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Fiducial Drell-Yan production at the LHC improved by transverse-momentum resummation at N^{4}LL+N^{3}LO

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Abstract

Drell-Yan production is one of the precision cornerstones of the LHC, serving as calibration for measurements such as the W-boson mass. Its extreme precision at the level of 1% challenges theory predictions at the highest level. We present the first independent calculation of Drell-Yan production at order α_s^3 in transverse-momentum (q_T) resummation improved perturbation theory. Our calculation reaches the state-of-the-art through inclusion of the recently published four loop rapidity anomalous dimension and three loop massive axial-vector contributions. We compare to the most recent data from CMS with fiducial and differential cross-section predictions and find excellent agreement at the percent level. Our resummed calculation including the matching to Z+jet production at NNLO is publicly available in the upcoming `CuTe-MCFM` 10.3 release and allows for theory-data comparison at an unprecedented level.

1. Introduction

Drell-Yan (Z-boson) production is among the most important standard candles of the high-energy LHC physics program due to its very precise measurement at the level of one percent [1–4]. It is used for the extraction of the strong coupling [5, 6], fitting of parton distribution functions [7, 8] that further constrain and determine Standard Model (SM) input parameters, and is also a crucial ingredient of the W-boson mass determination [9–11].

The current precision in QCD for Drell-Yan predictions is at the level of α_s^3 both fully differentially [12–15] and more inclusively [16, 17]. Calculations at this order have been performed at fixed order (N^{3}LO) and including the effects of transverse momentum (q_T) resummation up to N^{3}LL logarithmic accuracy. Currently all fully differential calculations at the level of α_s^3 employ transverse momentum subtractions or transverse momentum resummation. They have been enabled by the recent availability of the three-loop beam-functions [18–20], complete three-loop hard function [21–25] and the existence of a NNLO calculation of Z+jet production [26–30]. Beyond pure QCD corrections, the full set of two-loop mixed QCD$\otimes$EW corrections have been calculated very recently [31–33].

Traditionally there has been a focus on fixed-order calculations for total fiducial cross-sections, but now that relatively high perturbative orders have been reached, convergence issues of the perturbative series due to fiducial cuts have been identified, see e.g. refs. [34–36]. These issues trace back to a linear sensitivity of acceptance cuts to small transverse momenta, where fixed-order predictions are unreliable, leading to factorially divergent contributions [35]. It has shifted the focus towards resummation-improved results even for total fiducial cross-sections, which cure such problems without requiring any modification of analysis cuts.

All calculations matched to NNLO Z+jet fixed-order at large q_T have so far been based on the NNLOjet results [27]. Different implementations of q_T resummation and subtractions are built on top of this calculation. In particular, results for a matching to the resummation in DYTurbo [37] have been presented in ref. [13] where only non-singlet and vector singlet[1] contributions are included and truncation uncertainties are estimated by considering differences between successive orders. A matching to the RadISH resummation approach [14, 38] has been presented in refs. [12, 14] treating axial singlet contributions in the $m_t \to \infty$ EFT. This setup has subsequently been extended to calculate fiducial cross-sections also at fixed-order N^{3}LO, comparing the impact of power corrections through studying the difference between symmetric and product cuts [15] and

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[1] Perturbative corrections are typically separated into singlet and non-singlet contributions. For the singlet contributions the Z boson does not directly couple to the incoming quarks, but is separated through loops involving gluons. These contributions therefore only enter at higher orders.

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comparing with 13 TeV ATLAS data [4]. The RadISH based calculations provide uncertainty estimates for differential and fiducial results for the first time. Despite these studies, it is crucial to have an independent calculation of both the fixed-order components and the resummation implementation. While the NNLOjet calculation is tested by the correct approach of the triple singular limits through an implementation of (differential) q_T subtractions, it is important to also probe the finite contributions. As well as acting as a cross-check, an additional calculation also provides an independent estimate of uncertainties.

