# Simultaneous measurement of $\nu_\mu$ charged-current single $\pi^+$ production in CH, C, $H_2O$, Fe, and Pb targets in MINERvA

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## Abstract

Neutrino-induced charged-current single $\pi^+$ production in the $\Delta(1232)$ resonance region is of considerable interest to accelerator-based neutrino oscillation experiments. In this work, high statistics differential cross sections are reported for the semi-exclusive reaction $\nu_\mu A \to \mu^- \pi^+ +$ nucleon(s) on scintillator, carbon, water, iron, and lead targets recorded by MINERvA using a wide-band $\nu_\mu$ beam with $\langle E_\nu \rangle \approx 6$ GeV. Suppression of the cross section at low $Q^2$ and enhancement of low $T_\pi$ are observed in both light and heavy nuclear targets compared to phenomenological models used in current neutrino interaction generators. The cross-sections per nucleon for iron and lead compared to CH across the kinematic variables probed are 0.8 and 0.5 respectively, a scaling which is also not predicted by current generators.



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Charged pion production is an important process for accelerator-based oscillation experiments. For low energy experiments, T2K [1], T2HK [2], and SBN [3], this process is both a significant background to quasielastic scattering and is used as a signal reaction when the pion is identified. For higher energy experiments, NOvA [4] and DUNE [5], the process is a large fraction of the signal. Neutrino oscillation experiments rely on an accurate model of pion production to evaluate neutrino energy for events with charged pions. Although NOvA, T2K, and T2HK utilize relatively low-A nuclear media such as hydrocarbons or water, SBN and DUNE experiments rely on argon (A = 40) targets. In order for these experiments to make use of the wealth of neutrino interaction data obtained with light nuclei, a knowledge of neutrino cross-section scaling as a function of target A is required. This work tests scaling behavior by simultaneously measuring the differential cross sections of $\nu_\mu + A \to \mu^- + \pi^+ +$ nucleons as the target nucleon number is changed from carbon (A=12) to lead (A= 208). We also present cross section ratios to scintillator, where many systematic uncertainties cancel, allowing a precise measurement of the nuclear dependence.

Previous measurements of charged-current pion production [6–11] have found disagreements between data and models at both low $Q^2$ (the negative of the square of the four momentum transfer from the lepton) and in pion kinetic energy. Simulations of this process must model both the primary neutrino-nucleon interaction and a variety of nuclear effects, including interactions of produced hadrons within the nucleus. This convolution of primary production and nuclear modification makes it difficult to isolate the model features responsible for the disagreements. Measurements on multiple nuclei can help separate nuclear effects from other aspects of the interaction.

Signal reactions for this measurement are $\nu_\mu$ charged-current interactions which produce a single negatively charged muon and a single positively charged pion. Any number of baryons may be in the final state; however, no other mesons may be produced. To match MINERvA's acceptance, the muon is restricted in momentum ($p_\mu$) and angle with respect to the neutrino beam ($\theta_\mu$), such that $1.5 < p_\mu < 20$ GeV/c and $\theta_\mu < 13^\circ$. Pion kinetic energy ($T_\pi$) is restricted to $35 < T_\pi < 350$ MeV.

To enhance reactions involving a $\Delta$ baryon resonance, a selection is made on the invariant hadronic mass of the final state. We define $W_{\rm exp} = M^2 + 2ME_{\rm had} - Q^2$, where $E_{\rm had} = E_\nu - E_\mu$, $M$ is the average of the proton and neutron masses. $W_{\rm exp}$ is the invariant mass of the hadronic final state under the assumption that the target nucleon is at rest. $W_{\rm exp}$ is required to be

less than 1.4 GeV/$c^2$.

This analysis provides measurements in eight kinematic variables: the magnitude of the muon momentum and its longitudinal and transverse components ($p_\mu$, $p_{\mu,||}$, and $p_{\mu,T}$), the angle of the muon ($\theta_\mu$), the kinetic energy and angle of the charged pion with respect to the neutrino beam ($T_\pi$ and $\theta_\pi$), $Q^2$, and $W_{\rm exp}$ as defined by the kinematics of the final state particles.

These measurements use event samples collected at the MINERvA detector using the medium energy NuMI beam at Fermilab [12]. To create the neutrino beam, 120 GeV protons impact a graphite target and produce pions and kaons. Two magnetic horns focus these charged particles into a pipe where they decay into primarily muon neutrinos. A GEANT4 simulation of the NuMI beamline [13] predicts the neutrino flux using constraints from hadron production data. Previous MINERvA measurements of neutrino elastic scattering on atomic electrons, $\nu e^- \to \nu e^-$, constrain the normalization of the neutrino flux, reducing the uncertainty of the flux between 2 and 20 GeV from 7.8% to 3.9% [14, 15]. Another MINERvA measurement of inverse muon decay, $\nu_\mu e^- \to \mu^- \nu_e$, constrains the flux at higher neutrino energies [16]. These data represent an exposure of $10.6 \times 10^{20}$ protons on target, half of which were taken with the water target filled.

The MINERvA detector consists of a central polystyrene-based scintillator tracker with an upstream nuclear target region and downstream electromagnetic and hadronic calorimeters. The nuclear target region contains five planes of passive material comprised of carbon, water, iron, and lead targets. The scintillator tracking planes are 95% CH by weight. The passive targets are interspersed between regions of active tracking volumes. The finely segmented tracking volumes of the MINERvA detector consist of hexagonal planes of nestled triangular scintillator strips with a pitch of 1.7 cm , allowing for spatial reconstruction with a resolution of 3 mm [12]. The MINOS Near Detector, located 2 m downstream of the MINERvA detector measures charge and momentum of exiting muons [17].

The simulation of the MINERvA detector utilizes GEANT4 version 4.9.4p2 [18] with the QGSP_BERT physics list [19–21] to model the detector response. Calibrations using through-going muons provide the absolute energy scale [12]. Measurements on a scaled down version of the MINERvA detector in a charged particle test beam set the hadronic energy response [22]. Overlaying data onto simulated events captures the effects of overlapping activity due to additional beam interactions.

A modified version of the GENIE v2.12.6 event generator, denoted "MnvTune v4.3.1", supplies the neutrino interaction simulation [23]. For inelastic events with the invariant hadronic mass $W < 1.7$ GeV/$c^2$, resonance pion production assumes the Rein-Seghal model [24] with an axial mass of $M_A^{RES} = 1.12$ GeV/$c^2$ [25]. Deep inelastic scattering relies on the Bodek-Yang model [26] tuned to agree with external measurements of pion production and total cross sections. Coherent pion production is simulated with the Rein-Seghal [27, 28] model with corrections for muon mass. The nuclear medium is modeled as a relativistic Fermi gas (with Fermi momentum $p_F \sim 250$ MeV/$c$) [29] with an added Bodek-Ritchie high momentum tail [30, 31]. Simulation of hadron final state interactions within the nucleus are predicted by the INTRANUKE-hA package [23].

The GENIE simulation has been tuned to better reproduce MINERvA data and provide more accurate signal and background models. Modifications to the quasielastic process are described in Ref. [32, 33]. Pion production through baryon resonances is modified to match the $D_2$ bubble chamber data as in Ref. [34]. Coherent pion production is reweighted in both the energy of the pion $E_\pi$ and $\theta_\pi$ to agree with a recent measurement of coherent pion production [35]. The normalization of coherent pion production is increased by 43.7% to account for coherent interactions on hydrogen, known as diffractive pion production, based on the Kopeliovic model [36]. The changes above comprise "MnvTune v4.2.1".

However, signal events in the scintillator disagree with the prediction of this model. The model for all targets except hydrogen was corrected by matching the simulated cross section versus $Q^2$ to the data. This results in improved estimates of efficiencies and backgrounds. . The correction decreases the single pion production for $Q^2 < 0.1$ GeV$^2$/$c^2$ with a slow logarithmic increase for higher $Q^2$. Details of this tune are given in the supplemental material. This modification, combined with those of MnvTune v4.2.1, forms MnvTune v4.3.1.

Selected events are required to have at least two tracked particles that start in the correct target. One track must be identified as a negatively charged muon by the MINOS Near Detector. One remaining track must be identified as a non-interacting charged pion by matching a Michel electron to the endpoint of the track and having longitudinal energy deposition ($dE/dx$) consistent with a non-interacting pion. All remaining tracks must not have $dE/dx$ consistent with a pion. The charged pion tracking efficiency decreases at lower momentum and at angles perpendicular to the detector axis. The efficiency to reconstruct the Michel electron from the $\mu^+$ was predicted from the simulation and validated in the

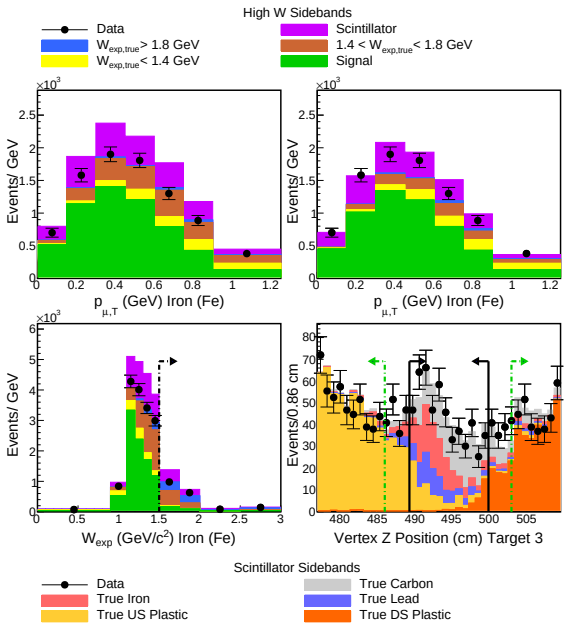


FIG. 1. $p_{\mu,T}$ distributions in iron for the signal regions before (top left) and after (top right) the simulation has been constrained with background estimates from data, the high $W$ sideband in $W_{exp}$ in iron (bottom left) and the scintillator sidebands in vertex z of target 3 (bottom left). The background in the bottom plots have not been constrained. The solid (dashed) arrows in the lower right plot delineate the signal (sideband) regions. Events labeled $W_{exp,true} < 1.4GeV$ are those that pass the signal $W_{exp,true}$ selection but fail other elements of the signal definition.

scintillator tracker using stopping muons produced in the rock upstream of the detector [37]. The $W_{\rm exp} < 1.4$ GeV/$c^2$ requirement strongly reduces the number of multipion events. Both $W_{\rm exp}$ and $Q^2$ are computed using the visible calorimetric energy in the detector to estimate $E_{\rm had}$ [37]. After all cuts there are 33,231 events in the tracker, and 1403 (1033) events in the iron (lead), and 295 (291) events in the carbon (water).

The two primary backgrounds are pion production events with $W_{\rm exp} > 1.4$ GeV/$c^2$ and pion production events that appear to originate from the passive nuclear targets but are actually produced in the adjacent scintillator. First, simulated events with reconstructed $W_{\rm exp} > 1.4$ GeV/$c^2$ in the scintillator target are weighted to match data using two scale factors, one for events with true $W_{\rm exp}$ between 1.4 and 1.8 GeV/$c^2$ and one for events with $W_{\rm exp}$ above 1.8 GeV/$c^2$. True $W_{\rm exp}$ is computed in the same manner as reconstructed $W_{\rm exp}$ but uses true simulated values. To improve the background prediction from events in scintillator adjacent to the passive target, pions with low $T_\pi$ or $\theta_\pi > 90^\circ$ also receive additional

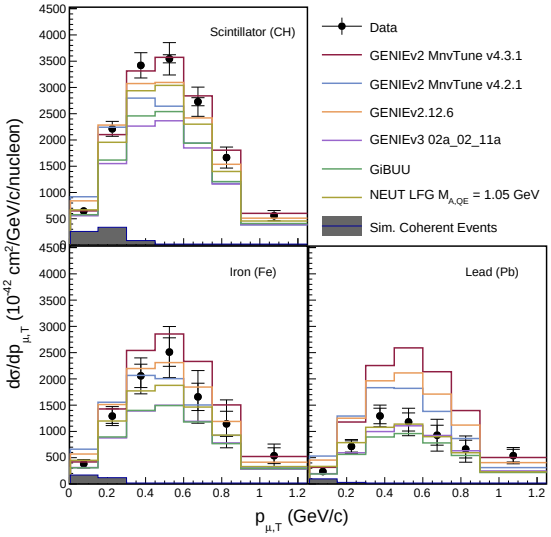


FIG. 2. Differential cross section $\frac{d\sigma}{dp_{\mu,T}}$ measured on scintillator, iron, and lead (solid points) compared to GENIE, GIBUU and NEUT generators, all of which are discrepant with one if not all nuclei.

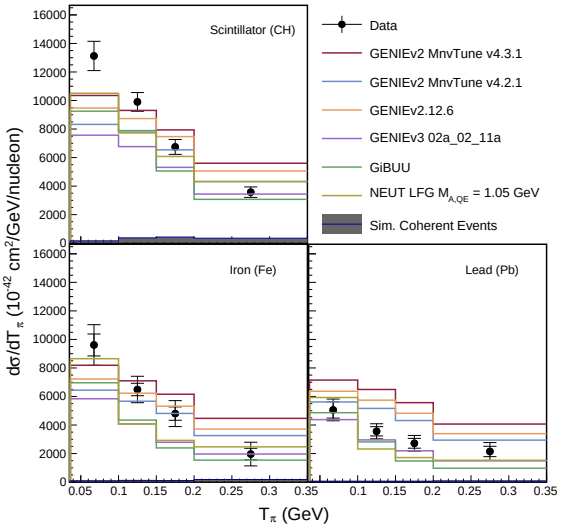


FIG. 3. Differential cross section $\frac{d\sigma}{dT_\pi}$ in scintillator, iron, and lead (solid points) compared to GENIE, GIBUU and NEUT generators, all of which are discrepant.

weights to improve agreement with the scintillator target data. Next, simulated events in each target are weighted to correct the prediction of adjacent scintillator backgrounds using events observed in data near the passive targets. Finally, a weight is applied for events with high reconstructed $W_{\rm exp}$, in the same manner as above, for each target material. Fig. 1

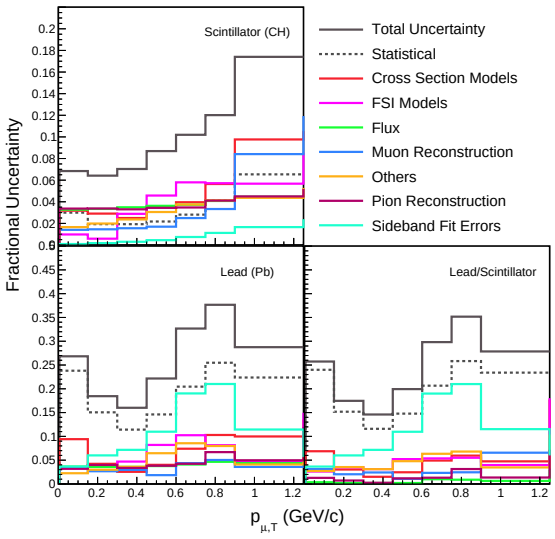


FIG. 4. Systematic uncertainties on the differential cross section $\frac{d\sigma}{dp_{\mu,T}}$ in lead and scintillator and uncertainties of the ratio of $\left(\frac{d\sigma_{Pb}}{dp_{\mu,T}}\right) / \left(\frac{d\sigma_{CH}}{dp_{\mu,T}}\right)$.

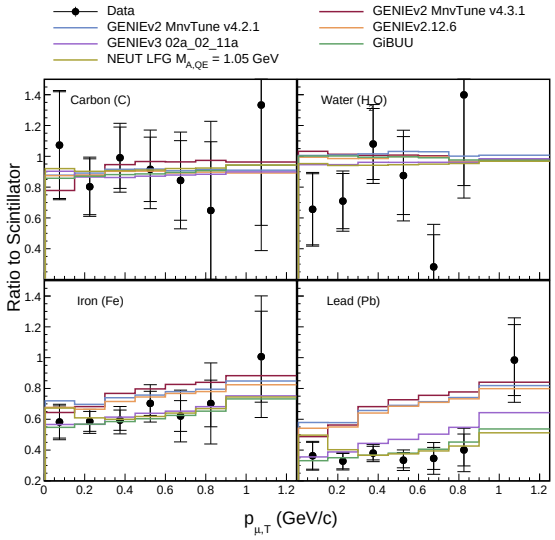


FIG. 5. Cross section ratios $(\frac{d\sigma_A}{dp_{\mu,T}})/(\frac{d\sigma_{CH}}{dp_{\mu,T}})$ for carbon, water, iron, and lead (solid points), as compared to GENIE, NEUT, and GIBUU.

shows a sample of data used in the background constraints, before and after constraints. The full set of weights are given in the supplemental material.

Kinematic smearing due to the detector in the background-subtracted distributions is removed with iterative D'Agostini unfolding [38, 39] as implemented in the RooUnfold [40] framework. The number of iterations was chosen by studying the fidelity of unfolding ran-

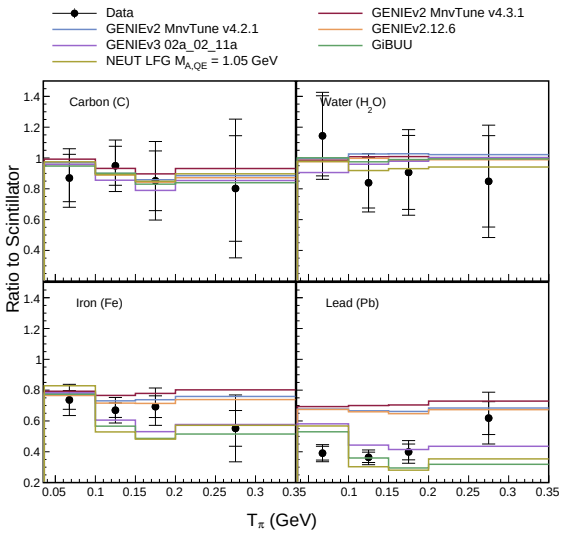


FIG. 6. Cross section ratios $(\frac{d\sigma_A}{dT_{\pi^+}})/(\frac{d\sigma_{CH}}{dT_{\pi^+}})$ for carbon, water, iron, and lead (solid points) compared to predictions from GENIE, NEUT, and GIBUU. The predictions from GIBUU and NEUT agree better with the measurements than those from GENIE.

domly thrown pseudodata samples generated from alternate physics models. The event distributions are corrected for selection efficiency and acceptance, which is 4.5% (1%) on average in the tracker (passive targets). Efficiencies are available in the supplement. The differential cross sections are obtained by normalizing the resulting event rates by the integrated neutrino flux and number of target nucleons.

One complication of forming ratios of cross sections between the passive targets and the tracker is that the fluxes have small differences due to the distributions of mass relative to the beam axis, as shown in the supplemental material. This is corrected by measuring the cross section on scintillator in several regions of the detector, each integrated over a slightly different neutrino energy distribution. Linear combinations of these regional cross sections are used to form a cross section integrated over a flux that matches the relevant target. These cross sections are used to form the ratio of target-to-tracker cross-section ratios [37, 41].

Figure 2 shows the differential cross section $\frac{d\sigma}{dp_{\mu,T}}$ on materials with highest statistics (iron, lead, and scintillator). Lower statistics measurements on water and carbon are available in the supplement. The statistical uncertainty dominates in all measurements, except in the scintillator. An example of the uncertainties on the cross sections are

shown in Fig. 4; uncertainties in the cross section models which enter via efficiency and unfolding are the largest, but are lower than the statistical uncertainties in all targets but the scintillator. Statistical uncertainties in the sideband constraints are also significant for the passive targets. Systematic uncertainties are smaller in the cross section ratios, which benefit from partial cancellation of the flux, cross section model, and reconstruction uncertainties.

Ratios of cross sections to that in scintillator are shown for $\frac{d\sigma}{dp_{\mu,T}}$ and $\frac{d\sigma}{dT_\pi}$ in Figs. 5 and 6. The cross-section ratios of carbon or water to scintillator can be characterized by a scale factor of unity, while ratios between iron or lead to scintillator are constant at 0.8 and 0.5 respectively, with no large modifications to distribution shapes for either $p_{\mu,T}$ or $T_\pi$. Cross sections and cross section ratios to scintillator in other kinematic variables are tabulated and shown in the supplemental material. Ratios in the other variables also exhibit this scaling behavior.

None of the six generator models investigated capture the evolution of absolute cross section. The generators NEUT[42] and GIBUU[43] do correctly predict the cross section ratios between Fe and Pb and scintillator, as shown in Figs. 5 and 6, while the GENIE predictions do not agree. The GENIE models use pion- and nucleon-nucleus scattering data to implement single-step absorption and other scattering processes (INTRANUKE-hA) [23], whereas NEUT, NuWRO and GIBUU employ different microscopic transport algorithms to simulate pion and nucleon intranuclear scattering.

As previously noted, the scintillator cross section showed a large discrepancy with the MINERvA-tuned GENIE model, which is corrected with a weight as a function of $Q^2$. In Fig. 2, the differential cross section as a function of the related observable $p_{\mu,T}$ $\left(Q^2 \approx {p_{\mu,T}}^2 \left[1 + \mathcal{O}(\frac{E_{\rm had}}{E_\mu})\right]\right)$ shows poor agreement both with untuned GENIE models (GENIEv2.12.6) and with tuned models without this weighting function (GENIEv2 MnvTune v4.2.1). The shape of the data is in better agreement with the weighted model (GENIEv2 MnvTune v4.3.1) in all targets, even though the weight is derived using only the scintillator measurement. The absolute normalization of the cross section is not well described by MnvTune v4.3.1 in Iron or Lead.

Similarly, the ratios of $d\sigma/dp_{\mu,T}$ between the targets and scintillator are consistent with being independent of $p_{\mu,T}$. A feature of both the GENIE v3 and GiBUU models is that they predict the large suppression lead relative to other targets. However, as seen in Fig. 6,

these models also predict a larger ratio at low $T_\pi$ in the heavy targets, a feature which is not seen in the data.

In summary, these data on single pion production on a wide variety of nuclei provide a new way to test nuclear models through their nuclear dependence. We observe a large modification to the predicted $Q^2$ distribution of single $\pi^+$ events that it can be consistently seen in all nuclei. These results give guidance in how the wealth of data on scintillator targets can be applied to models for oxygen and argon for future neutrino experiments.

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# I. SUPPLEMENTAL MATERIAL

## A. Flux Differences between Targets and Tracker before Reweighting of Tracker

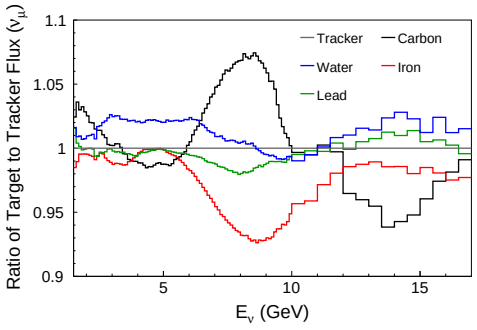



FIG. Supp.1. Ratio of neutrino fluxes in carbon, water, iron, and lead to scintillator as a function of energy before reweighting of the scintillator sample denominator.

### B. Single $\pi^+$ $Q^2$ tune function

The disagreement between the initial tuned GENIE model described above and the scintillator data was addressed by applying a weighting function, $w(Q^2)$ to all non-coherent single pion production on nuclei other than hydrogen. Coherent pion production and production on hydrogen are excluded from the correction because they are independently constrained by the results of Ref. [35] and Ref. [34], respectively. The predicted coherent and hydrogen cross section on scintillator in the data was subtracted from the measurement in the data, and compared to the same quantity in the simulation, and the ratio was fit to find $w(Q^2)$.

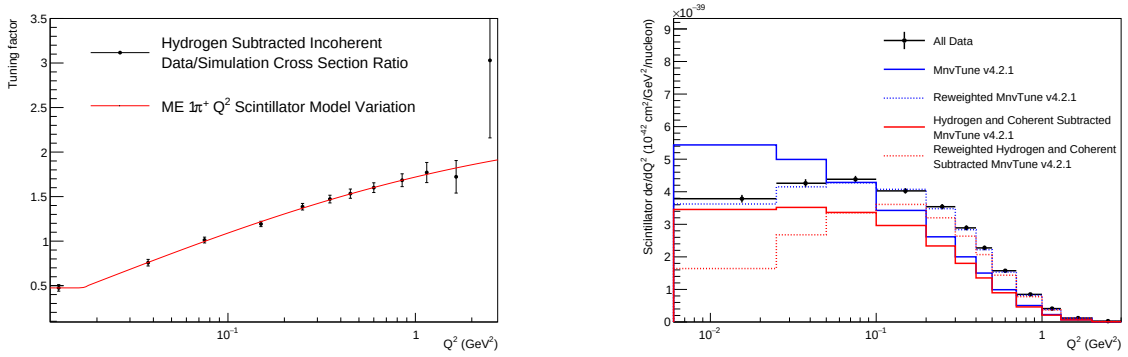
This fit has the form shown in equation 1, where Erf is the Gaussian error function, and the fitted parameters are given in Tab. Supp.I. Figure Supp.2 shows the cross section ratio as well as the fit.

These suppression at low $Q^2$ relative to high $Q^2$ is a feature that has been observed in different data sets, including MINOS on iron [44] and in other MINERvA pion measurements on scintillator [8, 10, 11, 45]. Models have some differences in prediction, although generally do not predict this large suppression. The high $Q^2$ shape evolution of different versions and tunes of GENIE goes from GENIEv2 MnvTune v4.2.1 (steepest), to untuned GENIE v2.12.6 to GENIE v3 02a_02_11a to GENIEv2 MnvTune v4.3.1 (flattest) via changes to the axial form factor and, in the case of GENIE v3, also the vector form factor and FSI processes. Like the tunes of Refs. 45 and 44, the MnvTune v4.3.1 adds the *ad hoc* additional shape at low $Q^2$ evident in these data.

$$w(Q^2) = a + b \times \text{Max}\left[0, \text{Erf}\left[x_3 \ln\left(\frac{Q^2}{x_1}\right)\right]\right] \tag{1}$$

| Parameter | Fitted Value |
|---|---|
| $a$ | $0.475 \pm 0.037$ |
| $b$ | $1.837 \pm 0.537$ |
| $x_1$ | $0.017 \pm 0.003$ |
| $x_3$ | $0.171 \pm 0.069$ |
| $\chi^2$/ndf | 1.58/11 |

TABLE Supp.I. Fitted parameters of the $Q^2$ model variation




(a) (b)

FIG. Supp.2. (a) Hydrogen Subtracted Incoherent Data/Simulation Cross Section Ratio and the $Q^2$ weighting function (FHC nu-e constraint). (b) Unweighted and Reweighted Total and Hydrogen Subtracted Incoherent Simulated Cross Section Compared to Data Cross Section.

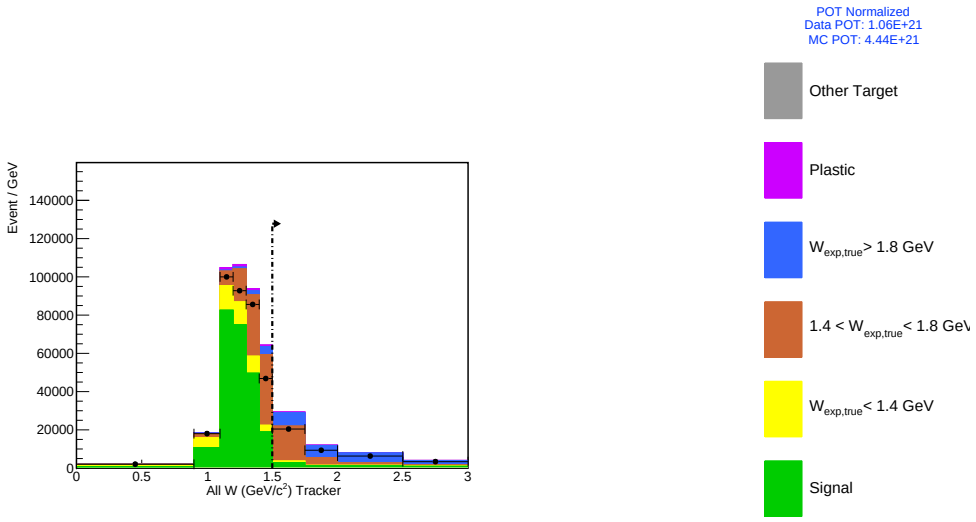
### C. Sideband scale factors

| Target | $1.4 < W_{\text{exp,true}} < 1.8$ GeV/$c^2$ | $1.8 < W_{\text{exp,true}}$ GeV/$c^2$ | $\chi^2$/ndf |
|---|---|---|---|
| Carbon | $0.58 \pm 0.44$ | $1.71 \pm 0.71$ | 0.147/4 |
| Water | $1.53 \pm 0.30$ | $0.00 \pm 0.90$ | 18.956/4 |
| Iron | $0.48 \pm 0.25$ | $1.05 \pm 0.29$ | 1.705/4 |
| Lead | $0.09 \pm 0.16$ | $1.06 \pm 0.20$ | 0.324/4 |
| Scintillator | $0.58 \pm 0.02$ | $0.80 \pm 0.02$ | 3.192/4 |

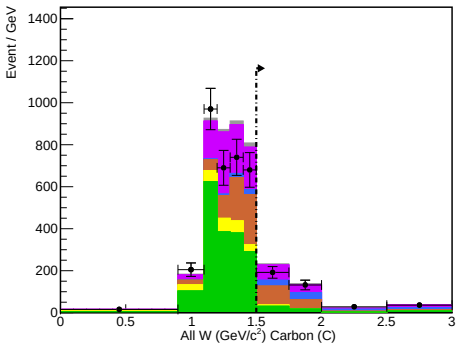
TABLE Supp.II. $W_{exp}$ sideband scales found from the fit in the high $W > 1.4$ GeV sideband for all targets.

| Target | Upstream Scale | Upstream $\chi^2$/ndf | Downstream Scale | Downstream $\chi^2$/ndf |
|---|---|---|---|---|
| Target 1 | $0.69 \pm 0.13$ | 1.45/6 | $0.93 \pm 0.07$ | 2.76/6 |
| Target 2 | $0.99 \pm 0.05$ | 3.90/7 | $0.88 \pm 0.07$ | 12.00/6 |
| Target 3 | $0.92 \pm 0.04$ | 6.93/10 | $0.88 \pm 0.06$ | 4.03/7 |
| Target 4 | $0.92 \pm 0.06$ | 4.18/5 | $0.83 \pm 0.05$ | 13.70/7 |
| Target 5 | $0.92 \pm 0.05$ | 9.89/7 | $0.86 \pm 0.05$ | 11.89/7 |

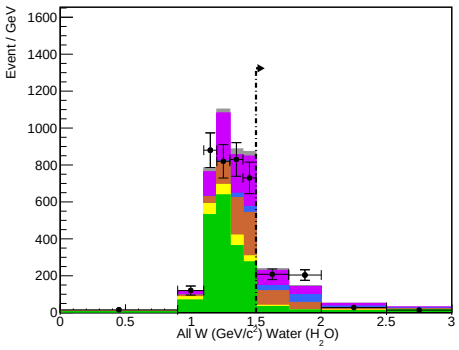
TABLE Supp.III. Upstream and downstream scintillator target sideband scales for all nuclear targets



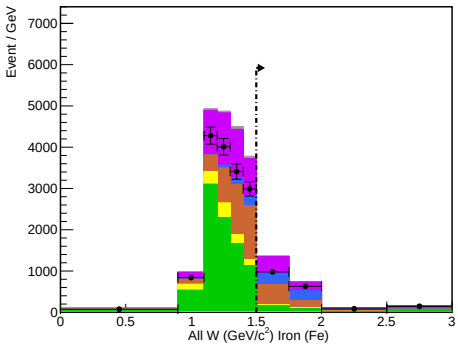

(a) Scintillator



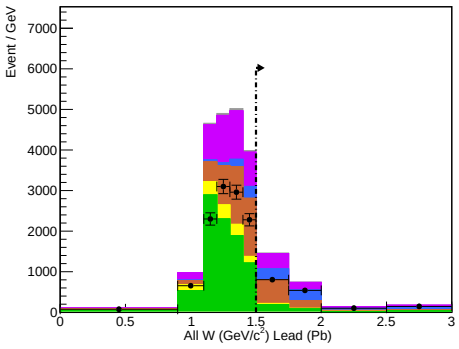

(b) Carbon




(c) Water

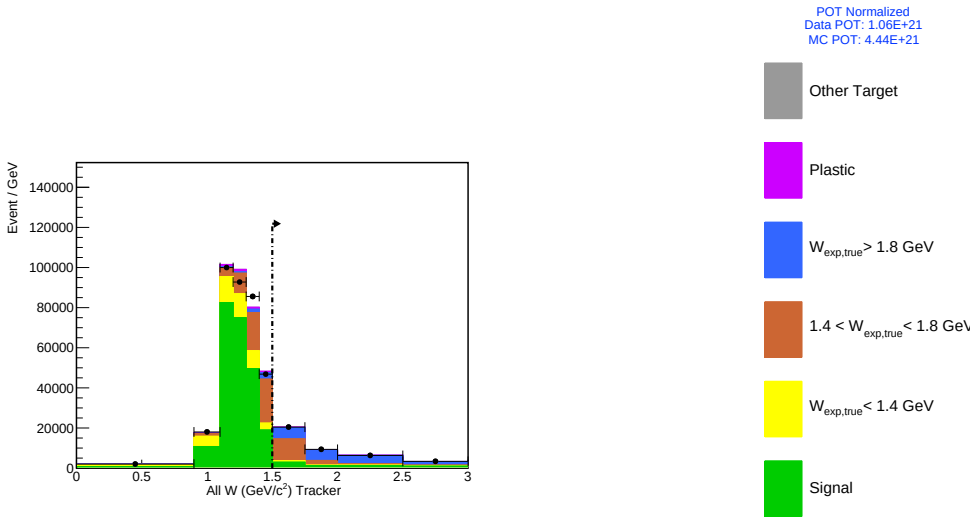



(d) Iron

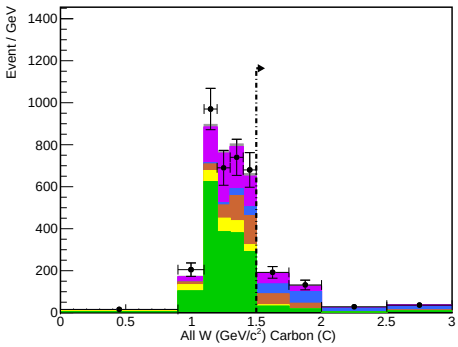



(e) Lead

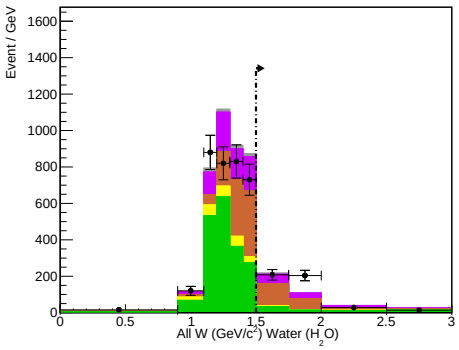
FIG. Supp.3. $W_{exp}$ distribution of selected events before constraining the simulation using background estimates from data. The dashed arrow delineates the high $W_{exp}$ sideband region.



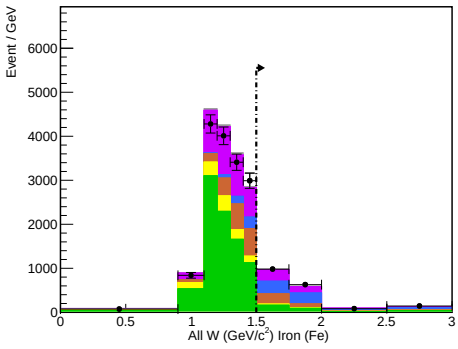

(a) Scintillator



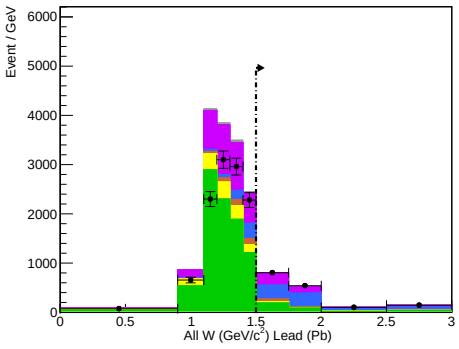

(b) Carbon




(c) Water




(d) Iron




(e) Lead

FIG. Supp.4. $W_{exp}$ distribution of selected events after constraining the simulation using background estimates from data. The dashed arrow delineates the high $W_{exp}$ sideband region.

### D. Coherent Pion Production Weights

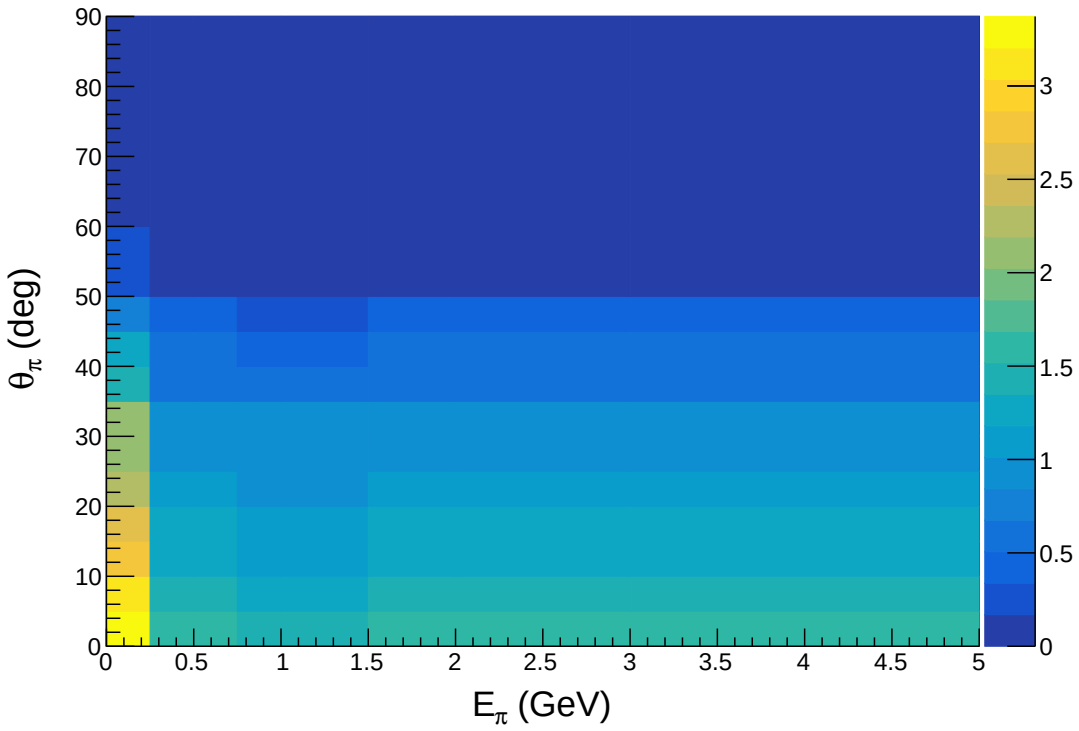
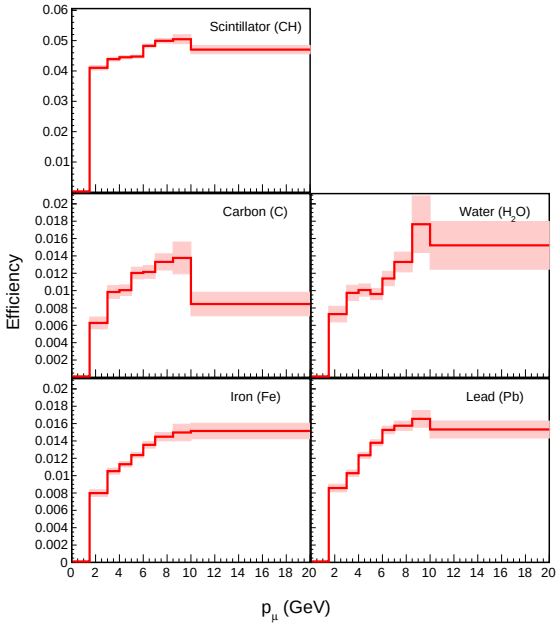



FIG. Supp.5. Plot of weights applied to coherent pion production events, given the kinematics of the coherent $\pi^+$, derived from ME MINERvA data [35]

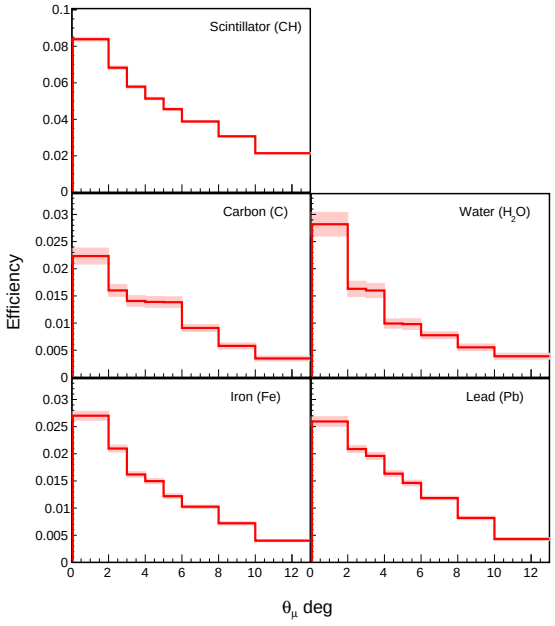
| $E_\pi$ (GeV) / $\theta_\pi$ (deg) | 0.0-0.25 | 0.25-0.5 | 0.5-0.75 | 0.75-1.0 | 1.0-1.5 | 1.5-2.0 | 2.0-3.0 | 3.0-5.0 |
|---|---|---|---|---|---|---|---|---|
| 0-5 | 3.372 | 1.593 | 1.593 | 1.369 | 1.360 | 1.593 | 1.593 | 1.593 |
| 5-10 | 3.083 | 1.456 | 1.456 | 1.251 | 1.243 | 1.456 | 1.456 | 1.456 |
| 10-15 | 2.742 | 1.296 | 1.296 | 1.113 | 1.106 | 1.296 | 1.296 | 1.296 |
| 15-20 | 2.577 | 1.218 | 1.218 | 1.046 | 1.039 | 1.218 | 1.218 | 1.218 |
| 20-25 | 2.345 | 1.108 | 1.108 | 0.952 | 0.945 | 1.108 | 1.108 | 1.108 |
| 25-30 | 2.116 | 1.000 | 1.000 | 0.859 | 0.853 | 1.000 | 1.000 | 1.000 |
| 30-35 | 2.116 | 1.000 | 1.000 | 0.859 | 0.853 | 1.000 | 1.000 | 1.000 |
| 35-40 | 1.422 | 0.672 | 0.672 | 0.577 | 0.573 | 0.672 | 0.672 | 0.672 |
| 40-45 | 1.238 | 0.585 | 0.585 | 0.503 | 0.499 | 0.585 | 0.585 | 0.585 |
| 45-50 | 0.713 | 0.337 | 0.337 | 0.289 | 0.288 | 0.337 | 0.337 | 0.337 |
| 50-60 | 0.263 | 0.124 | 0.124 | 0.107 | 0.106 | 0.124 | 0.124 | 0.124 |
| 60-70 | 0.049 | 0.023 | 0.023 | 0.020 | 0.020 | 0.023 | 0.023 | 0.023 |
| 70-80 | 0.049 | 0.023 | 0.023 | 0.020 | 0.020 | 0.023 | 0.023 | 0.023 |
| 80-90 | 0.049 | 0.023 | 0.023 | 0.020 | 0.020 | 0.023 | 0.023 | 0.023 |

TABLE Supp.IV. Tabulation of weights applied to coherent pion production events, given the kinematics of the coherent $\pi^+$, derived from ME MINERvA data [35]

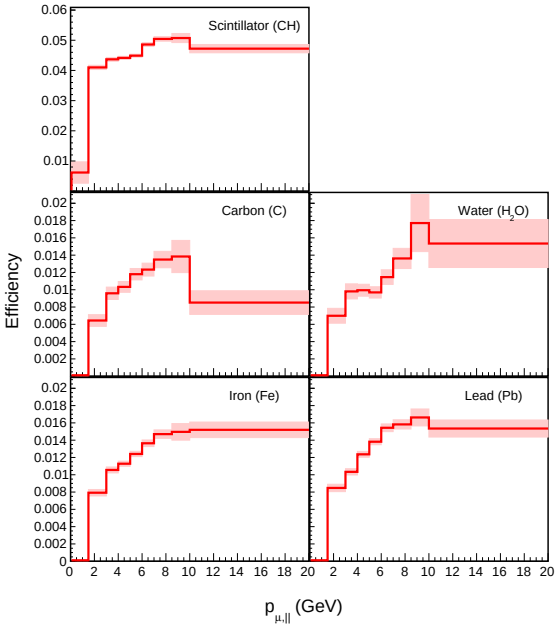
### E. Efficiencies



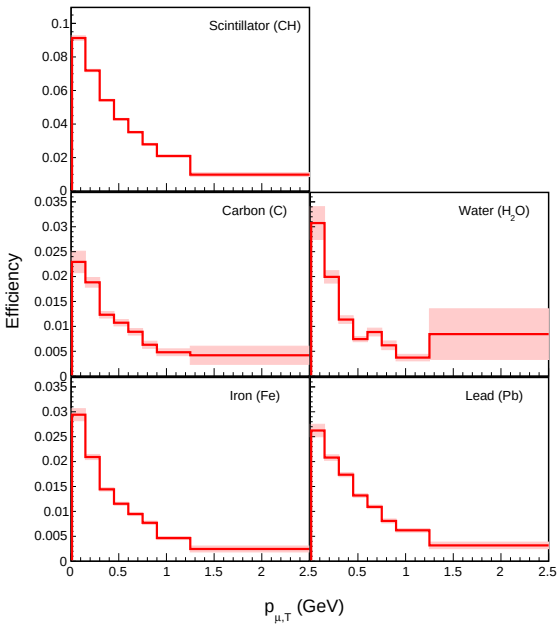

(a) Efficiency versus muon momentum




(b) Efficiency versus muon angle

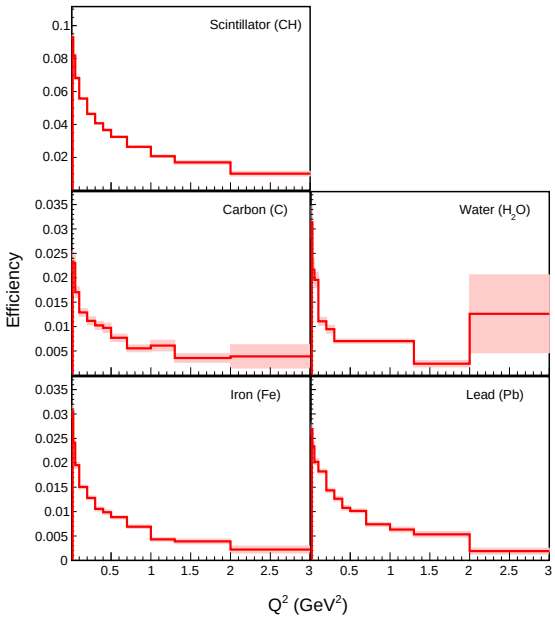



(c) Efficiency versus longitudinal muon momentum

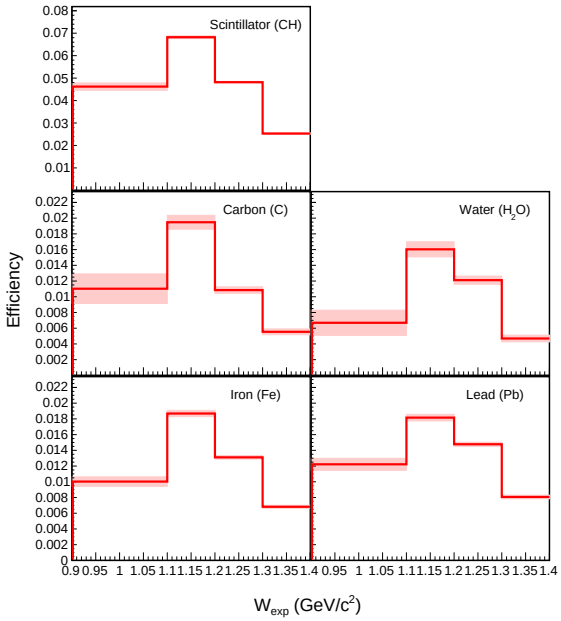



(d) Efficiency versus transverse muon momentum

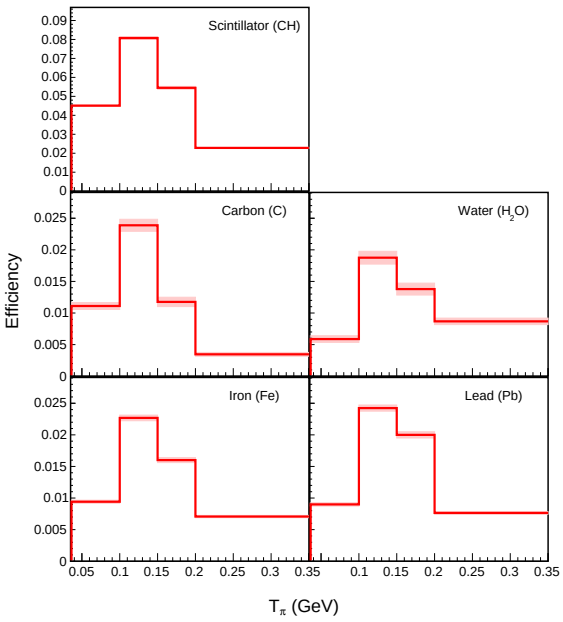
FIG. Supp.6. Efficiencies for scintillator (top left), carbon (middle left), water (middle right), iron (bottom left) and lead (bottom right). The shaded error bands show total uncertainties, including systematics and simulated statistics.



