T} \left(\frac{d\rho_{N_A}}{dt} + 3H(\rho_{N_A} + P_{N_A}) \right) a^3 \\ &= \frac{\langle \Gamma_{N_A} \rangle \rho_{N_A}}{T} a^3. \end{aligned} \quad (\text{S.1})$$

In the last equality, we simplify the expression by applying Eq. 7, leaving only a term dependent on the decay of N_A . We can also use the relationship between entropy density and temperature,

$$s(T) = \frac{2\pi^2}{45} g_{*,s} T^3, \quad (\text{S.2})$$

to express the time derivative of the total entropy in another way:

$$\begin{aligned} \frac{dS}{dt} &= \frac{d(sa^3)}{dt} = 3\dot{a}a^2s + a^3\dot{s} \\ &= 3\dot{a}a^2s + a^3 \frac{2\pi^2}{45} (3\dot{T}T^2g_{*,s} + \dot{g}_{*,s}T^3) \\ &= 3sa^3 \left(\frac{1}{a} \frac{da}{dt} + \frac{1}{T} \frac{dT}{dt} + \frac{1}{3g_{*,s}} \frac{dg_{*,s}}{dt} \right) \\ &= 3Hsa^3 \left(1 + \frac{a}{T} \frac{dT}{da} + \frac{1}{3} \frac{T}{g_{*,s}} \frac{dg_{*,s}}{dT} \frac{a}{T} \frac{dT}{da} \right) \\ &= 3Hsa^3 \left(1 + \frac{d \ln T}{d \ln a} \left[1 + \frac{1}{3} \frac{d \ln g_{*,s}}{d \ln T} \right] \right). \end{aligned} \quad (\text{S.3})$$

By equating Eqs. S.1 and S.3, we obtain a differential equation describing the temperature evolution of either sector with a :

$$\frac{d \ln T}{d \ln a} = -\frac{1 - \frac{\langle \Gamma_{N_A} \rangle \rho_{N_A}}{3HsT}}{1 + \frac{1}{3} \frac{d \ln g_{*,s}}{d \ln T}}. \quad (\text{S.4})$$

In the limit of $\rho_{N_A} = 0$ and constant $g_{*,s}$, Eq. S.4 becomes -1 , corresponding to the standard relationship for radiation, $T \propto a^{-1}$. Note that this derivation is valid only after the SM and twin sectors have decoupled.

Supplemental Material: Thermalizing the N_A Population

As noted in the main text, there is a generic tension in this model between thermalizing the N_A population (which is assumed in the first step of our cosmic timeline in Fig. 1), and having these states decay out of equilibrium into SM particles (which is required to ensure a large entropy transfer and yield an acceptably small contribution to N_{eff}). Neither active-sterile oscillations nor Higgs-mediated Yukawa interactions can produce N_A in equilibrium abundance in the parameter space of interest. To illustrate this tension, consider that thermalizing N_A through the oscillation channel requires the oscillation production rate to exceed the rate of Hubble expansion:

$$\Gamma_{\nu_A \rightarrow N_A} \sim \sin^2 2\theta_A^m G_F^2 T^5 \gtrsim \frac{T^2}{M_{\text{Pl}}} \sim H. \quad (\text{S.5})$$

Due to matter effects, $\Gamma_{\nu_A \rightarrow N_A}$ is maximized at a temperature of $T_{\text{max}}^{N_A} \approx 0.72 M_W^{2/3} M_{N_A}^{1/3}$ [48]. Requiring $\Gamma_{\nu_A \rightarrow N_A} > H$ at T_{max} , and using the seesaw relation for the active neutrino mass, results in the following condition:

$$m_{\nu_A} \gtrsim \frac{540 M_W^2}{M_{\text{Pl}}} \approx 3 \times 10^{-4} \text{eV}, \quad (\text{S.6})$$

which is excluded by measurements of ΔN_{eff} and therefore appears in the excluded region of Fig. 2 for all choices, independent of M_{N_A} . A similar argument can be made for Higgs-mediated scattering channels for N_A production in the unbroken electroweak phase (e.g., $\nu_A t \rightarrow N_A t$ through t -channel Higgs exchange). Thus, some additional interaction is necessary to establish a thermal initial condition for N_A ; no such requirements apply for N_B whose abundance is generated through sub-Hubble active-sterile oscillations in the twin sector.

Throughout our analysis, we assume that N_A freezes out from some unspecified interactions at a sufficiently high temperature and that N_B experiences no such interactions. Since we have explicitly introduced explicit \mathbb{Z}_2 breaking for neutrino parameters in this model, it is straightforward to add an additional new state that couples to SM sector neutrinos to bring N_A into equilibrium at early times and whose impact on later cosmological epochs is negligible (e.g., because it decays long before N_A decouples). Furthermore, since these additional states only introduce \mathbb{Z}_2 breaking in the neutrino sector, they have a negligible impact on the other desirable features of twin Higgs models (e.g., cancellation of the quadratically divergent top loop contribution to the Higgs mass).

In the main text we noted that the ultimate constraints in Fig 2 are dependent on the choice of this new interaction. For example, this is relevant if N_A freezes out at a temperature lower than its mass, since it will be exponentially suppressed as a cold relic and its decays will have limited impact on the ΔN_{eff} problem.