In this paper we present both a publicly available calculation of Z-boson production as well as differential and fiducial cross-sections at the state-of-the-art level N^{4}LL+N^{3}LO. To reach this accuracy we include the four loop rapidity anomalous dimension [39, 40], pushing the logarithmic accuracy of our results to N^{4}LL for the first time. We also include the massive three-loop axial singlet contributions [25] without the need for approximations. We compare at α_s^3 accuracy with the CMS 13 TeV precision measurement. All parts, both resummation and fixed-order are publicly available in the next `CuTe-MCFM` release 10.3. Public codes are crucial to ensure reproducibility, allow the community to perform independent checks, to calculate predictions with different parameters, and provide the basis for future theoretical improvements as strongly advocated by our community [41].

In section 2 we provide technical details of our calculation before presenting results in section 3 and concluding in section 4 with an outlook.

2. Calculation

We consider QCD corrections to the process $q + \bar{q} \to Z/\gamma(\to\ l^- + l^+)$. Our calculation in `CuTe-MCFM` [42, 43] matches resummation at the level of N^{4}LL to α_s^3 fixed-order Z+jet production. Apart from missing N^{3}LO PDF effects we achieve full α_s^3 fixed-order and transverse momentum renormalization-group-improved (RG-improved) logarithmic accuracy by counting $\log(q_T^2/Q^2) \sim 1/\alpha_s$.[2] Our calculation involves many contributions at the fixed-order and at the resummation level, which we discuss separately below.

Resummation. The resummation is based on the SCET formalism derived in refs. [44–46] and originally implemented as `CuTe-MCFM` in ref. [42] to N^{3}LL. Large logarithms $\log(q_T^2/Q^2)$ are resummed through RG evolution of hard- and beam functions in a small-q_T factorization theorem. Rapidity logarithms are directly exponentiated through the collinear-anomaly formalism.

At large q_T the small-q_T factorization theorem becomes invalid and one has to switch to fixed-order predictions. We switch using a transition function that smoothly interpolates between resummation and fixed-order without disturbing subleading power corrections, as detailed in ref. [42]. Within this procedure the overlap between fixed-order and resummation has to be subtracted by expanding the resummation to a fixed-order. This difference is referred to as matching corrections. For Z boson production they quickly approach zero for $q_T \to 0$ and remain at the few percent level up to ~ 30 GeV.

Three loop transverse momentum dependent beam functions have been calculated in refs. [18–20] and implemented in ref. [47] in `CuTe-MCFM`. Together with the α_s^3 hard function this enables resummation at the level of N^{3}LL'. The resummation of linear power corrections [34] has been included in `CuTe-MCFM` since its initial implementation through a recoil prescription [48]. They are crucial to improve the resummation itself as well as the numerical stability by allowing a larger matching cutoff (the value of q_T below which matching corrections are set to zero).

In this study we have upgraded the resummation to the logarithmic accuracy of N^{4}LL through the inclusion of the four loop rapidity anomalous dimension [39, 40]. While the five loop cusp anomalous dimension is also a necessary ingredient, it only enters through the hard function evolution and is numerically completely negligible. Already at a lower order the hard function evolution is precise at the level of one per-mille. We nevertheless include the hard function evolution taking four loop collinear anomalous dimensions from ref. [49] and a five loop cusp estimate from ref. [50] that agrees with our own Padé approximant estimate. The five loop beta function is taken from ref. [51].

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[2] While we are neglecting N^{3}LO PDFs for full N^{4}LL+N^{3}LO accuracy, it has been customary in the literature to refer to predictions as N^{3}LO despite the lack of these corrections.

Transverse momentum Fourier conjugate logarithms $L_\perp \sim \log(x_T^2\mu^2)$ appearing in the factorization theorem would traditionally be integrated over the full range of x_T. This requires the introduction of a prescription to avoid the Landau pole. Following the SCET resummation formalism of ref. [44, 45] this is not necessary as scales are always set in the perturbative regime. The formalism further employs an improved power counting $L_\perp \sim 1/\sqrt{\alpha_s}$ that is crucial to improve the resummation at small q_T [45]. At N^{4}LL the three-loop beamfunctions as calculated in ref. [18–20] are then not sufficient for improved α_s^3 accuracy. Using the beamfunction RGEs we reconstructed the logarithmic beamfunction terms up to order $\alpha_s^6 L_\perp^6$, $\alpha_s^4 L_\perp^4$ and $\alpha_s^4 L_\perp^2$. We performed the Mellin convolutions of beam function kernels and splitting functions up to three loops [52, 53] using the MT package [54].