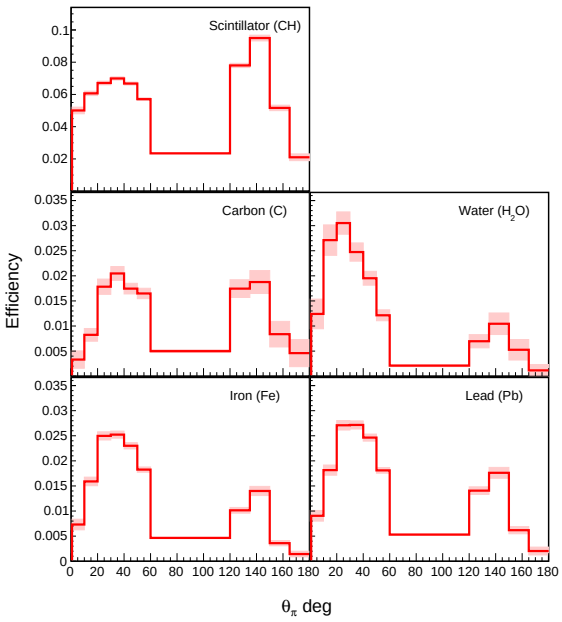

(a) Efficiency versus $Q^2$




(b) Efficiency versus $W$



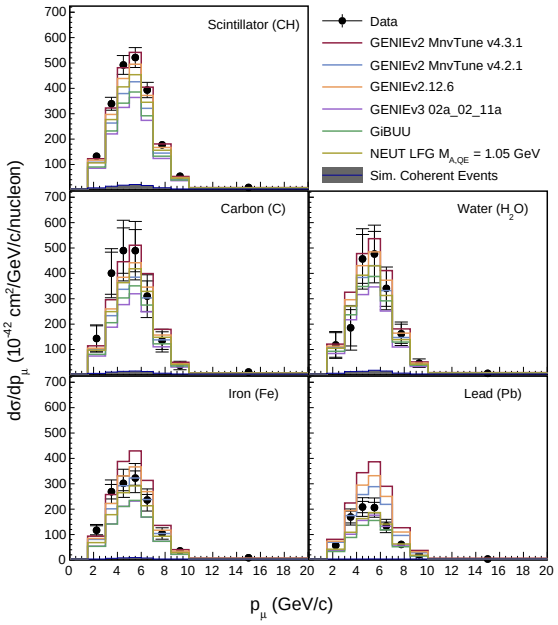

(c) Efficiency versus pion kinetic energy



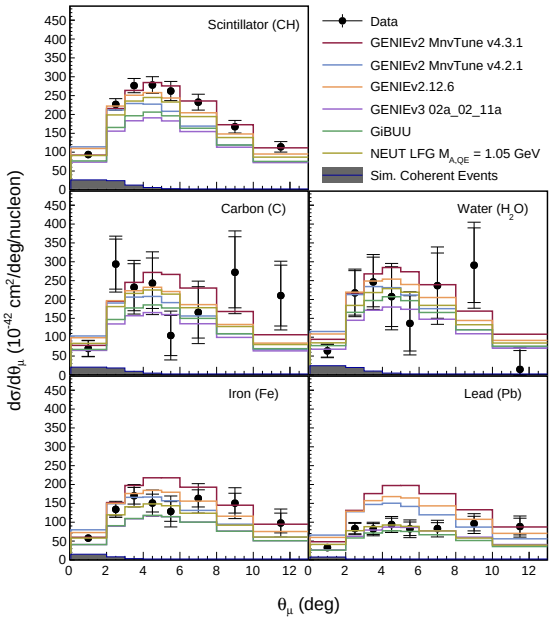

(d) Efficiency versus pion angle

FIG. Supp.7. Efficiencies for scintillator (top left), carbon (middle left), water (middle right), iron (bottom left) and lead (bottom right). The shaded error bands show total uncertainties, including systematics and simulated statistics.

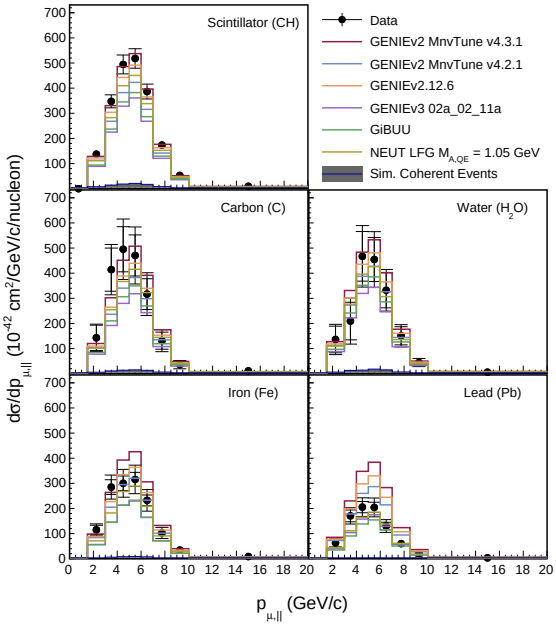
## F. Absolute Cross Section Plots



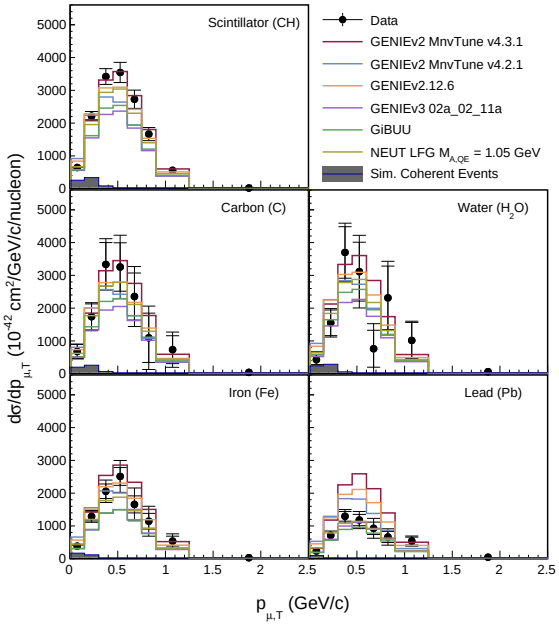

(a) Differential cross section $\frac{d\sigma}{dp_\mu}$ for scintillator, carbon, water, iron, and lead.




(b) Differential cross section $\frac{d\sigma}{d\theta_\mu}$ for scintillator, carbon, water, iron, and lead.

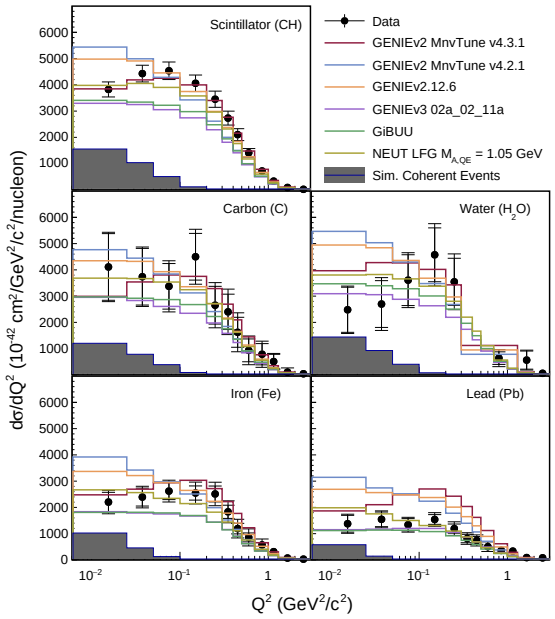



(c) Differential cross section $\frac{d\sigma}{dp_{\mu,||}}$ for scintillator, carbon, water, iron, and lead.

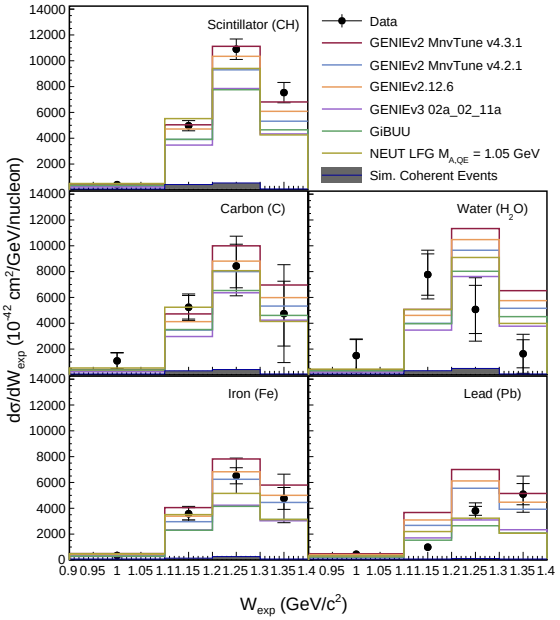



(d) Differential cross section $\frac{d\sigma}{dp_{\mu,T}}$ for scintillator, carbon, water, iron, and lead.

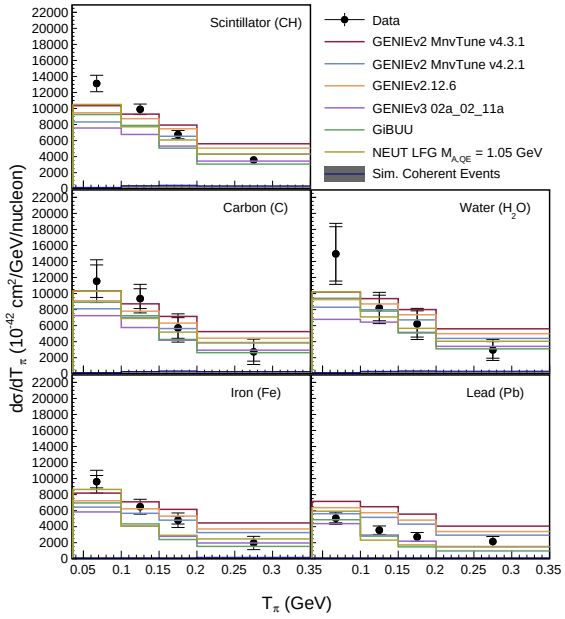
FIG. Supp.8. Differential cross sections versus muon variables.



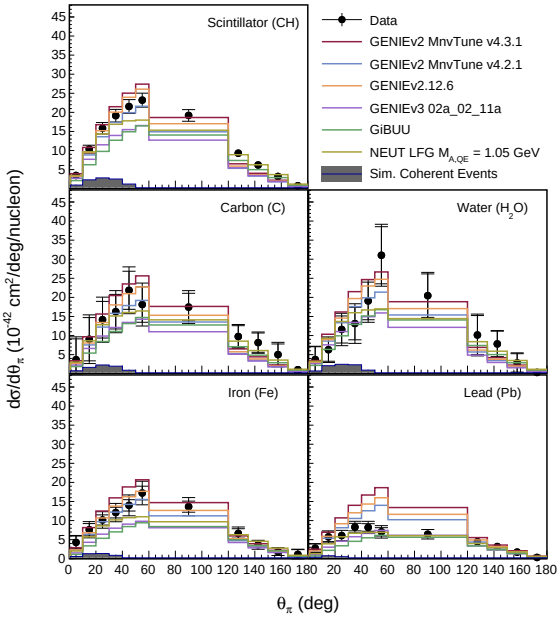

(a) Differential cross section $\frac{d\sigma}{dQ^2}$ for scintillator, carbon, water, iron, and lead.




(b) Differential cross section $\frac{d\sigma}{dW_{exp}}$ for scintillator, carbon, water, iron, and lead.



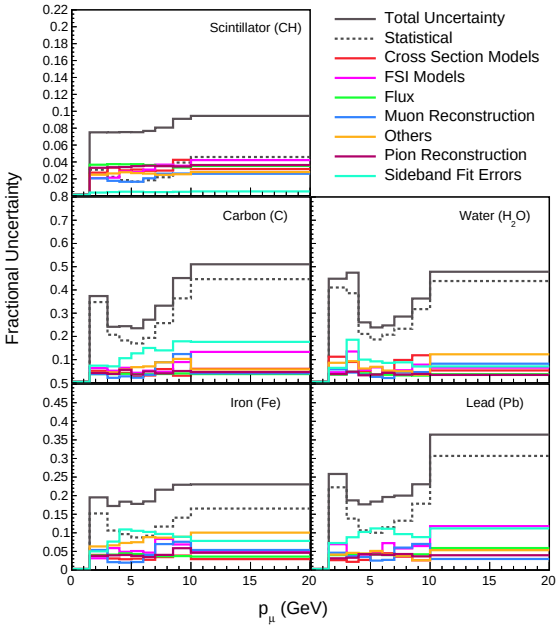

(c) Differential cross section $\frac{d\sigma}{dT_\pi}$ for scintillator, carbon, water, iron, and lead.



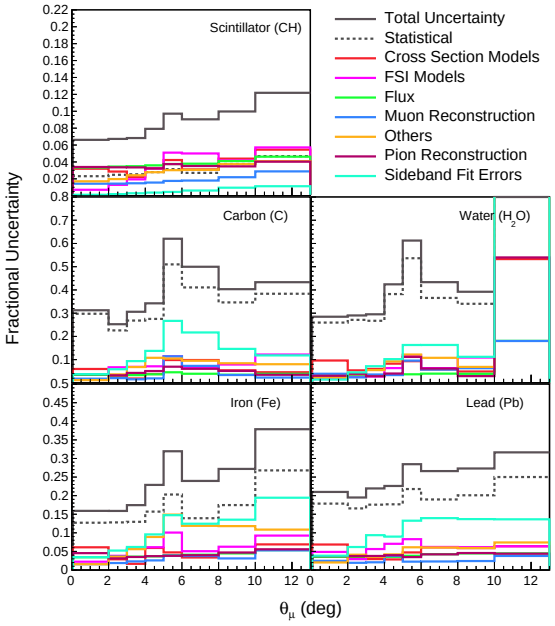

(d) Differential cross section $\frac{d\sigma}{d\theta_\pi}$ for scintillator, carbon, water, iron, and lead.

FIG. Supp.9. Differential cross sections versus other variables.

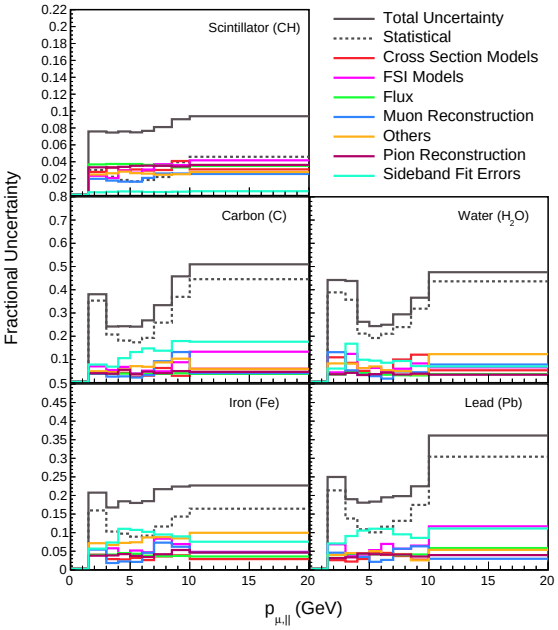
## G. Absolute Cross Section Error Summaries



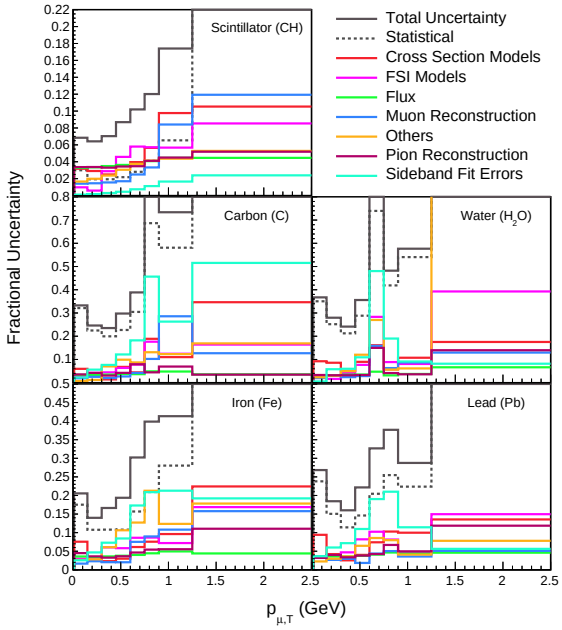

(a) Fractional uncertainties of $\frac{d\sigma}{dp_\mu}$.




(b) Fractional uncertainties of $\frac{d\sigma}{d\theta_\mu}$.

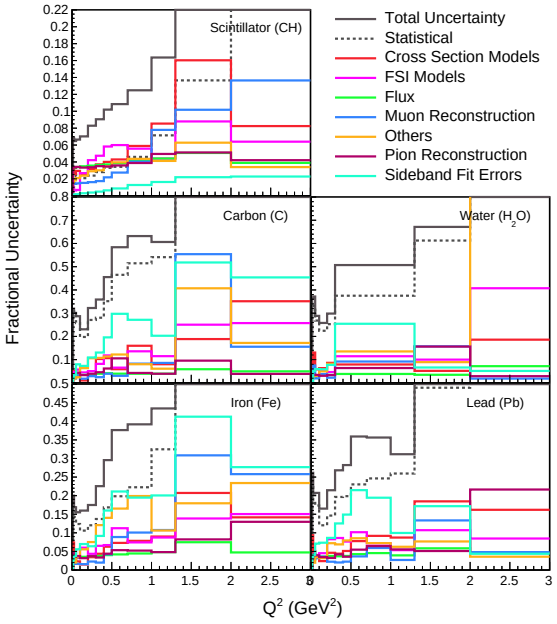



(c) Fractional uncertainties of $\frac{d\sigma}{dp_{\mu,||}}$.

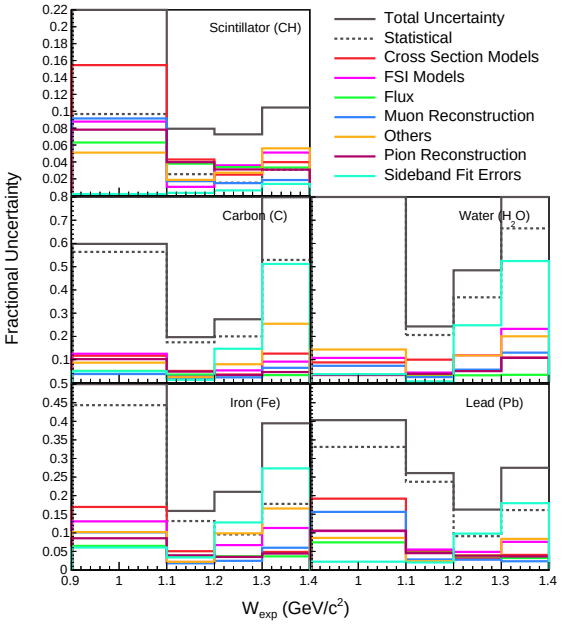



(d) Fractional uncertainties of $\frac{d\sigma}{dp_{\mu,T}}$ .

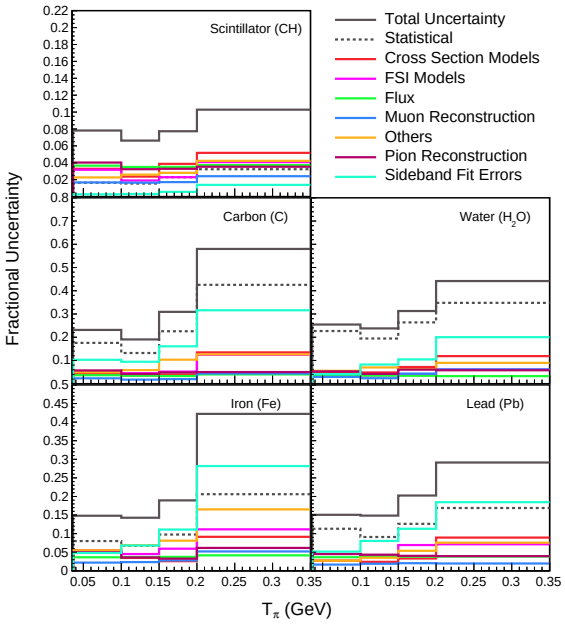
FIG. Supp.10. Systematic uncertainties of differential cross sections for scintillator (top left), carbon (middle left), water (middle right), iron (bottom left) and lead (bottom right)



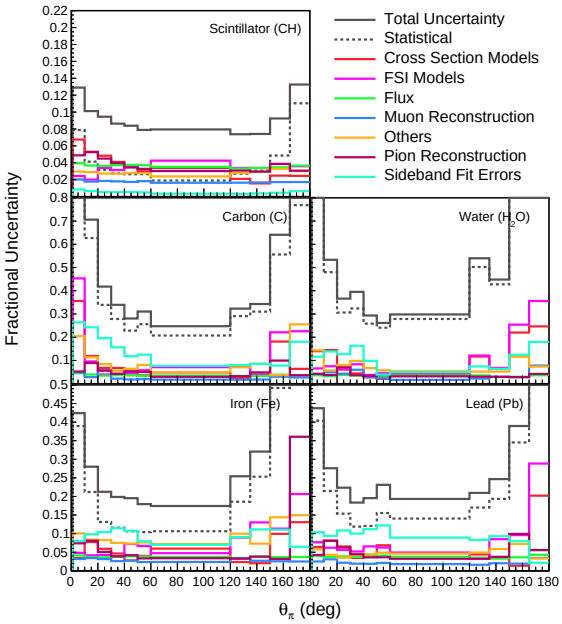

(a) Fractional uncertainties of $\frac{d\sigma}{dQ^2}$.




(b) Fractional Uncertainties $\frac{d\sigma}{dW_{exp}}$.




(c) Fractional Uncertainties $\frac{d\sigma}{dT_\pi}$.

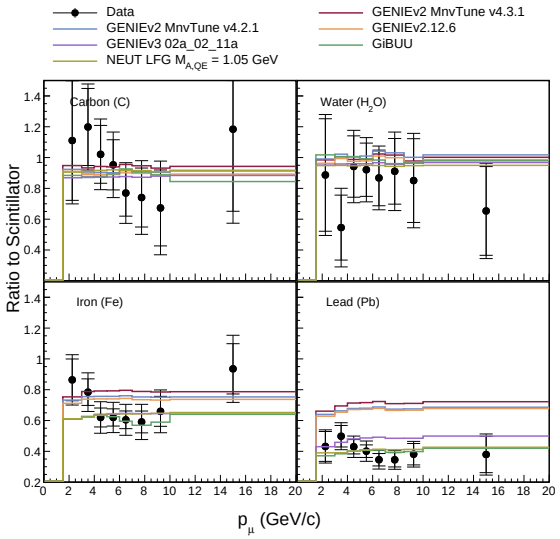



(d) Fractional Uncertainties $\frac{d\sigma}{d\theta_\pi}$.

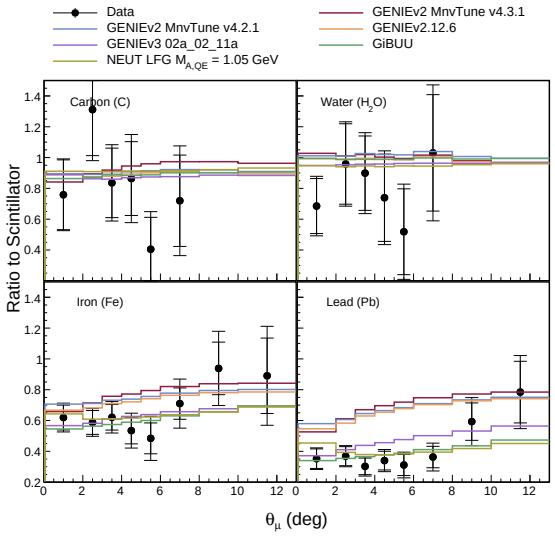
FIG. Supp.11. Systematic uncertainties of differential cross sections for scintillator (top left), carbon (middle left), water (middle right), iron (bottom left) and lead (bottom right)

### H. Cross Section Ratio Plots

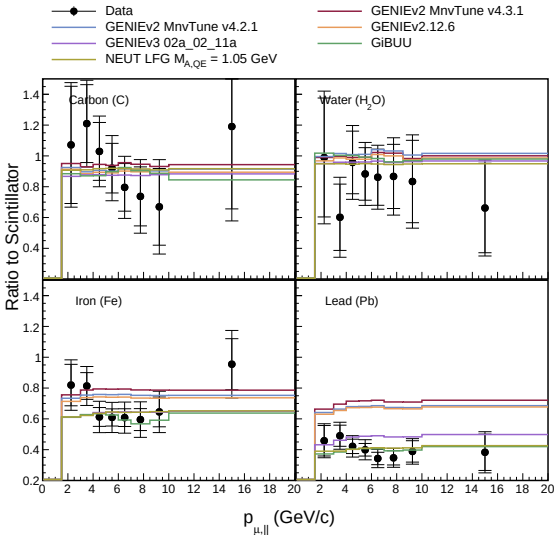
One of the striking features in these comparisons is that there are large difference in the ratios between the GENIEv2 and GENIEv3 models for the iron and lead targets. These are primarily from different the FSI effects in the generators. The increased pion scattering and absorption rates in heavy nuclei in GENIEv3 appear to better describe our data.



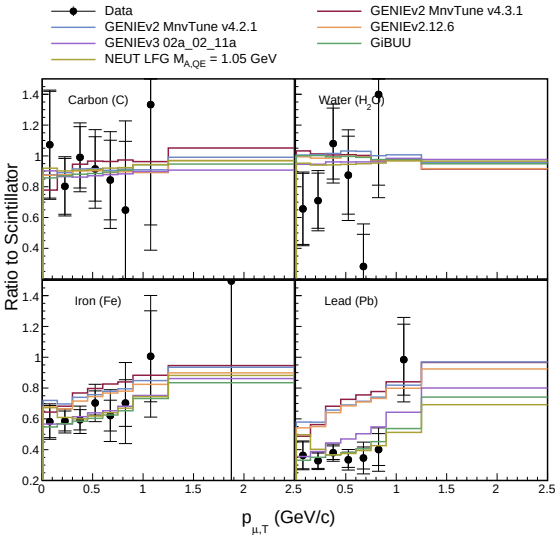

(a) Cross section ratios $\left(\frac{d\sigma_A}{dp_\mu}\right) / \left(\frac{d\sigma_{CH}}{dp_\mu}\right)$ for carbon, water, iron, and lead.




(b) Cross section ratios $\left(\frac{d\sigma_A}{d\theta_\mu}\right) / \left(\frac{d\sigma_{CH}}{d\theta_\mu}\right)$ for carbon, water, iron, and lead.

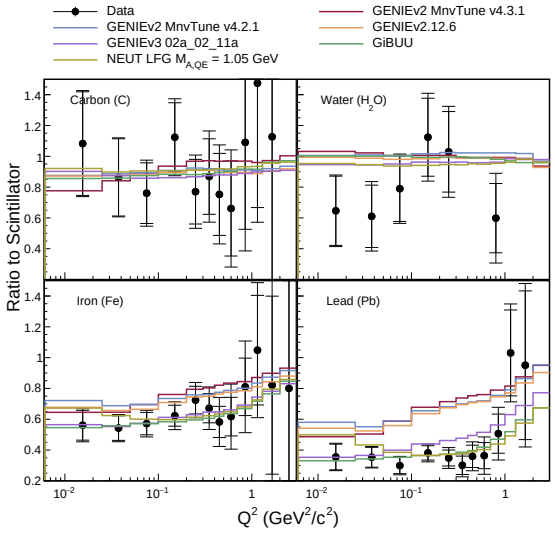



(c) Cross section ratios $\left(\frac{d\sigma_A}{dp_{\mu,||}}\right) / \left(\frac{d\sigma_{CH}}{dp_{\mu,||}}\right)$ for carbon, water, iron, and lead.

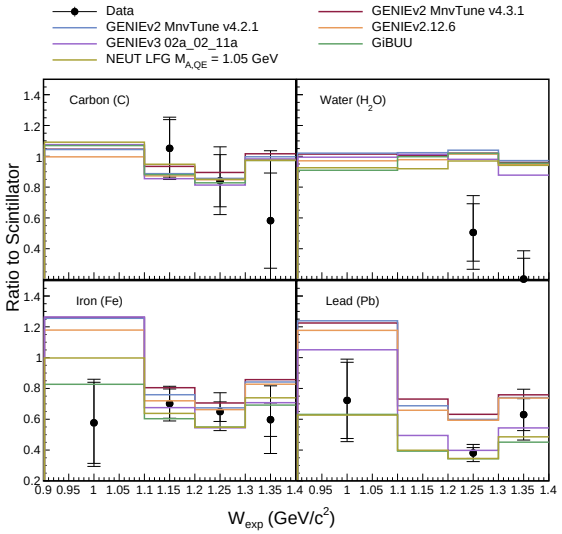



(d) Cross section ratios $\left(\frac{d\sigma_A}{dp_{\mu,T}}\right) / \left(\frac{d\sigma_{CH}}{dp_{\mu,T}}\right)$ for carbon, water, iron, and lead.

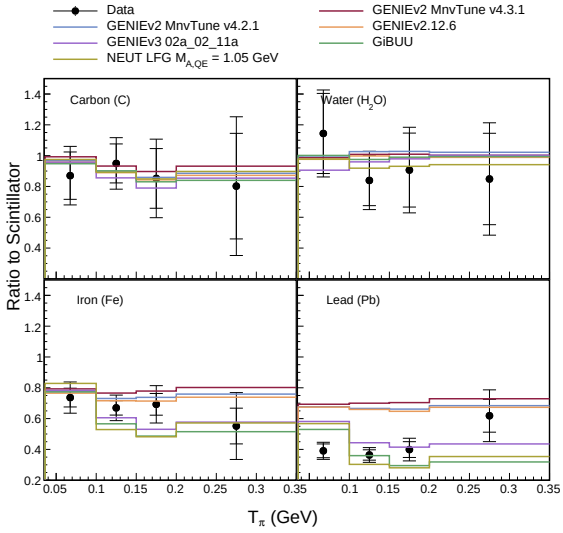
FIG. Supp.12. Cross section ratios versus muon variables.



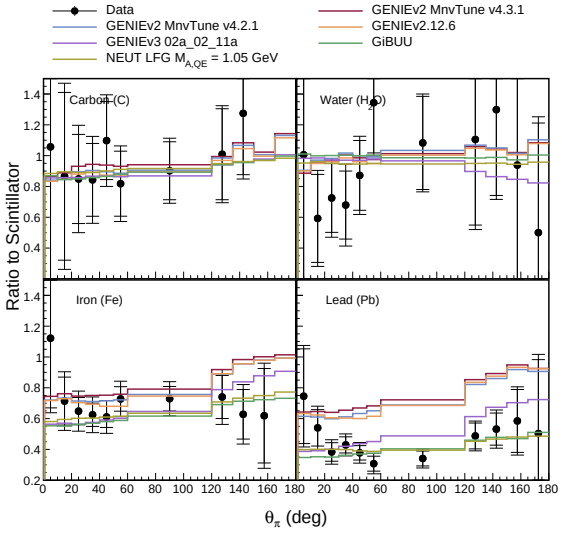

(a) Cross section ratios $\left(\frac{d\sigma_A}{dQ^2}\right) / \left(\frac{d\sigma_{CH}}{dQ^2}\right)$ for carbon, water, iron, and lead.




(b) Cross section ratios $\left(\frac{d\sigma_A}{dW_{exp}}\right) / \left(\frac{d\sigma_{CH}}{dW_{exp}}\right)$ for carbon, water, iron, and lead.



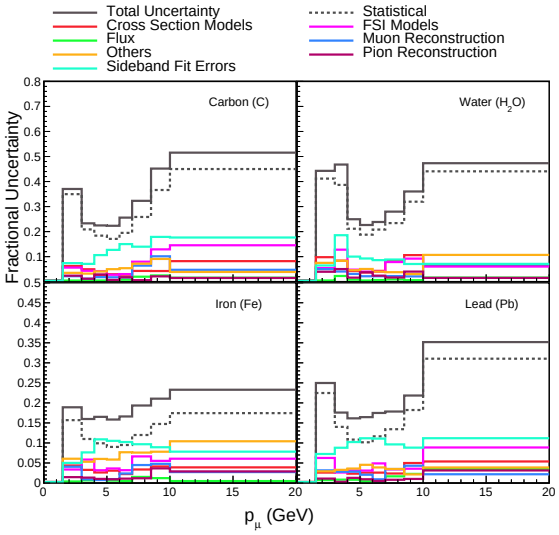

(c) Cross section ratios $\left(\frac{d\sigma_A}{dT_\pi}\right) / \left(\frac{d\sigma_{CH}}{dT_\pi}\right)$ for carbon, water, iron, and lead.



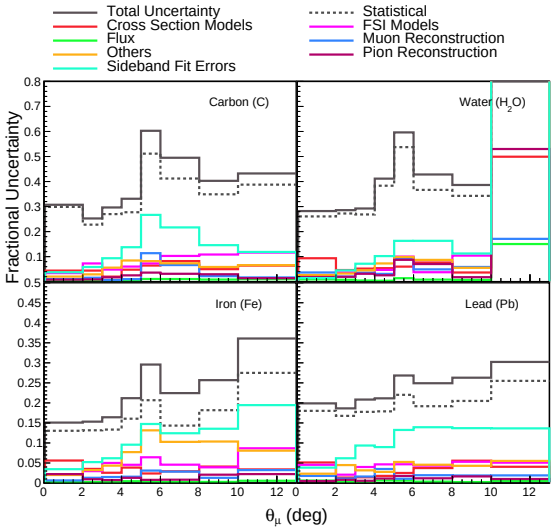

(d) Cross section ratios $\left(\frac{d\sigma_A}{d\theta_\pi}\right) / \left(\frac{d\sigma_{CH}}{d\theta_\pi}\right)$ for carbon, water, iron, and lead.

FIG. Supp.13. Cross section ratios versus other variables.

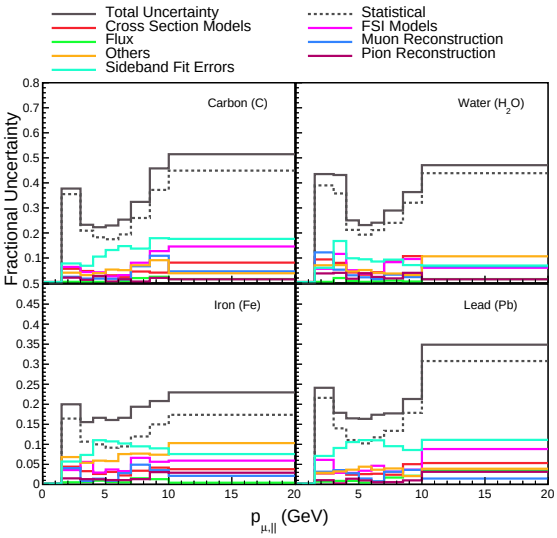
## I. Cross Section Ratio Error Summaries



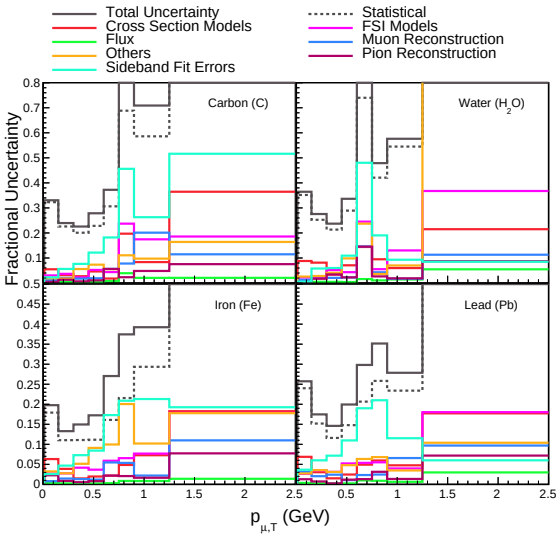

(a) Fractional uncertainties $\left(\frac{d\sigma_A}{dp_\mu}\right) / \left(\frac{d\sigma_{CH}}{dp_\mu}\right)$.




(b) Fractional uncertainties $\left(\frac{d\sigma_A}{d\theta_\mu}\right) / \left(\frac{d\sigma_{CH}}{d\theta_\mu}\right)$.

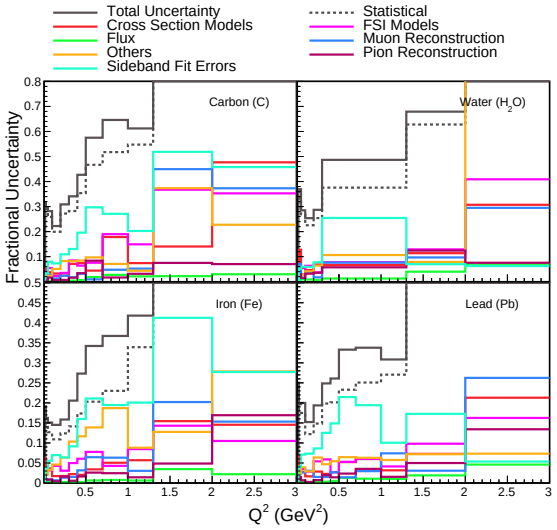



(c) Fractional uncertainties $\left(\frac{d\sigma_A}{dp_{\mu,||}}\right) / \left(\frac{d\sigma_{CH}}{dp_{\mu,||}}\right)$.

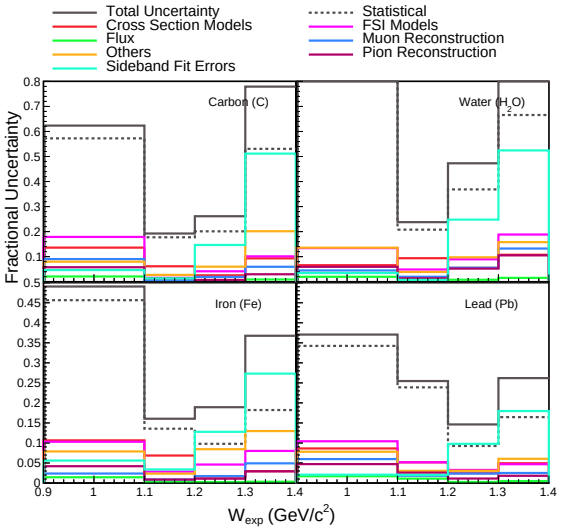



(d) Fractional uncertainties $\left(\frac{d\sigma_A}{dp_{\mu,T}}\right) / \left(\frac{d\sigma_{CH}}{dp_{\mu,T}}\right)$.

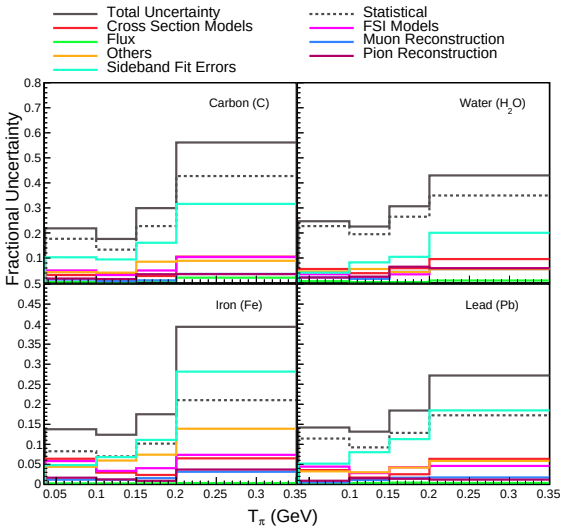
FIG. Supp.14. Systematic uncertainties of cross section ratios for carbon/scintillator (top left), water/scintillator (top right), iron/scintillator (bottom left) and lead/scintillator (bottom right)



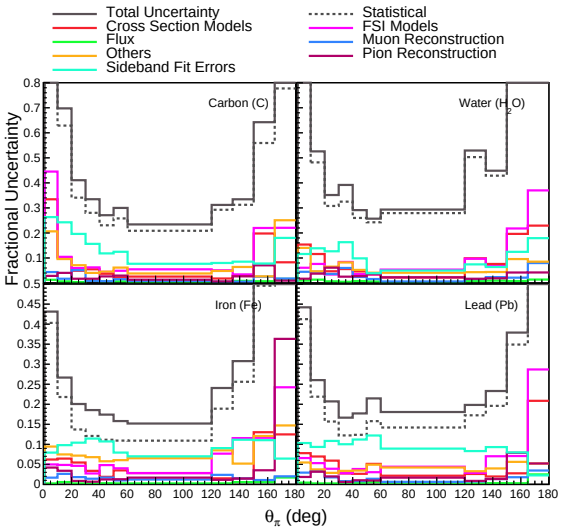

(a) Fractional uncertainties $\left(\frac{d\sigma_A}{dQ^2}\right) / \left(\frac{d\sigma_{CH}}{dQ^2}\right)$.




(b) Fractional uncertainties $\left(\frac{d\sigma_A}{dW_{exp}}\right) / \left(\frac{d\sigma_{CH}}{dW_{exp}}\right)$.




(c) Fractional uncertainties $\left(\frac{d\sigma_A}{dT_\pi}\right) / \left(\frac{d\sigma_{CH}}{dT_\pi}\right)$.




(d) Fractional uncertainties $\left(\frac{d\sigma_A}{d\theta_\pi}\right) / \left(\frac{d\sigma_{CH}}{d\theta_\pi}\right)$.

FIG. Supp.15. Systematic uncertainties of cross section ratios for carbon/scintillator (top left), water/scintillator (top right), iron/scintillator (bottom left) and lead/scintillator (bottom right)

### J. Cross Section Tables

TABLE Supp.V. Measured cross section as function of $p_\mu$ on scintillator, in units of $10^{-42}$ cm$^2$/GeV/c/nucleon, and the absolute and fractional cross section uncertainties

| Bin edges (GeV/c) | $\sigma$ | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 1.5-3.0 | 132.41 | 9.94 | 0.031 | 0.068 | 0.037 |
| 3.0-4.0 | 339.29 | 25.45 | 0.022 | 0.072 | 0.037 |
| 4.0-5.0 | 492.23 | 37.09 | 0.019 | 0.073 | 0.037 |
| 5.0-6.0 | 521.41 | 39.22 | 0.017 | 0.073 | 0.037 |
| 6.0-7.0 | 393.48 | 30.19 | 0.018 | 0.074 | 0.036 |
| 7.0-8.5 | 177.43 | 14.31 | 0.022 | 0.078 | 0.035 |
| 8.5-10.0 | 53.66 | 4.89 | 0.039 | 0.082 | 0.033 |
| 10.0-20.0 | 9.51 | 0.90 | 0.046 | 0.083 | 0.035 |

TABLE Supp.VI. Statistical covariance matrix of measured cross section as function of $p_\mu$ on scintillator, in units of $10^{-84}$ (cm$^2$/GeV/c/nucleon)$^2$

| Bin edges (GeV/c) | 1.5-3.0 | 3.0-4.0 | 4.0-5.0 | 5.0-6.0 | 6.0-7.0 | 7.0-8.5 | 8.5-10.0 | 10.0-20.0 |
|---|---|---|---|---|---|---|---|---|
| 1.5-3.0 | 16.658 | 4.105 | 0.935 | 0.338 | 0.170 | 0.049 | 0.020 | 0.002 |
| 3.0-4.0 | 4.105 | 57.479 | 12.979 | 3.077 | 0.916 | 0.225 | 0.043 | 0.006 |
| 4.0-5.0 | 0.935 | 12.979 | 83.056 | 19.009 | 4.249 | 0.707 | 0.145 | 0.016 |
| 5.0-6.0 | 0.338 | 3.077 | 19.009 | 76.961 | 17.936 | 2.974 | 0.414 | 0.049 |
| 6.0-7.0 | 0.170 | 0.916 | 4.249 | 17.936 | 52.187 | 8.527 | 1.138 | 0.061 |
| 7.0-8.5 | 0.049 | 0.225 | 0.707 | 2.974 | 8.527 | 15.067 | 1.952 | 0.104 |
| 8.5-10.0 | 0.020 | 0.043 | 0.145 | 0.414 | 1.138 | 1.952 | 4.444 | 0.153 |
| 10.0-20.0 | 0.002 | 0.006 | 0.016 | 0.049 | 0.061 | 0.104 | 0.153 | 0.190 |

TABLE Supp.VII. Systematic covariance matrix of measured cross section in $p_\mu$ on scintillator, in units of $10^{-84}$ $(\mathrm{cm}^2/\mathrm{GeV/c/nucleon})^2$

| Bin edges (GeV/c) | 1.5-3.0 | 3.0-4.0 | 4.0-5.0 | 5.0-6.0 | 6.0-7.0 | 7.0-8.5 | 8.5-10.0 | 10.0-20.0 |
|---|---|---|---|---|---|---|---|---|
| 1.5-3.0 | 82.125 | 213.956 | 314.097 | 329.205 | 240.409 | 106.454 | 32.919 | 5.696 |
| 3.0-4.0 | 213.956 | 590.438 | 854.841 | 906.030 | 662.006 | 293.496 | 94.406 | 15.869 |
| 4.0-5.0 | 314.097 | 854.841 | 1292.343 | 1361.955 | 1020.144 | 458.089 | 144.301 | 24.963 |
| 5.0-6.0 | 329.205 | 906.030 | 1361.955 | 1461.454 | 1099.429 | 498.336 | 156.016 | 26.910 |
| 6.0-7.0 | 240.409 | 662.006 | 1020.144 | 1099.429 | 859.179 | 394.495 | 122.519 | 21.348 |
| 7.0-8.5 | 106.454 | 293.496 | 458.089 | 498.336 | 394.495 | 189.709 | 57.596 | 10.355 |
| 8.5-10.0 | 32.919 | 94.406 | 144.301 | 156.016 | 122.519 | 57.596 | 19.425 | 3.242 |
| 10.0-20.0 | 5.696 | 15.869 | 24.963 | 26.910 | 21.348 | 10.355 | 3.242 | 0.618 |

TABLE Supp.VIII. Measured cross section as function of $\theta_\mu$ on scintillator, in units of $10^{-42}$ $\mathrm{cm}^2$/degree/nucleon, and the absolute and fractional cross section uncertainties

| Bin edges (degree) | $\sigma$ | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 0-2 | 93.64 | 6.19 | 0.023 | 0.062 | 0.033 |
| 2-3 | 226.67 | 15.25 | 0.025 | 0.063 | 0.035 |
| 3-4 | 276.53 | 18.89 | 0.025 | 0.063 | 0.035 |
| 4-5 | 278.12 | 22.02 | 0.028 | 0.074 | 0.036 |
| 5-6 | 262.12 | 25.46 | 0.031 | 0.092 | 0.038 |
| 6-8 | 232.61 | 21.04 | 0.027 | 0.086 | 0.038 |
| 8-10 | 167.48 | 16.70 | 0.038 | 0.092 | 0.041 |
| 10-13 | 114.14 | 13.91 | 0.047 | 0.112 | 0.046 |

TABLE Supp.IX. Statistical covariance matrix of measured cross section as function of $\theta_\mu$ on scintillator, in units of $10^{-84}$ (cm$^2$/degree/nucleon)$^2$

| Bin edges (degree) | 0-2 | 2-3 | 3-4 | 4-5 | 5-6 | 6-8 | 8-10 | 10-13 |
|---|---|---|---|---|---|---|---|---|
| 0-2 | 4.693 | -0.300 | -0.494 | -0.035 | -0.001 | -0.002 | -0.002 | -0.002 |
| 2-3 | -0.300 | 31.566 | 0.489 | -1.641 | -0.173 | -0.013 | -0.015 | -0.013 |
| 3-4 | -0.494 | 0.489 | 49.367 | 1.222 | -2.542 | -0.137 | 0.002 | -0.004 |
| 4-5 | -0.035 | -1.641 | 1.222 | 60.451 | 2.522 | -1.709 | -0.064 | 0.008 |
| 5-6 | -0.001 | -0.173 | -2.542 | 2.522 | 68.064 | -0.152 | -1.074 | -0.021 |
| 6-8 | -0.002 | -0.013 | -0.137 | -1.709 | -0.152 | 39.575 | -0.871 | -0.601 |
| 8-10 | -0.002 | -0.015 | 0.002 | -0.064 | -1.074 | -0.871 | 39.542 | -1.187 |
| 10-13 | -0.002 | -0.013 | -0.004 | 0.008 | -0.021 | -0.601 | -1.187 | 28.649 |

TABLE Supp.X. Systematic covariance matrix of measured cross section in $\theta_\mu$ on scintillator, in units of $10^{-84}$ (cm$^2$/degree/nucleon)$^2$

| Bin edges (degree) | 0-2 | 2-3 | 3-4 | 4-5 | 5-6 | 6-8 | 8-10 | 10-13 |
|---|---|---|---|---|---|---|---|---|
| 0-2 | 33.684 | 78.058 | 87.540 | 87.329 | 86.942 | 68.156 | 50.842 | 37.409 |
| 2-3 | 78.058 | 200.937 | 237.301 | 255.413 | 267.730 | 210.228 | 154.423 | 119.171 |
| 3-4 | 87.540 | 237.301 | 307.318 | 345.850 | 375.759 | 306.688 | 229.018 | 179.044 |
| 4-5 | 87.329 | 255.413 | 345.850 | 424.408 | 479.626 | 386.851 | 290.976 | 229.426 |
| 5-6 | 86.942 | 267.730 | 375.759 | 479.626 | 580.386 | 461.821 | 343.697 | 276.712 |
| 6-8 | 68.156 | 210.228 | 306.688 | 386.851 | 461.821 | 403.285 | 300.163 | 244.821 |
| 8-10 | 50.842 | 154.423 | 229.018 | 290.976 | 343.697 | 300.163 | 239.503 | 193.833 |
| 10-13 | 37.409 | 119.171 | 179.044 | 229.426 | 276.712 | 244.821 | 193.833 | 164.717 |