The hard function entering the factorization formula consists of $\overline{\text{MS}}$-renormalized virtual corrections. For Drell-Yan production one typically distinguishes between different classes of corrections based on the following decomposition. The Feynman rule vertex for the photon coupling to fermions is $-ieQ_f\gamma^\mu$, while the Z coupling is $-ie\gamma^\mu(v_L^f P_L + v_R^f P_R)$. In terms of vector and axial-vector components this decomposes as

$$(v_L^f P_L + v_R^f P_R) = \left(\frac{1}{2}v_L^f + \frac{1}{2}v_R^f\right) - \gamma_5\left(\frac{1}{2}v_L^f - \frac{1}{2}v_R^f\right). \tag{1}$$

The first term constitutes the vector coupling and is dressed by a vector form-factor F_V that encapsulates higher-order corrections. The second term constitutes the axial-vector coupling and is dressed by an axial-vector form-factor F_A. For a photon exchange $v_L = v_R = 1$ and $F_A = 0$. On the other hand, the coupling of Z bosons to quarks involves both a vector (F_V) and an axial-vector (F_A) form factor. A common approximation is to include only non-singlet contributions, which leads to $F_A = F_V$.

The three-loop corrections to the vector part have been known for a while now [21–23], while the three-loop corrections to the axial singlet part have only been computed recently in purely massless QCD [24] and with full top-quark mass dependence [25]. In our calculation we include the complete three-loop corrections with full top-quark mass dependence. While these contributions are small, the top-quark mass dependence does not decouple in either the $m_t \to \infty$ limit or the low-energy limit, in contrast to the vector case.

Fixed order. Our fixed-order NNLO Z+jet calculation is based on ref. [28], employing 1-jettiness subtractions [26, 55, 56]. For 1-jettiness subtractions at NNLO a crucial new ingredient compared to 0-jettiness is the NNLO soft function which has been calculated in refs. [57, 58]. Top-quark loop corrections to Z+jet and Z+2 jet production have been known analytically for some time [59] and are included in our calculation. Two-loop axial singlet contributions in the Z+jet hard function are unknown so far and have been neglected in our calculation.

We have performed extensive cross-checks of all elements of the calculation. We find numerical agreement between all bare amplitude expressions and Recola [60], and have reproduced the non-singlet hard function that was originally taken from the code PeTeR [61, 62] with an independent re-implementation from refs. [63–65]. We have thoroughly tested the implementation of the subtraction terms using the same methodology as in ref. [66]. Compared to the original implementation [28] we identified an inconsistency in a small number of subtraction terms and in the crossing of one-loop axial-vector helicity amplitudes. As a final check, we compared with fiducial results presented in ref. [67] for different partonic channels and find agreement.

Since our calculation is based on 1-jettiness slicing subtractions, unlike the local antenna subtractions used in the NNLOjet calculation [27], we have to pay attention to residual slicing cutoff effects. Jettiness slicing at the level of NNLO in association with one jet is widely believed to have reached its limits of applicability. But, as we demonstrate in this paper, optimized phase-space generation together with an efficient parallelization for the use of modern HPC resources [68] allows us to compute results at the level of N^{4}LL+N^{3}LO with negligible systematic cutoff uncertainties.

Nevertheless, we had to choose the q_T cutoff for the resummation matching corrections low enough that residual matching corrections can be neglected. The impact of this on fiducial results can be estimated by multiplying the resummed cross-section integrated up to the matching cutoff with the relative size of the neglected matching corrections. At α_s and α_s^2 matching corrections can be safely neglected below 1 GeV, but the numerical implementation allows for smaller cutoffs if necessary. For the α_s^3 coefficient we find that they can be neglected below 5 GeV with residual per-mille level effects at the order of the numerical integration uncertainty. This larger value is possible due to the inclusion of linear power corrections in our formalism. The size of the corrections is in line with the findings of previous studies [13, 15]. This results in an error that is below the quoted numerical precision of our fiducial results in the following (one pb). Similarly, the effect on all shown differential distributions in the following is at the per-mille level.

To reach the 5 GeV q_T cutoff we had to choose the 1-jettiness slicing parameter of the NNLO Z+jet fixed-order calculation small enough, which would otherwise lead to a mismatch in the matching corrections. For a q_T cutoff of 5 GeV the small size of the 1-jettiness parameter requires computing resources of about 6000 NERSC Perlmutter node hours for all fiducial and differential results presented in the following (we ran with 256 nodes for about one day). While a cutoff of 2 GeV to 3 GeV could likely be achieved with more resources (due to requiring a smaller jettiness parameter), the inclusion of subleading 1-jettiness power corrections, which have currently only been computed at a lower order [69], could be a more promising resource-saving approach.

3. Results

We present results at $\sqrt{s} = 13\,\text{TeV}$ using the `NNPDF4.0` PDF set at NNLO with $\alpha_s(m_Z) = 0.118$ [70]. Electroweak input parameters are chosen in the G_μ scheme with $m_Z = 91.1876\,\text{GeV}$, $m_W = 80.385\,\text{GeV}$, $\Gamma_Z = 2.4952\,\text{GeV}$ and $G_F = 1.166\,39 \times 10^{-5}\,\text{GeV}^{-2}$. We will denote the matched resummation accuracy with α_s for N^{2}LL+NLO, α_s^2 for N^{3}LL+NNLO and α_s^3 for N^{4}LL+N^{3}LO.

Our fiducial selection cuts in table 1 are chosen to compare with the most recent Z-boson precision measurement by CMS in ref. [3]. The choice of symmetric lepton cuts used in this analysis causes a poor perturbative convergence for fixed-order calculations and can also lead to numerical issues. However, the use of resummation resolves such issues [34–36].

In our calculation we distinguish between three scales for estimating uncertainties. We use a low (resummation) scale $\sim q_T$ (see ref. [42] for details) to which RGEs are evolved down from the hard scale chosen as $\sqrt{m_Z^2 + p_{T,Z}^2}$. The `CuTe-MCFM` resummation formalism [44–46] is originally derived using an analytic regulator to regulate rapidity divergences in the transverse position dependent PDFs (collinear anomaly formalism). This is opposed to using a rapidity regulator that introduces a rapidity scale [71]. We have re-introduced a scale estimating the effect of a different rapidity scale as suggested in ref. [72]. We vary these three scales independently to obtain a robust estimate of truncation uncertainties. Most importantly our formalism allows for the variation of the low scale, which dominates uncertainties at small q_T.

Apart from estimating truncation uncertainties through scale variation, we include matching uncertainties by varying the transition function in the region of about 40 GeV to 60 GeV. Those transition uncertainties are compatible with the resummation and fixed-order uncertainties at lower and larger q_T, respectively. This indicates that our transition region is chosen in a sensible region, because otherwise for larger q_T the resummation would break down, or for much smaller q_T the fixed-order prediction would break down and lead to significantly larger uncertainties.

In the following we symmetrize uncertainties for the resummation improved results in order to provide uncertainties at small q_T. While for Drell-Yan production our resummation formalism does not set the central scale of α_s below $\sim 2\,\text{GeV}$ [42], a downwards scale variation would probe close towards the non-perturbative regime. We therefore set a minimum scale of $\sim 2\,\text{GeV}$, hence making the downwards variation by a factor of 2 for $q_T < 4\,\text{GeV}$ ineffective. On the other hand the upwards variation still captures the uncertainty. Note that about 2% of the total fiducial cross-section comes from the region $q_T < 1\,\text{GeV}$ where one might expect additional non-perturbative effects of an unknown size.

The CMS collaboration [3] provides both differential results to compare with as well as a total fiducial cross-section measurement, that we discuss in turn below.

Table 1.: Fiducial cuts for $Z \to l^+l^-$ used in the CMS 13 TeV analysis [3].

Lepton cuts	$q_T^l > 25\,\text{GeV}, \lvert\eta^l\rvert < 2.4$
Separation cuts	$76.2\,\text{GeV} < m^{l^+l^-} < 106.2\,\text{GeV},$ $\lvert y^{l^+l^-}\rvert < 2.4$

Differential results. In fig. 1 we present the Z boson transverse momentum distribution at order α_s (N^{2}LL+NLO), α_s^2 (N^{3}LL+NNLO) and α_s^3 (N^{4}LL+N^{3}LO) and compare it to the CMS 13 TeV measurement [3] with the cuts shown in table 1.