TABLE Supp.XI. Measured cross section as function of $p_{\mu,||}$ on scintillator, in units of $10^{-42}$ cm$^2$/GeV/c/nucleon, and the absolute and fractional cross section uncertainties

| Bin edges (GeV/c) | $\sigma$ | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 1.5-3.0 | 137.27 | 10.42 | 0.030 | 0.070 | 0.037 |
| 3.0-4.0 | 347.62 | 25.93 | 0.022 | 0.071 | 0.037 |
| 4.0-5.0 | 494.41 | 37.52 | 0.018 | 0.074 | 0.037 |
| 5.0-6.0 | 518.20 | 38.78 | 0.017 | 0.073 | 0.037 |
| 6.0-7.0 | 386.60 | 29.60 | 0.018 | 0.074 | 0.036 |
| 7.0-8.5 | 173.45 | 14.06 | 0.022 | 0.078 | 0.035 |
| 8.5-10.0 | 52.71 | 4.76 | 0.040 | 0.081 | 0.033 |
| 10.0-20.0 | 9.44 | 0.89 | 0.046 | 0.082 | 0.035 |

TABLE Supp.XII. Statistical covariance matrix of measured cross section as function of $p_{\mu,||}$ on scintillator, in units of $10^{-84}$ (cm$^2$/GeV/c/nucleon)$^2$

| Bin edges (GeV/c) | 1.5-3.0 | 3.0-4.0 | 4.0-5.0 | 5.0-6.0 | 6.0-7.0 | 7.0-8.5 | 8.5-10.0 | 10.0-20.0 |
|---|---|---|---|---|---|---|---|---|
| 1.5-3.0 | 17.281 | 4.316 | 0.981 | 0.386 | 0.175 | 0.047 | 0.019 | 0.002 |
| 3.0-4.0 | 4.316 | 59.783 | 13.377 | 3.199 | 0.890 | 0.225 | 0.042 | 0.006 |
| 4.0-5.0 | 0.981 | 13.377 | 83.368 | 19.478 | 4.159 | 0.688 | 0.142 | 0.016 |
| 5.0-6.0 | 0.386 | 3.199 | 19.478 | 76.300 | 17.415 | 2.887 | 0.399 | 0.049 |
| 6.0-7.0 | 0.175 | 0.890 | 4.159 | 17.415 | 50.714 | 8.346 | 1.129 | 0.062 |
| 7.0-8.5 | 0.047 | 0.225 | 0.688 | 2.887 | 8.346 | 14.498 | 1.871 | 0.101 |
| 8.5-10.0 | 0.019 | 0.042 | 0.142 | 0.399 | 1.129 | 1.871 | 4.343 | 0.147 |
| 10.0-20.0 | 0.002 | 0.006 | 0.016 | 0.049 | 0.062 | 0.101 | 0.147 | 0.188 |

TABLE Supp.XIII. Systematic covariance matrix of measured cross section in $p_{\mu,||}$ on scintillator, in units of $10^{-84}$ (cm$^2$/GeV/c/nucleon)$^2$

| Bin edges (GeV/c) | 1.5-3.0 | 3.0-4.0 | 4.0-5.0 | 5.0-6.0 | 6.0-7.0 | 7.0-8.5 | 8.5-10.0 | 10.0-20.0 |
|---|---|---|---|---|---|---|---|---|
| 1.5-3.0 | 91.226 | 229.520 | 338.349 | 347.709 | 251.637 | 112.813 | 34.549 | 6.067 |
| 3.0-4.0 | 229.520 | 612.679 | 877.172 | 909.191 | 657.101 | 291.505 | 93.008 | 15.804 |
| 4.0-5.0 | 338.349 | 877.172 | 1324.084 | 1362.526 | 1012.135 | 457.507 | 142.290 | 25.029 |
| 5.0-6.0 | 347.709 | 909.191 | 1362.526 | 1427.562 | 1062.163 | 483.211 | 149.666 | 26.138 |
| 6.0-7.0 | 251.637 | 657.101 | 1012.135 | 1062.163 | 825.422 | 380.578 | 117.151 | 20.653 |
| 7.0-8.5 | 112.813 | 291.505 | 457.507 | 483.211 | 380.578 | 183.242 | 55.337 | 10.013 |
| 8.5-10.0 | 34.549 | 93.008 | 142.290 | 149.666 | 117.151 | 55.337 | 18.344 | 3.110 |
| 10.0-20.0 | 6.067 | 15.804 | 25.029 | 26.138 | 20.653 | 10.013 | 3.110 | 0.598 |

TABLE Supp.XIV. Measured cross section as function of $p_{\mu,T}$ on scintillator, in units of $10^{-42}$ cm$^2$/GeV/c/nucleon, and the absolute and fractional cross section uncertainties

| Bin edges (GeV/c) | $\sigma$ | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 0.00-0.15 | 646.46 | 44.23 | 0.030 | 0.062 | 0.033 |
| 0.15-0.30 | 2212.06 | 141.95 | 0.019 | 0.061 | 0.033 |
| 0.30-0.45 | 3421.09 | 240.61 | 0.019 | 0.068 | 0.035 |
| 0.45-0.60 | 3545.67 | 308.07 | 0.022 | 0.084 | 0.036 |
| 0.60-0.75 | 2728.36 | 278.16 | 0.028 | 0.098 | 0.037 |
| 0.75-0.90 | 1665.26 | 200.09 | 0.041 | 0.113 | 0.041 |
| 0.90-1.25 | 559.10 | 97.31 | 0.065 | 0.161 | 0.045 |
| 1.25-2.50 | 19.99 | 5.98 | 0.221 | 0.202 | 0.045 |

TABLE Supp.XV. Statistical covariance matrix of measured cross section as function of $p_{\mu,T}$ on scintillator, in units of $10^{-84}$ $(\mathrm{cm}^2/\mathrm{GeV/c/nucleon})^2$

| Bin edges (GeV/c) | 0.00-0.15 | 0.15-0.30 | 0.30-0.45 | 0.45-0.60 | 0.60-0.75 | 0.75-0.90 | 0.90-1.25 | 1.25-2.50 |
|---|---|---|---|---|---|---|---|---|
| 0.00-0.15 | 375.058 | -5.551 | -47.201 | -7.308 | -0.821 | -0.902 | -0.344 | -0.030 |
| 0.15-0.30 | -5.551 | 1821.647 | 31.115 | -156.758 | -39.058 | -9.338 | -2.217 | -0.043 |
| 0.30-0.45 | -47.201 | 31.115 | 4348.247 | 269.040 | -313.708 | -92.462 | -17.984 | -0.914 |
| 0.45-0.60 | -7.308 | -156.758 | 269.040 | 5947.956 | 700.721 | -342.839 | -78.672 | -2.713 |
| 0.60-0.75 | -0.821 | -39.058 | -313.708 | 700.721 | 5878.758 | 1013.662 | -145.751 | -2.860 |
| 0.75-0.90 | -0.902 | -9.338 | -92.462 | -342.839 | 1013.662 | 4679.844 | 465.915 | -9.180 |
| 0.90-1.25 | -0.344 | -2.217 | -17.984 | -78.672 | -145.751 | 465.915 | 1335.605 | 34.903 |
| 1.25-2.50 | -0.030 | -0.043 | -0.914 | -2.713 | -2.860 | -9.180 | 34.903 | 19.497 |

TABLE Supp.XVI. Systematic covariance matrix of measured cross section in $p_{\mu,T}$ on scintillator, in units of $10^{-84}$ (cm$^2$/GeV/c/nucleon)$^2$

| Bin edges (GeV/c) | 0.00-0.15 | 0.15-0.30 | 0.30-0.45 | 0.45-0.60 | 0.60-0.75 | 0.75-0.90 | 0.90-1.25 | 1.25-2.50 |
|---|---|---|---|---|---|---|---|---|
| 0.00-0.15 | 1581.186 | 5096.119 | 6401.031 | 6050.531 | 4214.151 | 2732.787 | 955.250 | 36.236 |
| 0.15-0.30 | 5096.119 | 18327.195 | 26728.571 | 28116.824 | 20742.179 | 13158.005 | 4370.207 | 161.226 |
| 0.30-0.45 | 6401.031 | 26728.571 | 53542.567 | 65838.516 | 54164.519 | 34609.724 | 11743.643 | 442.813 |
| 0.45-0.60 | 6050.531 | 28116.824 | 65838.516 | 88957.669 | 77342.326 | 50764.557 | 18249.458 | 689.458 |
| 0.60-0.75 | 4214.151 | 20742.179 | 54164.519 | 77342.326 | 71495.961 | 48206.936 | 18687.460 | 724.443 |
| 0.75-0.90 | 2732.787 | 13158.005 | 34609.724 | 50764.557 | 48206.936 | 35354.928 | 15091.296 | 591.143 |
| 0.90-1.25 | 955.250 | 4370.207 | 11743.643 | 18249.458 | 18687.460 | 15091.296 | 8133.526 | 341.419 |
| 1.25-2.50 | 36.236 | 161.226 | 442.813 | 689.458 | 724.443 | 591.143 | 341.419 | 16.250 |

TABLE Supp.XVII. Measured cross section as function of $Q^2$ on scintillator, in units of $10^{-42}$ $\mathrm{cm}^2/\mathrm{GeV}^2/c^2$/nucleon, and the absolute and fractional cross section uncertainties

| Bin edges ($\mathrm{GeV}^2/c^2$) | $\sigma$ | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 0.000-0.025 | 3825.00 | 261.09 | 0.029 | 0.062 | 0.033 |
| 0.025-0.050 | 4428.16 | 286.72 | 0.028 | 0.059 | 0.033 |
| 0.050-0.100 | 4535.82 | 301.52 | 0.023 | 0.062 | 0.034 |
| 0.100-0.200 | 4059.36 | 285.14 | 0.021 | 0.067 | 0.035 |
| 0.200-0.300 | 3452.35 | 286.17 | 0.024 | 0.079 | 0.036 |
| 0.300-0.400 | 2733.16 | 247.43 | 0.028 | 0.086 | 0.036 |
| 0.400-0.500 | 2103.42 | 214.43 | 0.033 | 0.096 | 0.037 |
| 0.500-0.700 | 1412.01 | 153.16 | 0.034 | 0.103 | 0.038 |
| 0.700-1.000 | 718.94 | 89.74 | 0.046 | 0.116 | 0.041 |
| 1.000-1.300 | 331.82 | 54.29 | 0.071 | 0.147 | 0.044 |
| 1.300-2.000 | 90.74 | 24.36 | 0.137 | 0.231 | 0.051 |
| 2.000-3.000 | 21.03 | 7.04 | 0.278 | 0.186 | 0.039 |

TABLE Supp.XVIII. Statistical covariance matrix of measured cross section as function of $Q^2$ on scintillator, in units of $10^{-84}$ $(\mathrm{cm}^2/\mathrm{GeV}^2/c^2/\mathrm{nucleon})^2$

| Bin edges ($\mathrm{GeV}^2/c^2$) | 0.000-0.025 | 0.025-0.050 | 0.050-0.100 | 0.100-0.200 | 0.200-0.300 | 0.300-0.400 | 0.400-0.500 | 0.500-0.700 | 0.700-1.000 | 1.000-1.300 | 1.300-2.000 | 2.000-3.000 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.000-0.025 | 12588.365 | 997.907 | -805.851 | -175.415 | -12.014 | -3.840 | -2.954 | -1.013 | -1.935 | -0.723 | -0.531 | -0.175 |
| 0.025-0.050 | 997.907 | 14929.823 | 1929.772 | -589.284 | -214.404 | -36.350 | -8.937 | -5.562 | -3.890 | -1.779 | -0.493 | -0.030 |
| 0.050-0.100 | -805.851 | 1929.772 | 10965.271 | 674.926 | -508.532 | -190.296 | -62.976 | -16.436 | -3.694 | -3.023 | -1.259 | -0.104 |
| 0.100-0.200 | -175.415 | -589.284 | 674.926 | 7118.253 | 814.386 | -378.057 | -250.650 | -76.736 | -19.479 | -5.880 | -2.364 | -0.485 |
| 0.200-0.300 | -12.014 | -214.404 | -508.532 | 814.386 | 6771.264 | 1416.831 | -132.953 | -214.847 | -56.635 | -19.106 | -2.782 | -1.670 |
| 0.300-0.400 | -3.840 | -36.350 | -190.296 | -378.057 | 1416.831 | 5865.008 | 1736.357 | -24.934 | -128.954 | -45.772 | -7.149 | -2.457 |
| 0.400-0.500 | -2.954 | -8.937 | -62.976 | -250.650 | -132.953 | 1736.357 | 4823.638 | 887.183 | -77.544 | -55.772 | -11.482 | 0.465 |
| 0.500-0.700 | -1.013 | -5.562 | -16.436 | -76.736 | -214.847 | -24.934 | 887.183 | 2338.753 | 352.481 | -49.685 | -20.726 | -1.956 |
| 0.700-1.000 | -1.935 | -3.890 | -3.694 | -19.479 | -56.635 | -128.954 | -77.544 | 352.481 | 1092.546 | 239.344 | 3.234 | -4.504 |
| 1.000-1.300 | -0.723 | -1.779 | -3.023 | -5.880 | -19.106 | -45.772 | -55.772 | -49.685 | 239.344 | 562.636 | 91.453 | 3.066 |
| 1.300-2.000 | -0.531 | -0.493 | -1.259 | -2.364 | -2.782 | -7.149 | -11.482 | -20.726 | 3.234 | 91.453 | 153.493 | 11.032 |
| 2.000-3.000 | -0.175 | -0.030 | -0.104 | -0.485 | -1.670 | -2.457 | 0.465 | -1.956 | -4.504 | 3.066 | 11.032 | 34.274 |

TABLE Supp.XIX. Systematic covariance matrix of measured cross section in $Q^2$ on scintillator, in units of $10^{-84}$ (cm$^2$/GeV$^2$/$c^2$/nucleon)$^2$

| Bin edges (GeV$^2$/$c^2$) | 0.000-0.025 | 0.025-0.050 | 0.050-0.100 | 0.100-0.200 | 0.200-0.300 | 0.300-0.400 | 0.400-0.500 | 0.500-0.700 | 0.700-1.000 | 1.000-1.300 | 1.300-2.000 | 2.000-3.000 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.000-0.025 | 55578.445 | 58860.557 | 60378.311 | 45220.647 | 34618.319 | 26584.728 | 20025.404 | 12986.738 | 6460.467 | 3981.279 | 829.538 | 205.817 |
| 0.025-0.050 | 58860.557 | 67278.874 | 71209.690 | 60105.923 | 50535.710 | 39818.233 | 31808.493 | 20735.959 | 10139.678 | 5164.600 | 1487.729 | 261.792 |
| 0.050-0.100 | 60378.311 | 71209.690 | 79947.393 | 69924.610 | 60147.726 | 47554.008 | 38176.351 | 25061.644 | 12064.101 | 6143.531 | 1774.373 | 308.808 |
| 0.100-0.200 | 45220.647 | 60105.923 | 69924.610 | 74185.188 | 71691.029 | 59199.546 | 49384.624 | 33309.494 | 16489.765 | 7198.661 | 2728.158 | 399.253 |
| 0.200-0.300 | 34618.319 | 50535.710 | 60147.726 | 71691.029 | 75124.673 | 63525.200 | 54225.183 | 37359.079 | 19135.220 | 8013.084 | 3474.311 | 468.633 |
| 0.300-0.400 | 26584.728 | 39818.233 | 47554.008 | 59199.546 | 63525.200 | 55354.698 | 46951.905 | 33113.395 | 17206.412 | 7264.286 | 3260.826 | 458.856 |
| 0.400-0.500 | 20025.404 | 31808.493 | 38176.351 | 49384.624 | 54225.183 | 46951.905 | 41157.215 | 28869.769 | 15207.015 | 6487.562 | 2953.839 | 418.405 |
| 0.500-0.700 | 12986.738 | 20735.959 | 25061.644 | 33309.494 | 37359.079 | 33113.395 | 28869.769 | 21118.541 | 11534.464 | 5238.876 | 2309.615 | 338.961 |
| 0.700-1.000 | 6460.467 | 10139.678 | 12064.101 | 16489.765 | 19135.220 | 17206.412 | 15207.015 | 11534.464 | 6960.978 | 3570.619 | 1531.486 | 228.184 |
| 1.000-1.300 | 3981.279 | 5164.600 | 6143.531 | 7198.661 | 8013.084 | 7264.286 | 6487.562 | 5238.876 | 3570.619 | 2385.019 | 913.026 | 154.129 |
| 1.300-2.000 | 829.538 | 1487.729 | 1774.373 | 2728.158 | 3474.311 | 3260.826 | 2953.839 | 2309.615 | 1531.486 | 913.026 | 439.729 | 62.254 |
| 2.000-3.000 | 205.817 | 261.792 | 308.808 | 399.253 | 468.633 | 458.856 | 418.405 | 338.961 | 228.184 | 154.129 | 62.254 | 15.245 |

TABLE Supp.XX. Measured cross section as function of $W_{exp}$ on scintillator, in units of $10^{-42}$ $cm^2/GeV/c^2$/nucleon, and the absolute and fractional cross section uncertainties

| Bin edges ($GeV/c^2$) | $\sigma$ | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 0.9-1.1 | 380.21 | 89.00 | 0.100 | 0.212 | 0.059 |
| 1.1-1.2 | 4969.82 | 394.14 | 0.026 | 0.075 | 0.038 |
| 1.2-1.3 | 10893.98 | 791.75 | 0.015 | 0.071 | 0.034 |
| 1.3-1.4 | 7534.12 | 786.69 | 0.031 | 0.100 | 0.033 |

TABLE Supp.XXI. Statistical covariance matrix of measured cross section as function of $W_{exp}$ on scintillator, in units of $10^{-84}$ $(cm^2/GeV/c^2/nucleon)^2$

| Bin edges ($GeV/c^2$) | 0.9-1.1 | 1.1-1.2 | 1.2-1.3 | 1.3-1.4 |
|---|---|---|---|---|
| 0.9-1.1 | 1434.912 | 298.236 | -1959.309 | -15.500 |
| 1.1-1.2 | 298.236 | 16108.522 | -6199.017 | -16343.752 |
| 1.2-1.3 | -1959.309 | -6199.017 | 28235.499 | 17223.717 |
| 1.3-1.4 | -15.500 | -16343.752 | 17223.717 | 53971.062 |

TABLE Supp.XXII. Systematic covariance matrix of measured cross section in $W_{exp}$ on scintillator, in units of $10^{-84}$ $(cm^2/GeV/c^2/nucleon)^2$

| Bin edges ($GeV/c^2$) | 0.9-1.1 | 1.1-1.2 | 1.2-1.3 | 1.3-1.4 |
|---|---|---|---|---|
| 0.9-1.1 | 6486.707 | 15615.022 | 35973.498 | 36397.745 |
| 1.1-1.2 | 15615.022 | 139234.924 | 211164.417 | 136341.941 |
| 1.2-1.3 | 35973.498 | 211164.417 | 598624.922 | 523114.324 |
| 1.3-1.4 | 36397.745 | 136341.941 | 523114.324 | 564914.576 |

TABLE Supp.XXIII. Measured cross section as function of $T_\pi$ on scintillator, in units of $10^{-42}$ cm$^2$/GeV/nucleon, and the absolute and fractional cross section uncertainties

| Bin edges (GeV) | $\sigma$ | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 0.035-0.100 | 13123.56 | 1025.94 | 0.016 | 0.076 | 0.037 |
| 0.100-0.150 | 9902.55 | 655.87 | 0.015 | 0.064 | 0.035 |
| 0.150-0.200 | 6746.23 | 521.56 | 0.023 | 0.074 | 0.035 |
| 0.200-0.350 | 3574.69 | 367.65 | 0.032 | 0.098 | 0.037 |

TABLE Supp.XXIV. Statistical covariance matrix of measured cross section as function of $T_\pi$ on scintillator, in units of $10^{-84}$ (cm$^2$/GeV/nucleon)$^2$

| Bin edges (GeV) | 0.035-0.100 | 0.100-0.150 | 0.150-0.200 | 0.200-0.350 |
|---|---|---|---|---|
| 0.035-0.100 | 46118.291 | 870.889 | -2121.179 | -1189.136 |
| 0.100-0.150 | 870.889 | 23295.488 | 2478.311 | -458.209 |
| 0.150-0.200 | -2121.179 | 2478.311 | 24009.621 | 3296.240 |
| 0.200-0.350 | -1189.136 | -458.209 | 3296.240 | 13381.849 |

TABLE Supp.XXV. Systematic covariance matrix of measured cross section in $T_\pi$ on scintillator, in units of $10^{-84}$ (cm$^2$/GeV/nucleon)$^2$

| Bin edges (GeV) | 0.035-0.100 | 0.100-0.150 | 0.150-0.200 | 0.200-0.350 |
|---|---|---|---|---|
| 0.035-0.100 | 1006425.993 | 600004.017 | 464834.038 | 298853.945 |
| 0.100-0.150 | 600004.017 | 406873.221 | 299782.750 | 190420.988 |
| 0.150-0.200 | 464834.038 | 299782.750 | 248015.149 | 165499.617 |
| 0.200-0.350 | 298853.945 | 190420.988 | 165499.617 | 121786.467 |

TABLE Supp.XXVI. Measured cross section as function of $\theta_\pi$ on scintillator, in units of $10^{-42}$ cm$^2$/degree/nucleon, and the absolute and fractional cross section uncertainties

| Bin edges (degree) | $\sigma$ | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 0-10 | 3.46 | 0.45 | 0.079 | 0.102 | 0.039 |
| 10-20 | 10.35 | 1.05 | 0.041 | 0.092 | 0.037 |
| 20-30 | 15.83 | 1.50 | 0.032 | 0.089 | 0.036 |
| 30-40 | 19.13 | 1.65 | 0.027 | 0.082 | 0.037 |
| 40-50 | 21.54 | 1.81 | 0.026 | 0.080 | 0.038 |
| 50-60 | 23.20 | 1.83 | 0.026 | 0.074 | 0.037 |
| 60-120 | 19.20 | 1.53 | 0.019 | 0.077 | 0.035 |
| 120-135 | 9.30 | 0.69 | 0.027 | 0.069 | 0.033 |
| 135-150 | 6.21 | 0.46 | 0.031 | 0.068 | 0.034 |
| 150-165 | 3.20 | 0.30 | 0.049 | 0.079 | 0.035 |
| 165-180 | 0.78 | 0.10 | 0.110 | 0.074 | 0.037 |

TABLE Supp.XXVII. Statistical covariance matrix of measured cross section as function of $\theta_\pi$ on scintillator, in units of $10^{-84}$ $(\mathrm{cm}^2/\mathrm{degree/nucleon})^2$

| Bin edges (degree) | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 | 60-120 | 120-135 | 135-150 | 150-165 | 165-180 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0-10 | 0.074 | 0.022 | -0.002 | -0.003 | -0.001 | -0.001 | -0.000 | -0.001 | -0.000 | -0.000 | 0.005 |
| 10-20 | 0.022 | 0.183 | 0.029 | -0.007 | -0.004 | -0.004 | -0.003 | -0.002 | -0.002 | 0.002 | 0.005 |
| 20-30 | -0.002 | 0.029 | 0.250 | 0.034 | -0.009 | -0.006 | -0.003 | -0.004 | -0.003 | 0.005 | -0.000 |
| 30-40 | -0.003 | -0.007 | 0.034 | 0.276 | 0.040 | -0.011 | -0.005 | -0.006 | 0.002 | 0.002 | -0.000 |
| 40-50 | -0.001 | -0.004 | -0.009 | 0.040 | 0.323 | 0.044 | -0.006 | -0.004 | 0.002 | -0.003 | -0.001 |
| 50-60 | -0.001 | -0.004 | -0.006 | -0.011 | 0.044 | 0.376 | 0.020 | 0.002 | -0.006 | -0.002 | -0.000 |
| 60-120 | -0.000 | -0.003 | -0.003 | -0.005 | -0.006 | 0.020 | 0.133 | 0.007 | -0.002 | 0.000 | 0.000 |
| 120-135 | -0.001 | -0.002 | -0.004 | -0.006 | -0.004 | 0.002 | 0.007 | 0.064 | 0.009 | -0.001 | 0.000 |
| 135-150 | -0.000 | -0.002 | -0.003 | 0.002 | 0.002 | -0.006 | -0.002 | 0.009 | 0.036 | 0.008 | 0.001 |
| 150-165 | -0.000 | 0.002 | 0.005 | 0.002 | -0.003 | -0.002 | 0.000 | -0.001 | 0.008 | 0.024 | 0.001 |
| 165-180 | 0.005 | 0.005 | -0.000 | -0.000 | -0.001 | -0.000 | 0.000 | 0.000 | 0.001 | 0.001 | 0.007 |

TABLE Supp.XXVIII. Systematic covariance matrix of measured cross section in $\theta_\pi$ on scintillator, in units of $10^{-84}$ (cm$^2$/degree/nucleon)$^2$

| Bin edges (degree) | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 | 60-120 | 120-135 | 135-150 | 150-165 | 165-180 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0-10 | 0.125 | 0.325 | 0.470 | 0.510 | 0.522 | 0.494 | 0.301 | 0.146 | 0.097 | 0.054 | 0.014 |
| 10-20 | 0.325 | 0.913 | 1.275 | 1.384 | 1.399 | 1.314 | 0.860 | 0.434 | 0.280 | 0.161 | 0.038 |
| 20-30 | 0.470 | 1.275 | 1.989 | 2.183 | 2.306 | 2.214 | 1.631 | 0.653 | 0.431 | 0.258 | 0.059 |
| 30-40 | 0.510 | 1.384 | 2.183 | 2.450 | 2.624 | 2.532 | 1.914 | 0.750 | 0.500 | 0.299 | 0.071 |
| 40-50 | 0.522 | 1.399 | 2.306 | 2.624 | 2.936 | 2.888 | 2.220 | 0.856 | 0.575 | 0.326 | 0.078 |
| 50-60 | 0.494 | 1.314 | 2.214 | 2.532 | 2.888 | 2.972 | 2.272 | 0.903 | 0.616 | 0.349 | 0.079 |
| 60-120 | 0.301 | 0.860 | 1.631 | 1.914 | 2.220 | 2.272 | 2.193 | 0.675 | 0.463 | 0.301 | 0.059 |
| 120-135 | 0.146 | 0.434 | 0.653 | 0.750 | 0.856 | 0.903 | 0.675 | 0.407 | 0.261 | 0.118 | 0.025 |
| 135-150 | 0.097 | 0.280 | 0.431 | 0.500 | 0.575 | 0.616 | 0.463 | 0.261 | 0.176 | 0.085 | 0.018 |
| 150-165 | 0.054 | 0.161 | 0.258 | 0.299 | 0.326 | 0.349 | 0.301 | 0.118 | 0.085 | 0.064 | 0.012 |
| 165-180 | 0.014 | 0.038 | 0.059 | 0.071 | 0.078 | 0.079 | 0.059 | 0.025 | 0.018 | 0.012 | 0.003 |

TABLE Supp.XXIX. Measured cross section as function of $p_\mu$ on carbon, in units of $10^{-42}$ cm$^2$/GeV/c/nucleon, and the absolute and fractional cross section uncertainties

| Bin edges (GeV/c) | $\sigma$ | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 1.5-3.0 | 143.43 | 53.59 | 0.348 | 0.137 | 0.038 |
| 3.0-4.0 | 400.66 | 96.71 | 0.207 | 0.124 | 0.038 |
| 4.0-5.0 | 489.84 | 119.50 | 0.183 | 0.162 | 0.039 |
| 5.0-6.0 | 489.52 | 115.11 | 0.170 | 0.163 | 0.037 |
| 6.0-7.0 | 310.21 | 84.33 | 0.194 | 0.191 | 0.037 |
| 7.0-8.5 | 135.12 | 44.89 | 0.257 | 0.210 | 0.038 |
| 8.5-10.0 | 36.97 | 16.68 | 0.364 | 0.267 | 0.041 |
| 10.0-20.0 | 11.32 | 5.78 | 0.446 | 0.247 | 0.038 |

TABLE Supp.XXX. Statistical covariance matrix of measured cross section as function of $p_\mu$ on carbon, in units of $10^{-84}$ (cm$^2$/GeV/c/nucleon)$^2$

| Bin edges (GeV/c) | 1.5-3.0 | 3.0-4.0 | 4.0-5.0 | 5.0-6.0 | 6.0-7.0 | 7.0-8.5 | 8.5-10.0 | 10.0-20.0 |
|---|---|---|---|---|---|---|---|---|
| 1.5-3.0 | 2487.306 | 783.530 | 123.415 | 78.780 | 75.516 | 20.167 | 1.410 | 1.759 |
| 3.0-4.0 | 783.530 | 6883.567 | 1843.185 | 398.312 | 105.427 | 9.190 | 0.161 | 0.000 |
| 4.0-5.0 | 123.415 | 1843.185 | 7998.915 | 2532.020 | 359.777 | 55.429 | 7.510 | 0.000 |
| 5.0-6.0 | 78.780 | 398.312 | 2532.020 | 6906.017 | 2408.268 | 351.079 | 61.864 | 4.991 |
| 6.0-7.0 | 75.516 | 105.427 | 359.777 | 2408.268 | 3610.355 | 1147.420 | 152.970 | 34.063 |
| 7.0-8.5 | 20.167 | 9.190 | 55.429 | 351.079 | 1147.420 | 1207.419 | 270.781 | 15.516 |
| 8.5-10.0 | 1.410 | 0.161 | 7.510 | 61.864 | 152.970 | 270.781 | 180.813 | 22.407 |
| 10.0-20.0 | 1.759 | 0.000 | 0.000 | 4.991 | 34.063 | 15.516 | 22.407 | 25.537 |

TABLE Supp.XXXI. Systematic covariance matrix of measured cross section in $p_\mu$ on carbon, in units of $10^{-84}$ (cm$^2$/GeV/c/nucleon)$^2$

| Bin edges (GeV/c) | 1.5-3.0 | 3.0-4.0 | 4.0-5.0 | 5.0-6.0 | 6.0-7.0 | 7.0-8.5 | 8.5-10.0 | 10.0-20.0 |
|---|---|---|---|---|---|---|---|---|
| 1.5-3.0 | 384.799 | 789.008 | 1193.866 | 1166.558 | 826.990 | 220.270 | 87.278 | 38.919 |
| 3.0-4.0 | 789.008 | 2468.666 | 3297.934 | 3441.037 | 2430.366 | 827.244 | 278.953 | 104.621 |
| 4.0-5.0 | 1193.866 | 3297.934 | 6281.017 | 5857.713 | 4182.090 | 1528.720 | 419.419 | 150.395 |
| 5.0-6.0 | 1166.558 | 3441.037 | 5857.713 | 6343.377 | 4461.196 | 1667.815 | 497.086 | 168.352 |
| 6.0-7.0 | 826.990 | 2430.366 | 4182.090 | 4461.196 | 3502.013 | 1401.819 | 439.792 | 128.544 |
| 7.0-8.5 | 220.270 | 827.244 | 1528.720 | 1667.815 | 1401.819 | 807.588 | 248.509 | 47.072 |
| 8.5-10.0 | 87.278 | 278.953 | 419.419 | 497.086 | 439.792 | 248.509 | 97.422 | 19.134 |
| 10.0-20.0 | 38.919 | 104.621 | 150.395 | 168.352 | 128.544 | 47.072 | 19.134 | 7.842 |

TABLE Supp.XXXII. Measured cross section as function of $\theta_\mu$ on carbon, in units of $10^{-42}$ cm$^2$/degree/nucleon, and the absolute and fractional cross section uncertainties

| Bin edges (degree) | $\sigma$ | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 0-2 | 69.93 | 21.86 | 0.297 | 0.096 | 0.034 |
| 2-3 | 293.81 | 74.18 | 0.226 | 0.113 | 0.033 |
| 3-4 | 232.51 | 71.15 | 0.268 | 0.147 | 0.033 |
| 4-5 | 243.12 | 83.24 | 0.275 | 0.204 | 0.037 |
| 5-6 | 104.44 | 64.79 | 0.510 | 0.353 | 0.046 |
| 6-8 | 165.82 | 82.86 | 0.411 | 0.285 | 0.039 |
| 8-10 | 272.11 | 109.74 | 0.346 | 0.207 | 0.035 |
| 10-13 | 210.36 | 91.15 | 0.384 | 0.202 | 0.039 |

TABLE Supp.XXXIII. Statistical covariance matrix of measured cross section as function of $\theta_\mu$ on carbon, in units of $10^{-84}$ (cm$^2$/degree/nucleon)$^2$

| Bin edges (degree) | 0-2 | 2-3 | 3-4 | 4-5 | 5-6 | 6-8 | 8-10 | 10-13 |
|---|---|---|---|---|---|---|---|---|
| 0-2 | 432.545 | -42.190 | -60.428 | -10.381 | 0.256 | 0.206 | -2.325 | -2.718 |
| 2-3 | -42.190 | 4402.124 | 227.802 | -355.022 | -55.531 | -0.703 | -21.100 | -2.610 |
| 3-4 | -60.428 | 227.802 | 3889.313 | 556.447 | -267.531 | -59.679 | -5.042 | -9.166 |
| 4-5 | -10.381 | -355.022 | 556.447 | 4479.508 | 604.254 | -323.825 | -38.103 | 3.448 |
| 5-6 | 0.256 | -55.531 | -267.531 | 604.254 | 2836.385 | 323.719 | -183.679 | -8.398 |
| 6-8 | 0.206 | -0.703 | -59.679 | -323.825 | 323.719 | 4640.159 | -126.552 | -144.285 |
| 8-10 | -2.325 | -21.100 | -5.042 | -38.103 | -183.679 | -126.552 | 8881.099 | -343.995 |
| 10-13 | -2.718 | -2.610 | -9.166 | 3.448 | -8.398 | -144.285 | -343.995 | 6510.178 |

TABLE Supp.XXXIV. Systematic covariance matrix of measured cross section in $\theta_\mu$ on carbon, in units of $10^{-84}$ (cm$^2$/degree/nucleon)$^2$

| Bin edges (degree) | 0-2 | 2-3 | 3-4 | 4-5 | 5-6 | 6-8 | 8-10 | 10-13 |
|---|---|---|---|---|---|---|---|---|
| 0-2 | 45.099 | 92.411 | 103.281 | 114.507 | 74.645 | 105.557 | 149.681 | 52.143 |
| 2-3 | 92.411 | 1100.609 | 841.832 | 1222.754 | 769.793 | 961.137 | 1118.738 | 1053.933 |
| 3-4 | 103.281 | 841.832 | 1172.797 | 1542.172 | 1062.769 | 1138.866 | 1384.286 | 1145.364 |
| 4-5 | 114.507 | 1222.754 | 1542.172 | 2449.024 | 1531.587 | 1737.054 | 2156.829 | 1604.998 |
| 5-6 | 74.645 | 769.793 | 1062.769 | 1531.587 | 1361.346 | 1240.504 | 1502.525 | 1102.167 |
| 6-8 | 105.557 | 961.137 | 1138.866 | 1737.054 | 1240.504 | 2226.414 | 1880.519 | 1195.813 |
| 8-10 | 149.681 | 1118.738 | 1384.286 | 2156.829 | 1502.525 | 1880.519 | 3161.621 | 1418.018 |
| 10-13 | 52.143 | 1053.933 | 1145.364 | 1604.998 | 1102.167 | 1195.813 | 1418.018 | 1798.854 |

TABLE Supp.XXXV. Measured cross section as function of $p_{\mu,||}$ on carbon, in units of $10^{-42}$ cm$^2$/GeV/c/nucleon, and the absolute and fractional cross section uncertainties

| Bin edges (GeV/c) | $\sigma$ | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 1.5-3.0 | 142.81 | 54.37 | 0.353 | 0.142 | 0.038 |
| 3.0-4.0 | 414.13 | 100.15 | 0.207 | 0.125 | 0.037 |
| 4.0-5.0 | 495.24 | 120.33 | 0.181 | 0.162 | 0.039 |
| 5.0-6.0 | 470.54 | 113.51 | 0.174 | 0.167 | 0.038 |
| 6.0-7.0 | 316.82 | 84.97 | 0.193 | 0.186 | 0.037 |
| 7.0-8.5 | 131.56 | 43.90 | 0.258 | 0.211 | 0.038 |
| 8.5-10.0 | 36.08 | 16.51 | 0.370 | 0.270 | 0.041 |
| 10.0-20.0 | 11.27 | 5.74 | 0.446 | 0.247 | 0.038 |

TABLE Supp.XXXVI. Statistical covariance matrix of measured cross section as function of $p_{\mu,||}$ on carbon, in units of $10^{-84}$ (cm$^2$/GeV/c/nucleon)$^2$

| Bin edges (GeV/c) | 1.5-3.0 | 3.0-4.0 | 4.0-5.0 | 5.0-6.0 | 6.0-7.0 | 7.0-8.5 | 8.5-10.0 | 10.0-20.0 |
|---|---|---|---|---|---|---|---|---|
| 1.5-3.0 | 2545.244 | 728.621 | 120.277 | 77.608 | 72.794 | 19.444 | 1.364 | 1.702 |
| 3.0-4.0 | 728.621 | 7370.998 | 1832.106 | 356.992 | 93.965 | 8.392 | 0.052 | 0.000 |
| 4.0-5.0 | 120.277 | 1832.106 | 8057.985 | 2544.775 | 374.057 | 53.227 | 7.161 | 0.000 |
| 5.0-6.0 | 77.608 | 356.992 | 2544.775 | 6684.440 | 2431.505 | 348.631 | 58.569 | 4.882 |
| 6.0-7.0 | 72.794 | 93.965 | 374.057 | 2431.505 | 3735.052 | 1088.284 | 140.039 | 33.362 |
| 7.0-8.5 | 19.444 | 8.392 | 53.227 | 348.631 | 1088.284 | 1156.326 | 260.739 | 15.362 |
| 8.5-10.0 | 1.364 | 0.052 | 7.161 | 58.569 | 140.039 | 260.739 | 177.793 | 22.270 |
| 10.0-20.0 | 1.702 | 0.000 | 0.000 | 4.882 | 33.362 | 15.362 | 22.270 | 25.211 |

TABLE Supp.XXXVII. Systematic covariance matrix of measured cross section in $p_{\mu,||}$ on carbon, in units of $10^{-84}$ (cm$^2$/GeV/c/nucleon)$^2$

| Bin edges (GeV/c) | 1.5-3.0 | 3.0-4.0 | 4.0-5.0 | 5.0-6.0 | 6.0-7.0 | 7.0-8.5 | 8.5-10.0 | 10.0-20.0 |
|---|---|---|---|---|---|---|---|---|
| 1.5-3.0 | 410.793 | 807.898 | 1244.792 | 1207.063 | 865.339 | 222.378 | 85.770 | 40.987 |
| 3.0-4.0 | 807.898 | 2659.590 | 3370.825 | 3460.466 | 2460.014 | 783.208 | 264.051 | 103.430 |
| 4.0-5.0 | 1244.792 | 3370.825 | 6421.428 | 5800.897 | 4234.157 | 1488.178 | 408.526 | 149.058 |
| 5.0-6.0 | 1207.063 | 3460.466 | 5800.897 | 6199.385 | 4432.492 | 1594.078 | 488.891 | 166.781 |
| 6.0-7.0 | 865.339 | 2460.014 | 4234.157 | 4432.492 | 3484.686 | 1332.120 | 428.733 | 129.620 |
| 7.0-8.5 | 222.378 | 783.208 | 1488.178 | 1594.078 | 1332.120 | 770.937 | 239.169 | 44.936 |
| 8.5-10.0 | 85.770 | 264.051 | 408.526 | 488.891 | 428.733 | 239.169 | 94.943 | 18.573 |
| 10.0-20.0 | 40.987 | 103.430 | 149.058 | 166.781 | 129.620 | 44.936 | 18.573 | 7.764 |

TABLE Supp.XXXVIII. Measured cross section as function of $p_{\mu,T}$ on carbon, in units of $10^{-42}$ cm$^2$/GeV/c/nucleon, and the absolute and fractional cross section uncertainties

| Bin edges (GeV/c) | $\sigma$ | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 0.00-0.15 | 677.76 | 225.45 | 0.321 | 0.087 | 0.034 |
| 0.15-0.30 | 1738.89 | 427.93 | 0.225 | 0.100 | 0.034 |
| 0.30-0.45 | 3334.80 | 784.27 | 0.200 | 0.124 | 0.035 |
| 0.45-0.60 | 3255.16 | 968.88 | 0.227 | 0.193 | 0.037 |
| 0.60-0.75 | 2354.67 | 914.93 | 0.305 | 0.241 | 0.037 |
| 0.75-0.90 | 1095.46 | 966.59 | 0.687 | 0.554 | 0.049 |
| 0.90-1.25 | 732.87 | 537.65 | 0.581 | 0.448 | 0.048 |
| 1.25-2.50 | 41.20 | 64.42 | 1.409 | 0.678 | 0.036 |

TABLE Supp.XXXIX. Statistical covariance matrix of measured cross section as function of $p_{\mu,T}$ on carbon, in units of $10^{-84}$ $(\mathrm{cm}^2/\mathrm{GeV/c/nucleon})^2$

| Bin edges (GeV/c) | 0.00-0.15 | 0.15-0.30 | 0.30-0.45 | 0.45-0.60 | 0.60-0.75 | 0.75-0.90 | 0.90-1.25 | 1.25-2.50 |
|---|---|---|---|---|---|---|---|---|
| 0.00-0.15 | 47345.844 | 489.515 | -9627.274 | -1191.565 | -50.835 | 695.864 | -132.553 | -2.979 |
| 0.15-0.30 | 489.515 | 152597.493 | 15878.159 | -24434.942 | -4415.122 | -2630.600 | -338.305 | -26.218 |
| 0.30-0.45 | -9627.274 | 15878.159 | 442805.481 | 56960.571 | -47584.361 | -18432.176 | -3106.294 | -512.456 |
| 0.45-0.60 | -1191.565 | -24434.942 | 56960.571 | 545345.745 | 82151.510 | -38191.909 | -22415.638 | 566.213 |
| 0.60-0.75 | -50.835 | -4415.122 | -47584.361 | 82151.510 | 514382.100 | 160232.420 | -4248.074 | -2161.159 |
| 0.75-0.90 | 695.864 | -2630.600 | -18432.176 | -38191.909 | 160232.420 | 565653.640 | 116267.363 | -433.914 |
| 0.90-1.25 | -132.553 | -338.305 | -3106.294 | -22415.638 | -4248.074 | 116267.363 | 181284.881 | -260.193 |
| 1.25-2.50 | -2.979 | -26.218 | -512.456 | 566.213 | -2161.159 | -433.914 | -260.193 | 3369.229 |

TABLE Supp.XL. Systematic covariance matrix of measured cross section in $p_{\mu,T}$ on carbon, in units of $10^{-84}$ (cm$^2$/GeV/c/nucleon)$^2$

| Bin edges (GeV/c) | 0.00-0.15 | 0.15-0.30 | 0.30-0.45 | 0.45-0.60 | 0.60-0.75 | 0.75-0.90 | 0.90-1.25 | 1.25-2.50 |
|---|---|---|---|---|---|---|---|---|
| 0.00-0.15 | 3483.817 | 7652.058 | 8866.626 | 10183.622 | 11849.515 | 6912.592 | 2468.776 | 265.614 |
| 0.15-0.30 | 7652.058 | 30530.330 | 51863.562 | 66554.567 | 71581.100 | 50684.480 | 18484.932 | 1839.939 |
| 0.30-0.45 | 8866.626 | 51863.562 | 172270.470 | 234351.239 | 208147.109 | 144317.649 | 56989.645 | 6228.014 |
| 0.45-0.60 | 10183.622 | 66554.567 | 234351.239 | 393376.163 | 322202.135 | 212486.586 | 91525.906 | 9190.991 |
| 0.60-0.75 | 11849.515 | 71581.100 | 208147.109 | 322202.135 | 322712.829 | 223779.297 | 80646.181 | 9352.017 |
| 0.75-0.90 | 6912.592 | 50684.480 | 144317.649 | 212486.586 | 223779.297 | 368643.495 | 152778.165 | 12440.530 |
| 0.90-1.25 | 2468.776 | 18484.932 | 56989.645 | 91525.906 | 80646.181 | 152778.165 | 107781.804 | 5769.398 |
| 1.25-2.50 | 265.614 | 1839.939 | 6228.014 | 9190.991 | 9352.017 | 12440.530 | 5769.398 | 780.369 |

TABLE Supp.XLI. Measured cross section as function of $Q^2$ on carbon, in units of $10^{-42}$ $\mathrm{cm}^2/\mathrm{GeV}^2/c^2$/nucleon, and the absolute and fractional cross section uncertainties

| Bin edges ($\mathrm{GeV}^2/c^2$) | $\sigma$ | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 0.000-0.025 | 4110.92 | 1313.92 | 0.309 | 0.082 | 0.034 |
| 0.025-0.050 | 3738.85 | 1125.78 | 0.289 | 0.086 | 0.035 |
| 0.050-0.100 | 3372.90 | 968.01 | 0.257 | 0.127 | 0.033 |
| 0.100-0.200 | 4499.60 | 1037.27 | 0.198 | 0.119 | 0.035 |
| 0.200-0.300 | 2654.05 | 855.20 | 0.271 | 0.175 | 0.036 |
| 0.300-0.400 | 2401.01 | 861.33 | 0.280 | 0.224 | 0.039 |
| 0.400-0.500 | 1622.89 | 738.87 | 0.352 | 0.289 | 0.042 |
| 0.500-0.700 | 934.89 | 545.84 | 0.465 | 0.353 | 0.040 |
| 0.700-1.000 | 786.73 | 497.09 | 0.515 | 0.366 | 0.039 |
| 1.000-1.300 | 508.34 | 308.25 | 0.541 | 0.274 | 0.040 |
| 1.300-2.000 | 104.02 | 162.03 | 1.255 | 0.923 | 0.059 |
| 2.000-3.000 | 43.44 | 76.15 | 1.619 | 0.673 | 0.050 |

TABLE Supp.XLII. Statistical covariance matrix of measured cross section as function of $Q^2$ on carbon, in units of $10^{-84}$ $(\mathrm{cm}^2/\mathrm{GeV}^2/c^2/\mathrm{nucleon})^2$

| Bin edges ($\mathrm{GeV}^2/c^2$) | 0.000-0.025 | 0.025-0.050 | 0.050-0.100 | 0.100-0.200 | 0.200-0.300 | 0.300-0.400 | 0.400-0.500 | 0.500-0.700 | 0.700-1.000 | 1.000-1.300 | 1.300-2.000 | 2.000-3.000 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.000-0.025 | 1612047.844 | 133164.747 | -109292.560 | -38037.712 | -210.854 | -289.388 | -431.586 | -381.587 | 1410.826 | -116.093 | -134.401 | -15.298 |
| 0.025-0.050 | 133164.747 | 1163631.354 | 296929.122 | -89658.591 | -29868.450 | -4855.439 | -2137.794 | -1577.626 | -2178.074 | -485.529 | -223.131 | 12.313 |
| 0.050-0.100 | -109292.560 | 296929.122 | 753462.588 | 147506.891 | -58070.191 | -21930.282 | -6742.827 | -1558.373 | 317.631 | -580.345 | -609.480 | -76.275 |
| 0.100-0.200 | -38037.712 | -89658.591 | 147506.891 | 790275.558 | 111043.502 | -56840.469 | -38915.673 | -5121.184 | -3538.403 | -2738.221 | -473.804 | -314.496 |
| 0.200-0.300 | -210.854 | -29868.450 | -58070.191 | 111043.502 | 516648.021 | 193831.617 | -9914.202 | -26131.027 | -9527.948 | -4605.887 | -2429.364 | -225.566 |
| 0.300-0.400 | -289.388 | -4855.439 | -21930.282 | -56840.469 | 193831.617 | 453257.205 | 226912.682 | 727.533 | -25604.708 | -10358.162 | 2497.816 | -92.988 |
| 0.400-0.500 | -431.586 | -2137.794 | -6742.827 | -38915.673 | -9914.202 | 226912.682 | 325672.916 | 114465.431 | -2753.532 | -9681.002 | -1603.063 | -511.652 |
| 0.500-0.700 | -381.587 | -1577.626 | -1558.373 | -5121.184 | -26131.027 | 727.533 | 114465.431 | 188992.547 | 64966.679 | -2230.532 | -223.786 | -666.490 |
| 0.700-1.000 | 1410.826 | -2178.074 | 317.631 | -3538.403 | -9527.948 | -25604.708 | -2753.532 | 64966.679 | 163968.352 | 51804.379 | 591.175 | -1196.626 |
| 1.000-1.300 | -116.093 | -485.529 | -580.345 | -2738.221 | -4605.887 | -10358.162 | -9681.002 | -2230.532 | 51804.379 | 75645.608 | 8609.031 | -672.953 |
| 1.300-2.000 | -134.401 | -223.131 | -609.480 | -473.804 | -2429.364 | 2497.816 | -1603.063 | -223.786 | 591.175 | 8609.031 | 17036.574 | 2099.067 |
| 2.000-3.000 | -15.298 | 12.313 | -76.275 | -314.496 | -225.566 | -92.988 | -511.652 | -666.490 | -1196.626 | -672.953 | 2099.067 | 4943.006 |

TABLE Supp.XLIII. Systematic covariance matrix of measured cross section in $Q^2$ on carbon, in units of $10^{-84}$ $(\mathrm{cm}^2/\mathrm{GeV}^2/c^2/\mathrm{nucleon})^2$

| Bin edges ($\mathrm{GeV}^2/c^2$) | 0.000-0.025 | 0.025-0.050 | 0.050-0.100 | 0.100-0.200 | 0.200-0.300 | 0.300-0.400 | 0.400-0.500 | 0.500-0.700 | 0.700-1.000 | 1.000-1.300 | 1.300-2.000 | 2.000-3.000 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.000-0.025 | 114340.749 | 63203.907 | 97883.668 | 69702.441 | 50788.032 | 44606.610 | 50119.145 | 27594.492 | 15002.767 | 9359.854 | 2927.239 | 1444.850 |
| 0.025-0.050 | 63203.907 | 103757.689 | 84392.474 | 109039.744 | 61996.950 | 53071.693 | 63755.278 | 53044.149 | 28270.909 | 14167.656 | 3437.170 | 2164.095 |
| 0.050-0.100 | 97883.668 | 84392.474 | 183575.702 | 170724.672 | 142519.813 | 153118.040 | 139533.540 | 99833.908 | 67543.256 | 30334.664 | 12124.226 | 4509.597 |
| 0.100-0.200 | 69702.441 | 109039.744 | 170724.672 | 285646.828 | 223066.738 | 226804.698 | 203762.849 | 146664.829 | 78950.233 | 38488.714 | 19109.338 | 6346.318 |
| 0.200-0.300 | 50788.032 | 61996.950 | 142519.813 | 223066.738 | 214718.388 | 225859.884 | 194853.625 | 120137.142 | 64661.826 | 32947.723 | 16705.759 | 4606.379 |
| 0.300-0.400 | 44606.610 | 53071.693 | 153118.040 | 226804.698 | 225859.884 | 288638.745 | 230674.942 | 132653.732 | 72541.441 | 34292.653 | 21857.936 | 5416.617 |
| 0.400-0.500 | 50119.145 | 63755.278 | 139533.540 | 203762.849 | 194853.625 | 230674.942 | 220250.698 | 124449.181 | 63894.843 | 32171.484 | 19459.809 | 5250.944 |
| 0.500-0.700 | 27594.492 | 53044.149 | 99833.908 | 146664.829 | 120137.142 | 132653.732 | 124449.181 | 108949.639 | 72984.339 | 34427.668 | 17917.460 | 5931.924 |
| 0.700-1.000 | 15002.767 | 28270.909 | 67543.256 | 78950.233 | 64661.826 | 72541.441 | 63894.843 | 72984.339 | 83134.974 | 34268.597 | 17608.447 | 5894.400 |
| 1.000-1.300 | 9359.854 | 14167.656 | 30334.664 | 38488.714 | 32947.723 | 34292.653 | 32171.484 | 34427.668 | 34268.597 | 19372.429 | 10260.940 | 3053.558 |
| 1.300-2.000 | 2927.239 | 3437.170 | 12124.226 | 19109.338 | 16705.759 | 21857.936 | 19459.809 | 17917.460 | 17608.447 | 10260.940 | 9218.640 | 1777.178 |
| 2.000-3.000 | 1444.850 | 2164.095 | 4509.597 | 6346.318 | 4606.379 | 5416.617 | 5250.944 | 5931.924 | 5894.400 | 3053.558 | 1777.178 | 855.347 |

TABLE Supp.XLIV. Measured cross section as function of $W_{exp}$ on carbon, in units of $10^{-42}$ $cm^2/GeV/c^2$/nucleon, and the absolute and fractional cross section uncertainties

| Bin edges ($GeV/c^2$) | $\sigma$ | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 0.9-1.1 | 935.92 | 613.41 | 0.627 | 0.191 | 0.043 |
| 1.1-1.2 | 5238.76 | 1031.06 | 0.175 | 0.091 | 0.036 |
| 1.2-1.3 | 8434.70 | 2309.31 | 0.200 | 0.187 | 0.033 |
| 1.3-1.4 | 4744.74 | 3791.22 | 0.529 | 0.599 | 0.033 |

TABLE Supp.XLV. Statistical covariance matrix of measured cross section as function of $W_{exp}$ on carbon, in units of $10^{-84}$ $(cm^2/GeV/c^2/nucleon)^2$

| Bin edges ($GeV/c^2$) | 0.9-1.1 | 1.1-1.2 | 1.2-1.3 | 1.3-1.4 |
|---|---|---|---|---|
| 0.9-1.1 | 344280.921 | -5955.534 | -433736.932 | -328835.505 |
| 1.1-1.2 | -5955.534 | 837000.556 | 82998.926 | -910335.711 |
| 1.2-1.3 | -433736.932 | 82998.926 | 2848202.794 | 1532250.771 |
| 1.3-1.4 | -328835.505 | -910335.711 | 1532250.771 | 6306221.739 |

TABLE Supp.XLVI. Systematic covariance matrix of measured cross section in $W_{exp}$ on carbon, in units of $10^{-84}$ $(cm^2/GeV/c^2/nucleon)^2$

| Bin edges ($GeV/c^2$) | 0.9-1.1 | 1.1-1.2 | 1.2-1.3 | 1.3-1.4 |
|---|---|---|---|---|
| 0.9-1.1 | 31991.340 | 34754.971 | 26908.976 | 39260.342 |
| 1.1-1.2 | 34754.971 | 226074.839 | 269792.794 | 290485.892 |
| 1.2-1.3 | 26908.976 | 269792.794 | 2484692.008 | 4127535.293 |
| 1.3-1.4 | 39260.342 | 290485.892 | 4127535.293 | 8067146.466 |

TABLE Supp.XLVII. Measured cross section as function of $T_\pi$ on carbon, in units of $10^{-42}$ cm$^2$/GeV/nucleon, and the absolute and fractional cross section uncertainties

| Bin edges (GeV) | $\sigma$ | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 0.035-0.100 | 11544.77 | 2671.16 | 0.176 | 0.151 | 0.036 |
| 0.100-0.150 | 9366.27 | 1783.83 | 0.132 | 0.137 | 0.034 |
| 0.150-0.200 | 5705.08 | 1763.73 | 0.226 | 0.211 | 0.033 |
| 0.200-0.350 | 2730.47 | 1583.72 | 0.425 | 0.394 | 0.040 |

TABLE Supp.XLVIII. Statistical covariance matrix of measured cross section as function of $T_\pi$ on carbon, in units of $10^{-84}$ (cm$^2$/GeV/nucleon)$^2$

| Bin edges (GeV) | 0.035-0.100 | 0.100-0.150 | 0.150-0.200 | 0.200-0.350 |
|---|---|---|---|---|
| 0.035-0.100 | 4110120.545 | 450633.521 | -366102.636 | -6362.214 |
| 0.100-0.150 | 450633.521 | 1529252.308 | 596865.376 | 246606.388 |
| 0.150-0.200 | -366102.636 | 596865.376 | 1657591.298 | 826260.188 |
| 0.200-0.350 | -6362.214 | 246606.388 | 826260.188 | 1349320.553 |

TABLE Supp.XLIX. Systematic covariance matrix of measured cross section in $T_\pi$ on carbon, in units of $10^{-84}$ (cm$^2$/GeV/nucleon)$^2$

| Bin edges (GeV) | 0.035-0.100 | 0.100-0.150 | 0.150-0.200 | 0.200-0.350 |
|---|---|---|---|---|
| 0.035-0.100 | 3024981.510 | 1978891.970 | 1702989.640 | 1319045.699 |
| 0.100-0.150 | 1978891.970 | 1652801.001 | 1398786.930 | 954689.298 |
| 0.150-0.200 | 1702989.640 | 1398786.930 | 1453140.559 | 1071815.858 |
| 0.200-0.350 | 1319045.699 | 954689.298 | 1071815.858 | 1158852.632 |

TABLE Supp.L. Measured cross section as function of $\theta_\pi$ on carbon, in units of $10^{-42}$ cm$^2$/degree/nucleon, and the absolute and fractional cross section uncertainties

| Bin edges (degree) | $\sigma$ | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 0-10 | 3.68 | 5.99 | 1.483 | 0.671 | 0.045 |
| 10-20 | 9.04 | 6.38 | 0.627 | 0.325 | 0.040 |
| 20-30 | 14.15 | 5.91 | 0.339 | 0.244 | 0.035 |
| 30-40 | 16.26 | 5.52 | 0.279 | 0.194 | 0.037 |
| 40-50 | 21.86 | 6.14 | 0.228 | 0.163 | 0.036 |
| 50-60 | 18.12 | 5.63 | 0.256 | 0.176 | 0.034 |
| 60-120 | 17.45 | 4.32 | 0.207 | 0.135 | 0.035 |
| 120-135 | 9.75 | 3.15 | 0.291 | 0.141 | 0.033 |
| 135-150 | 8.14 | 2.79 | 0.310 | 0.146 | 0.033 |
| 150-165 | 5.01 | 3.21 | 0.556 | 0.319 | 0.039 |
| 165-180 | 1.05 | 0.91 | 0.768 | 0.395 | 0.036 |

TABLE Supp.LI. Statistical covariance matrix of measured cross section as function of $\theta_\pi$ on carbon, in units of $10^{-84}$ (cm$^2$/degree/nucleon)$^2$

| Bin edges (degree) | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 | 60-120 | 120-135 | 135-150 | 150-165 | 165-180 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0-10 | 29.766 | 12.455 | 4.832 | -1.353 | 2.463 | -0.442 | -0.606 | -0.302 | -0.460 | -0.539 | 0.102 |
| 10-20 | 12.455 | 32.119 | 11.299 | -0.769 | -0.780 | -1.218 | -0.439 | 0.010 | -0.135 | 0.286 | 2.196 |
| 20-30 | 4.832 | 11.299 | 23.034 | 5.013 | -1.963 | -1.259 | -0.814 | -0.799 | -0.565 | 0.421 | 0.728 |
| 30-40 | -1.353 | -0.769 | 5.013 | 20.496 | 7.417 | -0.553 | -0.845 | -0.311 | 0.136 | -0.222 | -0.201 |
| 40-50 | 2.463 | -0.780 | -1.963 | 7.417 | 24.897 | 6.425 | -0.456 | -0.672 | -0.583 | -1.089 | -0.313 |
| 50-60 | -0.442 | -1.218 | -1.259 | -0.553 | 6.425 | 21.543 | 4.262 | -0.433 | -0.423 | -0.570 | -0.167 |
| 60-120 | -0.606 | -0.439 | -0.814 | -0.845 | -0.456 | 4.262 | 13.098 | 1.406 | -0.111 | -0.194 | -0.021 |
| 120-135 | -0.302 | 0.010 | -0.799 | -0.311 | -0.672 | -0.433 | 1.406 | 8.040 | 1.280 | 0.071 | 0.060 |
| 135-150 | -0.460 | -0.135 | -0.565 | 0.136 | -0.583 | -0.423 | -0.111 | 1.280 | 6.378 | 4.357 | 0.394 |
| 150-165 | -0.539 | 0.286 | 0.421 | -0.222 | -1.089 | -0.570 | -0.194 | 0.071 | 4.357 | 7.762 | 0.096 |
| 165-180 | 0.102 | 2.196 | 0.728 | -0.201 | -0.313 | -0.167 | -0.021 | 0.060 | 0.394 | 0.096 | 0.653 |

TABLE Supp.LII. Systematic covariance matrix of measured cross section in $\theta_\pi$ on carbon, in units of $10^{-84}$ (cm$^2$/degree/nucleon)$^2$

| Bin edges (degree) | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 | 60-120 | 120-135 | 135-150 | 150-165 | 165-180 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0-10 | 6.099 | 2.112 | 3.025 | 2.738 | 2.224 | 2.087 | 1.429 | 1.276 | 1.283 | 1.506 | 0.034 |
| 10-20 | 2.112 | 8.617 | 9.250 | 7.393 | 8.503 | 7.705 | 4.256 | 2.782 | 2.277 | 1.852 | 0.710 |
| 20-30 | 3.025 | 9.250 | 11.938 | 9.913 | 10.228 | 8.821 | 5.209 | 3.110 | 2.750 | 1.823 | 0.670 |
| 30-40 | 2.738 | 7.393 | 9.913 | 9.939 | 8.952 | 7.789 | 4.739 | 2.801 | 2.587 | 1.857 | 0.574 |
| 40-50 | 2.224 | 8.503 | 10.228 | 8.952 | 12.744 | 10.942 | 6.933 | 3.974 | 3.154 | 1.512 | 0.677 |
| 50-60 | 2.087 | 7.705 | 8.821 | 7.789 | 10.942 | 10.147 | 6.381 | 3.840 | 2.837 | 1.708 | 0.712 |
| 60-120 | 1.429 | 4.256 | 5.209 | 4.739 | 6.933 | 6.381 | 5.548 | 2.655 | 1.714 | 0.957 | 0.559 |
| 120-135 | 1.276 | 2.782 | 3.110 | 2.801 | 3.974 | 3.840 | 2.655 | 1.883 | 1.275 | 1.207 | 0.335 |
| 135-150 | 1.283 | 2.277 | 2.750 | 2.587 | 3.154 | 2.837 | 1.714 | 1.275 | 1.417 | 0.745 | 0.186 |
| 150-165 | 1.506 | 1.852 | 1.823 | 1.857 | 1.512 | 1.708 | 0.957 | 1.207 | 0.745 | 2.546 | 0.259 |
| 165-180 | 0.034 | 0.710 | 0.670 | 0.574 | 0.677 | 0.712 | 0.559 | 0.335 | 0.186 | 0.259 | 0.173 |

TABLE Supp.LIII. Measured cross section as function of $p_\mu$ on water, in units of $10^{-42}$ $cm^2$/GeV/c/nucleon, and the absolute and fractional cross section uncertainties

| Bin edges (GeV/c) | $\sigma$ | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 1.5-3.0 | 117.95 | 52.87 | 0.410 | 0.180 | 0.035 |
| 3.0-4.0 | 185.58 | 88.01 | 0.386 | 0.276 | 0.048 |
| 4.0-5.0 | 457.01 | 118.99 | 0.211 | 0.153 | 0.041 |
| 5.0-6.0 | 476.40 | 113.68 | 0.187 | 0.148 | 0.037 |
| 6.0-7.0 | 340.82 | 84.27 | 0.207 | 0.135 | 0.035 |
| 7.0-8.5 | 162.09 | 46.27 | 0.233 | 0.165 | 0.032 |
| 8.5-10.0 | 46.21 | 16.77 | 0.317 | 0.176 | 0.031 |
| 10.0-20.0 | 6.09 | 2.91 | 0.438 | 0.192 | 0.037 |

TABLE Supp.LIV. Statistical covariance matrix of measured cross section as function of $p_\mu$ on water, in units of $10^{-84}$ $(cm^2/GeV/c/nucleon)^2$

| Bin edges (GeV/c) | 1.5-3.0 | 3.0-4.0 | 4.0-5.0 | 5.0-6.0 | 6.0-7.0 | 7.0-8.5 | 8.5-10.0 | 10.0-20.0 |
|---|---|---|---|---|---|---|---|---|
| 1.5-3.0 | 2344.004 | 445.008 | 69.019 | 37.629 | 28.309 | 10.974 | 8.907 | 0.011 |
| 3.0-4.0 | 445.008 | 5127.450 | 2079.943 | 557.499 | 60.656 | 35.261 | 3.708 | 1.132 |
| 4.0-5.0 | 69.019 | 2079.943 | 9264.813 | 3453.632 | 698.630 | 141.989 | 30.908 | 2.745 |
| 5.0-6.0 | 37.629 | 557.499 | 3453.632 | 7926.040 | 3136.162 | 481.482 | 55.213 | 7.682 |
| 6.0-7.0 | 28.309 | 60.656 | 698.630 | 3136.162 | 4992.546 | 1267.690 | 85.475 | 1.391 |
| 7.0-8.5 | 10.974 | 35.261 | 141.989 | 481.482 | 1267.690 | 1425.398 | 217.459 | 10.233 |
| 8.5-10.0 | 8.907 | 3.708 | 30.908 | 55.213 | 85.475 | 217.459 | 215.068 | 20.762 |
| 10.0-20.0 | 0.011 | 1.132 | 2.745 | 7.682 | 1.391 | 10.233 | 20.762 | 7.127 |

TABLE Supp.LV. Systematic covariance matrix of measured cross section in $p_\mu$ on water, in units of $10^{-84}$ $(\mathrm{cm}^2/\mathrm{GeV/c/nucleon})^2$

| Bin edges (GeV/c) | 1.5-3.0 | 3.0-4.0 | 4.0-5.0 | 5.0-6.0 | 6.0-7.0 | 7.0-8.5 | 8.5-10.0 | 10.0-20.0 |
|---|---|---|---|---|---|---|---|---|
| 1.5-3.0 | 451.412 | 564.022 | 898.038 | 799.031 | 682.157 | 419.447 | 119.619 | 9.347 |
| 3.0-4.0 | 564.022 | 2619.122 | 2753.930 | 2802.366 | 1580.375 | 770.858 | 208.530 | 26.336 |
| 4.0-5.0 | 898.038 | 2753.930 | 4893.632 | 4338.470 | 2681.647 | 1244.144 | 288.103 | 47.295 |
| 5.0-6.0 | 799.031 | 2802.366 | 4338.470 | 4998.182 | 2697.240 | 1079.987 | 305.276 | 52.912 |
| 6.0-7.0 | 682.157 | 1580.375 | 2681.647 | 2697.240 | 2109.131 | 974.516 | 247.648 | 36.237 |
| 7.0-8.5 | 419.447 | 770.858 | 1244.144 | 1079.987 | 974.516 | 715.744 | 182.693 | 18.250 |
| 8.5-10.0 | 119.619 | 208.530 | 288.103 | 305.276 | 247.648 | 182.693 | 66.308 | 4.025 |
| 10.0-20.0 | 9.347 | 26.336 | 47.295 | 52.912 | 36.237 | 18.250 | 4.025 | 1.361 |

TABLE Supp.LVI. Measured cross section as function of $\theta_\mu$ on water, in units of $10^{-42}$ $\mathrm{cm}^2$/degree/nucleon, and the absolute and fractional cross section uncertainties

| Bin edges (degree) | $\sigma$ | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 0-2 | 63.72 | 18.14 | 0.260 | 0.117 | 0.031 |
| 2-3 | 217.48 | 63.11 | 0.271 | 0.103 | 0.035 |
| 3-4 | 246.44 | 72.74 | 0.267 | 0.125 | 0.033 |
| 4-5 | 207.57 | 88.04 | 0.382 | 0.184 | 0.034 |
| 5-6 | 136.66 | 83.70 | 0.536 | 0.296 | 0.037 |
| 6-8 | 236.70 | 102.60 | 0.366 | 0.233 | 0.040 |
| 8-10 | 290.88 | 114.06 | 0.340 | 0.195 | 0.039 |
| 10-13 | 14.67 | 57.44 | 3.399 | 1.945 | 0.181 |

TABLE Supp.LVII. Statistical covariance matrix of measured cross section as function of $\theta_\mu$ on water, in units of $10^{-84}$ (cm$^2$/degree/nucleon)$^2$

| Bin edges (degree) | 0-2 | 2-3 | 3-4 | 4-5 | 5-6 | 6-8 | 8-10 | 10-13 |
|---|---|---|---|---|---|---|---|---|
| 0-2 | 273.470 | 54.585 | -101.543 | -17.484 | -0.727 | -4.685 | -0.455 | 0.011 |
| 2-3 | 54.585 | 3478.295 | 496.736 | -513.517 | -80.607 | -13.769 | -12.044 | 0.139 |
| 3-4 | -101.543 | 496.736 | 4336.745 | 942.105 | -394.960 | -115.405 | 5.655 | 0.149 |
| 4-5 | -17.484 | -513.517 | 942.105 | 6294.647 | 1188.026 | -463.504 | -39.117 | 0.892 |
| 5-6 | -0.727 | -80.607 | -394.960 | 1188.026 | 5373.660 | 607.700 | -342.608 | -2.822 |
| 6-8 | -4.685 | -13.769 | -115.405 | -463.504 | 607.700 | 7487.508 | -398.111 | -72.215 |
| 8-10 | -0.455 | -12.044 | 5.655 | -39.117 | -342.608 | -398.111 | 9799.885 | 587.996 |
| 10-13 | 0.011 | 0.139 | 0.149 | 0.892 | -2.822 | -72.215 | 587.996 | 2485.525 |

TABLE Supp.LVIII. Systematic covariance matrix of measured cross section in $\theta_\mu$ on water, in units of $10^{-84}$ (cm$^2$/degree/nucleon)$^2$

| Bin edges (degree) | 0-2 | 2-3 | 3-4 | 4-5 | 5-6 | 6-8 | 8-10 | 10-13 |
|---|---|---|---|---|---|---|---|---|
| 0-2 | 55.609 | 108.738 | 139.883 | 81.848 | 35.719 | 28.759 | 40.840 | -6.741 |
| 2-3 | 108.738 | 504.494 | 552.733 | 601.157 | 570.449 | 631.701 | 410.197 | 287.381 |
| 3-4 | 139.883 | 552.733 | 954.832 | 802.914 | 660.357 | 1116.479 | 944.109 | 450.452 |
| 4-5 | 81.848 | 601.157 | 802.914 | 1456.929 | 1320.732 | 1478.837 | 1299.382 | 751.433 |
| 5-6 | 35.719 | 570.449 | 660.357 | 1320.732 | 1632.620 | 1328.831 | 1269.663 | 881.853 |
| 6-8 | 28.759 | 631.701 | 1116.479 | 1478.837 | 1328.831 | 3040.213 | 1943.068 | 1180.314 |
| 8-10 | 40.840 | 410.197 | 944.109 | 1299.382 | 1269.663 | 1943.068 | 3209.779 | 1043.855 |
| 10-13 | -6.741 | 287.381 | 450.452 | 751.433 | 881.853 | 1180.314 | 1043.855 | 814.089 |

TABLE Supp.LIX. Measured cross section as function of $p_{\mu,||}$ on water, in units of $10^{-42}$ cm$^2$/GeV/c/nucleon, and the absolute and fractional cross section uncertainties

| Bin edges (GeV/c) | $\sigma$ | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 1.5-3.0 | 136.29 | 60.24 | 0.389 | 0.210 | 0.035 |
| 3.0-4.0 | 209.53 | 91.74 | 0.357 | 0.253 | 0.046 |
| 4.0-5.0 | 467.09 | 122.37 | 0.211 | 0.155 | 0.041 |
| 5.0-6.0 | 454.55 | 110.72 | 0.192 | 0.149 | 0.037 |
| 6.0-7.0 | 332.04 | 82.82 | 0.211 | 0.134 | 0.034 |
| 7.0-8.5 | 150.87 | 44.38 | 0.239 | 0.171 | 0.032 |
| 8.5-10.0 | 44.53 | 16.29 | 0.319 | 0.180 | 0.031 |
| 10.0-20.0 | 6.13 | 2.91 | 0.436 | 0.189 | 0.037 |

TABLE Supp.LX. Statistical covariance matrix of measured cross section as function of $p_{\mu,||}$ on water, in units of $10^{-84}$ (cm$^2$/GeV/c/nucleon)$^2$

| Bin edges (GeV/c) | 1.5-3.0 | 3.0-4.0 | 4.0-5.0 | 5.0-6.0 | 6.0-7.0 | 7.0-8.5 | 8.5-10.0 | 10.0-20.0 |
|---|---|---|---|---|---|---|---|---|
| 1.5-3.0 | 2806.437 | 504.890 | 73.486 | 37.165 | 29.685 | 10.703 | 9.867 | 0.011 |
| 3.0-4.0 | 504.890 | 5594.629 | 2192.711 | 520.469 | 52.881 | 35.865 | 3.809 | 1.132 |
| 4.0-5.0 | 73.486 | 2192.711 | 9743.624 | 3427.026 | 667.524 | 136.501 | 30.407 | 2.822 |
| 5.0-6.0 | 37.165 | 520.469 | 3427.026 | 7644.453 | 2938.764 | 447.993 | 53.709 | 7.692 |
| 6.0-7.0 | 29.685 | 52.881 | 667.524 | 2938.764 | 4887.686 | 1195.760 | 81.566 | 1.344 |
| 7.0-8.5 | 10.703 | 35.865 | 136.501 | 447.993 | 1195.760 | 1302.963 | 209.148 | 9.855 |
| 8.5-10.0 | 9.867 | 3.809 | 30.407 | 53.709 | 81.566 | 209.148 | 201.152 | 20.494 |
| 10.0-20.0 | 0.011 | 1.132 | 2.822 | 7.692 | 1.344 | 9.855 | 20.494 | 7.146 |

TABLE Supp.LXI. Systematic covariance matrix of measured cross section in $p_{\mu,||}$ on water, in units of $10^{-84}$ $(\mathrm{cm}^2/\mathrm{GeV/c/nucleon})^2$

| Bin edges (GeV/c) | 1.5-3.0 | 3.0-4.0 | 4.0-5.0 | 5.0-6.0 | 6.0-7.0 | 7.0-8.5 | 8.5-10.0 | 10.0-20.0 |
|---|---|---|---|---|---|---|---|---|
| 1.5-3.0 | 822.289 | 462.724 | 1028.704 | 801.765 | 665.651 | 470.679 | 127.720 | 9.731 |
| 3.0-4.0 | 462.724 | 2820.764 | 2891.273 | 2824.462 | 1605.847 | 795.496 | 223.430 | 27.984 |
| 4.0-5.0 | 1028.704 | 2891.273 | 5230.407 | 4401.956 | 2668.438 | 1195.644 | 289.928 | 49.303 |
| 5.0-6.0 | 801.765 | 2824.462 | 4401.956 | 4615.427 | 2549.872 | 1005.696 | 294.413 | 50.238 |
| 6.0-7.0 | 665.651 | 1605.847 | 2668.438 | 2549.872 | 1972.173 | 891.953 | 230.546 | 34.929 |
| 7.0-8.5 | 470.679 | 795.496 | 1195.644 | 1005.696 | 891.953 | 666.928 | 168.923 | 17.475 |
| 8.5-10.0 | 127.720 | 223.430 | 289.928 | 294.413 | 230.546 | 168.923 | 64.052 | 3.808 |
| 10.0-20.0 | 9.731 | 27.984 | 49.303 | 50.238 | 34.929 | 17.475 | 3.808 | 1.345 |

TABLE Supp.LXII. Measured cross section as function of $p_{\mu,T}$ on water, in units of $10^{-42}$ $\mathrm{cm}^2/\mathrm{GeV/c/nucleon}$, and the absolute and fractional cross section uncertainties

| Bin edges (GeV/c) | $\sigma$ | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 0.00-0.15 | 427.07 | 156.83 | 0.350 | 0.110 | 0.031 |
| 0.15-0.30 | 1552.75 | 434.21 | 0.252 | 0.122 | 0.030 |
| 0.30-0.45 | 3698.80 | 892.81 | 0.212 | 0.115 | 0.035 |
| 0.45-0.60 | 3113.42 | 1107.87 | 0.288 | 0.209 | 0.035 |
| 0.60-0.75 | 762.06 | 764.44 | 0.739 | 0.678 | 0.047 |
| 0.75-0.90 | 2313.61 | 1115.44 | 0.419 | 0.239 | 0.032 |
| 0.90-1.25 | 1015.62 | 585.68 | 0.541 | 0.201 | 0.038 |
| 1.25-2.50 | 61.79 | 101.64 | 0.926 | 1.360 | 0.066 |

TABLE Supp.LXIII. Statistical covariance matrix of measured cross section as function of $p_{\mu,T}$ on water, in units of $10^{-84}$ $(\mathrm{cm}^2/\mathrm{GeV/c/nucleon})^2$

| Bin edges (GeV/c) | 0.00-0.15 | 0.15-0.30 | 0.30-0.45 | 0.45-0.60 | 0.60-0.75 | 0.75-0.90 | 0.90-1.25 | 1.25-2.50 |
|---|---|---|---|---|---|---|---|---|
| 0.00-0.15 | 22384.290 | 309.360 | -7344.185 | -1230.080 | -94.202 | -98.135 | -6.266 | 1.649 |
| 0.15-0.30 | 309.360 | 152790.960 | 14256.803 | -30091.672 | -6430.353 | -2133.393 | -3061.313 | -32.386 |
| 0.30-0.45 | -7344.185 | 14256.803 | 617215.664 | 92003.011 | -20825.426 | -38780.595 | -3925.249 | -1075.423 |
| 0.45-0.60 | -1230.080 | -30091.672 | 92003.011 | 805716.499 | 171697.794 | -44210.017 | -7286.457 | 778.975 |
| 0.60-0.75 | -94.202 | -6430.353 | -20825.426 | 171697.794 | 317423.898 | 241253.824 | 4577.656 | -1266.640 |
| 0.75-0.90 | -98.135 | -2133.393 | -38780.595 | -44210.017 | 241253.824 | 938404.156 | 35594.495 | -1501.990 |
| 0.90-1.25 | -6.266 | -3061.313 | -3925.249 | -7286.457 | 4577.656 | 35594.495 | 301365.386 | 99.880 |
| 1.25-2.50 | 1.649 | -32.386 | -1075.423 | 778.975 | -1266.640 | -1501.990 | 99.880 | 3273.325 |

TABLE Supp.LXIV. Systematic covariance matrix of measured cross section in $p_{\mu,T}$ on water, in units of $10^{-84}$ (cm$^2$/GeV/c/nucleon)$^2$

| Bin edges (GeV/c) | 0.00-0.15 | 0.15-0.30 | 0.30-0.45 | 0.45-0.60 | 0.60-0.75 | 0.75-0.90 | 0.90-1.25 | 1.25-2.50 |
|---|---|---|---|---|---|---|---|---|
| 0.00-0.15 | 2212.331 | 7014.185 | 6289.850 | 3963.270 | -1784.760 | 340.915 | 597.445 | -91.941 |
| 0.15-0.30 | 7014.185 | 35749.178 | 50405.043 | 58067.522 | 33792.236 | 35975.418 | 6890.791 | 1602.846 |
| 0.30-0.45 | 6289.850 | 50405.043 | 179890.379 | 203145.934 | 115557.035 | 120354.622 | 27245.951 | 10487.937 |
| 0.45-0.60 | 3963.270 | 58067.522 | 203145.934 | 421666.486 | 253933.394 | 248251.930 | 32377.946 | 25258.683 |
| 0.60-0.75 | -1784.760 | 33792.236 | 115557.035 | 253933.394 | 266938.915 | 244228.251 | 45765.171 | 16013.665 |
| 0.75-0.90 | 340.915 | 35975.418 | 120354.622 | 248251.930 | 244228.251 | 305802.129 | 47834.834 | 14707.867 |
| 0.90-1.25 | 597.445 | 6890.791 | 27245.951 | 32377.946 | 45765.171 | 47834.834 | 41657.051 | 893.402 |
| 1.25-2.50 | -91.941 | 1602.846 | 10487.937 | 25258.683 | 16013.665 | 14707.867 | 893.402 | 7058.065 |

TABLE Supp.LXV. Measured cross section as function of $Q^2$ on water, in units of $10^{-42}$ $\mathrm{cm}^2/\mathrm{GeV}^2/c^2$/nucleon, and the absolute and fractional cross section uncertainties

| Bin edges ($\mathrm{GeV}^2/c^2$) | $\sigma$ | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 0.000-0.025 | 2480.15 | 898.11 | 0.346 | 0.106 | 0.031 |
| 0.025-0.050 | 2698.62 | 1003.28 | 0.330 | 0.171 | 0.033 |
| 0.050-0.100 | 3608.06 | 1040.14 | 0.268 | 0.106 | 0.028 |
| 0.100-0.200 | 4573.34 | 1179.68 | 0.224 | 0.127 | 0.034 |
| 0.200-0.300 | 3545.20 | 1057.43 | 0.253 | 0.158 | 0.034 |
| 0.300-1.300 | 638.77 | 323.90 | 0.375 | 0.341 | 0.038 |
| 1.300-2.000 | 563.66 | 378.38 | 0.613 | 0.275 | 0.035 |
| 2.000-3.000 | 60.16 | 107.17 | 0.885 | 1.546 | 0.072 |

TABLE Supp.LXVI. Statistical covariance matrix of measured cross section as function of $Q^2$ on water, in units of $10^{-84}$ $(\mathrm{cm}^2/\mathrm{GeV}^2/c^2/\mathrm{nucleon})^2$

| Bin edges ($\mathrm{GeV}^2/c^2$) | 0.000-0.025 | 0.025-0.050 | 0.050-0.100 | 0.100-0.200 | 0.200-0.300 | 0.300-1.300 | 1.300-2.000 | 2.000-3.000 |
|---|---|---|---|---|---|---|---|---|
| 0.000-0.025 | 737910.311 | 134711.025 | -89360.998 | -31088.514 | -2618.557 | -324.926 | -0.770 | -1.189 |
| 0.025-0.050 | 134711.025 | 793841.706 | 333689.764 | -89325.086 | -44223.884 | -952.144 | -494.547 | -5.120 |
| 0.050-0.100 | -89360.998 | 333689.764 | 936646.336 | 126010.540 | -86362.695 | -7058.249 | -2918.545 | -90.130 |
| 0.100-0.200 | -31088.514 | -89325.086 | 126010.540 | 1051659.826 | 307358.275 | -24755.356 | -6781.373 | -690.978 |
| 0.200-0.300 | -2618.557 | -44223.884 | -86362.695 | 307358.275 | 803030.467 | 25543.239 | -13033.818 | 172.523 |
| 0.300-1.300 | -324.926 | -952.144 | -7058.249 | -24755.356 | 25543.239 | 57416.252 | 1624.192 | -345.051 |
| 1.300-2.000 | -0.770 | -494.547 | -2918.545 | -6781.373 | -13033.818 | 1624.192 | 119223.261 | 93.238 |
| 2.000-3.000 | -1.189 | -5.120 | -90.130 | -690.978 | 172.523 | -345.051 | 93.238 | 2834.882 |

TABLE Supp.LXVII. Systematic covariance matrix of measured cross section in $Q^2$ on water, in units of $10^{-84}$ $(\mathrm{cm}^2/\mathrm{GeV}^2/c^2/\mathrm{nucleon})^2$

| Bin edges ($\mathrm{GeV}^2/c^2$) | 0.000-0.025 | 0.025-0.050 | 0.050-0.100 | 0.100-0.200 | 0.200-0.300 | 0.300-1.300 | 1.300-2.000 | 2.000-3.000 |
|---|---|---|---|---|---|---|---|---|
| 0.000-0.025 | 68685.777 | 99092.605 | 67287.625 | 59917.695 | 30844.096 | -1265.600 | 8783.647 | -575.413 |
| 0.025-0.050 | 99092.605 | 212727.324 | 129946.537 | 151239.000 | 86775.289 | 14457.069 | 1406.976 | -268.952 |
| 0.050-0.100 | 67287.625 | 129946.537 | 145254.442 | 131364.523 | 127635.806 | 43360.984 | 15481.951 | 5821.559 |
| 0.100-0.200 | 59917.695 | 151239.000 | 131364.523 | 339983.217 | 241123.764 | 58008.934 | 7571.832 | 14480.329 |
| 0.200-0.300 | 30844.096 | 86775.289 | 127635.806 | 241123.764 | 315118.422 | 84308.069 | 15162.848 | 18844.419 |
| 0.300-1.300 | -1265.600 | 14457.069 | 43360.984 | 58008.934 | 84308.069 | 47496.488 | 14950.425 | 7426.101 |
| 1.300-2.000 | 8783.647 | 1406.976 | 15481.951 | 7571.832 | 15162.848 | 14950.425 | 23946.121 | 3237.185 |
| 2.000-3.000 | -575.413 | -268.952 | 5821.559 | 14480.329 | 18844.419 | 7426.101 | 3237.185 | 8651.566 |

TABLE Supp.LXVIII. Measured cross section as function of $W_{exp}$ on water, in units of $10^{-42}$ $cm^2/GeV/c^2$/nucleon, and the absolute and fractional cross section uncertainties

| Bin edges ($GeV/c^2$) | $\sigma$ | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 0.9-1.1 | 1604.24 | 1413.62 | 0.847 | 0.242 | 0.037 |
| 1.1-1.2 | 7771.74 | 1887.69 | 0.206 | 0.129 | 0.034 |
| 1.2-1.3 | 5069.96 | 2456.47 | 0.368 | 0.315 | 0.033 |
| 1.3-1.4 | 1635.65 | 1510.53 | 0.665 | 0.641 | 0.035 |

TABLE Supp.LXIX. Statistical covariance matrix of measured cross section as function of $W_{exp}$ on water, in units of $10^{-84}$ $(cm^2/GeV/c^2/nucleon)^2$

| Bin edges ($GeV/c^2$) | 0.9-1.1 | 1.1-1.2 | 1.2-1.3 | 1.3-1.4 |
|---|---|---|---|---|
| 0.9-1.1 | 1847344.362 | -656709.599 | -543616.547 | -214453.239 |
| 1.1-1.2 | -656709.599 | 2563274.464 | -922303.488 | -458316.063 |
| 1.2-1.3 | -543616.547 | -922303.488 | 3484780.550 | 1620816.512 |
| 1.3-1.4 | -214453.239 | -458316.063 | 1620816.512 | 1183090.426 |

TABLE Supp.LXX. Systematic covariance matrix of measured cross section in $W_{exp}$ on water, in units of $10^{-84}$ $(cm^2/GeV/c^2/nucleon)^2$

| Bin edges ($GeV/c^2$) | 0.9-1.1 | 1.1-1.2 | 1.2-1.3 | 1.3-1.4 |
|---|---|---|---|---|
| 0.9-1.1 | 150974.241 | 50687.062 | 58336.608 | 39527.718 |
| 1.1-1.2 | 50687.062 | 1000092.261 | 133191.537 | -6137.137 |
| 1.2-1.3 | 58336.608 | 133191.537 | 2549465.255 | 1600707.642 |
| 1.3-1.4 | 39527.718 | -6137.137 | 1600707.642 | 1098601.315 |

TABLE Supp.LXXI. Measured cross section as function of $T_\pi$ on water, in units of $10^{-42}$ cm$^2$/GeV/nucleon, and the absolute and fractional cross section uncertainties

| Bin edges (GeV) | $\sigma$ | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 0.035-0.100 | 14951.68 | 3806.97 | 0.227 | 0.116 | 0.032 |
| 0.100-0.150 | 8200.70 | 1951.07 | 0.194 | 0.137 | 0.035 |
| 0.150-0.200 | 6207.98 | 1941.76 | 0.264 | 0.168 | 0.034 |
| 0.200-0.350 | 2966.39 | 1310.43 | 0.348 | 0.272 | 0.033 |

TABLE Supp.LXXII. Statistical covariance matrix of measured cross section as function of $T_\pi$ on water, in units of $10^{-84}$ (cm$^2$/GeV/nucleon)$^2$

| Bin edges (GeV) | 0.035-0.100 | 0.100-0.150 | 0.150-0.200 | 0.200-0.350 |
|---|---|---|---|---|
| 0.035-0.100 | 11487318.614 | 868059.738 | -673181.424 | -199663.456 |
| 0.100-0.150 | 868059.738 | 2541618.601 | 323855.531 | -105323.874 |
| 0.150-0.200 | -673181.424 | 323855.531 | 2681915.718 | 246201.197 |
| 0.200-0.350 | -199663.456 | -105323.874 | 246201.197 | 1065321.825 |

TABLE Supp.LXXIII. Systematic covariance matrix of measured cross section in $T_\pi$ on water, in units of $10^{-84}$ (cm$^2$/GeV/nucleon)$^2$

| Bin edges (GeV) | 0.035-0.100 | 0.100-0.150 | 0.150-0.200 | 0.200-0.350 |
|---|---|---|---|---|
| 0.035-0.100 | 3005725.726 | 1539098.927 | 915615.078 | 671957.761 |
| 0.100-0.150 | 1539098.927 | 1265067.788 | 828750.051 | 606538.898 |
| 0.150-0.200 | 915615.078 | 828750.051 | 1088524.206 | 692340.688 |
| 0.200-0.350 | 671957.761 | 606538.898 | 692340.688 | 651915.142 |

TABLE Supp.LXXIV. Measured cross section as function of $\theta_\pi$ on water, in units of $10^{-42}$ cm$^2$/degree/nucleon, and the absolute and fractional cross section uncertainties

| Bin edges (degree) | $\sigma$ | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 0-10 | 3.73 | 3.47 | 0.897 | 0.251 | 0.036 |
| 10-20 | 6.29 | 3.35 | 0.480 | 0.232 | 0.032 |
| 20-30 | 11.59 | 4.24 | 0.306 | 0.201 | 0.037 |
| 30-40 | 13.14 | 5.19 | 0.324 | 0.227 | 0.035 |
| 40-50 | 19.07 | 5.60 | 0.258 | 0.139 | 0.032 |
| 50-60 | 31.04 | 8.12 | 0.241 | 0.101 | 0.033 |
| 60-120 | 20.46 | 6.10 | 0.279 | 0.106 | 0.035 |
| 120-135 | 10.15 | 5.48 | 0.502 | 0.198 | 0.035 |
| 135-150 | 7.81 | 3.50 | 0.428 | 0.134 | 0.029 |
| 150-165 | 2.83 | 2.64 | 0.851 | 0.380 | 0.030 |
| 165-180 | 0.34 | 0.50 | 1.418 | 0.484 | 0.036 |

TABLE Supp.LXXV. Statistical covariance matrix of measured cross section as function of $\theta_\pi$ on water, in units of $10^{-84}$ $(\mathrm{cm}^2/\mathrm{degree/nucleon})^2$

| Bin edges (degree) | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 | 60-120 | 120-135 | 135-150 | 150-165 | 165-180 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0-10 | 11.170 | 2.498 | -1.025 | 0.144 | -0.368 | -0.392 | -0.315 | -0.144 | -0.110 | -0.033 | 0.073 |
| 10-20 | 2.498 | 9.112 | 2.790 | -0.615 | -0.690 | 0.290 | -1.038 | -0.391 | 0.180 | -0.065 | 0.913 |
| 20-30 | -1.025 | 2.790 | 12.590 | 2.532 | -0.610 | -1.929 | -0.328 | -0.119 | -0.338 | -0.210 | -0.030 |
| 30-40 | 0.144 | -0.615 | 2.532 | 18.112 | 4.371 | -1.711 | 1.058 | -1.156 | -0.422 | -0.142 | -0.096 |
| 40-50 | -0.368 | -0.690 | -0.610 | 4.371 | 24.296 | 7.335 | 0.049 | -0.432 | 0.437 | -0.855 | -0.024 |
| 50-60 | -0.392 | 0.290 | -1.929 | -1.711 | 7.335 | 55.983 | 12.473 | -1.267 | -2.303 | -1.089 | 0.105 |
| 60-120 | -0.315 | -1.038 | -0.328 | 1.058 | 0.049 | 12.473 | 32.466 | 2.691 | 0.102 | 0.512 | -0.095 |
| 120-135 | -0.144 | -0.391 | -0.119 | -1.156 | -0.432 | -1.267 | 2.691 | 25.957 | 4.903 | 0.174 | -0.033 |
| 135-150 | -0.110 | 0.180 | -0.338 | -0.422 | 0.437 | -2.303 | 0.102 | 4.903 | 11.147 | 4.975 | 0.011 |
| 150-165 | -0.033 | -0.065 | -0.210 | -0.142 | -0.855 | -1.089 | 0.512 | 0.174 | 4.975 | 5.814 | -0.011 |
| 165-180 | 0.073 | 0.913 | -0.030 | -0.096 | -0.024 | 0.105 | -0.095 | -0.033 | 0.011 | -0.011 | 0.226 |

TABLE Supp.LXXVI. Systematic covariance matrix of measured cross section in $\theta_\pi$ on water, in units of $10^{-84}$ (cm$^2$/degree/nucleon)$^2$

| Bin edges (degree) | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 | 60-120 | 120-135 | 135-150 | 150-165 | 165-180 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0-10 | 0.874 | 0.417 | 0.892 | 1.532 | 1.083 | 0.835 | 0.382 | 0.989 | 0.189 | -0.067 | 0.056 |
| 10-20 | 0.417 | 2.131 | 2.779 | 2.292 | 2.270 | 2.104 | 1.702 | 1.130 | 0.512 | 0.455 | 0.120 |
| 20-30 | 0.892 | 2.779 | 5.418 | 4.935 | 4.810 | 4.863 | 3.200 | 2.073 | 1.203 | 1.039 | 0.166 |
| 30-40 | 1.532 | 2.292 | 4.935 | 8.866 | 6.480 | 4.936 | 3.897 | 1.623 | 1.690 | 0.872 | 0.117 |
| 40-50 | 1.083 | 2.270 | 4.810 | 6.480 | 7.040 | 6.230 | 4.455 | 2.552 | 1.932 | 1.482 | 0.155 |
| 50-60 | 0.835 | 2.104 | 4.863 | 4.936 | 6.230 | 9.898 | 4.975 | 3.563 | 1.863 | 2.035 | 0.116 |
| 60-120 | 0.382 | 1.702 | 3.200 | 3.897 | 4.455 | 4.975 | 4.694 | 1.217 | 1.618 | 1.141 | 0.036 |
| 120-135 | 0.989 | 1.130 | 2.073 | 1.623 | 2.552 | 3.563 | 1.217 | 4.038 | 0.640 | 0.662 | 0.127 |
| 135-150 | 0.189 | 0.512 | 1.203 | 1.690 | 1.932 | 1.863 | 1.618 | 0.640 | 1.099 | 0.818 | -0.040 |
| 150-165 | -0.067 | 0.455 | 1.039 | 0.872 | 1.482 | 2.035 | 1.141 | 0.662 | 0.818 | 1.159 | -0.015 |
| 165-180 | 0.056 | 0.120 | 0.166 | 0.117 | 0.155 | 0.116 | 0.036 | 0.127 | -0.040 | -0.015 | 0.026 |

TABLE Supp.LXXVII. Measured cross section as function of $p_\mu$ on iron, in units of $10^{-42}$ cm$^2$/GeV/c/nucleon, and the absolute and fractional cross section uncertainties

| Bin edges (GeV/c) | $\sigma$ | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 1.5-3.0 | 116.83 | 22.82 | 0.152 | 0.123 | 0.039 |
| 3.0-4.0 | 268.85 | 46.29 | 0.106 | 0.136 | 0.041 |
| 4.0-5.0 | 301.84 | 55.47 | 0.097 | 0.156 | 0.045 |
| 5.0-6.0 | 322.48 | 57.36 | 0.088 | 0.155 | 0.043 |
| 6.0-7.0 | 236.20 | 43.60 | 0.092 | 0.160 | 0.038 |
| 7.0-8.5 | 104.17 | 22.50 | 0.117 | 0.182 | 0.037 |
| 8.5-10.0 | 35.62 | 8.17 | 0.141 | 0.181 | 0.037 |
| 10.0-20.0 | 8.56 | 1.97 | 0.165 | 0.160 | 0.036 |

TABLE Supp.LXXVIII. Statistical covariance matrix of measured cross section as function of $p_\mu$ on iron, in units of $10^{-84}$ (cm$^2$/GeV/c/nucleon)$^2$

| Bin edges (GeV/c) | 1.5-3.0 | 3.0-4.0 | 4.0-5.0 | 5.0-6.0 | 6.0-7.0 | 7.0-8.5 | 8.5-10.0 | 10.0-20.0 |
|---|---|---|---|---|---|---|---|---|
| 1.5-3.0 | 315.869 | 105.314 | 22.345 | 15.804 | 2.906 | 0.259 | 1.159 | 0.243 |
| 3.0-4.0 | 105.314 | 814.581 | 220.469 | 53.732 | 12.553 | 6.185 | 1.503 | 0.499 |
| 4.0-5.0 | 22.345 | 220.469 | 849.305 | 333.122 | 68.504 | 9.452 | 1.780 | 0.440 |
| 5.0-6.0 | 15.804 | 53.732 | 333.122 | 797.696 | 292.874 | 34.524 | 5.772 | 0.967 |
| 6.0-7.0 | 2.906 | 12.553 | 68.504 | 292.874 | 474.114 | 123.750 | 15.375 | 1.503 |
| 7.0-8.5 | 0.259 | 6.185 | 9.452 | 34.524 | 123.750 | 148.134 | 32.875 | 2.247 |
| 8.5-10.0 | 1.159 | 1.503 | 1.780 | 5.772 | 15.375 | 32.875 | 25.134 | 2.980 |
| 10.0-20.0 | 0.243 | 0.499 | 0.440 | 0.967 | 1.503 | 2.247 | 2.980 | 2.002 |

TABLE Supp.LXXIX. Systematic covariance matrix of measured cross section in $p_\mu$ on iron, in units of $10^{-84}$ $(\mathrm{cm}^2/\mathrm{GeV/c/nucleon})^2$

| Bin edges (GeV/c) | 1.5-3.0 | 3.0-4.0 | 4.0-5.0 | 5.0-6.0 | 6.0-7.0 | 7.0-8.5 | 8.5-10.0 | 10.0-20.0 |
|---|---|---|---|---|---|---|---|---|
| 1.5-3.0 | 205.057 | 411.198 | 522.643 | 525.671 | 357.844 | 147.293 | 50.865 | 11.924 |
| 3.0-4.0 | 411.198 | 1328.219 | 1622.646 | 1716.459 | 1223.345 | 569.854 | 192.684 | 40.675 |
| 4.0-5.0 | 522.643 | 1622.646 | 2228.071 | 2298.227 | 1665.093 | 757.015 | 247.266 | 52.499 |
| 5.0-6.0 | 525.671 | 1716.459 | 2298.227 | 2492.373 | 1826.680 | 857.130 | 280.167 | 59.245 |
| 6.0-7.0 | 357.844 | 1223.345 | 1665.093 | 1826.680 | 1426.702 | 679.134 | 220.834 | 47.548 |
| 7.0-8.5 | 147.293 | 569.854 | 757.015 | 857.130 | 679.134 | 358.258 | 117.038 | 23.759 |
| 8.5-10.0 | 50.865 | 192.684 | 247.266 | 280.167 | 220.834 | 117.038 | 41.591 | 8.067 |
| 10.0-20.0 | 11.924 | 40.675 | 52.499 | 59.245 | 47.548 | 23.759 | 8.067 | 1.888 |

TABLE Supp.LXXX. Measured cross section as function of $\theta_\mu$ on iron, in units of $10^{-42}$ $\mathrm{cm}^2$/degree/nucleon, and the absolute and fractional cross section uncertainties

| Bin edges (degree) | $\sigma$ | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 0-2 | 57.81 | 9.20 | 0.127 | 0.096 | 0.034 |
| 2-3 | 134.08 | 21.30 | 0.128 | 0.094 | 0.036 |
| 3-4 | 170.17 | 29.69 | 0.130 | 0.117 | 0.036 |
| 4-5 | 150.80 | 34.52 | 0.157 | 0.167 | 0.037 |
| 5-6 | 128.51 | 41.05 | 0.203 | 0.247 | 0.039 |
| 6-8 | 163.18 | 39.09 | 0.139 | 0.195 | 0.041 |
| 8-10 | 150.52 | 40.93 | 0.175 | 0.208 | 0.045 |
| 10-13 | 97.89 | 37.08 | 0.268 | 0.268 | 0.053 |

TABLE Supp.LXXXI. Statistical covariance matrix of measured cross section as function of $\theta_\mu$ on iron, in units of $10^{-84}$ (cm$^2$/degree/nucleon)$^2$

| Bin edges (degree) | 0-2 | 2-3 | 3-4 | 4-5 | 5-6 | 6-8 | 8-10 | 10-13 |
|---|---|---|---|---|---|---|---|---|
| 0-2 | 54.038 | 1.548 | -13.083 | -2.225 | 0.045 | 0.009 | -0.008 | -0.560 |
| 2-3 | 1.548 | 294.722 | 47.593 | -35.823 | -9.843 | -0.361 | -0.634 | -1.134 |
| 3-4 | -13.083 | 47.593 | 486.432 | 77.132 | -48.142 | -8.617 | -0.080 | -1.852 |
| 4-5 | -2.225 | -35.823 | 77.132 | 560.801 | 117.294 | -35.313 | -5.619 | 0.001 |
| 5-6 | 0.045 | -9.843 | -48.142 | 117.294 | 680.720 | 38.411 | -31.330 | -1.575 |
| 6-8 | 0.009 | -0.361 | -8.617 | -35.313 | 38.411 | 516.629 | -8.166 | -21.606 |
| 8-10 | -0.008 | -0.634 | -0.080 | -5.619 | -31.330 | -8.166 | 691.011 | -15.851 |
| 10-13 | -0.560 | -1.134 | -1.852 | 0.001 | -1.575 | -21.606 | -15.851 | 687.894 |

TABLE Supp.LXXXII. Systematic covariance matrix of measured cross section in $\theta_\mu$ on iron, in units of $10^{-84}$ (cm$^2$/degree/nucleon)$^2$

| Bin edges (degree) | 0-2 | 2-3 | 3-4 | 4-5 | 5-6 | 6-8 | 8-10 | 10-13 |
|---|---|---|---|---|---|---|---|---|
| 0-2 | 30.528 | 50.989 | 58.155 | 52.156 | 59.051 | 67.665 | 66.932 | 55.487 |
| 2-3 | 50.989 | 159.060 | 212.872 | 252.762 | 318.832 | 308.256 | 309.224 | 236.358 |
| 3-4 | 58.155 | 212.872 | 395.244 | 445.472 | 555.057 | 563.320 | 551.134 | 454.471 |
| 4-5 | 52.156 | 252.762 | 445.472 | 630.696 | 755.249 | 745.566 | 720.905 | 569.497 |
| 5-6 | 59.051 | 318.832 | 555.057 | 755.249 | 1004.354 | 944.544 | 929.399 | 724.968 |
| 6-8 | 67.665 | 308.256 | 563.320 | 745.566 | 944.544 | 1011.501 | 961.363 | 765.424 |
| 8-10 | 66.932 | 309.224 | 551.134 | 720.905 | 929.399 | 961.363 | 984.233 | 773.472 |
| 10-13 | 55.487 | 236.358 | 454.471 | 569.497 | 724.968 | 765.424 | 773.472 | 686.724 |

TABLE Supp.LXXXIII. Measured cross section as function of $p_{\mu,||}$ on iron, in units of $10^{-42}$ cm$^2$/GeV/c/nucleon, and the absolute and fractional cross section uncertainties

| Bin edges (GeV/c) | $\sigma$ | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 1.5-3.0 | 114.93 | 23.86 | 0.160 | 0.133 | 0.040 |
| 3.0-4.0 | 285.09 | 47.81 | 0.103 | 0.132 | 0.041 |
| 4.0-5.0 | 300.20 | 55.34 | 0.097 | 0.157 | 0.045 |
| 5.0-6.0 | 315.62 | 56.79 | 0.089 | 0.156 | 0.043 |
| 6.0-7.0 | 232.29 | 42.91 | 0.092 | 0.160 | 0.038 |
| 7.0-8.5 | 102.57 | 22.26 | 0.116 | 0.183 | 0.037 |
| 8.5-10.0 | 34.29 | 7.68 | 0.143 | 0.172 | 0.037 |
| 10.0-20.0 | 8.70 | 1.97 | 0.164 | 0.156 | 0.036 |

TABLE Supp.LXXXIV. Statistical covariance matrix of measured cross section as function of $p_{\mu,||}$ on iron, in units of $10^{-84}$ (cm$^2$/GeV/c/nucleon)$^2$

| Bin edges (GeV/c) | 1.5-3.0 | 3.0-4.0 | 4.0-5.0 | 5.0-6.0 | 6.0-7.0 | 7.0-8.5 | 8.5-10.0 | 10.0-20.0 |
|---|---|---|---|---|---|---|---|---|
| 1.5-3.0 | 336.966 | 105.591 | 21.222 | 16.807 | 2.719 | 0.226 | 1.198 | 0.310 |
| 3.0-4.0 | 105.591 | 860.377 | 225.133 | 54.398 | 11.506 | 5.621 | 1.457 | 0.513 |
| 4.0-5.0 | 21.222 | 225.133 | 852.767 | 332.012 | 69.832 | 9.101 | 1.624 | 0.369 |
| 5.0-6.0 | 16.807 | 54.398 | 332.012 | 789.184 | 285.431 | 34.752 | 6.190 | 0.965 |
| 6.0-7.0 | 2.719 | 11.506 | 69.832 | 285.431 | 456.056 | 120.873 | 15.158 | 1.557 |
| 7.0-8.5 | 0.226 | 5.621 | 9.101 | 34.752 | 120.873 | 142.721 | 30.721 | 2.031 |
| 8.5-10.0 | 1.198 | 1.457 | 1.624 | 6.190 | 15.158 | 30.721 | 24.113 | 2.917 |
| 10.0-20.0 | 0.310 | 0.513 | 0.369 | 0.965 | 1.557 | 2.031 | 2.917 | 2.045 |

TABLE Supp.LXXXV. Systematic covariance matrix of measured cross section in $p_{\mu,||}$ on iron, in units of $10^{-84}$ $(\mathrm{cm}^2/\mathrm{GeV/c/nucleon})^2$

| Bin edges (GeV/c) | 1.5-3.0 | 3.0-4.0 | 4.0-5.0 | 5.0-6.0 | 6.0-7.0 | 7.0-8.5 | 8.5-10.0 | 10.0-20.0 |
|---|---|---|---|---|---|---|---|---|
| 1.5-3.0 | 232.325 | 457.975 | 581.593 | 576.180 | 381.001 | 165.652 | 55.418 | 13.358 |
| 3.0-4.0 | 457.975 | 1425.191 | 1674.184 | 1767.028 | 1267.987 | 602.910 | 194.546 | 43.654 |
| 4.0-5.0 | 581.593 | 1674.184 | 2209.657 | 2242.767 | 1589.613 | 712.797 | 227.146 | 51.563 |
| 5.0-6.0 | 576.180 | 1767.028 | 2242.767 | 2435.665 | 1763.682 | 826.869 | 259.459 | 57.703 |
| 6.0-7.0 | 381.001 | 1267.987 | 1589.613 | 1763.682 | 1385.463 | 664.782 | 203.946 | 46.508 |
| 7.0-8.5 | 165.652 | 602.910 | 712.797 | 826.869 | 664.782 | 352.611 | 106.696 | 23.186 |
| 8.5-10.0 | 55.418 | 194.546 | 227.146 | 259.459 | 203.946 | 106.696 | 34.872 | 7.187 |
| 10.0-20.0 | 13.358 | 43.654 | 51.563 | 57.703 | 46.508 | 23.186 | 7.187 | 1.851 |

TABLE Supp.LXXXVI. Measured cross section as function of $p_{\mu,T}$ on iron, in units of $10^{-42}$ $\mathrm{cm}^2/\mathrm{GeV/c/nucleon}$, and the absolute and fractional cross section uncertainties

| Bin edges (GeV/c) | $\sigma$ | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 0.00-0.15 | 383.24 | 78.77 | 0.175 | 0.108 | 0.036 |
| 0.15-0.30 | 1292.12 | 180.71 | 0.108 | 0.089 | 0.034 |
| 0.30-0.45 | 2057.62 | 342.26 | 0.108 | 0.126 | 0.036 |
| 0.45-0.60 | 2510.77 | 486.56 | 0.108 | 0.161 | 0.038 |
| 0.60-0.75 | 1657.69 | 500.94 | 0.157 | 0.258 | 0.040 |
| 0.75-0.90 | 1144.28 | 456.20 | 0.209 | 0.339 | 0.044 |
| 0.90-1.25 | 537.52 | 222.14 | 0.280 | 0.304 | 0.049 |
| 1.25-2.50 | 30.88 | 29.66 | 0.858 | 0.432 | 0.044 |

TABLE Supp.LXXXVII. Statistical covariance matrix of measured cross section as function of $p_{\mu,T}$ on iron, in units of $10^{-84}$ $(\mathrm{cm}^2/\mathrm{GeV/c/nucleon})^2$

| Bin edges (GeV/c) | 0.00-0.15 | 0.15-0.30 | 0.30-0.45 | 0.45-0.60 | 0.60-0.75 | 0.75-0.90 | 0.90-1.25 | 1.25-2.50 |
|---|---|---|---|---|---|---|---|---|
| 0.00-0.15 | 4508.029 | 200.588 | -1057.726 | -151.445 | -11.493 | -13.006 | -3.431 | -0.124 |
| 0.15-0.30 | 200.588 | 19354.213 | 2200.287 | -3066.130 | -782.930 | -151.086 | -68.478 | -17.264 |
| 0.30-0.45 | -1057.726 | 2200.287 | 49488.545 | 5599.511 | -4144.878 | -1804.099 | -554.261 | -9.056 |
| 0.45-0.60 | -151.445 | -3066.130 | 5599.511 | 73629.427 | 11005.561 | -5077.959 | -1892.165 | -35.771 |
| 0.60-0.75 | -11.493 | -782.930 | -4144.878 | 11005.561 | 67610.981 | 16189.200 | -3072.318 | -34.839 |
| 0.75-0.90 | -13.006 | -151.086 | -1804.099 | -5077.959 | 16189.200 | 57364.592 | 12622.564 | -456.645 |
| 0.90-1.25 | -3.431 | -68.478 | -554.261 | -1892.165 | -3072.318 | 12622.564 | 22709.915 | 626.156 |
| 1.25-2.50 | -0.124 | -17.264 | -9.056 | -35.771 | -34.839 | -456.645 | 626.156 | 701.992 |

TABLE Supp.LXXXVIII. Systematic covariance matrix of measured cross section in $p_{\mu,T}$ on iron, in units of $10^{-84}$ (cm$^2$/GeV/c/nucleon)$^2$

| Bin edges (GeV/c) | 0.00-0.15 | 0.15-0.30 | 0.30-0.45 | 0.45-0.60 | 0.60-0.75 | 0.75-0.90 | 0.90-1.25 | 1.25-2.50 |
|---|---|---|---|---|---|---|---|---|
| 0.00-0.15 | 1697.438 | 3083.106 | 1352.142 | 982.632 | 1042.083 | -44.294 | 526.367 | 59.871 |
| 0.15-0.30 | 3083.106 | 13303.600 | 22528.156 | 32586.614 | 29441.668 | 25380.953 | 9590.877 | 498.537 |
| 0.30-0.45 | 1352.142 | 22528.156 | 67655.950 | 99661.336 | 101743.975 | 87615.854 | 33418.402 | 1559.410 |
| 0.45-0.60 | 982.632 | 32586.614 | 99661.336 | 163108.310 | 160333.855 | 143991.058 | 52913.450 | 2540.412 |
| 0.60-0.75 | 1042.083 | 29441.668 | 101743.975 | 160333.855 | 183327.323 | 157677.680 | 64102.260 | 3163.920 |
| 0.75-0.90 | -44.294 | 25380.953 | 87615.854 | 143991.058 | 157677.680 | 150753.848 | 59087.249 | 3359.485 |
| 0.90-1.25 | 526.367 | 9590.877 | 33418.402 | 52913.450 | 64102.260 | 59087.249 | 26635.988 | 1593.865 |
| 1.25-2.50 | 59.871 | 498.537 | 1559.410 | 2540.412 | 3163.920 | 3359.485 | 1593.865 | 177.810 |

TABLE Supp.LXXXIX. Measured cross section as function of $Q^2$ on iron, in units of $10^{-42}$ $\mathrm{cm}^2/\mathrm{GeV}^2/c^2$/nucleon, and the absolute and fractional cross section uncertainties

| Bin edges ($\mathrm{GeV}^2/c^2$) | $\sigma$ | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 0.000-0.025 | 2203.06 | 440.60 | 0.169 | 0.108 | 0.036 |
| 0.025-0.050 | 2393.39 | 403.92 | 0.143 | 0.090 | 0.036 |
| 0.050-0.100 | 2621.21 | 406.66 | 0.124 | 0.094 | 0.034 |
| 0.100-0.200 | 2548.13 | 407.20 | 0.106 | 0.120 | 0.036 |
| 0.200-0.300 | 2512.02 | 439.28 | 0.120 | 0.127 | 0.037 |
| 0.300-0.400 | 1831.97 | 413.22 | 0.137 | 0.179 | 0.038 |
| 0.400-0.500 | 1192.24 | 352.42 | 0.168 | 0.243 | 0.040 |
| 0.500-0.700 | 857.58 | 322.78 | 0.198 | 0.320 | 0.042 |
| 0.700-1.000 | 571.66 | 224.01 | 0.223 | 0.323 | 0.044 |
| 1.000-1.300 | 314.24 | 136.57 | 0.325 | 0.289 | 0.048 |
| 1.300-2.000 | 70.95 | 65.02 | 0.684 | 0.610 | 0.075 |
| 2.000-3.000 | 29.19 | 30.93 | 0.929 | 0.509 | 0.047 |

TABLE Supp.XC. Statistical covariance matrix of measured cross section as function of $Q^2$ on iron, in units of $10^{-84}$ $(\mathrm{cm}^2/\mathrm{GeV}^2/c^2/\mathrm{nucleon})^2$

| Bin edges ($\mathrm{GeV}^2/c^2$) | 0.000-0.025 | 0.025-0.050 | 0.050-0.100 | 0.100-0.200 | 0.200-0.300 | 0.300-0.400 | 0.400-0.500 | 0.500-0.700 | 0.700-1.000 | 1.000-1.300 | 1.300-2.000 | 2.000-3.000 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.000-0.025 | 137801.601 | 29581.907 | -13084.445 | -4487.854 | -351.306 | -215.348 | -2.885 | -7.405 | -35.435 | -7.505 | -0.332 | 0.429 |
| 0.025-0.050 | 29581.907 | 116449.941 | 35811.441 | -7818.177 | -4115.050 | -518.530 | -213.727 | -80.378 | -16.845 | -31.024 | -30.061 | -9.789 |
| 0.050-0.100 | -13084.445 | 35811.441 | 105091.742 | 16546.048 | -9383.267 | -3935.939 | -1127.083 | -283.563 | -95.588 | -32.006 | -96.217 | -56.591 |
| 0.100-0.200 | -4487.854 | -7818.177 | 16546.048 | 72901.198 | 16065.321 | -4398.396 | -4091.900 | -1297.178 | -553.793 | -505.000 | 45.509 | 40.002 |
| 0.200-0.300 | -351.306 | -4115.050 | -9383.267 | 16065.321 | 91346.779 | 24554.767 | 187.068 | -3523.817 | -1354.187 | -687.084 | -330.202 | -250.335 |
| 0.300-0.400 | -215.348 | -518.530 | -3935.939 | -4398.396 | 24554.767 | 63156.008 | 22886.304 | -1436.591 | -2212.579 | -645.433 | -61.612 | 260.211 |
| 0.400-0.500 | -2.885 | -213.727 | -1127.083 | -4091.900 | 187.068 | 22886.304 | 40333.847 | 13802.060 | -1188.428 | -1911.245 | -151.578 | 107.033 |
| 0.500-0.700 | -7.405 | -80.378 | -283.563 | -1297.178 | -3523.817 | -1436.591 | 13802.060 | 28940.832 | 6654.313 | -614.375 | -178.851 | -102.142 |
| 0.700-1.000 | -35.435 | -16.845 | -95.588 | -553.793 | -1354.187 | -2212.579 | -1188.428 | 6654.313 | 16190.031 | 6244.640 | 105.428 | -42.257 |
| 1.000-1.300 | -7.505 | -31.024 | -32.006 | -505.000 | -687.084 | -645.433 | -1911.245 | -614.375 | 6244.640 | 10412.006 | 1945.571 | -12.112 |
| 1.300-2.000 | -0.332 | -30.061 | -96.217 | 45.509 | -330.202 | -61.612 | -151.578 | -178.851 | 105.428 | 1945.571 | 2354.592 | 478.895 |
| 2.000-3.000 | 0.429 | -9.789 | -56.591 | 40.002 | -250.335 | 260.211 | 107.033 | -102.142 | -42.257 | -12.112 | 478.895 | 735.750 |

TABLE Supp.XCI. Systematic covariance matrix of measured cross section in $Q^2$ on iron, in units of $10^{-84}$ $(\mathrm{cm}^2/\mathrm{GeV}^2/c^2/\mathrm{nucleon})^2$

| Bin edges ($\mathrm{GeV}^2/c^2$) | 0.000-0.025 | 0.025-0.050 | 0.050-0.100 | 0.100-0.200 | 0.200-0.300 | 0.300-0.400 | 0.400-0.500 | 0.500-0.700 | 0.700-1.000 | 1.000-1.300 | 1.300-2.000 | 2.000-3.000 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.000-0.025 | 56330.153 | 41854.391 | 28615.528 | 11777.222 | 8987.773 | 2808.922 | 3952.899 | 1414.203 | -441.422 | 1719.485 | 404.092 | -120.953 |
| 0.025-0.050 | 41854.391 | 46697.584 | 43841.747 | 39293.903 | 35734.056 | 34708.185 | 28744.777 | 26472.560 | 16308.957 | 7832.234 | 2691.034 | 871.441 |
| 0.050-0.100 | 28615.528 | 43841.747 | 60283.640 | 67369.854 | 63426.373 | 62541.414 | 53859.168 | 50007.653 | 31908.075 | 14771.552 | 5183.326 | 1710.945 |
| 0.100-0.200 | 11777.222 | 39293.903 | 67369.854 | 92910.828 | 88557.671 | 89883.663 | 78305.677 | 73333.008 | 45901.172 | 20886.720 | 7572.366 | 2500.306 |
| 0.200-0.300 | 8987.773 | 35734.056 | 63426.373 | 88557.671 | 101623.257 | 98829.623 | 86403.079 | 79752.621 | 51111.939 | 22341.626 | 8140.579 | 2902.191 |
| 0.300-0.400 | 2808.922 | 34708.185 | 62541.414 | 89883.663 | 98829.623 | 107593.652 | 90251.175 | 81170.858 | 53030.603 | 22342.842 | 8164.355 | 3128.716 |
| 0.400-0.500 | 3952.899 | 28744.777 | 53859.168 | 78305.677 | 86403.079 | 90251.175 | 83869.271 | 76341.753 | 49259.257 | 22398.760 | 9132.650 | 3139.109 |
| 0.500-0.700 | 1414.203 | 26472.560 | 50007.653 | 73333.008 | 79752.621 | 81170.858 | 76341.753 | 75245.661 | 48525.106 | 22687.294 | 9375.036 | 3072.433 |
| 0.700-1.000 | -441.422 | 16308.957 | 31908.075 | 45901.172 | 51111.939 | 53030.603 | 49259.257 | 48525.106 | 33989.888 | 15320.991 | 6322.854 | 2224.255 |
| 1.000-1.300 | 1719.485 | 7832.234 | 14771.552 | 20886.720 | 22341.626 | 22342.842 | 22398.760 | 22687.294 | 15320.991 | 8239.954 | 3609.320 | 1099.543 |
| 1.300-2.000 | 404.092 | 2691.034 | 5183.326 | 7572.366 | 8140.579 | 8164.355 | 9132.650 | 9375.036 | 6322.854 | 3609.320 | 1872.378 | 567.369 |
| 2.000-3.000 | -120.953 | 871.441 | 1710.945 | 2500.306 | 2902.191 | 3128.716 | 3139.109 | 3072.433 | 2224.255 | 1099.543 | 567.369 | 221.141 |

TABLE Supp.XCII. Measured cross section as function of $W_{exp}$ on iron, in units of $10^{-42}$ $cm^2/GeV/c^2$/nucleon, and the absolute and fractional cross section uncertainties

| Bin edges ($GeV/c^2$) | $\sigma$ | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 0.9-1.1 | 311.04 | 166.76 | 0.474 | 0.250 | 0.058 |
| 1.1-1.2 | 3578.05 | 568.58 | 0.132 | 0.089 | 0.037 |
| 1.2-1.3 | 6517.70 | 1370.61 | 0.096 | 0.187 | 0.037 |
| 1.3-1.4 | 4762.99 | 1879.65 | 0.178 | 0.352 | 0.037 |

TABLE Supp.XCIII. Statistical covariance matrix of measured cross section as function of $W_{exp}$ on iron, in units of $10^{-84}$ $(cm^2/GeV/c^2/nucleon)^2$

| Bin edges ($GeV/c^2$) | 0.9-1.1 | 1.1-1.2 | 1.2-1.3 | 1.3-1.4 |
|---|---|---|---|---|
| 0.9-1.1 | 21751.952 | 8816.208 | -32096.118 | -11418.298 |
| 1.1-1.2 | 8816.208 | 221945.211 | -113809.407 | -238691.744 |
| 1.2-1.3 | -32096.118 | -113809.407 | 388748.322 | 294208.988 |
| 1.3-1.4 | -11418.298 | -238691.744 | 294208.988 | 717522.632 |

TABLE Supp.XCIV. Systematic covariance matrix of measured cross section in $W_{exp}$ on iron, in units of $10^{-84}$ $(cm^2/GeV/c^2/nucleon)^2$

| Bin edges ($GeV/c^2$) | 0.9-1.1 | 1.1-1.2 | 1.2-1.3 | 1.3-1.4 |
|---|---|---|---|---|
| 0.9-1.1 | 6057.167 | 7556.637 | 35126.356 | 46543.098 |
| 1.1-1.2 | 7556.637 | 101332.959 | -9217.793 | -109601.685 |
| 1.2-1.3 | 35126.356 | -9217.793 | 1489829.153 | 1981530.923 |
| 1.3-1.4 | 46543.098 | -109601.685 | 1981530.923 | 2815551.855 |

TABLE Supp.XCV. Measured cross section as function of $T_\pi$ on iron, in units of $10^{-42}$ cm$^2$/GeV/nucleon, and the absolute and fractional cross section uncertainties

| Bin edges (GeV) | $\sigma$ | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 0.035-0.100 | 9611.08 | 1423.80 | 0.080 | 0.125 | 0.036 |
| 0.100-0.150 | 6490.73 | 926.72 | 0.067 | 0.126 | 0.037 |
| 0.150-0.200 | 4801.53 | 909.68 | 0.098 | 0.162 | 0.037 |
| 0.200-0.350 | 1964.13 | 829.17 | 0.206 | 0.368 | 0.042 |

TABLE Supp.XCVI. Statistical covariance matrix of measured cross section as function of $T_\pi$ on iron, in units of $10^{-84}$ (cm$^2$/GeV/nucleon)$^2$

| Bin edges (GeV) | 0.035-0.100 | 0.100-0.150 | 0.150-0.200 | 0.200-0.350 |
|---|---|---|---|---|
| 0.035-0.100 | 592308.755 | 144789.490 | -51192.110 | -29375.688 |
| 0.100-0.150 | 144789.490 | 190306.593 | 62421.208 | -9576.103 |
| 0.150-0.200 | -51192.110 | 62421.208 | 219623.729 | 60254.833 |
| 0.200-0.350 | -29375.688 | -9576.103 | 60254.833 | 163896.809 |

TABLE Supp.XCVII. Systematic covariance matrix of measured cross section in $T_\pi$ on iron, in units of $10^{-84}$ (cm$^2$/GeV/nucleon)$^2$

| Bin edges (GeV) | 0.035-0.100 | 0.100-0.150 | 0.150-0.200 | 0.200-0.350 |
|---|---|---|---|---|
| 0.035-0.100 | 1434906.838 | 857837.924 | 660551.198 | 539981.697 |
| 0.100-0.150 | 857837.924 | 668506.151 | 580579.025 | 474120.643 |
| 0.150-0.200 | 660551.198 | 580579.025 | 607902.437 | 539205.808 |
| 0.200-0.350 | 539981.697 | 474120.643 | 539205.808 | 523625.100 |

TABLE Supp.XCVIII. Measured cross section as function of $\theta_\pi$ on iron, in units of $10^{-42}$ cm$^2$/degree/nucleon, and the absolute and fractional cross section uncertainties

| Bin edges (degree) | $\sigma$ | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 0-10 | 4.28 | 1.82 | 0.389 | 0.168 | 0.041 |
| 10-20 | 7.60 | 2.13 | 0.212 | 0.183 | 0.037 |
| 20-30 | 10.16 | 2.16 | 0.131 | 0.167 | 0.036 |
| 30-40 | 12.09 | 2.40 | 0.116 | 0.161 | 0.037 |
| 40-50 | 13.99 | 2.73 | 0.106 | 0.164 | 0.039 |
| 50-60 | 17.22 | 3.09 | 0.104 | 0.146 | 0.038 |
| 60-120 | 13.65 | 2.38 | 0.106 | 0.138 | 0.038 |
| 120-135 | 6.63 | 1.69 | 0.186 | 0.174 | 0.035 |
| 135-150 | 3.66 | 1.17 | 0.253 | 0.197 | 0.038 |
| 150-165 | 1.87 | 1.02 | 0.492 | 0.242 | 0.037 |
| 165-180 | 1.14 | 1.36 | 1.096 | 0.467 | 0.037 |

TABLE Supp.XCIX. Statistical covariance matrix of measured cross section as function of $\theta_\pi$ on iron, in units of $10^{-84}$ (cm$^2$/degree/nucleon)$^2$

| Bin edges (degree) | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 | 60-120 | 120-135 | 135-150 | 150-165 | 165-180 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0-10 | 2.778 | 0.967 | -0.097 | -0.023 | 0.009 | -0.055 | -0.044 | -0.026 | 0.019 | 0.025 | -0.002 |
| 10-20 | 0.967 | 2.596 | 0.658 | -0.123 | -0.184 | -0.099 | -0.044 | -0.046 | -0.028 | -0.018 | -0.009 |
| 20-30 | -0.097 | 0.658 | 1.783 | 0.606 | -0.124 | -0.171 | -0.084 | -0.066 | -0.008 | 0.012 | -0.018 |
| 30-40 | -0.023 | -0.123 | 0.606 | 1.968 | 0.855 | -0.186 | -0.160 | -0.086 | 0.021 | 0.026 | -0.030 |
| 40-50 | 0.009 | -0.184 | -0.124 | 0.855 | 2.216 | 1.051 | -0.123 | -0.053 | -0.005 | -0.029 | -0.025 |
| 50-60 | -0.055 | -0.099 | -0.171 | -0.186 | 1.051 | 3.212 | 0.896 | -0.133 | -0.121 | -0.077 | 0.003 |
| 60-120 | -0.044 | -0.044 | -0.084 | -0.160 | -0.123 | 0.896 | 2.110 | 0.287 | -0.036 | -0.009 | 0.051 |
| 120-135 | -0.026 | -0.046 | -0.066 | -0.086 | -0.053 | -0.133 | 0.287 | 1.519 | 0.433 | 0.039 | -0.130 |
| 135-150 | 0.019 | -0.028 | -0.008 | 0.021 | -0.005 | -0.121 | -0.036 | 0.433 | 0.859 | 0.425 | 0.122 |
| 150-165 | 0.025 | -0.018 | 0.012 | 0.026 | -0.029 | -0.077 | -0.009 | 0.039 | 0.425 | 0.842 | 0.660 |
| 165-180 | -0.002 | -0.009 | -0.018 | -0.030 | -0.025 | 0.003 | 0.051 | -0.130 | 0.122 | 0.660 | 1.562 |

TABLE Supp.C. Systematic covariance matrix of measured cross section in $\theta_\pi$ on iron, in units of $10^{-84}$ (cm$^2$/degree/nucleon)$^2$

| Bin edges (degree) | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 | 60-120 | 120-135 | 135-150 | 150-165 | 165-180 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0-10 | 0.519 | 0.817 | 0.997 | 1.074 | 1.244 | 1.284 | 0.896 | 0.666 | 0.352 | 0.211 | -0.162 |
| 10-20 | 0.817 | 1.934 | 2.154 | 2.341 | 2.542 | 2.253 | 1.340 | 1.150 | 0.608 | 0.430 | -0.239 |
| 20-30 | 0.997 | 2.154 | 2.885 | 3.148 | 3.495 | 3.379 | 2.171 | 1.630 | 0.949 | 0.583 | -0.198 |
| 30-40 | 1.074 | 2.341 | 3.148 | 3.808 | 4.236 | 4.240 | 2.849 | 1.777 | 1.019 | 0.621 | -0.090 |
| 40-50 | 1.244 | 2.542 | 3.495 | 4.236 | 5.253 | 5.456 | 3.638 | 2.285 | 1.325 | 0.718 | -0.088 |
| 50-60 | 1.284 | 2.253 | 3.379 | 4.240 | 5.456 | 6.333 | 4.491 | 2.476 | 1.388 | 0.696 | 0.002 |
| 60-120 | 0.896 | 1.340 | 2.171 | 2.849 | 3.638 | 4.491 | 3.552 | 1.600 | 0.835 | 0.391 | 0.091 |
| 120-135 | 0.666 | 1.150 | 1.630 | 1.777 | 2.285 | 2.476 | 1.600 | 1.327 | 0.754 | 0.425 | -0.112 |
| 135-150 | 0.352 | 0.608 | 0.949 | 1.019 | 1.325 | 1.388 | 0.835 | 0.754 | 0.520 | 0.270 | -0.067 |
| 150-165 | 0.211 | 0.430 | 0.583 | 0.621 | 0.718 | 0.696 | 0.391 | 0.425 | 0.270 | 0.205 | -0.028 |
| 165-180 | -0.162 | -0.239 | -0.198 | -0.090 | -0.088 | 0.002 | 0.091 | -0.112 | -0.067 | -0.028 | 0.284 |

TABLE Supp.CI. Measured cross section as function of $p_\mu$ on lead, in units of $10^{-42}$ cm$^2$/GeV/c/nucleon, and the absolute and fractional cross section uncertainties

| Bin edges (GeV/c) | $\sigma$ | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 1.5-3.0 | 57.22 | 14.78 | 0.222 | 0.131 | 0.040 |
| 3.0-4.0 | 169.13 | 31.67 | 0.138 | 0.126 | 0.042 |
| 4.0-5.0 | 208.37 | 36.80 | 0.107 | 0.141 | 0.043 |
| 5.0-6.0 | 206.20 | 37.95 | 0.100 | 0.154 | 0.042 |
| 6.0-7.0 | 133.72 | 26.25 | 0.115 | 0.159 | 0.040 |
| 7.0-8.5 | 61.02 | 12.21 | 0.133 | 0.150 | 0.039 |
| 8.5-10.0 | 20.70 | 4.78 | 0.178 | 0.146 | 0.042 |
| 10.0-20.0 | 3.54 | 1.29 | 0.307 | 0.196 | 0.059 |

TABLE Supp.CII. Statistical covariance matrix of measured cross section as function of $p_\mu$ on lead, in units of $10^{-84}$ (cm$^2$/GeV/c/nucleon)$^2$

| Bin edges (GeV/c) | 1.5-3.0 | 3.0-4.0 | 4.0-5.0 | 5.0-6.0 | 6.0-7.0 | 7.0-8.5 | 8.5-10.0 | 10.0-20.0 |
|---|---|---|---|---|---|---|---|---|
| 1.5-3.0 | 162.019 | 55.143 | 10.979 | 2.356 | 5.699 | 0.630 | 0.112 | 0.017 |
| 3.0-4.0 | 55.143 | 546.846 | 159.863 | 29.564 | 7.808 | 2.796 | 0.687 | 0.060 |
| 4.0-5.0 | 10.979 | 159.863 | 494.840 | 173.574 | 38.342 | 11.228 | 3.055 | 0.185 |
| 5.0-6.0 | 2.356 | 29.564 | 173.574 | 426.939 | 169.281 | 25.683 | 3.167 | 0.484 |
| 6.0-7.0 | 5.699 | 7.808 | 38.342 | 169.281 | 236.627 | 68.538 | 7.526 | 0.549 |
| 7.0-8.5 | 0.630 | 2.796 | 11.228 | 25.683 | 68.538 | 65.416 | 15.972 | 1.145 |
| 8.5-10.0 | 0.112 | 0.687 | 3.055 | 3.167 | 7.526 | 15.972 | 13.637 | 1.904 |
| 10.0-20.0 | 0.017 | 0.060 | 0.185 | 0.484 | 0.549 | 1.145 | 1.904 | 1.183 |

TABLE Supp.CIII. Systematic covariance matrix of measured cross section in $p_\mu$ on lead, in units of $10^{-84}$ $(\mathrm{cm}^2/\mathrm{GeV/c/nucleon})^2$

| Bin edges (GeV/c) | 1.5-3.0 | 3.0-4.0 | 4.0-5.0 | 5.0-6.0 | 6.0-7.0 | 7.0-8.5 | 8.5-10.0 | 10.0-20.0 |
|---|---|---|---|---|---|---|---|---|
| 1.5-3.0 | 56.444 | 138.926 | 180.808 | 195.336 | 116.599 | 37.107 | 11.447 | 3.040 |
| 3.0-4.0 | 138.926 | 456.300 | 585.215 | 618.568 | 366.751 | 130.067 | 36.735 | 9.805 |
| 4.0-5.0 | 180.808 | 585.215 | 859.583 | 899.855 | 540.719 | 203.785 | 51.933 | 14.063 |
| 5.0-6.0 | 195.336 | 618.568 | 899.855 | 1013.175 | 637.749 | 241.083 | 66.174 | 16.345 |
| 6.0-7.0 | 116.599 | 366.751 | 540.719 | 637.749 | 452.536 | 178.971 | 53.757 | 11.877 |
| 7.0-8.5 | 37.107 | 130.067 | 203.785 | 241.083 | 178.971 | 83.684 | 24.919 | 4.785 |
| 8.5-10.0 | 11.447 | 36.735 | 51.933 | 66.174 | 53.757 | 24.919 | 9.164 | 1.611 |
| 10.0-20.0 | 3.040 | 9.805 | 14.063 | 16.345 | 11.877 | 4.785 | 1.611 | 0.483 |

TABLE Supp.CIV. Measured cross section as function of $\theta_\mu$ on lead, in units of $10^{-42}$ $\mathrm{cm}^2$/degree/nucleon, and the absolute and fractional cross section uncertainties

| Bin edges (degree) | $\sigma$ | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 0-2 | 32.71 | 6.87 | 0.179 | 0.110 | 0.035 |
| 2-3 | 83.11 | 16.21 | 0.166 | 0.103 | 0.036 |
| 3-4 | 81.92 | 17.97 | 0.176 | 0.131 | 0.036 |
| 4-5 | 93.94 | 21.24 | 0.177 | 0.141 | 0.036 |
| 5-6 | 82.57 | 23.50 | 0.218 | 0.183 | 0.038 |
| 6-8 | 83.06 | 22.10 | 0.190 | 0.187 | 0.042 |
| 8-10 | 96.34 | 26.31 | 0.201 | 0.185 | 0.045 |
| 10-13 | 88.49 | 28.00 | 0.250 | 0.194 | 0.045 |

TABLE Supp.CV. Statistical covariance matrix of measured cross section as function of $\theta_\mu$ on lead, in units of $10^{-84}$ (cm$^2$/degree/nucleon)$^2$

| Bin edges (degree) | 0-2 | 2-3 | 3-4 | 4-5 | 5-6 | 6-8 | 8-10 | 10-13 |
|---|---|---|---|---|---|---|---|---|
| 0-2 | 34.123 | 1.170 | -7.061 | -1.132 | 0.053 | -0.004 | -0.048 | -0.190 |
| 2-3 | 1.170 | 189.638 | 30.525 | -21.629 | -6.990 | -0.080 | -0.095 | -0.407 |
| 3-4 | -7.061 | 30.525 | 207.225 | 34.916 | -21.579 | -4.114 | -0.110 | -0.681 |
| 4-5 | -1.132 | -21.629 | 34.916 | 275.591 | 60.008 | -17.741 | -2.978 | 0.399 |
| 5-6 | 0.053 | -6.990 | -21.579 | 60.008 | 323.349 | 15.500 | -15.806 | -1.288 |
| 6-8 | -0.004 | -0.080 | -4.114 | -17.741 | 15.500 | 248.413 | -1.075 | -14.438 |
| 8-10 | -0.048 | -0.095 | -0.110 | -2.978 | -15.806 | -1.075 | 375.065 | -9.371 |
| 10-13 | -0.190 | -0.407 | -0.681 | 0.399 | -1.288 | -14.438 | -9.371 | 489.807 |

TABLE Supp.CVI. Systematic covariance matrix of measured cross section in $\theta_\mu$ on lead, in units of $10^{-84}$ (cm$^2$/degree/nucleon)$^2$

| Bin edges (degree) | 0-2 | 2-3 | 3-4 | 4-5 | 5-6 | 6-8 | 8-10 | 10-13 |
|---|---|---|---|---|---|---|---|---|
| 0-2 | 13.053 | 21.354 | 24.029 | 25.945 | 26.727 | 23.505 | 22.209 | 25.444 |
| 2-3 | 21.354 | 73.124 | 82.686 | 85.119 | 105.230 | 101.031 | 112.547 | 94.467 |
| 3-4 | 24.029 | 82.686 | 115.798 | 129.470 | 151.397 | 152.158 | 161.481 | 151.063 |
| 4-5 | 25.945 | 85.119 | 129.470 | 175.384 | 180.866 | 177.610 | 196.907 | 183.731 |
| 5-6 | 26.727 | 105.230 | 151.397 | 180.866 | 229.048 | 221.694 | 233.313 | 216.679 |
| 6-8 | 23.505 | 101.031 | 152.158 | 177.610 | 221.694 | 240.058 | 258.273 | 236.181 |
| 8-10 | 22.209 | 112.547 | 161.481 | 196.907 | 233.313 | 258.273 | 317.068 | 269.857 |
| 10-13 | 25.444 | 94.467 | 151.063 | 183.731 | 216.679 | 236.181 | 269.857 | 294.391 |

TABLE Supp.CVII. Measured cross section as function of $p_{\mu,||}$ on lead, in units of $10^{-42}$ cm$^2$/GeV/c/nucleon, and the absolute and fractional cross section uncertainties

| Bin edges (GeV/c) | $\sigma$ | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 1.5-3.0 | 62.81 | 15.69 | 0.214 | 0.129 | 0.040 |
| 3.0-4.0 | 170.55 | 32.40 | 0.138 | 0.131 | 0.042 |
| 4.0-5.0 | 205.44 | 37.10 | 0.109 | 0.144 | 0.044 |
| 5.0-6.0 | 204.29 | 37.49 | 0.101 | 0.153 | 0.042 |
| 6.0-7.0 | 130.24 | 25.40 | 0.116 | 0.157 | 0.040 |
| 7.0-8.5 | 60.09 | 11.91 | 0.132 | 0.148 | 0.039 |
| 8.5-10.0 | 20.73 | 4.66 | 0.175 | 0.141 | 0.041 |
| 10.0-20.0 | 3.55 | 1.28 | 0.304 | 0.195 | 0.059 |

TABLE Supp.CVIII. Statistical covariance matrix of measured cross section as function of $p_{\mu,||}$ on lead, in units of $10^{-84}$ (cm$^2$/GeV/c/nucleon)$^2$

| Bin edges (GeV/c) | 1.5-3.0 | 3.0-4.0 | 4.0-5.0 | 5.0-6.0 | 6.0-7.0 | 7.0-8.5 | 8.5-10.0 | 10.0-20.0 |
|---|---|---|---|---|---|---|---|---|
| 1.5-3.0 | 180.102 | 54.791 | 10.594 | 2.700 | 5.881 | 0.643 | 0.111 | 0.018 |
| 3.0-4.0 | 54.791 | 553.360 | 147.805 | 33.256 | 8.554 | 4.122 | 0.910 | 0.062 |
| 4.0-5.0 | 10.594 | 147.805 | 501.413 | 175.103 | 38.949 | 10.598 | 3.017 | 0.187 |
| 5.0-6.0 | 2.700 | 33.256 | 175.103 | 423.991 | 159.229 | 24.427 | 3.065 | 0.470 |
| 6.0-7.0 | 5.881 | 8.554 | 38.949 | 159.229 | 227.923 | 68.353 | 7.886 | 0.515 |
| 7.0-8.5 | 0.643 | 4.122 | 10.598 | 24.427 | 68.353 | 62.634 | 15.833 | 1.209 |
| 8.5-10.0 | 0.111 | 0.910 | 3.017 | 3.065 | 7.886 | 15.833 | 13.109 | 1.902 |
| 10.0-20.0 | 0.018 | 0.062 | 0.187 | 0.470 | 0.515 | 1.209 | 1.902 | 1.169 |

TABLE Supp.CIX. Systematic covariance matrix of measured cross section in $p_{\mu,||}$ on lead, in units of $10^{-84}$ $(\mathrm{cm}^2/\mathrm{GeV/c/nucleon})^2$

| Bin edges (GeV/c) | 1.5-3.0 | 3.0-4.0 | 4.0-5.0 | 5.0-6.0 | 6.0-7.0 | 7.0-8.5 | 8.5-10.0 | 10.0-20.0 |
|---|---|---|---|---|---|---|---|---|
| 1.5-3.0 | 65.945 | 156.648 | 196.080 | 208.037 | 121.268 | 40.055 | 12.698 | 3.241 |
| 3.0-4.0 | 156.648 | 496.145 | 613.874 | 627.753 | 362.434 | 128.593 | 36.249 | 9.914 |
| 4.0-5.0 | 196.080 | 613.874 | 875.271 | 890.815 | 528.535 | 204.789 | 52.470 | 14.028 |
| 5.0-6.0 | 208.037 | 627.753 | 890.815 | 981.148 | 610.077 | 238.019 | 65.819 | 16.173 |
| 6.0-7.0 | 121.268 | 362.434 | 528.535 | 610.077 | 417.195 | 168.414 | 50.493 | 11.430 |
| 7.0-8.5 | 40.055 | 128.593 | 204.789 | 238.019 | 168.414 | 79.260 | 23.105 | 4.764 |
| 8.5-10.0 | 12.698 | 36.249 | 52.470 | 65.819 | 50.493 | 23.105 | 8.596 | 1.597 |
| 10.0-20.0 | 3.241 | 9.914 | 14.028 | 16.173 | 11.430 | 4.764 | 1.597 | 0.479 |

TABLE Supp.CX. Measured cross section as function of $p_{\mu,T}$ on lead, in units of $10^{-42}$ $\mathrm{cm}^2$/GeV/c/nucleon, and the absolute and fractional cross section uncertainties

| Bin edges (GeV/c) | $\sigma$ | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 0.00-0.15 | 236.50 | 63.46 | 0.238 | 0.124 | 0.036 |
| 0.15-0.30 | 713.67 | 131.67 | 0.151 | 0.107 | 0.036 |
| 0.30-0.45 | 1294.09 | 207.18 | 0.114 | 0.112 | 0.036 |
| 0.45-0.60 | 1179.81 | 261.53 | 0.146 | 0.167 | 0.038 |
| 0.60-0.75 | 928.01 | 303.17 | 0.205 | 0.255 | 0.041 |
| 0.75-0.90 | 663.67 | 249.94 | 0.255 | 0.277 | 0.047 |
| 0.90-1.25 | 538.56 | 154.85 | 0.224 | 0.181 | 0.044 |
| 1.25-2.50 | 46.26 | 28.11 | 0.548 | 0.262 | 0.046 |

TABLE Supp.CXI. Statistical covariance matrix of measured cross section as function of $p_{\mu,T}$ on lead, in units of $10^{-84}$ (cm$^2$/GeV/c/nucleon)$^2$

| Bin edges (GeV/c) | 0.00-0.15 | 0.15-0.30 | 0.30-0.45 | 0.45-0.60 | 0.60-0.75 | 0.75-0.90 | 0.90-1.25 | 1.25-2.50 |
|---|---|---|---|---|---|---|---|---|
| 0.00-0.15 | 3168.718 | 396.989 | -682.793 | -135.891 | -4.150 | -15.259 | -2.058 | -2.887 |
| 0.15-0.30 | 396.989 | 11544.339 | 1232.806 | -1676.433 | -454.364 | -54.702 | -41.615 | -5.107 |
| 0.30-0.45 | -682.793 | 1232.806 | 21839.737 | 3944.473 | -2826.174 | -928.275 | -216.527 | -6.595 |
| 0.45-0.60 | -135.891 | -1676.433 | 3944.473 | 29715.318 | 7674.514 | -1779.356 | -1087.723 | -45.942 |
| 0.60-0.75 | -4.150 | -454.364 | -2826.174 | 7674.514 | 36030.549 | 7778.024 | -1015.831 | -92.489 |
| 0.75-0.90 | -15.259 | -54.702 | -928.275 | -1779.356 | 7778.024 | 28589.494 | 5302.666 | -387.123 |
| 0.90-1.25 | -2.058 | -41.615 | -216.527 | -1087.723 | -1015.831 | 5302.666 | 14519.371 | 454.036 |
| 1.25-2.50 | -2.887 | -5.107 | -6.595 | -45.942 | -92.489 | -387.123 | 454.036 | 643.079 |

TABLE Supp.CXII. Systematic covariance matrix of measured cross section in $p_{\mu,T}$ on lead, in units of $10^{-84}$ (cm$^2$/GeV/c/nucleon)$^2$

| Bin edges (GeV/c) | 0.00-0.15 | 0.15-0.30 | 0.30-0.45 | 0.45-0.60 | 0.60-0.75 | 0.75-0.90 | 0.90-1.25 | 1.25-2.50 |
|---|---|---|---|---|---|---|---|---|
| 0.00-0.15 | 858.314 | 1541.536 | 1477.698 | 858.624 | 566.947 | 760.244 | 425.940 | 31.396 |
| 0.15-0.30 | 1541.536 | 5793.906 | 9326.513 | 10708.725 | 10733.551 | 8355.208 | 3623.301 | 211.474 |
| 0.30-0.45 | 1477.698 | 9326.513 | 21083.833 | 26557.906 | 27334.872 | 20448.934 | 8612.808 | 492.206 |
| 0.45-0.60 | 858.624 | 10708.725 | 26557.906 | 38682.204 | 41349.523 | 30321.221 | 12514.317 | 663.020 |
| 0.60-0.75 | 566.947 | 10733.551 | 27334.872 | 41349.523 | 55883.369 | 40454.619 | 18278.196 | 831.407 |
| 0.75-0.90 | 760.244 | 8355.208 | 20448.934 | 30321.221 | 40454.619 | 33878.710 | 16027.219 | 756.951 |
| 0.90-1.25 | 425.940 | 3623.301 | 8612.808 | 12514.317 | 18278.196 | 16027.219 | 9459.079 | 337.098 |
| 1.25-2.50 | 31.396 | 211.474 | 492.206 | 663.020 | 831.407 | 756.951 | 337.098 | 147.264 |

TABLE Supp.CXIII. Measured cross section as function of $Q^2$ on lead, in units of $10^{-42}$ $\mathrm{cm}^2/\mathrm{GeV}^2/c^2$/nucleon, and the absolute and fractional cross section uncertainties

| Bin edges ($\mathrm{GeV}^2/c^2$) | $\sigma$ | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 0.000-0.025 | 1377.45 | 360.28 | 0.232 | 0.120 | 0.036 |
| 0.025-0.050 | 1545.90 | 318.03 | 0.180 | 0.099 | 0.035 |
| 0.050-0.100 | 1340.67 | 278.06 | 0.173 | 0.114 | 0.036 |
| 0.100-0.200 | 1540.32 | 254.33 | 0.120 | 0.113 | 0.036 |
| 0.200-0.300 | 1192.87 | 255.96 | 0.146 | 0.157 | 0.038 |
| 0.300-0.400 | 814.78 | 218.51 | 0.196 | 0.183 | 0.037 |
| 0.400-0.500 | 742.47 | 209.57 | 0.199 | 0.201 | 0.038 |
| 0.500-0.700 | 505.16 | 181.41 | 0.230 | 0.276 | 0.043 |
| 0.700-1.000 | 362.55 | 129.20 | 0.246 | 0.258 | 0.045 |
| 1.000-1.300 | 334.27 | 104.12 | 0.260 | 0.172 | 0.039 |
| 1.300-2.000 | 89.19 | 52.40 | 0.490 | 0.324 | 0.058 |
| 2.000-3.000 | 79.01 | 76.23 | 0.918 | 0.296 | 0.045 |

TABLE Supp.CXIV. Statistical covariance matrix of measured cross section as function of $Q^2$ on lead, in units of $10^{-84}$ $(\mathrm{cm}^2/\mathrm{GeV}^2/c^2/\mathrm{nucleon})^2$

| Bin edges ($\mathrm{GeV}^2/c^2$) | 0.000-0.025 | 0.025-0.050 | 0.050-0.100 | 0.100-0.200 | 0.200-0.300 | 0.300-0.400 | 0.400-0.500 | 0.500-0.700 | 0.700-1.000 | 1.000-1.300 | 1.300-2.000 | 2.000-3.000 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.000-0.025 | 102365.910 | 23995.884 | -8159.314 | -2931.052 | -204.852 | -174.866 | -70.549 | 16.612 | -62.726 | 20.284 | -16.180 | -17.044 |
| 0.025-0.050 | 23995.884 | 77779.070 | 23456.388 | -4519.319 | -2625.286 | -478.484 | -167.234 | -45.000 | -40.480 | -32.874 | -9.654 | -3.802 |
| 0.050-0.100 | -8159.314 | 23456.388 | 54101.847 | 9375.054 | -4821.719 | -1589.678 | -712.645 | -145.199 | -39.017 | -73.589 | -28.538 | 1.758 |
| 0.100-0.200 | -2931.052 | -4519.319 | 9375.054 | 34308.499 | 8233.980 | -2135.072 | -2573.844 | -749.455 | -244.834 | -143.762 | -5.293 | -19.055 |
| 0.200-0.300 | -204.852 | -2625.286 | -4821.719 | 8233.980 | 30482.401 | 12298.839 | 895.149 | -1464.890 | -410.208 | -23.786 | -38.294 | -19.719 |
| 0.300-0.400 | -174.866 | -478.484 | -1589.678 | -2135.072 | 12298.839 | 25629.989 | 14476.309 | -2.271 | -1103.469 | -592.153 | -234.311 | -10.325 |
| 0.400-0.500 | -70.549 | -167.234 | -712.645 | -2573.844 | 895.149 | 14476.309 | 21746.274 | 8145.249 | -370.599 | -900.004 | -166.443 | -70.245 |
| 0.500-0.700 | 16.612 | -45.000 | -145.199 | -749.455 | -1464.890 | -2.271 | 8145.249 | 13530.088 | 3797.890 | -616.488 | -17.299 | -106.552 |
| 0.700-1.000 | -62.726 | -40.480 | -39.017 | -244.834 | -410.208 | -1103.469 | -370.599 | 3797.890 | 7975.856 | 2679.136 | -63.894 | -319.962 |
| 1.000-1.300 | 20.284 | -32.874 | -73.589 | -143.762 | -23.786 | -592.153 | -900.004 | -616.488 | 2679.136 | 7526.346 | 591.026 | -223.317 |
| 1.300-2.000 | -16.180 | -9.654 | -28.538 | -5.293 | -38.294 | -234.311 | -166.443 | -17.299 | -63.894 | 591.026 | 1912.384 | 306.063 |
| 2.000-3.000 | -17.044 | -3.802 | 1.758 | -19.055 | -19.719 | -10.325 | -70.245 | -106.552 | -319.962 | -223.317 | 306.063 | 5262.012 |

TABLE Supp.CXV. Systematic covariance matrix of measured cross section in $Q^2$ on lead, in units of $10^{-84}$ $(\mathrm{cm}^2/\mathrm{GeV}^2/c^2/\mathrm{nucleon})^2$

| Bin edges ($\mathrm{GeV}^2/c^2$) | 0.000-0.025 | 0.025-0.050 | 0.050-0.100 | 0.100-0.200 | 0.200-0.300 | 0.300-0.400 | 0.400-0.500 | 0.500-0.700 | 0.700-1.000 | 1.000-1.300 | 1.300-2.000 | 2.000-3.000 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.000-0.025 | 27438.389 | 20614.170 | 15391.629 | 11891.348 | 4765.830 | 3573.036 | 3746.310 | 3030.552 | 2372.444 | 2054.434 | -71.575 | 595.811 |
| 0.025-0.050 | 20614.170 | 23365.053 | 19586.410 | 18309.054 | 13307.121 | 10080.808 | 10051.827 | 9511.742 | 5927.853 | 4205.073 | 556.662 | 1133.213 |
| 0.050-0.100 | 15391.629 | 19586.410 | 23217.530 | 24616.084 | 22049.533 | 16620.749 | 16862.697 | 14612.344 | 8927.013 | 5825.351 | 994.784 | 995.743 |
| 0.100-0.200 | 11891.348 | 18309.054 | 24616.084 | 30375.661 | 27380.065 | 22412.748 | 22350.023 | 19200.507 | 11450.749 | 6802.159 | 1607.278 | 1210.251 |
| 0.200-0.300 | 4765.830 | 13307.121 | 22049.533 | 27380.065 | 35034.638 | 24271.171 | 23806.637 | 20699.693 | 12851.084 | 7331.150 | 2153.086 | 978.034 |
| 0.300-0.400 | 3573.036 | 10080.808 | 16620.749 | 22412.748 | 24271.171 | 22118.452 | 21160.786 | 18779.817 | 11030.151 | 6022.207 | 2193.369 | 719.037 |
| 0.400-0.500 | 3746.310 | 10051.827 | 16862.697 | 22350.023 | 23806.637 | 21160.786 | 22174.645 | 19688.088 | 11706.085 | 6521.884 | 2398.046 | 567.351 |
| 0.500-0.700 | 3030.552 | 9511.742 | 14612.344 | 19200.507 | 20699.693 | 18779.817 | 19688.088 | 19381.297 | 12112.459 | 6186.160 | 2733.678 | 593.115 |
| 0.700-1.000 | 2372.444 | 5927.853 | 8927.013 | 11450.749 | 12851.084 | 11030.151 | 11706.085 | 12112.459 | 8715.631 | 4576.629 | 2066.718 | 543.318 |
| 1.000-1.300 | 2054.434 | 4205.073 | 5825.351 | 6802.159 | 7331.150 | 6022.207 | 6521.884 | 6186.160 | 4576.629 | 3313.826 | 1091.299 | 224.801 |
| 1.300-2.000 | -71.575 | 556.662 | 994.784 | 1607.278 | 2153.086 | 2193.369 | 2398.046 | 2733.678 | 2066.718 | 1091.299 | 833.186 | 15.014 |
| 2.000-3.000 | 595.811 | 1133.213 | 995.743 | 1210.251 | 978.034 | 719.037 | 567.351 | 593.115 | 543.318 | 224.801 | 15.014 | 548.266 |

TABLE Supp.CXVI. Measured cross section as function of $W_{exp}$ on lead, in units of $10^{-42}$ $cm^2/GeV/c^2$/nucleon, and the absolute and fractional cross section uncertainties

| Bin edges ($GeV/c^2$) | $\sigma$ | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 0.9-1.1 | 324.04 | 158.71 | 0.422 | 0.248 | 0.055 |
| 1.1-1.2 | 986.81 | 257.24 | 0.237 | 0.108 | 0.046 |
| 1.2-1.3 | 3802.07 | 617.99 | 0.091 | 0.135 | 0.037 |
| 1.3-1.4 | 5092.70 | 1400.14 | 0.161 | 0.223 | 0.032 |

TABLE Supp.CXVII. Statistical covariance matrix of measured cross section as function of $W_{exp}$ on lead, in units of $10^{-84}$ $(cm^2/GeV/c^2/nucleon)^2$

| Bin edges ($GeV/c^2$) | 0.9-1.1 | 1.1-1.2 | 1.2-1.3 | 1.3-1.4 |
|---|---|---|---|---|
| 0.9-1.1 | 18723.909 | 10639.010 | -18411.776 | -28099.433 |
| 1.1-1.2 | 10639.010 | 54830.704 | 23209.963 | -124904.569 |
| 1.2-1.3 | -18411.776 | 23209.963 | 119494.363 | 55595.419 |
| 1.3-1.4 | -28099.433 | -124904.569 | 55595.419 | 674041.129 |

TABLE Supp.CXVIII. Systematic covariance matrix of measured cross section in $W_{exp}$ on lead, in units of $10^{-84}$ $(cm^2/GeV/c^2/nucleon)^2$

| Bin edges ($GeV/c^2$) | 0.9-1.1 | 1.1-1.2 | 1.2-1.3 | 1.3-1.4 |
|---|---|---|---|---|
| 0.9-1.1 | 6465.812 | 5506.681 | 2629.126 | 6224.264 |
| 1.1-1.2 | 5506.681 | 11343.237 | 18638.835 | 14533.237 |
| 1.2-1.3 | 2629.126 | 18638.835 | 262420.462 | 543487.733 |
| 1.3-1.4 | 6224.264 | 14533.237 | 543487.733 | 1286338.954 |

TABLE Supp.CXIX. Measured cross section as function of $T_\pi$ on lead, in units of $10^{-42}$ cm$^2$/GeV/nucleon, and the absolute and fractional cross section uncertainties

| Bin edges (GeV) | $\sigma$ | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 0.035-0.100 | 5058.30 | 762.04 | 0.113 | 0.099 | 0.037 |
| 0.100-0.150 | 3560.00 | 528.69 | 0.091 | 0.117 | 0.038 |
| 0.150-0.200 | 2718.88 | 550.67 | 0.126 | 0.158 | 0.038 |
| 0.200-0.350 | 2145.01 | 624.92 | 0.169 | 0.237 | 0.038 |

TABLE Supp.CXX. Statistical covariance matrix of measured cross section as function of $T_\pi$ on lead, in units of $10^{-84}$ (cm$^2$/GeV/nucleon)$^2$

| Bin edges (GeV) | 0.035-0.100 | 0.100-0.150 | 0.150-0.200 | 0.200-0.350 |
|---|---|---|---|---|
| 0.035-0.100 | 327493.820 | 72027.359 | -23454.488 | -22608.464 |
| 0.100-0.150 | 72027.359 | 104866.238 | 29191.098 | -8581.637 |
| 0.150-0.200 | -23454.488 | 29191.098 | 117994.805 | 28803.449 |
| 0.200-0.350 | -22608.464 | -8581.637 | 28803.449 | 131287.372 |

TABLE Supp.CXXI. Systematic covariance matrix of measured cross section in $T_\pi$ on lead, in units of $10^{-84}$ (cm$^2$/GeV/nucleon)$^2$

| Bin edges (GeV) | 0.035-0.100 | 0.100-0.150 | 0.150-0.200 | 0.200-0.350 |
|---|---|---|---|---|
| 0.035-0.100 | 253210.315 | 191336.076 | 166383.958 | 172831.262 |
| 0.100-0.150 | 191336.076 | 174649.446 | 165733.149 | 176988.883 |
| 0.150-0.200 | 166383.958 | 165733.149 | 185243.795 | 198764.255 |
| 0.200-0.350 | 172831.262 | 176988.883 | 198764.255 | 259233.214 |

TABLE Supp.CXXII. Measured cross section as function of $\theta_\pi$ on lead, in units of $10^{-42}$ cm$^2$/degree/nucleon, and the absolute and fractional cross section uncertainties

| Bin edges (degree) | $\sigma$ | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 0-10 | 2.76 | 1.21 | 0.404 | 0.168 | 0.040 |
| 10-20 | 5.72 | 1.58 | 0.215 | 0.173 | 0.038 |
| 20-30 | 6.08 | 1.36 | 0.154 | 0.162 | 0.039 |
| 30-40 | 8.29 | 1.52 | 0.119 | 0.140 | 0.038 |
| 40-50 | 8.20 | 1.60 | 0.121 | 0.154 | 0.039 |
| 50-60 | 7.04 | 1.63 | 0.155 | 0.171 | 0.041 |
| 60-120 | 6.42 | 1.24 | 0.141 | 0.133 | 0.038 |
| 120-135 | 4.46 | 0.93 | 0.170 | 0.122 | 0.035 |
| 135-150 | 3.19 | 0.79 | 0.193 | 0.153 | 0.034 |
| 150-165 | 1.74 | 0.68 | 0.345 | 0.180 | 0.036 |
| 165-180 | 0.34 | 0.34 | 0.945 | 0.363 | 0.043 |

TABLE Supp.CXXIII. Statistical covariance matrix of measured cross section as function of $\theta_\pi$ on lead, in units of $10^{-84}$ $(\mathrm{cm}^2/\mathrm{degree/nucleon})^2$

| Bin edges (degree) | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 | 60-120 | 120-135 | 135-150 | 150-165 | 165-180 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0-10 | 1.239 | 0.505 | 0.026 | -0.019 | -0.031 | -0.034 | -0.027 | -0.016 | -0.013 | 0.027 | -0.003 |
| 10-20 | 0.505 | 1.513 | 0.471 | -0.038 | -0.117 | -0.084 | -0.043 | -0.035 | -0.030 | -0.002 | -0.004 |
| 20-30 | 0.026 | 0.471 | 0.873 | 0.417 | -0.011 | -0.092 | -0.050 | -0.030 | -0.010 | 0.011 | 0.003 |
| 30-40 | -0.019 | -0.038 | 0.417 | 0.973 | 0.511 | 0.052 | -0.092 | -0.054 | 0.002 | 0.000 | -0.012 |
| 40-50 | -0.031 | -0.117 | -0.011 | 0.511 | 0.978 | 0.615 | 0.085 | -0.018 | -0.021 | -0.012 | -0.014 |
| 50-60 | -0.034 | -0.084 | -0.092 | 0.052 | 0.615 | 1.199 | 0.446 | -0.006 | -0.069 | -0.025 | -0.013 |
| 60-120 | -0.027 | -0.043 | -0.050 | -0.092 | 0.085 | 0.446 | 0.814 | 0.157 | 0.007 | -0.005 | 0.026 |
| 120-135 | -0.016 | -0.035 | -0.030 | -0.054 | -0.018 | -0.006 | 0.157 | 0.576 | 0.237 | 0.006 | 0.061 |
| 135-150 | -0.013 | -0.030 | -0.010 | 0.002 | -0.021 | -0.069 | 0.007 | 0.237 | 0.381 | 0.185 | 0.011 |
| 150-165 | 0.027 | -0.002 | 0.011 | 0.000 | -0.012 | -0.025 | -0.005 | 0.006 | 0.185 | 0.360 | -0.003 |
| 165-180 | -0.003 | -0.004 | 0.003 | -0.012 | -0.014 | -0.013 | 0.026 | 0.061 | 0.011 | -0.003 | 0.101 |

## K. Tables of Ratios of cross sections

TABLE Supp.CXXIV. Systematic covariance matrix of measured cross section in $\theta_\pi$ on lead, in units of $10^{-84}$ (cm$^2$/degree/nucleon)$^2$

| Bin edges (degree) | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 | 60-120 | 120-135 | 135-150 | 150-165 | 165-180 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0-10 | 0.214 | 0.349 | 0.382 | 0.450 | 0.476 | 0.398 | 0.283 | 0.215 | 0.192 | 0.089 | -0.001 |
| 10-20 | 0.349 | 0.981 | 0.884 | 0.904 | 0.855 | 0.675 | 0.447 | 0.387 | 0.329 | 0.152 | -0.018 |
| 20-30 | 0.382 | 0.884 | 0.974 | 1.055 | 1.026 | 0.838 | 0.531 | 0.455 | 0.369 | 0.175 | -0.004 |
| 30-40 | 0.450 | 0.904 | 1.055 | 1.344 | 1.395 | 1.255 | 0.828 | 0.522 | 0.446 | 0.207 | 0.020 |
| 40-50 | 0.476 | 0.855 | 1.026 | 1.395 | 1.592 | 1.447 | 0.979 | 0.552 | 0.484 | 0.231 | 0.010 |
| 50-60 | 0.398 | 0.675 | 0.838 | 1.255 | 1.447 | 1.456 | 0.971 | 0.433 | 0.386 | 0.183 | 0.037 |
| 60-120 | 0.283 | 0.447 | 0.531 | 0.828 | 0.979 | 0.971 | 0.725 | 0.301 | 0.294 | 0.128 | 0.020 |
| 120-135 | 0.215 | 0.387 | 0.455 | 0.522 | 0.552 | 0.433 | 0.301 | 0.297 | 0.238 | 0.117 | -0.004 |
| 135-150 | 0.192 | 0.329 | 0.369 | 0.446 | 0.484 | 0.386 | 0.294 | 0.238 | 0.239 | 0.122 | -0.008 |
| 150-165 | 0.089 | 0.152 | 0.175 | 0.207 | 0.231 | 0.183 | 0.128 | 0.117 | 0.122 | 0.098 | -0.004 |
| 165-180 | -0.001 | -0.018 | -0.004 | 0.020 | 0.010 | 0.037 | 0.020 | -0.004 | -0.008 | -0.004 | 0.015 |

TABLE Supp.CXXVI. Statistical covariance matrix of measured cross section ratio of carbon over scintillator in $p_\mu$

| Bin edges (GeV/c) | 1.5-3.0 | 3.0-4.0 | 4.0-5.0 | 5.0-6.0 | 6.0-7.0 | 7.0-8.5 | 8.5-10.0 | 10.0-20.0 |
|---|---|---|---|---|---|---|---|---|
| 1.5-3.0 | 0.1507 | 0.0181 | 0.0020 | 0.0012 | 0.0015 | 0.0009 | 0.0002 | 0.0014 |
| 3.0-4.0 | 0.0181 | 0.0626 | 0.0115 | 0.0023 | 0.0008 | 0.0002 | 0.0000 | 0.0000 |
| 4.0-5.0 | 0.0020 | 0.0115 | 0.0352 | 0.0103 | 0.0019 | 0.0006 | 0.0003 | 0.0000 |
| 5.0-6.0 | 0.0012 | 0.0023 | 0.0103 | 0.0265 | 0.0116 | 0.0037 | 0.0022 | 0.0010 |
| 6.0-7.0 | 0.0015 | 0.0008 | 0.0019 | 0.0116 | 0.0225 | 0.0156 | 0.0069 | 0.0088 |
| 7.0-8.5 | 0.0009 | 0.0002 | 0.0006 | 0.0037 | 0.0156 | 0.0366 | 0.0270 | 0.0089 |
| 8.5-10.0 | 0.0002 | 0.0000 | 0.0003 | 0.0022 | 0.0069 | 0.0270 | 0.0607 | 0.0427 |
| 10.0-20.0 | 0.0014 | 0.0000 | 0.0000 | 0.0010 | 0.0088 | 0.0089 | 0.0427 | 0.2838 |

TABLE Supp.CXXV. Measured cross section ratio of carbon over scintillator in $p_\mu$, and the absolute and fractional cross section ratio uncertainties

| Bin edges (GeV/c) | Ratio | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 1.5-3.0 | 1.11 | 0.411 | 0.350 | 0.122 | 0.004 |
| 3.0-4.0 | 1.20 | 0.279 | 0.209 | 0.104 | 0.003 |
| 4.0-5.0 | 1.02 | 0.229 | 0.184 | 0.129 | 0.005 |
| 5.0-6.0 | 0.95 | 0.213 | 0.171 | 0.144 | 0.002 |
| 6.0-7.0 | 0.77 | 0.197 | 0.195 | 0.166 | 0.008 |
| 7.0-8.5 | 0.74 | 0.239 | 0.258 | 0.194 | 0.020 |
| 8.5-10.0 | 0.67 | 0.304 | 0.366 | 0.265 | 0.025 |
| 10.0-20.0 | 1.18 | 0.610 | 0.450 | 0.251 | 0.016 |

TABLE Supp.CXXVII. Systematic covariance matrix of measured cross section ratio of carbon over scintillator in $p_\mu$

| Bin edges (GeV/c) | 1.5-3.0 | 3.0-4.0 | 4.0-5.0 | 5.0-6.0 | 6.0-7.0 | 7.0-8.5 | 8.5-10.0 | 10.0-20.0 |
|---|---|---|---|---|---|---|---|---|
| 1.5-3.0 | 0.018 | 0.013 | 0.012 | 0.013 | 0.012 | 0.008 | 0.014 | 0.032 |
| 3.0-4.0 | 0.013 | 0.015 | 0.012 | 0.014 | 0.013 | 0.010 | 0.014 | 0.030 |
| 4.0-5.0 | 0.012 | 0.012 | 0.017 | 0.016 | 0.015 | 0.013 | 0.013 | 0.028 |
| 5.0-6.0 | 0.013 | 0.014 | 0.016 | 0.019 | 0.016 | 0.014 | 0.016 | 0.032 |
| 6.0-7.0 | 0.012 | 0.013 | 0.015 | 0.016 | 0.016 | 0.015 | 0.017 | 0.030 |
| 7.0-8.5 | 0.008 | 0.010 | 0.013 | 0.014 | 0.015 | 0.021 | 0.022 | 0.026 |
| 8.5-10.0 | 0.014 | 0.014 | 0.013 | 0.016 | 0.017 | 0.022 | 0.032 | 0.038 |
| 10.0-20.0 | 0.032 | 0.030 | 0.028 | 0.032 | 0.030 | 0.026 | 0.038 | 0.089 |

TABLE Supp.CXXVIII. Measured cross section ratio of carbon over scintillator in $\theta_\mu$, and the absolute and fractional cross section ratio uncertainties

| Bin edges (degree) | Ratio | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 0-2 | 0.76 | 0.233 | 0.299 | 0.072 | 0.004 |
| 2-3 | 1.31 | 0.331 | 0.228 | 0.109 | 0.004 |
| 3-4 | 0.84 | 0.248 | 0.270 | 0.123 | 0.004 |
| 4-5 | 0.86 | 0.286 | 0.277 | 0.181 | 0.002 |
| 5-6 | 0.40 | 0.244 | 0.511 | 0.319 | 0.011 |
| 6-8 | 0.72 | 0.356 | 0.412 | 0.274 | 0.011 |
| 8-10 | 1.58 | 0.636 | 0.349 | 0.201 | 0.009 |
| 10-13 | 1.88 | 0.811 | 0.388 | 0.191 | 0.010 |

TABLE Supp.CXXIX. Statistical covariance matrix of measured cross section ratio of carbon over scintillator in $\theta_\mu$

| Bin edges (degree) | 0-2 | 2-3 | 3-4 | 4-5 | 5-6 | 6-8 | 8-10 | 10-13 |
|---|---|---|---|---|---|---|---|---|
| 0-2 | 0.0513 | -0.0020 | -0.0024 | -0.0004 | 0.0000 | 0.0000 | -0.0001 | -0.0003 |
| 2-3 | -0.0020 | 0.0892 | 0.0037 | -0.0056 | -0.0010 | -0.0000 | -0.0005 | -0.0001 |
| 3-4 | -0.0024 | 0.0037 | 0.0509 | 0.0071 | -0.0037 | -0.0009 | -0.0001 | -0.0003 |
| 4-5 | -0.0004 | -0.0056 | 0.0071 | 0.0574 | 0.0083 | -0.0050 | -0.0008 | 0.0001 |
| 5-6 | 0.0000 | -0.0010 | -0.0037 | 0.0083 | 0.0429 | 0.0054 | -0.0041 | -0.0003 |
| 6-8 | 0.0000 | -0.0000 | -0.0009 | -0.0050 | 0.0054 | 0.0879 | -0.0032 | -0.0056 |
| 8-10 | -0.0001 | -0.0005 | -0.0001 | -0.0008 | -0.0041 | -0.0032 | 0.3035 | -0.0178 |
| 10-13 | -0.0003 | -0.0001 | -0.0003 | 0.0001 | -0.0003 | -0.0056 | -0.0178 | 0.5289 |

TABLE Supp.CXXX. Systematic covariance matrix of measured cross section ratio of carbon over scintillator in $\theta_\mu$

| Bin edges (degree) | 0-2 | 2-3 | 3-4 | 4-5 | 5-6 | 6-8 | 8-10 | 10-13 |
|---|---|---|---|---|---|---|---|---|
| 0-2 | 0.003 | 0.001 | 0.002 | 0.002 | 0.002 | 0.002 | 0.006 | 0.001 |
| 2-3 | 0.001 | 0.020 | 0.009 | 0.015 | 0.010 | 0.014 | 0.023 | 0.035 |
| 3-4 | 0.002 | 0.009 | 0.010 | 0.014 | 0.011 | 0.012 | 0.019 | 0.024 |
| 4-5 | 0.002 | 0.015 | 0.014 | 0.024 | 0.016 | 0.021 | 0.036 | 0.037 |
| 5-6 | 0.002 | 0.010 | 0.011 | 0.016 | 0.017 | 0.016 | 0.026 | 0.026 |
| 6-8 | 0.002 | 0.014 | 0.012 | 0.021 | 0.016 | 0.039 | 0.042 | 0.040 |
| 8-10 | 0.006 | 0.023 | 0.019 | 0.036 | 0.026 | 0.042 | 0.101 | 0.062 |
| 10-13 | 0.001 | 0.035 | 0.024 | 0.037 | 0.026 | 0.040 | 0.062 | 0.129 |

TABLE Supp.CXXXI. Measured cross section ratio of carbon over scintillator in $p_{\mu,||}$, and the absolute and fractional cross section ratio uncertainties

| Bin edges (GeV/c) | Ratio | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 1.5-3.0 | 1.07 | 0.404 | 0.355 | 0.128 | 0.005 |
| 3.0-4.0 | 1.21 | 0.281 | 0.209 | 0.102 | 0.003 |
| 4.0-5.0 | 1.03 | 0.229 | 0.183 | 0.128 | 0.005 |
| 5.0-6.0 | 0.92 | 0.212 | 0.175 | 0.149 | 0.002 |
| 6.0-7.0 | 0.80 | 0.202 | 0.194 | 0.163 | 0.007 |
| 7.0-8.5 | 0.74 | 0.239 | 0.260 | 0.194 | 0.020 |
| 8.5-10.0 | 0.67 | 0.306 | 0.372 | 0.267 | 0.025 |
| 10.0-20.0 | 1.19 | 0.613 | 0.449 | 0.251 | 0.016 |

TABLE Supp.CXXXII. Statistical covariance matrix of measured cross section ratio of carbon over scintillator in $p_{\mu,||}$

| Bin edges (GeV/c) | 1.5-3.0 | 3.0-4.0 | 4.0-5.0 | 5.0-6.0 | 6.0-7.0 | 7.0-8.5 | 8.5-10.0 | 10.0-20.0 |
|---|---|---|---|---|---|---|---|---|
| 1.5-3.0 | 0.1447 | 0.0160 | 0.0019 | 0.0011 | 0.0014 | 0.0008 | 0.0002 | 0.0013 |
| 3.0-4.0 | 0.0160 | 0.0639 | 0.0111 | 0.0020 | 0.0007 | 0.0001 | 0.0000 | 0.0000 |
| 4.0-5.0 | 0.0019 | 0.0111 | 0.0353 | 0.0103 | 0.0020 | 0.0006 | 0.0003 | 0.0000 |
| 5.0-6.0 | 0.0011 | 0.0020 | 0.0103 | 0.0259 | 0.0119 | 0.0038 | 0.0021 | 0.0010 |
| 6.0-7.0 | 0.0014 | 0.0007 | 0.0020 | 0.0119 | 0.0239 | 0.0153 | 0.0065 | 0.0089 |
| 7.0-8.5 | 0.0008 | 0.0001 | 0.0006 | 0.0038 | 0.0153 | 0.0367 | 0.0271 | 0.0091 |
| 8.5-10.0 | 0.0002 | 0.0000 | 0.0003 | 0.0021 | 0.0065 | 0.0271 | 0.0620 | 0.0436 |
| 10.0-20.0 | 0.0013 | 0.0000 | 0.0000 | 0.0010 | 0.0089 | 0.0091 | 0.0436 | 0.2857 |

TABLE Supp.CXXXIII. Systematic covariance matrix of measured cross section ratio of carbon over scintillator in $p_{\mu,||}$

| Bin edges (GeV/c) | 1.5-3.0 | 3.0-4.0 | 4.0-5.0 | 5.0-6.0 | 6.0-7.0 | 7.0-8.5 | 8.5-10.0 | 10.0-20.0 |
|---|---|---|---|---|---|---|---|---|
| 1.5-3.0 | 0.019 | 0.012 | 0.013 | 0.013 | 0.013 | 0.009 | 0.014 | 0.033 |
| 3.0-4.0 | 0.012 | 0.015 | 0.012 | 0.014 | 0.013 | 0.009 | 0.013 | 0.028 |
| 4.0-5.0 | 0.013 | 0.012 | 0.017 | 0.016 | 0.015 | 0.013 | 0.013 | 0.028 |
| 5.0-6.0 | 0.013 | 0.014 | 0.016 | 0.019 | 0.017 | 0.014 | 0.016 | 0.032 |
| 6.0-7.0 | 0.013 | 0.013 | 0.015 | 0.017 | 0.017 | 0.015 | 0.017 | 0.031 |
| 7.0-8.5 | 0.009 | 0.009 | 0.013 | 0.014 | 0.015 | 0.020 | 0.022 | 0.025 |
| 8.5-10.0 | 0.014 | 0.013 | 0.013 | 0.016 | 0.017 | 0.022 | 0.032 | 0.038 |
| 10.0-20.0 | 0.033 | 0.028 | 0.028 | 0.032 | 0.031 | 0.025 | 0.038 | 0.090 |

TABLE Supp.CXXXIV. Measured cross section ratio of carbon over scintillator in $p_{\mu,T}$, and the absolute and fractional cross section ratio uncertainties

| Bin edges (GeV/c) | Ratio | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 0.00-0.15 | 1.07 | 0.355 | 0.323 | 0.070 | 0.003 |
| 0.15-0.30 | 0.80 | 0.192 | 0.226 | 0.080 | 0.002 |
| 0.30-0.45 | 0.99 | 0.224 | 0.201 | 0.103 | 0.003 |
| 0.45-0.60 | 0.92 | 0.255 | 0.228 | 0.160 | 0.004 |
| 0.60-0.75 | 0.84 | 0.314 | 0.306 | 0.212 | 0.008 |
| 0.75-0.90 | 0.65 | 0.579 | 0.688 | 0.569 | 0.039 |
| 0.90-1.25 | 1.33 | 0.945 | 0.586 | 0.399 | 0.021 |
| 1.25-2.50 | 1.97 | 3.141 | 1.434 | 0.693 | 0.021 |

TABLE Supp.CXXXV. Statistical covariance matrix of measured cross section ratio of carbon over scintillator in $p_{\mu,T}$

| Bin edges (GeV/c) | 0.00-0.15 | 0.15-0.30 | 0.30-0.45 | 0.45-0.60 | 0.60-0.75 | 0.75-0.90 | 0.90-1.25 | 1.25-2.50 |
|---|---|---|---|---|---|---|---|---|
| 0.00-0.15 | 0.1200 | 0.0004 | -0.0045 | -0.0005 | -0.0000 | 0.0007 | -0.0004 | -0.0002 |
| 0.15-0.30 | 0.0004 | 0.0328 | 0.0022 | -0.0032 | -0.0007 | -0.0007 | -0.0003 | -0.0006 |
| 0.30-0.45 | -0.0045 | 0.0022 | 0.0396 | 0.0048 | -0.0051 | -0.0032 | -0.0017 | -0.0073 |
| 0.45-0.60 | -0.0005 | -0.0032 | 0.0048 | 0.0437 | 0.0083 | -0.0064 | -0.0115 | 0.0076 |
| 0.60-0.75 | -0.0000 | -0.0007 | -0.0051 | 0.0083 | 0.0667 | 0.0340 | -0.0028 | -0.0370 |
| 0.75-0.90 | 0.0007 | -0.0007 | -0.0032 | -0.0064 | 0.0340 | 0.1992 | 0.1252 | -0.0123 |
| 0.90-1.25 | -0.0004 | -0.0003 | -0.0017 | -0.0115 | -0.0028 | 0.1252 | 0.6101 | -0.0227 |
| 1.25-2.50 | -0.0002 | -0.0006 | -0.0073 | 0.0076 | -0.0370 | -0.0123 | -0.0227 | 7.9959 |

TABLE Supp.CXXXVI. Systematic covariance matrix of measured cross section ratio of carbon over scintillator in $p_{\mu,T}$

| Bin edges (GeV/c) | 0.00-0.15 | 0.15-0.30 | 0.30-0.45 | 0.45-0.60 | 0.60-0.75 | 0.75-0.90 | 0.90-1.25 | 1.25-2.50 |
|---|---|---|---|---|---|---|---|---|
| 0.00-0.15 | 0.006 | 0.003 | 0.002 | 0.002 | 0.004 | 0.003 | 0.002 | 0.020 |
| 0.15-0.30 | 0.003 | 0.004 | 0.004 | 0.005 | 0.008 | 0.011 | 0.014 | 0.041 |
| 0.30-0.45 | 0.002 | 0.004 | 0.010 | 0.013 | 0.014 | 0.025 | 0.031 | 0.097 |
| 0.45-0.60 | 0.002 | 0.005 | 0.013 | 0.021 | 0.022 | 0.034 | 0.043 | 0.131 |
| 0.60-0.75 | 0.004 | 0.008 | 0.014 | 0.022 | 0.032 | 0.047 | 0.048 | 0.164 |
| 0.75-0.90 | 0.003 | 0.011 | 0.025 | 0.034 | 0.047 | 0.136 | 0.162 | 0.355 |
| 0.90-1.25 | 0.002 | 0.014 | 0.031 | 0.043 | 0.048 | 0.162 | 0.283 | 0.461 |
| 1.25-2.50 | 0.020 | 0.041 | 0.097 | 0.131 | 0.164 | 0.355 | 0.461 | 1.868 |

TABLE Supp.CXXXVII. Measured cross section ratio of carbon over scintillator in $Q^2$, and the absolute and fractional cross section ratio uncertainties

| Bin edges ($\mathrm{GeV}^2/c^2$) | Ratio | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 0.000-0.025 | 1.08 | 0.344 | 0.311 | 0.068 | 0.003 |
| 0.025-0.050 | 0.86 | 0.256 | 0.290 | 0.062 | 0.003 |
| 0.050-0.100 | 0.76 | 0.214 | 0.259 | 0.111 | 0.003 |
| 0.100-0.200 | 1.12 | 0.250 | 0.199 | 0.099 | 0.003 |
| 0.200-0.300 | 0.77 | 0.238 | 0.272 | 0.146 | 0.004 |
| 0.300-0.400 | 0.87 | 0.296 | 0.282 | 0.191 | 0.003 |
| 0.400-0.500 | 0.75 | 0.321 | 0.354 | 0.239 | 0.005 |
| 0.500-0.700 | 0.66 | 0.380 | 0.467 | 0.336 | 0.018 |
| 0.700-1.000 | 1.09 | 0.704 | 0.518 | 0.387 | 0.029 |
| 1.000-1.300 | 1.47 | 0.903 | 0.548 | 0.274 | 0.022 |
| 1.300-2.000 | 1.13 | 1.733 | 1.264 | 0.878 | 0.022 |
| 2.000-3.000 | 2.46 | 4.688 | 1.695 | 0.871 | 0.033 |

TABLE Supp.CXXXVIII. Statistical covariance matrix of measured cross section ratio of carbon over scintillator in $Q^2$

| Bin edges (GeV$^2/c^2$) | 0.000-0.025 | 0.025-0.050 | 0.050-0.100 | 0.100-0.200 | 0.200-0.300 | 0.300-0.400 | 0.400-0.500 | 0.500-0.700 | 0.700-1.000 | 1.000-1.300 | 1.300-2.000 | 2.000-3.000 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.000-0.025 | 0.1131 | 0.0081 | -0.0065 | -0.0025 | -0.0000 | -0.0000 | -0.0001 | -0.0001 | 0.0005 | -0.0001 | -0.0004 | -0.0002 |
| 0.025-0.050 | 0.0081 | 0.0629 | 0.0155 | -0.0052 | -0.0020 | -0.0004 | -0.0002 | -0.0003 | -0.0007 | -0.0003 | -0.0006 | 0.0002 |
| 0.050-0.100 | -0.0065 | 0.0155 | 0.0388 | 0.0083 | -0.0038 | -0.0018 | -0.0007 | -0.0002 | 0.0001 | -0.0004 | -0.0015 | -0.0010 |
| 0.100-0.200 | -0.0025 | -0.0052 | 0.0083 | 0.0501 | 0.0080 | -0.0051 | -0.0045 | -0.0009 | -0.0012 | -0.0020 | -0.0013 | -0.0044 |
| 0.200-0.300 | -0.0000 | -0.0020 | -0.0038 | 0.0080 | 0.0440 | 0.0203 | -0.0013 | -0.0054 | -0.0038 | -0.0039 | -0.0076 | -0.0037 |
| 0.300-0.400 | -0.0000 | -0.0004 | -0.0018 | -0.0051 | 0.0203 | 0.0601 | 0.0381 | 0.0002 | -0.0128 | -0.0109 | 0.0098 | -0.0019 |
| 0.400-0.500 | -0.0001 | -0.0002 | -0.0007 | -0.0045 | -0.0013 | 0.0381 | 0.0709 | 0.0375 | -0.0018 | -0.0130 | -0.0081 | -0.0134 |
| 0.500-0.700 | -0.0001 | -0.0003 | -0.0002 | -0.0009 | -0.0054 | 0.0002 | 0.0375 | 0.0953 | 0.0637 | -0.0046 | -0.0017 | -0.0267 |
| 0.700-1.000 | 0.0005 | -0.0007 | 0.0001 | -0.0012 | -0.0038 | -0.0128 | -0.0018 | 0.0637 | 0.3184 | 0.2083 | 0.0089 | -0.0939 |
| 1.000-1.300 | -0.0001 | -0.0003 | -0.0004 | -0.0020 | -0.0039 | -0.0109 | -0.0130 | -0.0046 | 0.2083 | 0.6520 | 0.2704 | -0.1106 |
| 1.300-2.000 | -0.0004 | -0.0006 | -0.0015 | -0.0013 | -0.0076 | 0.0098 | -0.0081 | -0.0017 | 0.0089 | 0.2704 | 2.0261 | 1.2873 |
| 2.000-3.000 | -0.0002 | 0.0002 | -0.0010 | -0.0044 | -0.0037 | -0.0019 | -0.0134 | -0.0267 | -0.0939 | -0.1106 | 1.2873 | 17.3908 |

TABLE Supp.CXXXIX. Systematic covariance matrix of measured cross section ratio of carbon over scintillator in $Q^2$

| Bin edges (GeV$^2/c^2$) | 0.000-0.025 | 0.025-0.050 | 0.050-0.100 | 0.100-0.200 | 0.200-0.300 | 0.300-0.400 | 0.400-0.500 | 0.500-0.700 | 0.700-1.000 | 1.000-1.300 | 1.300-2.000 | 2.000-3.000 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.000-0.025 | 0.005 | 0.001 | 0.003 | 0.002 | 0.002 | 0.002 | 0.004 | 0.002 | -0.001 | 0.003 | -0.000 | 0.023 |
| 0.025-0.050 | 0.001 | 0.003 | 0.002 | 0.003 | 0.001 | 0.001 | 0.003 | 0.005 | 0.005 | 0.007 | 0.006 | 0.034 |
| 0.050-0.100 | 0.003 | 0.002 | 0.007 | 0.007 | 0.007 | 0.009 | 0.011 | 0.013 | 0.017 | 0.019 | 0.030 | 0.070 |
| 0.100-0.200 | 0.002 | 0.003 | 0.007 | 0.012 | 0.011 | 0.013 | 0.015 | 0.020 | 0.024 | 0.028 | 0.054 | 0.124 |
| 0.200-0.300 | 0.002 | 0.001 | 0.007 | 0.011 | 0.013 | 0.016 | 0.017 | 0.019 | 0.025 | 0.029 | 0.057 | 0.105 |
| 0.300-0.400 | 0.002 | 0.001 | 0.009 | 0.013 | 0.016 | 0.028 | 0.026 | 0.027 | 0.034 | 0.035 | 0.087 | 0.137 |
| 0.400-0.500 | 0.004 | 0.003 | 0.011 | 0.015 | 0.017 | 0.026 | 0.032 | 0.031 | 0.035 | 0.036 | 0.083 | 0.144 |
| 0.500-0.700 | 0.002 | 0.005 | 0.013 | 0.020 | 0.019 | 0.027 | 0.031 | 0.049 | 0.072 | 0.066 | 0.129 | 0.244 |
| 0.700-1.000 | -0.001 | 0.005 | 0.017 | 0.024 | 0.025 | 0.034 | 0.035 | 0.072 | 0.178 | 0.145 | 0.257 | 0.462 |
| 1.000-1.300 | 0.003 | 0.007 | 0.019 | 0.028 | 0.029 | 0.035 | 0.036 | 0.066 | 0.145 | 0.163 | 0.271 | 0.506 |
| 1.300-2.000 | -0.000 | 0.006 | 0.030 | 0.054 | 0.057 | 0.087 | 0.083 | 0.129 | 0.257 | 0.271 | 0.979 | 0.596 |
| 2.000-3.000 | 0.023 | 0.034 | 0.070 | 0.124 | 0.105 | 0.137 | 0.144 | 0.244 | 0.462 | 0.506 | 0.596 | 4.588 |

TABLE Supp.CXL. Measured cross section ratio of carbon over scintillator in $W_{exp}$, and the absolute and fractional cross section ratio uncertainties

| Bin edges (GeV/c$^2$) | Ratio | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 0.9-1.1 | 1.66 | 1.128 | 0.636 | 0.237 | 0.019 |
| 1.1-1.2 | 1.05 | 0.202 | 0.177 | 0.075 | 0.002 |
| 1.2-1.3 | 0.84 | 0.220 | 0.201 | 0.167 | 0.006 |
| 1.3-1.4 | 0.58 | 0.454 | 0.531 | 0.571 | 0.009 |

TABLE Supp.CXLI. Statistical covariance matrix of measured cross section ratio of carbon over scintillator in $W_{exp}$

| Bin edges (GeV/c$^2$) | 0.9-1.1 | 1.1-1.2 | 1.2-1.3 | 1.3-1.4 |
|---|---|---|---|---|
| 0.9-1.1 | 1.1178 | -0.0021 | -0.0769 | -0.0717 |
| 1.1-1.2 | -0.0021 | 0.0348 | 0.0017 | -0.0224 |
| 1.2-1.3 | -0.0769 | 0.0017 | 0.0286 | 0.0188 |
| 1.3-1.4 | -0.0717 | -0.0224 | 0.0188 | 0.0955 |

TABLE Supp.CXLII. Systematic covariance matrix of measured cross section ratio of carbon over scintillator in $W_{exp}$

| Bin edges (GeV/c$^2$) | 0.9-1.1 | 1.1-1.2 | 1.2-1.3 | 1.3-1.4 |
|---|---|---|---|---|
| 0.9-1.1 | 0.155 | 0.004 | 0.005 | 0.002 |
| 1.1-1.2 | 0.004 | 0.006 | 0.002 | 0.004 |
| 1.2-1.3 | 0.005 | 0.002 | 0.020 | 0.044 |
| 1.3-1.4 | 0.002 | 0.004 | 0.044 | 0.110 |

TABLE Supp.CXLIII. Measured cross section ratio of carbon over scintillator in $T_\pi$, and the absolute and fractional cross section ratio uncertainties

| Bin edges (GeV) | Ratio | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 0.035-0.100 | 0.87 | 0.190 | 0.177 | 0.128 | 0.002 |
| 0.100-0.150 | 0.95 | 0.167 | 0.133 | 0.115 | 0.003 |
| 0.150-0.200 | 0.85 | 0.255 | 0.227 | 0.194 | 0.008 |
| 0.200-0.350 | 0.80 | 0.450 | 0.427 | 0.364 | 0.021 |

TABLE Supp.CXLIV. Statistical covariance matrix of measured cross section ratio of carbon over scintillator in $T_\pi$

| Bin edges (GeV) | 0.035-0.100 | 0.100-0.150 | 0.150-0.200 | 0.200-0.350 |
|---|---|---|---|---|
| 0.035-0.100 | 0.0236 | 0.0034 | -0.0041 | -0.0001 |
| 0.100-0.150 | 0.0034 | 0.0160 | 0.0090 | 0.0073 |
| 0.150-0.200 | -0.0041 | 0.0090 | 0.0375 | 0.0362 |
| 0.200-0.350 | -0.0001 | 0.0073 | 0.0362 | 0.1174 |

TABLE Supp.CXLV. Systematic covariance matrix of measured cross section ratio of carbon over scintillator in $T_\pi$

| Bin edges (GeV) | 0.035-0.100 | 0.100-0.150 | 0.150-0.200 | 0.200-0.350 |
|---|---|---|---|---|
| 0.035-0.100 | 0.012 | 0.011 | 0.015 | 0.024 |
| 0.100-0.150 | 0.011 | 0.012 | 0.017 | 0.023 |
| 0.150-0.200 | 0.015 | 0.017 | 0.027 | 0.039 |
| 0.200-0.350 | 0.024 | 0.023 | 0.039 | 0.085 |

TABLE Supp.CXLVI. Measured cross section ratio of carbon over scintillator in $\theta_\pi$, and the absolute and fractional cross section ratio uncertainties

| Bin edges (degree) | Ratio | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 0-10 | 1.06 | 1.715 | 1.486 | 0.652 | 0.013 |
| 10-20 | 0.87 | 0.604 | 0.629 | 0.302 | 0.007 |
| 20-30 | 0.85 | 0.348 | 0.341 | 0.229 | 0.004 |
| 30-40 | 0.84 | 0.282 | 0.280 | 0.183 | 0.009 |
| 40-50 | 1.10 | 0.297 | 0.231 | 0.142 | 0.007 |
| 50-60 | 0.82 | 0.245 | 0.258 | 0.153 | 0.007 |
| 60-120 | 0.90 | 0.211 | 0.209 | 0.106 | 0.003 |
| 120-135 | 1.01 | 0.315 | 0.293 | 0.108 | 0.003 |
| 135-150 | 1.27 | 0.426 | 0.312 | 0.119 | 0.009 |
| 150-165 | 1.63 | 1.047 | 0.559 | 0.316 | 0.007 |
| 165-180 | 1.69 | 1.465 | 0.777 | 0.389 | 0.017 |

TABLE Supp.CXLVII. Statistical covariance matrix of measured cross section ratio of carbon over scintillator in $\theta_\pi$

| Bin edges (degree) | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 | 60-120 | 120-135 | 135-150 | 150-165 | 165-180 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0-10 | 2.4667 | 0.3428 | 0.0833 | -0.0202 | 0.0355 | -0.0057 | -0.0090 | -0.0090 | -0.0207 | -0.0504 | 0.0468 |
| 10-20 | 0.3428 | 0.2965 | 0.0649 | -0.0038 | -0.0037 | -0.0053 | -0.0022 | 0.0001 | -0.0020 | 0.0089 | 0.3371 |
| 20-30 | 0.0833 | 0.0649 | 0.0837 | 0.0156 | -0.0059 | -0.0034 | -0.0025 | -0.0050 | -0.0053 | 0.0082 | 0.0700 |
| 30-40 | -0.0202 | -0.0038 | 0.0156 | 0.0558 | 0.0193 | -0.0013 | -0.0023 | -0.0017 | 0.0011 | -0.0037 | -0.0167 |
| 40-50 | 0.0355 | -0.0037 | -0.0059 | 0.0193 | 0.0640 | 0.0146 | -0.0012 | -0.0035 | -0.0046 | -0.0178 | -0.0252 |
| 50-60 | -0.0057 | -0.0053 | -0.0034 | -0.0013 | 0.0146 | 0.0445 | 0.0099 | -0.0020 | -0.0030 | -0.0084 | -0.0121 |
| 60-120 | -0.0090 | -0.0022 | -0.0025 | -0.0023 | -0.0012 | 0.0099 | 0.0354 | 0.0075 | -0.0009 | -0.0033 | -0.0018 |
| 120-135 | -0.0090 | 0.0001 | -0.0050 | -0.0017 | -0.0035 | -0.0020 | 0.0075 | 0.0872 | 0.0207 | 0.0024 | 0.0100 |
| 135-150 | -0.0207 | -0.0020 | -0.0053 | 0.0011 | -0.0046 | -0.0030 | -0.0009 | 0.0207 | 0.1586 | 0.2220 | 0.0990 |
| 150-165 | -0.0504 | 0.0089 | 0.0082 | -0.0037 | -0.0178 | -0.0084 | -0.0033 | 0.0024 | 0.2220 | 0.8311 | 0.0499 |
| 165-180 | 0.0468 | 0.3371 | 0.0700 | -0.0167 | -0.0252 | -0.0121 | -0.0018 | 0.0100 | 0.0990 | 0.0499 | 1.7155 |

TABLE Supp.CXLVIII. Systematic covariance matrix of measured cross section ratio of carbon over scintillator in $\theta_\pi$

| Bin edges (degree) | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 | 60-120 | 120-135 | 135-150 | 150-165 | 165-180 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0-10 | 0.475 | 0.048 | 0.047 | 0.036 | 0.027 | 0.023 | 0.023 | 0.021 | 0.036 | 0.129 | 0.004 |
| 10-20 | 0.048 | 0.068 | 0.045 | 0.032 | 0.036 | 0.028 | 0.018 | 0.022 | 0.027 | 0.043 | 0.102 |
| 20-30 | 0.047 | 0.045 | 0.038 | 0.027 | 0.025 | 0.019 | 0.013 | 0.015 | 0.021 | 0.030 | 0.057 |
| 30-40 | 0.036 | 0.032 | 0.027 | 0.024 | 0.018 | 0.014 | 0.010 | 0.011 | 0.017 | 0.030 | 0.045 |
| 40-50 | 0.027 | 0.036 | 0.025 | 0.018 | 0.024 | 0.019 | 0.011 | 0.014 | 0.017 | 0.018 | 0.051 |
| 50-60 | 0.023 | 0.028 | 0.019 | 0.014 | 0.019 | 0.016 | 0.009 | 0.012 | 0.014 | 0.019 | 0.042 |
| 60-120 | 0.023 | 0.018 | 0.013 | 0.010 | 0.011 | 0.009 | 0.009 | 0.009 | 0.008 | 0.017 | 0.043 |
| 120-135 | 0.021 | 0.022 | 0.015 | 0.011 | 0.014 | 0.012 | 0.009 | 0.012 | 0.011 | 0.023 | 0.047 |
| 135-150 | 0.036 | 0.027 | 0.021 | 0.017 | 0.017 | 0.014 | 0.008 | 0.011 | 0.023 | 0.014 | 0.028 |
| 150-165 | 0.129 | 0.043 | 0.030 | 0.030 | 0.018 | 0.019 | 0.017 | 0.023 | 0.014 | 0.265 | 0.099 |
| 165-180 | 0.004 | 0.102 | 0.057 | 0.045 | 0.051 | 0.042 | 0.043 | 0.047 | 0.028 | 0.099 | 0.430 |

TABLE Supp.CXLIX. Measured cross section ratio of water over scintillator in $p_\mu$, and the absolute and fractional cross section ratio uncertainties

| Bin edges (GeV/c) | Ratio | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 1.5-3.0 | 0.89 | 0.393 | 0.412 | 0.163 | 0.005 |
| 3.0-4.0 | 0.55 | 0.255 | 0.387 | 0.264 | 0.023 |
| 4.0-5.0 | 0.94 | 0.235 | 0.211 | 0.132 | 0.008 |
| 5.0-6.0 | 0.92 | 0.209 | 0.188 | 0.127 | 0.005 |
| 6.0-7.0 | 0.87 | 0.207 | 0.208 | 0.116 | 0.006 |
| 7.0-8.5 | 0.91 | 0.255 | 0.234 | 0.153 | 0.009 |
| 8.5-10.0 | 0.85 | 0.307 | 0.320 | 0.167 | 0.006 |
| 10.0-20.0 | 0.65 | 0.309 | 0.441 | 0.172 | 0.016 |

TABLE Supp.CL. Statistical covariance matrix of measured cross section ratio of water over scintillator in $p_\mu$

| Bin edges (GeV/c) | 1.5-3.0 | 3.0-4.0 | 4.0-5.0 | 5.0-6.0 | 6.0-7.0 | 7.0-8.5 | 8.5-10.0 | 10.0-20.0 |
|---|---|---|---|---|---|---|---|---|
| 1.5-3.0 | 0.1333 | 0.0098 | 0.0011 | 0.0005 | 0.0005 | 0.0005 | 0.0012 | 0.0000 |
| 3.0-4.0 | 0.0098 | 0.0444 | 0.0126 | 0.0032 | 0.0005 | 0.0006 | 0.0002 | 0.0004 |
| 4.0-5.0 | 0.0011 | 0.0126 | 0.0397 | 0.0138 | 0.0037 | 0.0016 | 0.0012 | 0.0006 |
| 5.0-6.0 | 0.0005 | 0.0032 | 0.0138 | 0.0299 | 0.0154 | 0.0052 | 0.0020 | 0.0016 |
| 6.0-7.0 | 0.0005 | 0.0005 | 0.0037 | 0.0154 | 0.0326 | 0.0181 | 0.0040 | 0.0004 |
| 7.0-8.5 | 0.0005 | 0.0006 | 0.0016 | 0.0052 | 0.0181 | 0.0454 | 0.0225 | 0.0062 |
| 8.5-10.0 | 0.0012 | 0.0002 | 0.0012 | 0.0020 | 0.0040 | 0.0225 | 0.0739 | 0.0410 |
| 10.0-20.0 | 0.0000 | 0.0004 | 0.0006 | 0.0016 | 0.0004 | 0.0062 | 0.0410 | 0.0830 |

TABLE Supp.CLI. Systematic covariance matrix of measured cross section ratio of water over scintillator in $p_\mu$

| Bin edges (GeV/c) | 1.5-3.0 | 3.0-4.0 | 4.0-5.0 | 5.0-6.0 | 6.0-7.0 | 7.0-8.5 | 8.5-10.0 | 10.0-20.0 |
|---|---|---|---|---|---|---|---|---|
| 1.5-3.0 | 0.021 | 0.009 | 0.009 | 0.007 | 0.010 | 0.015 | 0.014 | 0.005 |
| 3.0-4.0 | 0.009 | 0.021 | 0.013 | 0.013 | 0.009 | 0.010 | 0.009 | 0.006 |
| 4.0-5.0 | 0.009 | 0.013 | 0.016 | 0.012 | 0.010 | 0.011 | 0.009 | 0.007 |
| 5.0-6.0 | 0.007 | 0.013 | 0.012 | 0.014 | 0.009 | 0.008 | 0.008 | 0.007 |
| 6.0-7.0 | 0.010 | 0.009 | 0.010 | 0.009 | 0.010 | 0.011 | 0.010 | 0.007 |
| 7.0-8.5 | 0.015 | 0.010 | 0.011 | 0.008 | 0.011 | 0.020 | 0.016 | 0.008 |
| 8.5-10.0 | 0.014 | 0.009 | 0.009 | 0.008 | 0.010 | 0.016 | 0.020 | 0.006 |
| 10.0-20.0 | 0.005 | 0.006 | 0.007 | 0.007 | 0.007 | 0.008 | 0.006 | 0.013 |

TABLE Supp.CLII. Measured cross section ratio of water over scintillator in $\theta_\mu$, and the absolute and fractional cross section ratio uncertainties

| Bin edges (degree) | Ratio | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 0-2 | 0.69 | 0.193 | 0.261 | 0.108 | 0.009 |
| 2-3 | 0.96 | 0.274 | 0.272 | 0.086 | 0.007 |
| 3-4 | 0.90 | 0.263 | 0.269 | 0.116 | 0.005 |
| 4-5 | 0.74 | 0.304 | 0.383 | 0.150 | 0.004 |
| 5-6 | 0.52 | 0.309 | 0.537 | 0.258 | 0.014 |
| 6-8 | 1.03 | 0.441 | 0.367 | 0.221 | 0.008 |
| 8-10 | 1.78 | 0.686 | 0.343 | 0.178 | 0.007 |
| 10-13 | 0.13 | 0.512 | 3.400 | 1.891 | 0.150 |

TABLE Supp.CLIII. Statistical covariance matrix of measured cross section ratio of water over scintillator in $\theta_\mu$

| Bin edges (degree) | 0-2 | 2-3 | 3-4 | 4-5 | 5-6 | 6-8 | 8-10 | 10-13 |
|---|---|---|---|---|---|---|---|---|
| 0-2 | 0.0319 | 0.0026 | -0.0040 | -0.0007 | -0.0000 | -0.0002 | -0.0000 | 0.0000 |
| 2-3 | 0.0026 | 0.0681 | 0.0080 | -0.0081 | -0.0013 | -0.0003 | -0.0003 | 0.0000 |
| 3-4 | -0.0040 | 0.0080 | 0.0582 | 0.0122 | -0.0055 | -0.0018 | 0.0001 | 0.0000 |
| 4-5 | -0.0007 | -0.0081 | 0.0122 | 0.0803 | 0.0161 | -0.0072 | -0.0009 | 0.0000 |
| 5-6 | -0.0000 | -0.0013 | -0.0055 | 0.0161 | 0.0776 | 0.0100 | -0.0079 | -0.0001 |
| 6-8 | -0.0002 | -0.0003 | -0.0018 | -0.0072 | 0.0100 | 0.1428 | -0.0106 | -0.0028 |
| 8-10 | -0.0000 | -0.0003 | 0.0001 | -0.0009 | -0.0079 | -0.0106 | 0.3704 | 0.0322 |
| 10-13 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | -0.0001 | -0.0028 | 0.0322 | 0.2003 |

TABLE Supp.CLIV. Systematic covariance matrix of measured cross section ratio of water over scintillator in $\theta_\mu$

| Bin edges (degree) | 0-2 | 2-3 | 3-4 | 4-5 | 5-6 | 6-8 | 8-10 | 10-13 |
|---|---|---|---|---|---|---|---|---|
| 0-2 | 0.005 | 0.004 | 0.004 | 0.001 | 0.000 | 0.000 | 0.001 | -0.002 |
| 2-3 | 0.004 | 0.007 | 0.006 | 0.005 | 0.006 | 0.009 | 0.005 | 0.008 |
| 3-4 | 0.004 | 0.006 | 0.011 | 0.007 | 0.006 | 0.015 | 0.018 | 0.010 |
| 4-5 | 0.001 | 0.005 | 0.007 | 0.012 | 0.012 | 0.016 | 0.015 | 0.018 |
| 5-6 | 0.000 | 0.006 | 0.006 | 0.012 | 0.018 | 0.015 | 0.017 | 0.024 |
| 6-8 | 0.000 | 0.009 | 0.015 | 0.016 | 0.015 | 0.052 | 0.039 | 0.035 |
| 8-10 | 0.001 | 0.005 | 0.018 | 0.015 | 0.017 | 0.039 | 0.100 | 0.037 |
| 10-13 | -0.002 | 0.008 | 0.010 | 0.018 | 0.024 | 0.035 | 0.037 | 0.062 |

TABLE Supp.CLV. Measured cross section ratio of water over scintillator in $p_{\mu,||}$, and the absolute and fractional cross section ratio uncertainties

| Bin edges (GeV/c) | Ratio | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 1.5-3.0 | 0.99 | 0.431 | 0.390 | 0.193 | 0.005 |
| 3.0-4.0 | 0.60 | 0.260 | 0.358 | 0.241 | 0.020 |
| 4.0-5.0 | 0.96 | 0.240 | 0.212 | 0.133 | 0.008 |
| 5.0-6.0 | 0.88 | 0.204 | 0.193 | 0.128 | 0.005 |
| 6.0-7.0 | 0.86 | 0.208 | 0.211 | 0.116 | 0.007 |
| 7.0-8.5 | 0.87 | 0.251 | 0.240 | 0.162 | 0.010 |
| 8.5-10.0 | 0.83 | 0.303 | 0.321 | 0.171 | 0.006 |
| 10.0-20.0 | 0.66 | 0.312 | 0.439 | 0.171 | 0.016 |

TABLE Supp.CLVI. Statistical covariance matrix of measured cross section ratio of water over scintillator in $p_{\mu,||}$

| Bin edges (GeV/c) | 1.5-3.0 | 3.0-4.0 | 4.0-5.0 | 5.0-6.0 | 6.0-7.0 | 7.0-8.5 | 8.5-10.0 | 10.0-20.0 |
|---|---|---|---|---|---|---|---|---|
| 1.5-3.0 | 0.1490 | 0.0105 | 0.0011 | 0.0005 | 0.0006 | 0.0004 | 0.0013 | 0.0000 |
| 3.0-4.0 | 0.0105 | 0.0463 | 0.0129 | 0.0029 | 0.0004 | 0.0006 | 0.0002 | 0.0004 |
| 4.0-5.0 | 0.0011 | 0.0129 | 0.0413 | 0.0136 | 0.0036 | 0.0016 | 0.0012 | 0.0006 |
| 5.0-6.0 | 0.0005 | 0.0029 | 0.0136 | 0.0291 | 0.0148 | 0.0050 | 0.0020 | 0.0016 |
| 6.0-7.0 | 0.0006 | 0.0004 | 0.0036 | 0.0148 | 0.0332 | 0.0178 | 0.0040 | 0.0004 |
| 7.0-8.5 | 0.0004 | 0.0006 | 0.0016 | 0.0050 | 0.0178 | 0.0434 | 0.0225 | 0.0061 |
| 8.5-10.0 | 0.0013 | 0.0002 | 0.0012 | 0.0020 | 0.0040 | 0.0225 | 0.0715 | 0.0414 |
| 10.0-20.0 | 0.0000 | 0.0004 | 0.0006 | 0.0016 | 0.0004 | 0.0061 | 0.0414 | 0.0843 |

TABLE Supp.CLVII. Systematic covariance matrix of measured cross section ratio of water over scintillator in $p_{\mu,||}$

| Bin edges (GeV/c) | 1.5-3.0 | 3.0-4.0 | 4.0-5.0 | 5.0-6.0 | 6.0-7.0 | 7.0-8.5 | 8.5-10.0 | 10.0-20.0 |
|---|---|---|---|---|---|---|---|---|
| 1.5-3.0 | 0.037 | 0.006 | 0.010 | 0.007 | 0.010 | 0.018 | 0.016 | 0.006 |
| 3.0-4.0 | 0.006 | 0.021 | 0.013 | 0.013 | 0.009 | 0.010 | 0.009 | 0.006 |
| 4.0-5.0 | 0.010 | 0.013 | 0.016 | 0.012 | 0.010 | 0.010 | 0.009 | 0.007 |
| 5.0-6.0 | 0.007 | 0.013 | 0.012 | 0.013 | 0.009 | 0.008 | 0.008 | 0.007 |
| 6.0-7.0 | 0.010 | 0.009 | 0.010 | 0.009 | 0.010 | 0.011 | 0.009 | 0.007 |
| 7.0-8.5 | 0.018 | 0.010 | 0.010 | 0.008 | 0.011 | 0.020 | 0.016 | 0.008 |
| 8.5-10.0 | 0.016 | 0.009 | 0.009 | 0.008 | 0.009 | 0.016 | 0.020 | 0.006 |
| 10.0-20.0 | 0.006 | 0.006 | 0.007 | 0.007 | 0.007 | 0.008 | 0.006 | 0.013 |

TABLE Supp.CLVIII. Measured cross section ratio of water over scintillator in $p_{\mu,T}$, and the absolute and fractional cross section ratio uncertainties

| Bin edges (GeV/c) | Ratio | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 0.00-0.15 | 0.66 | 0.239 | 0.352 | 0.096 | 0.011 |
| 0.15-0.30 | 0.71 | 0.196 | 0.253 | 0.110 | 0.005 |
| 0.30-0.45 | 1.08 | 0.256 | 0.213 | 0.103 | 0.005 |
| 0.45-0.60 | 0.88 | 0.294 | 0.289 | 0.172 | 0.004 |
| 0.60-0.75 | 0.28 | 0.276 | 0.740 | 0.641 | 0.016 |
| 0.75-0.90 | 1.40 | 0.670 | 0.421 | 0.228 | 0.010 |
| 0.90-1.25 | 1.88 | 1.082 | 0.545 | 0.187 | 0.014 |
| 1.25-2.50 | 2.94 | 4.698 | 0.959 | 1.279 | 0.055 |

TABLE Supp.CLIX. Statistical covariance matrix of measured cross section ratio of water over scintillator in $p_{\mu,T}$

| Bin edges (GeV/c) | 0.00-0.15 | 0.15-0.30 | 0.30-0.45 | 0.45-0.60 | 0.60-0.75 | 0.75-0.90 | 0.90-1.25 | 1.25-2.50 |
|---|---|---|---|---|---|---|---|---|
| 0.00-0.15 | 0.0533 | 0.0002 | -0.0033 | -0.0005 | -0.0001 | -0.0001 | -0.0000 | 0.0001 |
| 0.15-0.30 | 0.0002 | 0.0321 | 0.0019 | -0.0039 | -0.0011 | -0.0006 | -0.0026 | -0.0007 |
| 0.30-0.45 | -0.0033 | 0.0019 | 0.0531 | 0.0075 | -0.0023 | -0.0068 | -0.0021 | -0.0149 |
| 0.45-0.60 | -0.0005 | -0.0039 | 0.0075 | 0.0640 | 0.0179 | -0.0075 | -0.0038 | 0.0104 |
| 0.60-0.75 | -0.0001 | -0.0011 | -0.0023 | 0.0179 | 0.0436 | 0.0540 | 0.0031 | -0.0223 |
| 0.75-0.90 | -0.0001 | -0.0006 | -0.0068 | -0.0075 | 0.0540 | 0.3466 | 0.0398 | -0.0432 |
| 0.90-1.25 | -0.0000 | -0.0026 | -0.0021 | -0.0038 | 0.0031 | 0.0398 | 1.0485 | 0.0088 |
| 1.25-2.50 | 0.0001 | -0.0007 | -0.0149 | 0.0104 | -0.0223 | -0.0432 | 0.0088 | 7.9464 |

TABLE Supp.CLX. Systematic covariance matrix of measured cross section ratio of water over scintillator in $p_{\mu,T}$

| Bin edges (GeV/c) | 0.00-0.15 | 0.15-0.30 | 0.30-0.45 | 0.45-0.60 | 0.60-0.75 | 0.75-0.90 | 0.90-1.25 | 1.25-2.50 |
|---|---|---|---|---|---|---|---|---|
| 0.00-0.15 | 0.004 | 0.004 | 0.001 | 0.000 | -0.001 | -0.001 | 0.000 | -0.015 |
| 0.15-0.30 | 0.004 | 0.006 | 0.004 | 0.004 | 0.004 | 0.008 | 0.008 | 0.006 |
| 0.30-0.45 | 0.001 | 0.004 | 0.012 | 0.011 | 0.009 | 0.019 | 0.021 | 0.059 |
| 0.45-0.60 | 0.000 | 0.004 | 0.011 | 0.023 | 0.020 | 0.029 | 0.008 | 0.206 |
| 0.60-0.75 | -0.001 | 0.004 | 0.009 | 0.020 | 0.033 | 0.043 | 0.014 | 0.205 |
| 0.75-0.90 | -0.001 | 0.008 | 0.019 | 0.029 | 0.043 | 0.102 | 0.050 | 0.192 |
| 0.90-1.25 | 0.000 | 0.008 | 0.021 | 0.008 | 0.014 | 0.050 | 0.123 | -0.226 |
| 1.25-2.50 | -0.015 | 0.006 | 0.059 | 0.206 | 0.205 | 0.192 | -0.226 | 14.125 |

TABLE Supp.CLXI. Measured cross section ratio of water over scintillator in $Q^2$, and the absolute and fractional cross section ratio uncertainties

| Bin edges ($\mathrm{GeV}^2/c^2$) | Ratio | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 0.000-0.025 | 0.65 | 0.232 | 0.348 | 0.091 | 0.010 |
| 0.025-0.050 | 0.61 | 0.225 | 0.331 | 0.163 | 0.005 |
| 0.050-0.100 | 0.79 | 0.225 | 0.269 | 0.093 | 0.008 |
| 0.100-0.200 | 1.12 | 0.285 | 0.225 | 0.117 | 0.005 |
| 0.200-0.300 | 1.03 | 0.295 | 0.254 | 0.133 | 0.003 |
| 0.300-1.300 | 0.60 | 0.291 | 0.376 | 0.309 | 0.007 |
| 1.300-2.000 | 6.21 | 4.215 | 0.628 | 0.257 | 0.028 |
| 2.000-3.000 | 2.11 | 3.800 | 0.967 | 1.520 | 0.077 |

TABLE Supp.CLXII. Statistical covariance matrix of measured cross section ratio of water over scintillator in $Q^2$

| Bin edges (GeV$^2/c^2$) | 0.000-0.025 | 0.025-0.050 | 0.050-0.100 | 0.100-0.200 | 0.200-0.300 | 0.300-1.300 | 1.300-2.000 | 2.000-3.000 |
|---|---|---|---|---|---|---|---|---|
| 0.000-0.025 | 0.0505 | 0.0079 | -0.0051 | -0.0020 | -0.0002 | -0.0001 | -0.0000 | -0.0000 |
| 0.025-0.050 | 0.0079 | 0.0409 | 0.0165 | -0.0050 | -0.0029 | -0.0002 | -0.0012 | -0.0000 |
| 0.050-0.100 | -0.0051 | 0.0165 | 0.0452 | 0.0068 | -0.0055 | -0.0014 | -0.0070 | -0.0007 |
| 0.100-0.200 | -0.0020 | -0.0050 | 0.0068 | 0.0641 | 0.0219 | -0.0057 | -0.0184 | -0.0060 |
| 0.200-0.300 | -0.0002 | -0.0029 | -0.0055 | 0.0219 | 0.0683 | 0.0069 | -0.0417 | 0.0018 |
| 0.300-1.300 | -0.0001 | -0.0002 | -0.0014 | -0.0057 | 0.0069 | 0.0505 | 0.0168 | -0.0113 |
| 1.300-2.000 | -0.0000 | -0.0012 | -0.0070 | -0.0184 | -0.0417 | 0.0168 | 15.2090 | 0.0360 |
| 2.000-3.000 | -0.0000 | -0.0000 | -0.0007 | -0.0060 | 0.0018 | -0.0113 | 0.0360 | 4.1625 |

TABLE Supp.CLXIII. Systematic covariance matrix of measured cross section ratio of water over scintillator in $Q^2$

| Bin edges ($\mathrm{GeV}^2/c^2$) | 0.000-0.025 | 0.025-0.050 | 0.050-0.100 | 0.100-0.200 | 0.200-0.300 | 0.300-1.300 | 1.300-2.000 | 2.000-3.000 |
|---|---|---|---|---|---|---|---|---|
| 0.000-0.025 | 0.003 | 0.005 | 0.003 | 0.002 | 0.001 | -0.001 | 0.009 | -0.009 |
| 0.025-0.050 | 0.005 | 0.010 | 0.005 | 0.007 | 0.004 | 0.003 | 0.017 | -0.002 |
| 0.050-0.100 | 0.003 | 0.005 | 0.005 | 0.004 | 0.004 | 0.007 | 0.016 | 0.014 |
| 0.100-0.200 | 0.002 | 0.007 | 0.004 | 0.017 | 0.012 | 0.011 | 0.031 | 0.059 |
| 0.200-0.300 | 0.001 | 0.004 | 0.004 | 0.012 | 0.019 | 0.016 | 0.019 | 0.082 |
| 0.300-1.300 | -0.001 | 0.003 | 0.007 | 0.011 | 0.016 | 0.034 | 0.075 | 0.147 |
| 1.300-2.000 | 0.009 | 0.017 | 0.016 | 0.031 | 0.019 | 0.075 | 2.555 | 0.282 |
| 2.000-3.000 | -0.009 | -0.002 | 0.014 | 0.059 | 0.082 | 0.147 | 0.282 | 10.275 |

TABLE Supp.CLXIV. Measured cross section ratio of water over scintillator in $W_{exp}$, and the absolute and fractional cross section ratio uncertainties

| Bin edges (GeV/c$^2$) | Ratio | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 0.9-1.1 | 2.88 | 2.560 | 0.852 | 0.250 | 0.023 |
| 1.1-1.2 | 1.54 | 0.367 | 0.208 | 0.115 | 0.007 |
| 1.2-1.3 | 0.51 | 0.239 | 0.369 | 0.296 | 0.008 |
| 1.3-1.4 | 0.20 | 0.184 | 0.666 | 0.613 | 0.015 |

TABLE Supp.CLXV. Statistical covariance matrix of measured cross section ratio of water over scintillator in $W_{exp}$

| Bin edges (GeV/c$^2$) | 0.9-1.1 | 1.1-1.2 | 1.2-1.3 | 1.3-1.4 |
|---|---|---|---|---|
| 0.9-1.1 | 6.0348 | -0.2345 | -0.0974 | -0.0478 |
| 1.1-1.2 | -0.2345 | 0.1030 | -0.0183 | -0.0113 |
| 1.2-1.3 | -0.0974 | -0.0183 | 0.0347 | 0.0201 |
| 1.3-1.4 | -0.0478 | -0.0113 | 0.0201 | 0.0183 |

TABLE Supp.CLXVI. Systematic covariance matrix of measured cross section ratio of water over scintillator in $W_{exp}$

| Bin edges (GeV/c$^2$) | 0.9-1.1 | 1.1-1.2 | 1.2-1.3 | 1.3-1.4 |
|---|---|---|---|---|
| 0.9-1.1 | 0.521 | 0.012 | 0.008 | 0.002 |
| 1.1-1.2 | 0.012 | 0.032 | 0.000 | -0.001 |
| 1.2-1.3 | 0.008 | 0.000 | 0.022 | 0.018 |
| 1.3-1.4 | 0.002 | -0.001 | 0.018 | 0.015 |

TABLE Supp.CLXVII. Measured cross section ratio of water over scintillator in $T_\pi$, and the absolute and fractional cross section ratio uncertainties

| Bin edges (GeV) | Ratio | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 0.035-0.100 | 1.14 | 0.282 | 0.227 | 0.096 | 0.007 |
| 0.100-0.150 | 0.84 | 0.189 | 0.195 | 0.113 | 0.002 |
| 0.150-0.200 | 0.91 | 0.278 | 0.265 | 0.154 | 0.003 |
| 0.200-0.350 | 0.85 | 0.365 | 0.350 | 0.250 | 0.010 |

TABLE Supp.CLXVIII. Statistical covariance matrix of measured cross section ratio of water over scintillator in $T_\pi$

| Bin edges (GeV) | 0.035-0.100 | 0.100-0.150 | 0.150-0.200 | 0.200-0.350 |
|---|---|---|---|---|
| 0.035-0.100 | 0.0676 | 0.0068 | -0.0075 | -0.0044 |
| 0.100-0.150 | 0.0068 | 0.0268 | 0.0048 | -0.0031 |
| 0.150-0.200 | -0.0075 | 0.0048 | 0.0576 | 0.0103 |
| 0.200-0.350 | -0.0044 | -0.0031 | 0.0103 | 0.0880 |

TABLE Supp.CLXIX. Systematic covariance matrix of measured cross section ratio of water over scintillator in $T_\pi$

| Bin edges (GeV) | 0.035-0.100 | 0.100-0.150 | 0.150-0.200 | 0.200-0.350 |
|---|---|---|---|---|
| 0.035-0.100 | 0.012 | 0.007 | 0.007 | 0.011 |
| 0.100-0.150 | 0.007 | 0.009 | 0.009 | 0.014 |
| 0.150-0.200 | 0.007 | 0.009 | 0.020 | 0.023 |
| 0.200-0.350 | 0.011 | 0.014 | 0.023 | 0.045 |

TABLE Supp.CLXX. Measured cross section ratio of water over scintillator in $\theta_\pi$, and the absolute and fractional cross section ratio uncertainties

| Bin edges (degree) | Ratio | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 0-10 | 1.01 | 0.940 | 0.901 | 0.251 | 0.010 |
| 10-20 | 0.59 | 0.312 | 0.482 | 0.211 | 0.013 |
| 20-30 | 0.73 | 0.255 | 0.308 | 0.170 | 0.007 |
| 30-40 | 0.68 | 0.266 | 0.325 | 0.219 | 0.005 |
| 40-50 | 0.87 | 0.254 | 0.260 | 0.131 | 0.008 |
| 50-60 | 1.34 | 0.345 | 0.243 | 0.085 | 0.006 |
| 60-120 | 1.08 | 0.318 | 0.279 | 0.090 | 0.003 |
| 120-135 | 1.11 | 0.585 | 0.503 | 0.165 | 0.007 |
| 135-150 | 1.30 | 0.582 | 0.429 | 0.132 | 0.009 |
| 150-165 | 0.94 | 0.860 | 0.853 | 0.336 | 0.008 |
| 165-180 | 0.50 | 0.752 | 1.423 | 0.487 | 0.014 |

TABLE Supp.CLXXI. Statistical covariance matrix of measured cross section ratio of water over scintillator in $\theta_\pi$

| Bin edges (degree) | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 | 60-120 | 120-135 | 135-150 | 150-165 | 165-180 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0-10 | 0.8207 | 0.0635 | -0.0173 | 0.0020 | -0.0045 | -0.0046 | -0.0045 | -0.0042 | -0.0049 | -0.0030 | 0.0294 |
| 10-20 | 0.0635 | 0.0816 | 0.0165 | -0.0030 | -0.0030 | 0.0012 | -0.0052 | -0.0040 | 0.0028 | -0.0020 | 0.1282 |
| 20-30 | -0.0173 | 0.0165 | 0.0498 | 0.0082 | -0.0017 | -0.0052 | -0.0011 | -0.0008 | -0.0035 | -0.0044 | -0.0028 |
| 30-40 | 0.0020 | -0.0030 | 0.0082 | 0.0487 | 0.0103 | -0.0038 | 0.0029 | -0.0065 | -0.0036 | -0.0024 | -0.0074 |
| 40-50 | -0.0045 | -0.0030 | -0.0017 | 0.0103 | 0.0514 | 0.0145 | 0.0001 | -0.0021 | 0.0033 | -0.0130 | -0.0017 |
| 50-60 | -0.0046 | 0.0012 | -0.0052 | -0.0038 | 0.0145 | 0.1063 | 0.0286 | -0.0060 | -0.0166 | -0.0156 | 0.0068 |
| 60-120 | -0.0045 | -0.0052 | -0.0011 | 0.0029 | 0.0001 | 0.0286 | 0.0914 | 0.0155 | 0.0009 | 0.0090 | -0.0075 |
| 120-135 | -0.0042 | -0.0040 | -0.0008 | -0.0065 | -0.0021 | -0.0060 | 0.0155 | 0.3089 | 0.0888 | 0.0063 | -0.0054 |
| 135-150 | -0.0049 | 0.0028 | -0.0035 | -0.0036 | 0.0033 | -0.0166 | 0.0009 | 0.0888 | 0.3099 | 0.2742 | 0.0028 |
| 150-165 | -0.0030 | -0.0020 | -0.0044 | -0.0024 | -0.0130 | -0.0156 | 0.0090 | 0.0063 | 0.2742 | 0.6408 | -0.0054 |
| 165-180 | 0.0294 | 0.1282 | -0.0028 | -0.0074 | -0.0017 | 0.0068 | -0.0075 | -0.0054 | 0.0028 | -0.0054 | 0.5064 |

TABLE Supp.CLXXII. Systematic covariance matrix of measured cross section ratio of water over scintillator in $\theta_\pi$

| Bin edges (degree) | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 | 60-120 | 120-135 | 135-150 | 150-165 | 165-180 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0-10 | 0.064 | 0.006 | 0.011 | 0.021 | 0.013 | 0.009 | 0.006 | 0.021 | 0.005 | -0.007 | 0.017 |
| 10-20 | 0.006 | 0.016 | 0.012 | 0.008 | 0.007 | 0.004 | 0.006 | 0.009 | 0.006 | 0.012 | 0.014 |
| 20-30 | 0.011 | 0.012 | 0.015 | 0.013 | 0.010 | 0.007 | 0.006 | 0.010 | 0.008 | 0.014 | 0.010 |
| 30-40 | 0.021 | 0.008 | 0.013 | 0.022 | 0.014 | 0.009 | 0.008 | 0.007 | 0.013 | 0.009 | 0.006 |
| 40-50 | 0.013 | 0.007 | 0.010 | 0.014 | 0.013 | 0.009 | 0.008 | 0.009 | 0.012 | 0.013 | 0.005 |
| 50-60 | 0.009 | 0.004 | 0.007 | 0.009 | 0.009 | 0.013 | 0.007 | 0.010 | 0.009 | 0.014 | -0.000 |
| 60-120 | 0.006 | 0.006 | 0.006 | 0.008 | 0.008 | 0.007 | 0.009 | 0.006 | 0.009 | 0.003 | 0.000 |
| 120-135 | 0.021 | 0.009 | 0.010 | 0.007 | 0.009 | 0.010 | 0.006 | 0.033 | 0.003 | 0.016 | 0.016 |
| 135-150 | 0.005 | 0.006 | 0.008 | 0.013 | 0.012 | 0.009 | 0.009 | 0.003 | 0.029 | 0.032 | -0.013 |
| 150-165 | -0.007 | 0.012 | 0.014 | 0.009 | 0.013 | 0.014 | 0.003 | 0.016 | 0.032 | 0.100 | -0.008 |
| 165-180 | 0.017 | 0.014 | 0.010 | 0.006 | 0.005 | -0.000 | 0.000 | 0.016 | -0.013 | -0.008 | 0.059 |

TABLE Supp.CLXXIII. Measured cross section ratio of iron over scintillator in $p_\mu$, and the absolute and fractional cross section ratio uncertainties

| Bin edges (GeV/c) | Ratio | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 1.5-3.0 | 0.86 | 0.163 | 0.157 | 0.105 | 0.004 |
| 3.0-4.0 | 0.78 | 0.125 | 0.110 | 0.116 | 0.007 |
| 4.0-5.0 | 0.62 | 0.102 | 0.099 | 0.132 | 0.009 |
| 5.0-6.0 | 0.62 | 0.098 | 0.090 | 0.130 | 0.006 |
| 6.0-7.0 | 0.60 | 0.100 | 0.095 | 0.136 | 0.005 |
| 7.0-8.5 | 0.59 | 0.114 | 0.120 | 0.152 | 0.015 |
| 8.5-10.0 | 0.66 | 0.139 | 0.147 | 0.150 | 0.012 |
| 10.0-20.0 | 0.94 | 0.218 | 0.174 | 0.154 | 0.005 |

TABLE Supp.CLXXIV. Statistical covariance matrix of measured cross section ratio of iron over scintillator in $p_\mu$

| Bin edges (GeV/c) | 1.5-3.0 | 3.0-4.0 | 4.0-5.0 | 5.0-6.0 | 6.0-7.0 | 7.0-8.5 | 8.5-10.0 | 10.0-20.0 |
|---|---|---|---|---|---|---|---|---|
| 1.5-3.0 | 0.0184 | 0.0023 | 0.0003 | 0.0002 | 0.0001 | 0.0000 | 0.0002 | 0.0002 |
| 3.0-4.0 | 0.0023 | 0.0074 | 0.0013 | 0.0003 | 0.0001 | 0.0001 | 0.0001 | 0.0002 |
| 4.0-5.0 | 0.0003 | 0.0013 | 0.0038 | 0.0013 | 0.0004 | 0.0001 | 0.0001 | 0.0001 |
| 5.0-6.0 | 0.0002 | 0.0003 | 0.0013 | 0.0031 | 0.0014 | 0.0004 | 0.0002 | 0.0002 |
| 6.0-7.0 | 0.0001 | 0.0001 | 0.0004 | 0.0014 | 0.0033 | 0.0018 | 0.0007 | 0.0004 |
| 7.0-8.5 | 0.0000 | 0.0001 | 0.0001 | 0.0004 | 0.0018 | 0.0050 | 0.0035 | 0.0014 |
| 8.5-10.0 | 0.0002 | 0.0001 | 0.0001 | 0.0002 | 0.0007 | 0.0035 | 0.0095 | 0.0060 |
| 10.0-20.0 | 0.0002 | 0.0002 | 0.0001 | 0.0002 | 0.0004 | 0.0014 | 0.0060 | 0.0266 |

TABLE Supp.CLXXV. Systematic covariance matrix of measured cross section ratio of iron over scintillator in $p_\mu$

| Bin edges (GeV/c) | 1.5-3.0 | 3.0-4.0 | 4.0-5.0 | 5.0-6.0 | 6.0-7.0 | 7.0-8.5 | 8.5-10.0 | 10.0-20.0 |
|---|---|---|---|---|---|---|---|---|
| 1.5-3.0 | 0.008 | 0.006 | 0.005 | 0.005 | 0.005 | 0.004 | 0.005 | 0.008 |
| 3.0-4.0 | 0.006 | 0.008 | 0.007 | 0.007 | 0.007 | 0.007 | 0.008 | 0.011 |
| 4.0-5.0 | 0.005 | 0.007 | 0.007 | 0.006 | 0.006 | 0.006 | 0.007 | 0.009 |
| 5.0-6.0 | 0.005 | 0.007 | 0.006 | 0.007 | 0.006 | 0.007 | 0.007 | 0.010 |
| 6.0-7.0 | 0.005 | 0.007 | 0.006 | 0.006 | 0.007 | 0.007 | 0.007 | 0.010 |
| 7.0-8.5 | 0.004 | 0.007 | 0.006 | 0.007 | 0.007 | 0.008 | 0.008 | 0.011 |
| 8.5-10.0 | 0.005 | 0.008 | 0.007 | 0.007 | 0.007 | 0.008 | 0.010 | 0.012 |
| 10.0-20.0 | 0.008 | 0.011 | 0.009 | 0.010 | 0.010 | 0.011 | 0.012 | 0.021 |

TABLE Supp.CLXXVI. Measured cross section ratio of iron over scintillator in $\theta_\mu$, and the absolute and fractional cross section ratio uncertainties

| Bin edges (degree) | Ratio | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 0-2 | 0.62 | 0.093 | 0.130 | 0.076 | 0.002 |
| 2-3 | 0.59 | 0.090 | 0.132 | 0.078 | 0.002 |
| 3-4 | 0.62 | 0.102 | 0.133 | 0.096 | 0.001 |
| 4-5 | 0.53 | 0.113 | 0.161 | 0.138 | 0.004 |
| 5-6 | 0.48 | 0.143 | 0.207 | 0.212 | 0.003 |
| 6-8 | 0.71 | 0.159 | 0.143 | 0.173 | 0.002 |
| 8-10 | 0.94 | 0.241 | 0.182 | 0.181 | 0.003 |
| 10-13 | 0.89 | 0.321 | 0.275 | 0.234 | 0.006 |

TABLE Supp.CLXXVII. Statistical covariance matrix of measured cross section ratio of iron over scintillator in $\theta_\mu$

| Bin edges (degree) | 0-2 | 2-3 | 3-4 | 4-5 | 5-6 | 6-8 | 8-10 | 10-13 |
|---|---|---|---|---|---|---|---|---|
| 0-2 | 0.0065 | 0.0001 | -0.0005 | -0.0001 | 0.0000 | 0.0000 | -0.0000 | -0.0001 |
| 2-3 | 0.0001 | 0.0060 | 0.0008 | -0.0006 | -0.0002 | -0.0000 | -0.0000 | -0.0000 |
| 3-4 | -0.0005 | 0.0008 | 0.0069 | 0.0010 | -0.0007 | -0.0001 | -0.0000 | -0.0001 |
| 4-5 | -0.0001 | -0.0006 | 0.0010 | 0.0074 | 0.0016 | -0.0005 | -0.0001 | 0.0000 |
| 5-6 | 0.0000 | -0.0002 | -0.0007 | 0.0016 | 0.0100 | 0.0006 | -0.0007 | -0.0001 |
| 6-8 | 0.0000 | -0.0000 | -0.0001 | -0.0005 | 0.0006 | 0.0104 | -0.0002 | -0.0009 |
| 8-10 | -0.0000 | -0.0000 | -0.0000 | -0.0001 | -0.0007 | -0.0002 | 0.0291 | -0.0009 |
| 10-13 | -0.0001 | -0.0000 | -0.0001 | 0.0000 | -0.0001 | -0.0009 | -0.0009 | 0.0599 |

TABLE Supp.CLXXVIII. Systematic covariance matrix of measured cross section ratio of iron over scintillator in $\theta_\mu$

| Bin edges (degree) | 0-2 | 2-3 | 3-4 | 4-5 | 5-6 | 6-8 | 8-10 | 10-13 |
|---|---|---|---|---|---|---|---|---|
| 0-2 | 0.002 | 0.001 | 0.001 | 0.001 | 0.002 | 0.002 | 0.003 | 0.004 |
| 2-3 | 0.001 | 0.002 | 0.002 | 0.003 | 0.004 | 0.004 | 0.006 | 0.007 |
| 3-4 | 0.001 | 0.002 | 0.004 | 0.004 | 0.005 | 0.006 | 0.009 | 0.011 |
| 4-5 | 0.001 | 0.003 | 0.004 | 0.005 | 0.007 | 0.008 | 0.011 | 0.013 |
| 5-6 | 0.002 | 0.004 | 0.005 | 0.007 | 0.010 | 0.011 | 0.016 | 0.018 |
| 6-8 | 0.002 | 0.004 | 0.006 | 0.008 | 0.011 | 0.015 | 0.019 | 0.023 |
| 8-10 | 0.003 | 0.006 | 0.009 | 0.011 | 0.016 | 0.019 | 0.029 | 0.032 |
| 10-13 | 0.004 | 0.007 | 0.011 | 0.013 | 0.018 | 0.023 | 0.032 | 0.043 |

TABLE Supp.CLXXIX. Measured cross section ratio of iron over scintillator in $p_{\mu,||}$, and the absolute and fractional cross section ratio uncertainties

| Bin edges (GeV/c) | Ratio | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 1.5-3.0 | 0.82 | 0.164 | 0.164 | 0.114 | 0.005 |
| 3.0-4.0 | 0.81 | 0.126 | 0.107 | 0.113 | 0.006 |
| 4.0-5.0 | 0.61 | 0.102 | 0.100 | 0.133 | 0.009 |
| 5.0-6.0 | 0.61 | 0.098 | 0.091 | 0.132 | 0.006 |
| 6.0-7.0 | 0.61 | 0.101 | 0.095 | 0.137 | 0.005 |
| 7.0-8.5 | 0.60 | 0.115 | 0.120 | 0.152 | 0.015 |
| 8.5-10.0 | 0.65 | 0.134 | 0.150 | 0.144 | 0.013 |
| 10.0-20.0 | 0.95 | 0.219 | 0.174 | 0.150 | 0.004 |

TABLE Supp.CLXXX. Statistical covariance matrix of measured cross section ratio of iron over scintillator in $p_{\mu,||}$

| Bin edges (GeV/c) | 1.5-3.0 | 3.0-4.0 | 4.0-5.0 | 5.0-6.0 | 6.0-7.0 | 7.0-8.5 | 8.5-10.0 | 10.0-20.0 |
|---|---|---|---|---|---|---|---|---|
| 1.5-3.0 | 0.0181 | 0.0021 | 0.0003 | 0.0002 | 0.0001 | 0.0000 | 0.0002 | 0.0002 |
| 3.0-4.0 | 0.0021 | 0.0075 | 0.0013 | 0.0003 | 0.0001 | 0.0001 | 0.0001 | 0.0002 |
| 4.0-5.0 | 0.0003 | 0.0013 | 0.0037 | 0.0013 | 0.0004 | 0.0001 | 0.0001 | 0.0001 |
| 5.0-6.0 | 0.0002 | 0.0003 | 0.0013 | 0.0031 | 0.0014 | 0.0004 | 0.0002 | 0.0002 |
| 6.0-7.0 | 0.0001 | 0.0001 | 0.0004 | 0.0014 | 0.0033 | 0.0018 | 0.0007 | 0.0004 |
| 7.0-8.5 | 0.0000 | 0.0001 | 0.0001 | 0.0004 | 0.0018 | 0.0051 | 0.0034 | 0.0013 |
| 8.5-10.0 | 0.0002 | 0.0001 | 0.0001 | 0.0002 | 0.0007 | 0.0034 | 0.0093 | 0.0060 |
| 10.0-20.0 | 0.0002 | 0.0002 | 0.0001 | 0.0002 | 0.0004 | 0.0013 | 0.0060 | 0.0275 |

TABLE Supp.CLXXXI. Systematic covariance matrix of measured cross section ratio of iron over scintillator in $p_{\mu,||}$

| Bin edges (GeV/c) | 1.5-3.0 | 3.0-4.0 | 4.0-5.0 | 5.0-6.0 | 6.0-7.0 | 7.0-8.5 | 8.5-10.0 | 10.0-20.0 |
|---|---|---|---|---|---|---|---|---|
| 1.5-3.0 | 0.009 | 0.006 | 0.006 | 0.005 | 0.005 | 0.005 | 0.006 | 0.009 |
| 3.0-4.0 | 0.006 | 0.008 | 0.007 | 0.007 | 0.007 | 0.008 | 0.008 | 0.012 |
| 4.0-5.0 | 0.006 | 0.007 | 0.007 | 0.006 | 0.006 | 0.006 | 0.006 | 0.009 |
| 5.0-6.0 | 0.005 | 0.007 | 0.006 | 0.006 | 0.006 | 0.007 | 0.007 | 0.009 |
| 6.0-7.0 | 0.005 | 0.007 | 0.006 | 0.006 | 0.007 | 0.007 | 0.007 | 0.010 |
| 7.0-8.5 | 0.005 | 0.008 | 0.006 | 0.007 | 0.007 | 0.008 | 0.008 | 0.011 |
| 8.5-10.0 | 0.006 | 0.008 | 0.006 | 0.007 | 0.007 | 0.008 | 0.009 | 0.011 |
| 10.0-20.0 | 0.009 | 0.012 | 0.009 | 0.009 | 0.010 | 0.011 | 0.011 | 0.021 |

TABLE Supp.CLXXXII. Measured cross section ratio of iron over scintillator in $p_{\mu,T}$, and the absolute and fractional cross section ratio uncertainties

| Bin edges (GeV/c) | Ratio | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 0.00-0.15 | 0.58 | 0.115 | 0.179 | 0.084 | 0.003 |
| 0.15-0.30 | 0.59 | 0.078 | 0.110 | 0.074 | 0.001 |
| 0.30-0.45 | 0.59 | 0.089 | 0.111 | 0.100 | 0.001 |
| 0.45-0.60 | 0.70 | 0.121 | 0.111 | 0.132 | 0.001 |
| 0.60-0.75 | 0.62 | 0.168 | 0.161 | 0.217 | 0.003 |
| 0.75-0.90 | 0.70 | 0.263 | 0.215 | 0.306 | 0.008 |
| 0.90-1.25 | 1.01 | 0.395 | 0.293 | 0.260 | 0.008 |
| 1.25-2.50 | 1.50 | 1.483 | 0.912 | 0.390 | 0.014 |

TABLE Supp.CLXXXIII. Statistical covariance matrix of measured cross section ratio of iron over scintillator in $p_{\mu,T}$

| Bin edges (GeV/c) | 0.00-0.15 | 0.15-0.30 | 0.30-0.45 | 0.45-0.60 | 0.60-0.75 | 0.75-0.90 | 0.90-1.25 | 1.25-2.50 |
|---|---|---|---|---|---|---|---|---|
| 0.00-0.15 | 0.0109 | 0.0001 | -0.0005 | -0.0001 | -0.0000 | -0.0000 | -0.0000 | -0.0000 |
| 0.15-0.30 | 0.0001 | 0.0042 | 0.0003 | -0.0004 | -0.0001 | -0.0000 | -0.0001 | -0.0004 |
| 0.30-0.45 | -0.0005 | 0.0003 | 0.0043 | 0.0005 | -0.0004 | -0.0003 | -0.0003 | -0.0001 |
| 0.45-0.60 | -0.0001 | -0.0004 | 0.0005 | 0.0061 | 0.0012 | -0.0009 | -0.0010 | -0.0005 |
| 0.60-0.75 | -0.0000 | -0.0001 | -0.0004 | 0.0012 | 0.0100 | 0.0037 | -0.0022 | -0.0006 |
| 0.75-0.90 | -0.0000 | -0.0000 | -0.0003 | -0.0009 | 0.0037 | 0.0229 | 0.0145 | -0.0136 |
| 0.90-1.25 | -0.0000 | -0.0001 | -0.0003 | -0.0010 | -0.0022 | 0.0145 | 0.0872 | 0.0567 |
| 1.25-2.50 | -0.0000 | -0.0004 | -0.0001 | -0.0005 | -0.0006 | -0.0136 | 0.0567 | 1.8586 |

TABLE Supp.CLXXXIV. Systematic covariance matrix of measured cross section ratio of iron over scintillator in $p_{\mu,T}$

| Bin edges (GeV/c) | 0.00-0.15 | 0.15-0.30 | 0.30-0.45 | 0.45-0.60 | 0.60-0.75 | 0.75-0.90 | 0.90-1.25 | 1.25-2.50 |
|---|---|---|---|---|---|---|---|---|
| 0.00-0.15 | 0.002 | 0.001 | 0.000 | 0.000 | 0.001 | -0.000 | 0.002 | 0.001 |
| 0.15-0.30 | 0.001 | 0.002 | 0.002 | 0.003 | 0.004 | 0.006 | 0.008 | 0.011 |
| 0.30-0.45 | 0.000 | 0.002 | 0.004 | 0.005 | 0.007 | 0.011 | 0.013 | 0.018 |
| 0.45-0.60 | 0.000 | 0.003 | 0.005 | 0.009 | 0.011 | 0.019 | 0.021 | 0.032 |
| 0.60-0.75 | 0.001 | 0.004 | 0.007 | 0.011 | 0.018 | 0.027 | 0.032 | 0.043 |
| 0.75-0.90 | -0.000 | 0.006 | 0.011 | 0.019 | 0.027 | 0.046 | 0.051 | 0.078 |
| 0.90-1.25 | 0.002 | 0.008 | 0.013 | 0.021 | 0.032 | 0.051 | 0.069 | 0.100 |
| 1.25-2.50 | 0.001 | 0.011 | 0.018 | 0.032 | 0.043 | 0.078 | 0.100 | 0.340 |

TABLE Supp.CLXXXV. Measured cross section ratio of iron over scintillator in $Q^2$, and the absolute and fractional cross section ratio uncertainties

| Bin edges ($\mathrm{GeV}^2/c^2$) | Ratio | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 0.000-0.025 | 0.56 | 0.108 | 0.172 | 0.084 | 0.003 |
| 0.025-0.050 | 0.54 | 0.088 | 0.147 | 0.071 | 0.003 |
| 0.050-0.100 | 0.57 | 0.086 | 0.127 | 0.082 | 0.002 |
| 0.100-0.200 | 0.62 | 0.090 | 0.109 | 0.096 | 0.001 |
| 0.200-0.300 | 0.72 | 0.115 | 0.124 | 0.099 | 0.002 |
| 0.300-0.400 | 0.67 | 0.138 | 0.141 | 0.149 | 0.002 |
| 0.400-0.500 | 0.58 | 0.159 | 0.174 | 0.211 | 0.002 |
| 0.500-0.700 | 0.62 | 0.211 | 0.203 | 0.275 | 0.005 |
| 0.700-1.000 | 0.81 | 0.298 | 0.230 | 0.286 | 0.007 |
| 1.000-1.300 | 1.05 | 0.438 | 0.339 | 0.244 | 0.006 |
| 1.300-2.000 | 0.82 | 0.721 | 0.704 | 0.524 | 0.033 |
| 2.000-3.000 | 0.80 | 0.898 | 1.010 | 0.489 | 0.021 |

TABLE Supp.CLXXXVI. Statistical covariance matrix of measured cross section ratio of iron over scintillator in $Q^2$

| Bin edges (GeV$^2/c^2$) | 0.000-0.025 | 0.025-0.050 | 0.050-0.100 | 0.100-0.200 | 0.200-0.300 | 0.300-0.400 | 0.400-0.500 | 0.500-0.700 | 0.700-1.000 | 1.000-1.300 | 1.300-2.000 | 2.000-3.000 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.000-0.025 | 0.0094 | 0.0017 | -0.0007 | -0.0003 | -0.0000 | -0.0000 | -0.0000 | -0.0000 | -0.0000 | -0.0000 | -0.0000 | 0.0000 |
| 0.025-0.050 | 0.0017 | 0.0063 | 0.0018 | -0.0004 | -0.0003 | -0.0000 | -0.0000 | -0.0000 | -0.0000 | -0.0000 | -0.0001 | -0.0001 |
| 0.050-0.100 | -0.0007 | 0.0018 | 0.0053 | 0.0009 | -0.0006 | -0.0003 | -0.0001 | -0.0000 | -0.0000 | -0.0000 | -0.0002 | -0.0003 |
| 0.100-0.200 | -0.0003 | -0.0004 | 0.0009 | 0.0046 | 0.0011 | -0.0004 | -0.0005 | -0.0002 | -0.0002 | -0.0004 | 0.0001 | 0.0003 |
| 0.200-0.300 | -0.0000 | -0.0003 | -0.0006 | 0.0011 | 0.0081 | 0.0026 | 0.0000 | -0.0007 | -0.0006 | -0.0007 | -0.0011 | -0.0020 |
| 0.300-0.400 | -0.0000 | -0.0000 | -0.0003 | -0.0004 | 0.0026 | 0.0090 | 0.0041 | -0.0004 | -0.0011 | -0.0008 | -0.0003 | 0.0026 |
| 0.400-0.500 | -0.0000 | -0.0000 | -0.0001 | -0.0005 | 0.0000 | 0.0041 | 0.0102 | 0.0049 | -0.0008 | -0.0031 | -0.0009 | 0.0014 |
| 0.500-0.700 | -0.0000 | -0.0000 | -0.0000 | -0.0002 | -0.0007 | -0.0004 | 0.0049 | 0.0157 | 0.0068 | -0.0015 | -0.0015 | -0.0020 |
| 0.700-1.000 | -0.0000 | -0.0000 | -0.0000 | -0.0002 | -0.0006 | -0.0011 | -0.0008 | 0.0068 | 0.0348 | 0.0295 | 0.0017 | -0.0016 |
| 1.000-1.300 | -0.0000 | -0.0000 | -0.0000 | -0.0004 | -0.0007 | -0.0008 | -0.0031 | -0.0015 | 0.0295 | 0.1264 | 0.0751 | -0.0011 |
| 1.300-2.000 | -0.0000 | -0.0001 | -0.0002 | 0.0001 | -0.0011 | -0.0003 | -0.0009 | -0.0015 | 0.0017 | 0.0751 | 0.3339 | 0.1520 |
| 2.000-3.000 | 0.0000 | -0.0001 | -0.0003 | 0.0003 | -0.0020 | 0.0026 | 0.0014 | -0.0020 | -0.0016 | -0.0011 | 0.1520 | 0.6530 |

TABLE Supp.CLXXXVII. Systematic covariance matrix of measured cross section ratio of iron over scintillator in $Q^2$

| Bin edges (GeV$^2/c^2$) | 0.000-0.025 | 0.025-0.050 | 0.050-0.100 | 0.100-0.200 | 0.200-0.300 | 0.300-0.400 | 0.400-0.500 | 0.500-0.700 | 0.700-1.000 | 1.000-1.300 | 1.300-2.000 | 2.000-3.000 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.000-0.025 | 0.002 | 0.001 | 0.001 | 0.000 | 0.000 | -0.000 | 0.000 | 0.000 | -0.000 | 0.002 | 0.002 | -0.002 |
| 0.025-0.050 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.002 | 0.002 | 0.003 | 0.004 | 0.005 | 0.007 | 0.004 |
| 0.050-0.100 | 0.001 | 0.001 | 0.002 | 0.002 | 0.003 | 0.004 | 0.004 | 0.006 | 0.008 | 0.009 | 0.012 | 0.010 |
| 0.100-0.200 | 0.000 | 0.001 | 0.002 | 0.004 | 0.004 | 0.005 | 0.006 | 0.009 | 0.012 | 0.014 | 0.019 | 0.015 |
| 0.200-0.300 | 0.000 | 0.001 | 0.003 | 0.004 | 0.005 | 0.007 | 0.008 | 0.011 | 0.015 | 0.015 | 0.021 | 0.020 |
| 0.300-0.400 | -0.000 | 0.002 | 0.004 | 0.005 | 0.007 | 0.010 | 0.011 | 0.015 | 0.020 | 0.021 | 0.028 | 0.030 |
| 0.400-0.500 | 0.000 | 0.002 | 0.004 | 0.006 | 0.008 | 0.011 | 0.015 | 0.019 | 0.026 | 0.027 | 0.042 | 0.037 |
| 0.500-0.700 | 0.000 | 0.003 | 0.006 | 0.009 | 0.011 | 0.015 | 0.019 | 0.029 | 0.037 | 0.038 | 0.060 | 0.049 |
| 0.700-1.000 | -0.000 | 0.004 | 0.008 | 0.012 | 0.015 | 0.020 | 0.026 | 0.037 | 0.054 | 0.051 | 0.077 | 0.069 |
| 1.000-1.300 | 0.002 | 0.005 | 0.009 | 0.014 | 0.015 | 0.021 | 0.027 | 0.038 | 0.051 | 0.066 | 0.091 | 0.066 |
| 1.300-2.000 | 0.002 | 0.007 | 0.012 | 0.019 | 0.021 | 0.028 | 0.042 | 0.060 | 0.077 | 0.091 | 0.185 | 0.122 |
| 2.000-3.000 | -0.002 | 0.004 | 0.010 | 0.015 | 0.020 | 0.030 | 0.037 | 0.049 | 0.069 | 0.066 | 0.122 | 0.153 |

TABLE Supp.CLXXXVIII. Measured cross section ratio of iron over scintillator in $W_{exp}$, and the absolute and fractional cross section ratio uncertainties

| Bin edges (GeV/c$^2$) | Ratio | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 0.9-1.1 | 0.56 | 0.291 | 0.488 | 0.170 | 0.013 |
| 1.1-1.2 | 0.70 | 0.113 | 0.136 | 0.086 | 0.002 |
| 1.2-1.3 | 0.65 | 0.123 | 0.098 | 0.162 | 0.002 |
| 1.3-1.4 | 0.60 | 0.220 | 0.182 | 0.319 | 0.003 |

TABLE Supp.CLXXXIX. Statistical covariance matrix of measured cross section ratio of iron over scintillator in $W_{exp}$

| Bin edges (GeV/c$^2$) | 0.9-1.1 | 1.1-1.2 | 1.2-1.3 | 1.3-1.4 |
|---|---|---|---|---|
| 0.9-1.1 | 0.0757 | 0.0031 | -0.0058 | -0.0026 |
| 1.1-1.2 | 0.0031 | 0.0090 | -0.0022 | -0.0059 |
| 1.2-1.3 | -0.0058 | -0.0022 | 0.0040 | 0.0037 |
| 1.3-1.4 | -0.0026 | -0.0059 | 0.0037 | 0.0118 |

TABLE Supp.CXC. Systematic covariance matrix of measured cross section ratio of iron over scintillator in $W_{exp}$

| Bin edges (GeV/c$^2$) | 0.9-1.1 | 1.1-1.2 | 1.2-1.3 | 1.3-1.4 |
|---|---|---|---|---|
| 0.9-1.1 | 0.009 | 0.002 | 0.004 | 0.006 |
| 1.1-1.2 | 0.002 | 0.004 | -0.001 | -0.004 |
| 1.2-1.3 | 0.004 | -0.001 | 0.011 | 0.019 |
| 1.3-1.4 | 0.006 | -0.004 | 0.019 | 0.036 |

TABLE Supp.CXCI. Measured cross section ratio of iron over scintillator in $T_\pi$, and the absolute and fractional cross section ratio uncertainties

| Bin edges (GeV) | Ratio | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 0.035-0.100 | 0.74 | 0.101 | 0.083 | 0.110 | 0.001 |
| 0.100-0.150 | 0.67 | 0.083 | 0.070 | 0.102 | 0.002 |
| 0.150-0.200 | 0.69 | 0.121 | 0.102 | 0.142 | 0.001 |
| 0.200-0.350 | 0.55 | 0.217 | 0.210 | 0.332 | 0.003 |

TABLE Supp.CXCII. Statistical covariance matrix of measured cross section ratio of iron over scintillator in $T_\pi$

| Bin edges (GeV) | 0.035-0.100 | 0.100-0.150 | 0.150-0.200 | 0.200-0.350 |
|---|---|---|---|---|
| 0.035-0.100 | 0.0037 | 0.0011 | -0.0006 | -0.0006 |
| 0.100-0.150 | 0.0011 | 0.0022 | 0.0009 | -0.0003 |
| 0.150-0.200 | -0.0006 | 0.0009 | 0.0050 | 0.0024 |
| 0.200-0.350 | -0.0006 | -0.0003 | 0.0024 | 0.0134 |

TABLE Supp.CXCIII. Systematic covariance matrix of measured cross section ratio of iron over scintillator in $T_\pi$

| Bin edges (GeV) | 0.035-0.100 | 0.100-0.150 | 0.150-0.200 | 0.200-0.350 |
|---|---|---|---|---|
| 0.035-0.100 | 0.007 | 0.004 | 0.005 | 0.008 |
| 0.100-0.150 | 0.004 | 0.005 | 0.006 | 0.010 |
| 0.150-0.200 | 0.005 | 0.006 | 0.010 | 0.017 |
| 0.200-0.350 | 0.008 | 0.010 | 0.017 | 0.034 |

TABLE Supp.CXCIV. Measured cross section ratio of iron over scintillator in $\theta_\pi$, and the absolute and fractional cross section ratio uncertainties

| Bin edges (degree) | Ratio | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 0-10 | 1.12 | 0.484 | 0.403 | 0.153 | 0.002 |
| 10-20 | 0.71 | 0.190 | 0.218 | 0.153 | 0.006 |
| 20-30 | 0.65 | 0.130 | 0.137 | 0.147 | 0.001 |
| 30-40 | 0.62 | 0.116 | 0.121 | 0.141 | 0.001 |
| 40-50 | 0.61 | 0.106 | 0.111 | 0.134 | 0.003 |
| 50-60 | 0.73 | 0.115 | 0.109 | 0.114 | 0.001 |
| 60-120 | 0.73 | 0.111 | 0.109 | 0.106 | 0.002 |
| 120-135 | 0.74 | 0.178 | 0.189 | 0.149 | 0.002 |
| 135-150 | 0.63 | 0.193 | 0.256 | 0.171 | 0.006 |
| 150-165 | 0.62 | 0.341 | 0.496 | 0.242 | 0.006 |
| 165-180 | 1.64 | 1.981 | 1.107 | 0.482 | 0.017 |

TABLE Supp.CXCV. Statistical covariance matrix of measured cross section ratio of iron over scintillator in $\theta_\pi$

| Bin edges (degree) | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 | 60-120 | 120-135 | 135-150 | 150-165 | 165-180 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0-10 | 0.2045 | 0.0238 | -0.0016 | -0.0003 | 0.0001 | -0.0006 | -0.0006 | -0.0007 | 0.0009 | 0.0022 | -0.0007 |
| 10-20 | 0.0238 | 0.0242 | 0.0039 | -0.0006 | -0.0008 | -0.0004 | -0.0002 | -0.0005 | -0.0005 | -0.0005 | -0.0012 |
| 20-30 | -0.0016 | 0.0039 | 0.0079 | 0.0020 | -0.0003 | -0.0005 | -0.0003 | -0.0005 | -0.0001 | 0.0003 | -0.0016 |
| 30-40 | -0.0003 | -0.0006 | 0.0020 | 0.0057 | 0.0019 | -0.0004 | -0.0004 | -0.0005 | 0.0002 | 0.0004 | -0.0022 |
| 40-50 | 0.0001 | -0.0008 | -0.0003 | 0.0019 | 0.0046 | 0.0019 | -0.0003 | -0.0003 | -0.0000 | -0.0004 | -0.0016 |
| 50-60 | -0.0006 | -0.0004 | -0.0005 | -0.0004 | 0.0019 | 0.0063 | 0.0020 | -0.0006 | -0.0009 | -0.0011 | 0.0002 |
| 60-120 | -0.0006 | -0.0002 | -0.0003 | -0.0004 | -0.0003 | 0.0020 | 0.0063 | 0.0017 | -0.0003 | -0.0002 | 0.0039 |
| 120-135 | -0.0007 | -0.0005 | -0.0005 | -0.0005 | -0.0003 | -0.0006 | 0.0017 | 0.0196 | 0.0083 | 0.0014 | -0.0208 |
| 135-150 | 0.0009 | -0.0005 | -0.0001 | 0.0002 | -0.0000 | -0.0009 | -0.0003 | 0.0083 | 0.0259 | 0.0242 | 0.0300 |
| 150-165 | 0.0022 | -0.0005 | 0.0003 | 0.0004 | -0.0004 | -0.0011 | -0.0002 | 0.0014 | 0.0242 | 0.0940 | 0.3147 |
| 165-180 | -0.0007 | -0.0012 | -0.0016 | -0.0022 | -0.0016 | 0.0002 | 0.0039 | -0.0208 | 0.0300 | 0.3147 | 3.3000 |

TABLE Supp.CXCVI. Systematic covariance matrix of measured cross section ratio of iron over scintillator in $\theta_\pi$

| Bin edges (degree) | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 | 60-120 | 120-135 | 135-150 | 150-165 | 165-180 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0-10 | 0.030 | 0.014 | 0.012 | 0.011 | 0.010 | 0.011 | 0.011 | 0.014 | 0.010 | 0.012 | -0.043 |
| 10-20 | 0.014 | 0.012 | 0.009 | 0.008 | 0.008 | 0.007 | 0.006 | 0.009 | 0.007 | 0.011 | -0.015 |
| 20-30 | 0.012 | 0.009 | 0.009 | 0.008 | 0.007 | 0.006 | 0.006 | 0.009 | 0.008 | 0.010 | -0.007 |
| 30-40 | 0.011 | 0.008 | 0.008 | 0.008 | 0.007 | 0.006 | 0.006 | 0.008 | 0.006 | 0.009 | 0.000 |
| 40-50 | 0.010 | 0.008 | 0.007 | 0.007 | 0.007 | 0.006 | 0.005 | 0.008 | 0.007 | 0.008 | -0.003 |
| 50-60 | 0.011 | 0.007 | 0.006 | 0.006 | 0.006 | 0.007 | 0.006 | 0.008 | 0.007 | 0.006 | 0.001 |
| 60-120 | 0.011 | 0.006 | 0.006 | 0.006 | 0.005 | 0.006 | 0.006 | 0.007 | 0.005 | 0.006 | 0.005 |
| 120-135 | 0.014 | 0.009 | 0.009 | 0.008 | 0.008 | 0.008 | 0.007 | 0.012 | 0.010 | 0.011 | -0.009 |
| 135-150 | 0.010 | 0.007 | 0.008 | 0.006 | 0.007 | 0.007 | 0.005 | 0.010 | 0.011 | 0.011 | -0.011 |
| 150-165 | 0.012 | 0.011 | 0.010 | 0.009 | 0.008 | 0.006 | 0.006 | 0.011 | 0.011 | 0.022 | -0.003 |
| 165-180 | -0.043 | -0.015 | -0.007 | 0.000 | -0.003 | 0.001 | 0.005 | -0.009 | -0.011 | -0.003 | 0.626 |

TABLE Supp.CXCVII. Measured cross section ratio of lead over scintillator in $p_\mu$, and the absolute and fractional cross section ratio uncertainties

| Bin edges (GeV/c) | Ratio | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 1.5-3.0 | 0.43 | 0.108 | 0.225 | 0.109 | 0.007 |
| 3.0-4.0 | 0.50 | 0.088 | 0.140 | 0.106 | 0.008 |
| 4.0-5.0 | 0.43 | 0.069 | 0.108 | 0.120 | 0.008 |
| 5.0-6.0 | 0.40 | 0.066 | 0.102 | 0.129 | 0.004 |
| 6.0-7.0 | 0.35 | 0.060 | 0.117 | 0.130 | 0.006 |
| 7.0-8.5 | 0.34 | 0.062 | 0.134 | 0.117 | 0.017 |
| 8.5-10.0 | 0.38 | 0.083 | 0.182 | 0.120 | 0.023 |
| 10.0-20.0 | 0.38 | 0.133 | 0.310 | 0.166 | 0.035 |

TABLE Supp.CXCVIII. Statistical covariance matrix of measured cross section ratio of lead over scintillator in $p_\mu$

| Bin edges (GeV/c) | 1.5-3.0 | 3.0-4.0 | 4.0-5.0 | 5.0-6.0 | 6.0-7.0 | 7.0-8.5 | 8.5-10.0 | 10.0-20.0 |
|---|---|---|---|---|---|---|---|---|
| 1.5-3.0 | 0.0094 | 0.0012 | 0.0002 | 0.0000 | 0.0001 | 0.0000 | 0.0000 | 0.0000 |
| 3.0-4.0 | 0.0012 | 0.0049 | 0.0010 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 |
| 4.0-5.0 | 0.0002 | 0.0010 | 0.0022 | 0.0007 | 0.0002 | 0.0001 | 0.0001 | 0.0000 |
| 5.0-6.0 | 0.0000 | 0.0002 | 0.0007 | 0.0017 | 0.0009 | 0.0003 | 0.0001 | 0.0001 |
| 6.0-7.0 | 0.0001 | 0.0001 | 0.0002 | 0.0009 | 0.0016 | 0.0010 | 0.0004 | 0.0002 |
| 7.0-8.5 | 0.0000 | 0.0000 | 0.0001 | 0.0003 | 0.0010 | 0.0021 | 0.0017 | 0.0007 |
| 8.5-10.0 | 0.0000 | 0.0000 | 0.0001 | 0.0001 | 0.0004 | 0.0017 | 0.0048 | 0.0038 |
| 10.0-20.0 | 0.0000 | 0.0000 | 0.0000 | 0.0001 | 0.0002 | 0.0007 | 0.0038 | 0.0139 |

TABLE Supp.CXCIX. Systematic covariance matrix of measured cross section ratio of lead over scintillator in $p_\mu$

| Bin edges (GeV/c) | 1.5-3.0 | 3.0-4.0 | 4.0-5.0 | 5.0-6.0 | 6.0-7.0 | 7.0-8.5 | 8.5-10.0 | 10.0-20.0 |
|---|---|---|---|---|---|---|---|---|
| 1.5-3.0 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.001 | 0.001 | 0.002 |
| 3.0-4.0 | 0.002 | 0.003 | 0.002 | 0.002 | 0.002 | 0.001 | 0.002 | 0.002 |
| 4.0-5.0 | 0.002 | 0.002 | 0.003 | 0.003 | 0.002 | 0.002 | 0.001 | 0.002 |
| 5.0-6.0 | 0.002 | 0.002 | 0.003 | 0.003 | 0.002 | 0.002 | 0.002 | 0.002 |
| 6.0-7.0 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 |
| 7.0-8.5 | 0.001 | 0.001 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 |
| 8.5-10.0 | 0.001 | 0.002 | 0.001 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 |
| 10.0-20.0 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.004 |

TABLE Supp.CC. Measured cross section ratio of lead over scintillator in $\theta_\mu$, and the absolute and fractional cross section ratio uncertainties

| Bin edges (degree) | Ratio | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 0-2 | 0.35 | 0.070 | 0.180 | 0.084 | 0.006 |
| 2-3 | 0.37 | 0.069 | 0.168 | 0.081 | 0.003 |
| 3-4 | 0.30 | 0.063 | 0.178 | 0.109 | 0.003 |
| 4-5 | 0.34 | 0.072 | 0.179 | 0.113 | 0.006 |
| 5-6 | 0.31 | 0.084 | 0.220 | 0.154 | 0.006 |
| 6-8 | 0.36 | 0.090 | 0.192 | 0.159 | 0.003 |
| 8-10 | 0.59 | 0.156 | 0.205 | 0.165 | 0.002 |
| 10-13 | 0.78 | 0.237 | 0.255 | 0.162 | 0.005 |

TABLE Supp.CCI. Statistical covariance matrix of measured cross section ratio of lead over scintillator in $\theta_\mu$

| Bin edges (degree) | 0-2 | 2-3 | 3-4 | 4-5 | 5-6 | 6-8 | 8-10 | 10-13 |
|---|---|---|---|---|---|---|---|---|
| 0-2 | 0.0040 | 0.0001 | -0.0003 | -0.0000 | 0.0000 | -0.0000 | -0.0000 | -0.0000 |
| 2-3 | 0.0001 | 0.0038 | 0.0005 | -0.0003 | -0.0001 | -0.0000 | -0.0000 | -0.0000 |
| 3-4 | -0.0003 | 0.0005 | 0.0029 | 0.0005 | -0.0003 | -0.0001 | -0.0000 | -0.0000 |
| 4-5 | -0.0000 | -0.0003 | 0.0005 | 0.0037 | 0.0008 | -0.0003 | -0.0001 | 0.0000 |
| 5-6 | 0.0000 | -0.0001 | -0.0003 | 0.0008 | 0.0047 | 0.0003 | -0.0004 | -0.0000 |
| 6-8 | -0.0000 | -0.0000 | -0.0001 | -0.0003 | 0.0003 | 0.0048 | -0.0000 | -0.0006 |
| 8-10 | -0.0000 | -0.0000 | -0.0000 | -0.0001 | -0.0004 | -0.0000 | 0.0148 | -0.0005 |
| 10-13 | -0.0000 | -0.0000 | -0.0000 | 0.0000 | -0.0000 | -0.0006 | -0.0005 | 0.0400 |

TABLE Supp.CCII. Systematic covariance matrix of measured cross section ratio of lead over scintillator in $\theta_\mu$

| Bin edges (degree) | 0-2 | 2-3 | 3-4 | 4-5 | 5-6 | 6-8 | 8-10 | 10-13 |
|---|---|---|---|---|---|---|---|---|
| 0-2 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.002 |
| 2-3 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.002 | 0.002 |
| 3-4 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.002 | 0.002 | 0.003 |
| 4-5 | 0.001 | 0.001 | 0.001 | 0.001 | 0.002 | 0.002 | 0.003 | 0.004 |
| 5-6 | 0.001 | 0.001 | 0.001 | 0.002 | 0.002 | 0.003 | 0.004 | 0.005 |
| 6-8 | 0.001 | 0.001 | 0.002 | 0.002 | 0.003 | 0.003 | 0.005 | 0.006 |
| 8-10 | 0.001 | 0.002 | 0.002 | 0.003 | 0.004 | 0.005 | 0.010 | 0.010 |
| 10-13 | 0.002 | 0.002 | 0.003 | 0.004 | 0.005 | 0.006 | 0.010 | 0.016 |

TABLE Supp.CCIII. Measured cross section ratio of lead over scintillator in $p_{\mu,||}$, and the absolute and fractional cross section ratio uncertainties

| Bin edges (GeV/c) | Ratio | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 1.5-3.0 | 0.46 | 0.110 | 0.216 | 0.107 | 0.007 |
| 3.0-4.0 | 0.49 | 0.088 | 0.140 | 0.111 | 0.009 |
| 4.0-5.0 | 0.42 | 0.069 | 0.111 | 0.122 | 0.009 |
| 5.0-6.0 | 0.40 | 0.065 | 0.102 | 0.128 | 0.004 |
| 6.0-7.0 | 0.34 | 0.060 | 0.117 | 0.128 | 0.007 |
| 7.0-8.5 | 0.35 | 0.061 | 0.134 | 0.116 | 0.017 |
| 8.5-10.0 | 0.39 | 0.083 | 0.179 | 0.116 | 0.021 |
| 10.0-20.0 | 0.38 | 0.134 | 0.308 | 0.164 | 0.034 |

TABLE Supp.CCIV. Statistical covariance matrix of measured cross section ratio of lead over scintillator in $p_{\mu,||}$

| Bin edges (GeV/c) | 1.5-3.0 | 3.0-4.0 | 4.0-5.0 | 5.0-6.0 | 6.0-7.0 | 7.0-8.5 | 8.5-10.0 | 10.0-20.0 |
|---|---|---|---|---|---|---|---|---|
| 1.5-3.0 | 0.0098 | 0.0011 | 0.0002 | 0.0000 | 0.0001 | 0.0000 | 0.0000 | 0.0000 |
| 3.0-4.0 | 0.0011 | 0.0047 | 0.0009 | 0.0002 | 0.0001 | 0.0001 | 0.0000 | 0.0000 |
| 4.0-5.0 | 0.0002 | 0.0009 | 0.0022 | 0.0007 | 0.0002 | 0.0001 | 0.0001 | 0.0000 |
| 5.0-6.0 | 0.0000 | 0.0002 | 0.0007 | 0.0017 | 0.0008 | 0.0003 | 0.0001 | 0.0001 |
| 6.0-7.0 | 0.0001 | 0.0001 | 0.0002 | 0.0008 | 0.0016 | 0.0010 | 0.0004 | 0.0001 |
| 7.0-8.5 | 0.0000 | 0.0001 | 0.0001 | 0.0003 | 0.0010 | 0.0021 | 0.0017 | 0.0008 |
| 8.5-10.0 | 0.0000 | 0.0000 | 0.0001 | 0.0001 | 0.0004 | 0.0017 | 0.0048 | 0.0038 |
| 10.0-20.0 | 0.0000 | 0.0000 | 0.0000 | 0.0001 | 0.0001 | 0.0008 | 0.0038 | 0.0139 |

TABLE Supp.CCV. Systematic covariance matrix of measured cross section ratio of lead over scintillator in $p_{\mu,||}$

| Bin edges (GeV/c) | 1.5-3.0 | 3.0-4.0 | 4.0-5.0 | 5.0-6.0 | 6.0-7.0 | 7.0-8.5 | 8.5-10.0 | 10.0-20.0 |
|---|---|---|---|---|---|---|---|---|
| 1.5-3.0 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.001 | 0.001 | 0.002 |
| 3.0-4.0 | 0.002 | 0.003 | 0.003 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 |
| 4.0-5.0 | 0.002 | 0.003 | 0.003 | 0.002 | 0.002 | 0.002 | 0.001 | 0.002 |
| 5.0-6.0 | 0.002 | 0.002 | 0.002 | 0.003 | 0.002 | 0.002 | 0.002 | 0.002 |
| 6.0-7.0 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 |
| 7.0-8.5 | 0.001 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.001 | 0.002 |
| 8.5-10.0 | 0.001 | 0.002 | 0.001 | 0.002 | 0.002 | 0.001 | 0.002 | 0.002 |
| 10.0-20.0 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.004 |

TABLE Supp.CCVI. Measured cross section ratio of lead over scintillator in $p_{\mu,T}$, and the absolute and fractional cross section ratio uncertainties

| Bin edges (GeV/c) | Ratio | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 0.00-0.15 | 0.36 | 0.093 | 0.240 | 0.093 | 0.004 |
| 0.15-0.30 | 0.33 | 0.057 | 0.152 | 0.086 | 0.001 |
| 0.30-0.45 | 0.38 | 0.056 | 0.116 | 0.089 | 0.001 |
| 0.45-0.60 | 0.33 | 0.067 | 0.148 | 0.134 | 0.002 |
| 0.60-0.75 | 0.35 | 0.103 | 0.207 | 0.215 | 0.010 |
| 0.75-0.90 | 0.40 | 0.141 | 0.258 | 0.239 | 0.009 |
| 0.90-1.25 | 0.98 | 0.274 | 0.234 | 0.151 | 0.006 |
| 1.25-2.50 | 2.14 | 1.439 | 0.601 | 0.306 | 0.030 |

TABLE Supp.CCVII. Statistical covariance matrix of measured cross section ratio of lead over scintillator in $p_{\mu,T}$

| Bin edges (GeV/c) | 0.00-0.15 | 0.15-0.30 | 0.30-0.45 | 0.45-0.60 | 0.60-0.75 | 0.75-0.90 | 0.90-1.25 | 1.25-2.50 |
|---|---|---|---|---|---|---|---|---|
| 0.00-0.15 | 0.0076 | 0.0003 | -0.0003 | -0.0001 | -0.0000 | -0.0000 | -0.0000 | -0.0002 |
| 0.15-0.30 | 0.0003 | 0.0025 | 0.0002 | -0.0002 | -0.0001 | -0.0000 | -0.0000 | -0.0001 |
| 0.30-0.45 | -0.0003 | 0.0002 | 0.0019 | 0.0003 | -0.0003 | -0.0002 | -0.0001 | -0.0001 |
| 0.45-0.60 | -0.0001 | -0.0002 | 0.0003 | 0.0024 | 0.0008 | -0.0003 | -0.0006 | -0.0006 |
| 0.60-0.75 | -0.0000 | -0.0001 | -0.0003 | 0.0008 | 0.0051 | 0.0018 | -0.0007 | -0.0016 |
| 0.75-0.90 | -0.0000 | -0.0000 | -0.0002 | -0.0003 | 0.0018 | 0.0107 | 0.0059 | -0.0108 |
| 0.90-1.25 | -0.0000 | -0.0000 | -0.0001 | -0.0006 | -0.0007 | 0.0059 | 0.0530 | 0.0383 |
| 1.25-2.50 | -0.0002 | -0.0001 | -0.0001 | -0.0006 | -0.0016 | -0.0108 | 0.0383 | 1.6442 |

TABLE Supp.CCVIII. Systematic covariance matrix of measured cross section ratio of lead over scintillator in $p_{\mu,T}$

| Bin edges (GeV/c) | 0.00-0.15 | 0.15-0.30 | 0.30-0.45 | 0.45-0.60 | 0.60-0.75 | 0.75-0.90 | 0.90-1.25 | 1.25-2.50 |
|---|---|---|---|---|---|---|---|---|
| 0.00-0.15 | 0.001 | 0.001 | 0.000 | 0.000 | 0.000 | 0.001 | 0.003 | 0.004 |
| 0.15-0.30 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.002 | 0.003 | 0.002 |
| 0.30-0.45 | 0.000 | 0.001 | 0.001 | 0.001 | 0.002 | 0.003 | 0.004 | 0.006 |
| 0.45-0.60 | 0.000 | 0.001 | 0.001 | 0.002 | 0.003 | 0.004 | 0.005 | 0.006 |
| 0.60-0.75 | 0.000 | 0.001 | 0.002 | 0.003 | 0.006 | 0.007 | 0.007 | 0.005 |
| 0.75-0.90 | 0.001 | 0.002 | 0.003 | 0.004 | 0.007 | 0.009 | 0.011 | 0.008 |
| 0.90-1.25 | 0.003 | 0.003 | 0.004 | 0.005 | 0.007 | 0.011 | 0.022 | 0.029 |
| 1.25-2.50 | 0.004 | 0.002 | 0.006 | 0.006 | 0.005 | 0.008 | 0.029 | 0.427 |

TABLE Supp.CCIX. Measured cross section ratio of lead over scintillator in $Q^2$, and the absolute and fractional cross section ratio uncertainties

| Bin edges ($\mathrm{GeV}^2/c^2$) | Ratio | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 0.000-0.025 | 0.36 | 0.089 | 0.234 | 0.089 | 0.003 |
| 0.025-0.050 | 0.35 | 0.069 | 0.183 | 0.073 | 0.001 |
| 0.050-0.100 | 0.30 | 0.060 | 0.175 | 0.096 | 0.002 |
| 0.100-0.200 | 0.38 | 0.058 | 0.122 | 0.091 | 0.001 |
| 0.200-0.300 | 0.35 | 0.068 | 0.148 | 0.125 | 0.002 |
| 0.300-0.400 | 0.30 | 0.075 | 0.198 | 0.150 | 0.003 |
| 0.400-0.500 | 0.36 | 0.094 | 0.201 | 0.166 | 0.005 |
| 0.500-0.700 | 0.36 | 0.121 | 0.233 | 0.238 | 0.010 |
| 0.700-1.000 | 0.51 | 0.171 | 0.251 | 0.226 | 0.007 |
| 1.000-1.300 | 1.03 | 0.318 | 0.271 | 0.148 | 0.009 |
| 1.300-2.000 | 0.95 | 0.531 | 0.508 | 0.231 | 0.017 |
| 2.000-3.000 | 3.01 | 3.221 | 0.989 | 0.412 | 0.050 |

TABLE Supp.CCX. Statistical covariance matrix of measured cross section ratio of lead over scintillator in $Q^2$

| Bin edges ($\mathrm{GeV}^2/c^2$) | 0.000-0.025 | 0.025-0.050 | 0.050-0.100 | 0.100-0.200 | 0.200-0.300 | 0.300-0.400 | 0.400-0.500 | 0.500-0.700 | 0.700-1.000 | 1.000-1.300 | 1.300-2.000 | 2.000-3.000 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.000-0.025 | 0.0069 | 0.0014 | -0.0005 | -0.0002 | -0.0000 | -0.0000 | -0.0000 | 0.0000 | -0.0000 | 0.0000 | -0.0000 | -0.0002 |
| 0.025-0.050 | 0.0014 | 0.0042 | 0.0012 | -0.0003 | -0.0002 | -0.0000 | -0.0000 | -0.0000 | -0.0000 | -0.0000 | -0.0000 | -0.0000 |
| 0.050-0.100 | -0.0005 | 0.0012 | 0.0027 | 0.0005 | -0.0003 | -0.0001 | -0.0001 | -0.0000 | -0.0000 | -0.0001 | -0.0001 | 0.0000 |
| 0.100-0.200 | -0.0002 | -0.0003 | 0.0005 | 0.0022 | 0.0006 | -0.0002 | -0.0003 | -0.0001 | -0.0001 | -0.0001 | -0.0000 | -0.0002 |
| 0.200-0.300 | -0.0000 | -0.0002 | -0.0003 | 0.0006 | 0.0027 | 0.0013 | 0.0001 | -0.0003 | -0.0002 | -0.0000 | -0.0001 | -0.0002 |
| 0.300-0.400 | -0.0000 | -0.0000 | -0.0001 | -0.0002 | 0.0013 | 0.0036 | 0.0026 | -0.0000 | -0.0006 | -0.0007 | -0.0009 | -0.0001 |
| 0.400-0.500 | -0.0000 | -0.0000 | -0.0001 | -0.0003 | 0.0001 | 0.0026 | 0.0052 | 0.0028 | -0.0003 | -0.0013 | -0.0009 | -0.0013 |
| 0.500-0.700 | 0.0000 | -0.0000 | -0.0000 | -0.0001 | -0.0003 | -0.0000 | 0.0028 | 0.0072 | 0.0038 | -0.0014 | -0.0001 | -0.0029 |
| 0.700-1.000 | -0.0000 | -0.0000 | -0.0000 | -0.0001 | -0.0002 | -0.0006 | -0.0003 | 0.0038 | 0.0161 | 0.0115 | -0.0010 | -0.0170 |
| 1.000-1.300 | 0.0000 | -0.0000 | -0.0001 | -0.0001 | -0.0000 | -0.0007 | -0.0013 | -0.0014 | 0.0115 | 0.0777 | 0.0194 | -0.0262 |
| 1.300-2.000 | -0.0000 | -0.0000 | -0.0001 | -0.0000 | -0.0001 | -0.0009 | -0.0009 | -0.0001 | -0.0010 | 0.0194 | 0.2330 | 0.1241 |
| 2.000-3.000 | -0.0002 | -0.0000 | 0.0000 | -0.0002 | -0.0002 | -0.0001 | -0.0013 | -0.0029 | -0.0170 | -0.0262 | 0.1241 | 8.8446 |

TABLE Supp.CCXI. Systematic covariance matrix of measured cross section ratio of lead over scintillator in $Q^2$

| Bin edges ($\mathrm{GeV}^2/c^2$) | 0.000-0.025 | 0.025-0.050 | 0.050-0.100 | 0.100-0.200 | 0.200-0.300 | 0.300-0.400 | 0.400-0.500 | 0.500-0.700 | 0.700-1.000 | 1.000-1.300 | 1.300-2.000 | 2.000-3.000 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.000-0.025 | 0.001 | 0.001 | 0.001 | 0.001 | 0.000 | 0.000 | 0.001 | 0.001 | 0.001 | 0.002 | 0.001 | 0.008 |
| 0.025-0.050 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.002 | 0.002 | 0.005 |
| 0.050-0.100 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.002 | 0.002 | 0.004 | 0.002 | 0.005 |
| 0.100-0.200 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.002 | 0.002 | 0.003 | 0.005 | 0.004 | 0.012 |
| 0.200-0.300 | 0.000 | 0.001 | 0.001 | 0.001 | 0.002 | 0.002 | 0.002 | 0.003 | 0.004 | 0.004 | 0.004 | 0.002 |
| 0.300-0.400 | 0.000 | 0.001 | 0.001 | 0.001 | 0.002 | 0.002 | 0.003 | 0.003 | 0.004 | 0.005 | 0.006 | 0.008 |
| 0.400-0.500 | 0.001 | 0.001 | 0.001 | 0.002 | 0.002 | 0.003 | 0.004 | 0.005 | 0.006 | 0.007 | 0.009 | 0.012 |
| 0.500-0.700 | 0.001 | 0.001 | 0.002 | 0.002 | 0.003 | 0.003 | 0.005 | 0.007 | 0.009 | 0.008 | 0.014 | 0.005 |
| 0.700-1.000 | 0.001 | 0.001 | 0.002 | 0.003 | 0.004 | 0.004 | 0.006 | 0.009 | 0.013 | 0.011 | 0.019 | 0.011 |
| 1.000-1.300 | 0.002 | 0.002 | 0.004 | 0.005 | 0.004 | 0.005 | 0.007 | 0.008 | 0.011 | 0.023 | 0.013 | 0.061 |
| 1.300-2.000 | 0.001 | 0.002 | 0.002 | 0.004 | 0.004 | 0.006 | 0.009 | 0.014 | 0.019 | 0.013 | 0.048 | 0.000 |
| 2.000-3.000 | 0.008 | 0.005 | 0.005 | 0.012 | 0.002 | 0.008 | 0.012 | 0.005 | 0.011 | 0.061 | 0.000 | 1.531 |

TABLE Supp.CCXII. Measured cross section ratio of lead over scintillator in $W_{exp}$, and the absolute and fractional cross section ratio uncertainties

| Bin edges (GeV/c$^2$) | Ratio | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 0.9-1.1 | 0.58 | 0.267 | 0.432 | 0.148 | 0.013 |
| 1.1-1.2 | 0.20 | 0.051 | 0.239 | 0.088 | 0.011 |
| 1.2-1.3 | 0.38 | 0.056 | 0.092 | 0.113 | 0.003 |
| 1.3-1.4 | 0.63 | 0.165 | 0.164 | 0.204 | 0.005 |

TABLE Supp.CCXIII. Statistical covariance matrix of measured cross section ratio of lead over scintillator in $W_{exp}$

| Bin edges (GeV/c$^2$) | 0.9-1.1 | 1.1-1.2 | 1.2-1.3 | 1.3-1.4 |
|---|---|---|---|---|
| 0.9-1.1 | 0.0636 | 0.0039 | -0.0033 | -0.0063 |
| 1.1-1.2 | 0.0039 | 0.0023 | 0.0005 | -0.0031 |
| 1.2-1.3 | -0.0033 | 0.0005 | 0.0012 | 0.0007 |
| 1.3-1.4 | -0.0063 | -0.0031 | 0.0007 | 0.0107 |

TABLE Supp.CCXIV. Systematic covariance matrix of measured cross section ratio of lead over scintillator in $W_{exp}$

| Bin edges (GeV/c$^2$) | 0.9-1.1 | 1.1-1.2 | 1.2-1.3 | 1.3-1.4 |
|---|---|---|---|---|
| 0.9-1.1 | 0.007 | 0.001 | -0.000 | -0.001 |
| 1.1-1.2 | 0.001 | 0.000 | -0.000 | -0.001 |
| 1.2-1.3 | -0.000 | -0.000 | 0.002 | 0.005 |
| 1.3-1.4 | -0.001 | -0.001 | 0.005 | 0.016 |

TABLE Supp.CCXV. Measured cross section ratio of lead over scintillator in $T_\pi$, and the absolute and fractional cross section ratio uncertainties

| Bin edges (GeV) | Ratio | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 0.035-0.100 | 0.39 | 0.055 | 0.114 | 0.084 | 0.002 |
| 0.100-0.150 | 0.36 | 0.048 | 0.092 | 0.094 | 0.003 |
| 0.150-0.200 | 0.40 | 0.074 | 0.128 | 0.132 | 0.004 |
| 0.200-0.350 | 0.62 | 0.168 | 0.172 | 0.210 | 0.003 |

TABLE Supp.CCXVI. Statistical covariance matrix of measured cross section ratio of lead over scintillator in $T_\pi$

| Bin edges (GeV) | 0.035-0.100 | 0.100-0.150 | 0.150-0.200 | 0.200-0.350 |
|---|---|---|---|---|
| 0.035-0.100 | 0.0020 | 0.0006 | -0.0003 | -0.0005 |
| 0.100-0.150 | 0.0006 | 0.0011 | 0.0004 | -0.0003 |
| 0.150-0.200 | -0.0003 | 0.0004 | 0.0026 | 0.0012 |
| 0.200-0.350 | -0.0005 | -0.0003 | 0.0012 | 0.0114 |

TABLE Supp.CCXVII. Systematic covariance matrix of measured cross section ratio of lead over scintillator in $T_\pi$

| Bin edges (GeV) | 0.035-0.100 | 0.100-0.150 | 0.150-0.200 | 0.200-0.350 |
|---|---|---|---|---|
| 0.035-0.100 | 0.001 | 0.001 | 0.001 | 0.003 |
| 0.100-0.150 | 0.001 | 0.001 | 0.002 | 0.004 |
| 0.150-0.200 | 0.001 | 0.002 | 0.003 | 0.006 |
| 0.200-0.350 | 0.003 | 0.004 | 0.006 | 0.017 |

TABLE Supp.CCXVIII. Measured cross section ratio of lead over scintillator in $\theta_\pi$, and the absolute and fractional cross section ratio uncertainties

| Bin edges (degree) | Ratio | Abs. Unc. | Frac. Stat. Unc. | Frac. Sys. Unc. | Frac. Flux Unc. |
|---|---|---|---|---|---|
| 0-10 | 0.75 | 0.329 | 0.412 | 0.159 | 0.004 |
| 10-20 | 0.54 | 0.141 | 0.219 | 0.141 | 0.002 |
| 20-30 | 0.38 | 0.079 | 0.157 | 0.135 | 0.001 |
| 30-40 | 0.43 | 0.071 | 0.122 | 0.113 | 0.004 |
| 40-50 | 0.38 | 0.067 | 0.124 | 0.127 | 0.004 |
| 50-60 | 0.31 | 0.066 | 0.158 | 0.145 | 0.005 |
| 60-120 | 0.34 | 0.062 | 0.142 | 0.112 | 0.004 |
| 120-135 | 0.49 | 0.097 | 0.172 | 0.098 | 0.001 |
| 135-150 | 0.53 | 0.124 | 0.196 | 0.126 | 0.003 |
| 150-165 | 0.58 | 0.222 | 0.349 | 0.148 | 0.003 |
| 165-180 | 0.50 | 0.514 | 0.953 | 0.365 | 0.025 |

TABLE Supp.CCXIX. Statistical covariance matrix of measured cross section ratio of lead over scintillator in $\theta_\pi$

| Bin edges (degree) | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 | 60-120 | 120-135 | 135-150 | 150-165 | 165-180 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0-10 | 0.0945 | 0.0129 | 0.0004 | -0.0003 | -0.0004 | -0.0004 | -0.0004 | -0.0005 | -0.0006 | 0.0025 | -0.0011 |
| 10-20 | 0.0129 | 0.0140 | 0.0028 | -0.0002 | -0.0005 | -0.0003 | -0.0002 | -0.0004 | -0.0005 | -0.0000 | -0.0006 |
| 20-30 | 0.0004 | 0.0028 | 0.0036 | 0.0014 | -0.0000 | -0.0003 | -0.0002 | -0.0002 | -0.0001 | 0.0002 | 0.0003 |
| 30-40 | -0.0003 | -0.0002 | 0.0014 | 0.0027 | 0.0012 | 0.0001 | -0.0003 | -0.0003 | 0.0000 | 0.0000 | -0.0009 |
| 40-50 | -0.0004 | -0.0005 | -0.0000 | 0.0012 | 0.0022 | 0.0012 | 0.0002 | -0.0001 | -0.0002 | -0.0002 | -0.0010 |
| 50-60 | -0.0004 | -0.0003 | -0.0003 | 0.0001 | 0.0012 | 0.0024 | 0.0010 | -0.0000 | -0.0005 | -0.0004 | -0.0009 |
| 60-120 | -0.0004 | -0.0002 | -0.0002 | -0.0003 | 0.0002 | 0.0010 | 0.0023 | 0.0009 | 0.0001 | -0.0001 | 0.0021 |
| 120-135 | -0.0005 | -0.0004 | -0.0002 | -0.0003 | -0.0001 | -0.0000 | 0.0009 | 0.0071 | 0.0043 | 0.0002 | 0.0100 |
| 135-150 | -0.0006 | -0.0005 | -0.0001 | 0.0000 | -0.0002 | -0.0005 | 0.0001 | 0.0043 | 0.0109 | 0.0104 | 0.0028 |
| 150-165 | 0.0025 | -0.0000 | 0.0002 | 0.0000 | -0.0002 | -0.0004 | -0.0001 | 0.0002 | 0.0104 | 0.0416 | -0.0017 |
| 165-180 | -0.0011 | -0.0006 | 0.0003 | -0.0009 | -0.0010 | -0.0009 | 0.0021 | 0.0100 | 0.0028 | -0.0017 | 0.2299 |

TABLE Supp.CCXX. Systematic covariance matrix of measured cross section ratio of lead over scintillator in $\theta_\pi$

| Bin edges (degree) | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 | 60-120 | 120-135 | 135-150 | 150-165 | 165-180 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0-10 | 0.014 | 0.007 | 0.005 | 0.005 | 0.004 | 0.003 | 0.003 | 0.004 | 0.006 | 0.004 | -0.001 |
| 10-20 | 0.007 | 0.006 | 0.003 | 0.003 | 0.003 | 0.002 | 0.002 | 0.002 | 0.003 | 0.003 | -0.003 |
| 20-30 | 0.005 | 0.003 | 0.003 | 0.002 | 0.002 | 0.002 | 0.001 | 0.002 | 0.003 | 0.002 | -0.001 |
| 30-40 | 0.005 | 0.003 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.003 | 0.002 | 0.000 |
| 40-50 | 0.004 | 0.003 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.003 | 0.002 | -0.001 |
| 50-60 | 0.003 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.001 | 0.001 | 0.002 | 0.001 | 0.002 |
| 60-120 | 0.003 | 0.002 | 0.001 | 0.002 | 0.002 | 0.001 | 0.001 | 0.001 | 0.002 | 0.001 | -0.000 |
| 120-135 | 0.004 | 0.002 | 0.002 | 0.002 | 0.002 | 0.001 | 0.001 | 0.002 | 0.003 | 0.003 | -0.000 |
| 135-150 | 0.006 | 0.003 | 0.003 | 0.003 | 0.003 | 0.002 | 0.002 | 0.003 | 0.005 | 0.004 | -0.003 |
| 150-165 | 0.004 | 0.003 | 0.002 | 0.002 | 0.002 | 0.001 | 0.001 | 0.003 | 0.004 | 0.007 | -0.005 |
| 165-180 | -0.001 | -0.003 | -0.001 | 0.000 | -0.001 | 0.002 | -0.000 | -0.000 | -0.003 | -0.005 | 0.034 |