Overall there is an excellent agreement between theory and data at the highest order. Going from α_s^2 to α_s^3 decreases uncertainties and improves agreement with data noticeably at both large and small q_T. In the first bin $0\,\text{GeV} < q_T < 1\,\text{GeV}$ we notice a relatively large difference to the data, but this is also where one would expect a non-negligible contribution from non-perturbative effects.

For the Φ^* distribution shown in fig. 2 results are overall very similar. For the transverse momentum distribution we neglect matching corrections at α_s^3 below $q_T < 5\,\text{GeV}$. Here we correspondingly neglect them below $\Phi^* < 5\,\text{GeV}/m_Z \sim 0.05$ and at lower orders below $\Phi^* < 1\,\text{GeV}/m_Z \sim 0.01$, an overall per-mille level effect in that region.

Since our resummation implementation is fully differential in the electroweak final state we can naturally also present the transverse momentum distribution of the final state lepton, see fig. 3. This is plagued by a Jacobian peak at fixed-order and crucially requires resummation. The higher-order α_s^3 corrections further stabilize the results with smaller uncertainties.

Total fiducial cross-section. In table 2 we present total fiducial cross sections. Uncertainties of the fixed-order NNLO (α_s^2) result, obtained by taking the envelope of a variation of renormalization and factorization scales by a factor of two, are particularly small at the level of 0.5%. The resummation improved results are obtained by integrating over the matched q_T spectrum shown in fig. 1. Uncertainties of the resummation improved predictions are obtained by taking the envelope of the variation of hard, low and rapidity scales in the fixed-order and resummation region. Separately we quote a matching uncertainty by varying the transition function that switches from fixed-order to resummed predictions in the transition region between 40 GeV to 60 GeV.

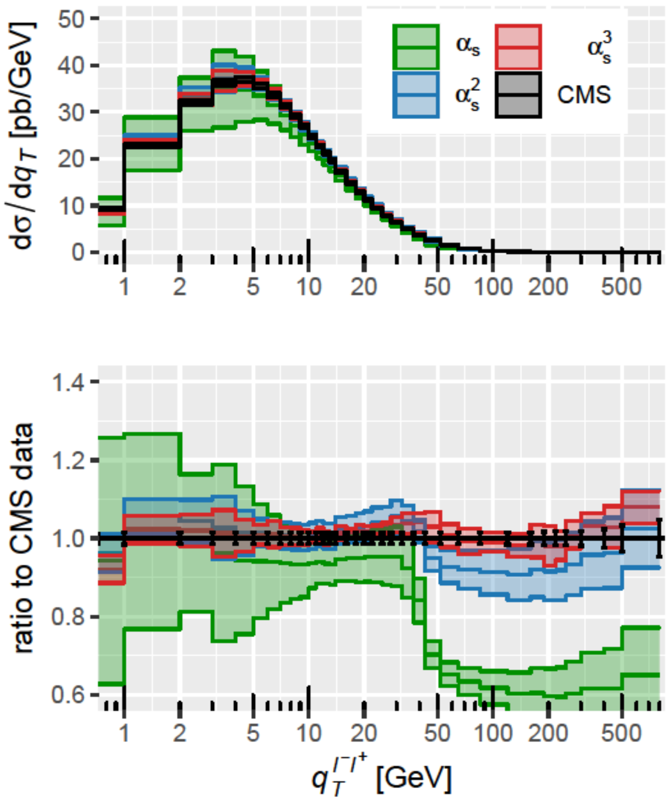

Figure 1.: Differential transverse-momentum resummation improved predictions for the $q_T^{l^-l^+}$ distribution at order α_s, α_s^2 and α_s^3.

The resummation improved result at α_s has large uncertainties that stem from an insufficient order of the resummation (N^{2}LL), which still has substantial uncertainties in the Sudakov peak region (c.f. fig. 1). The results quickly stabilize, with less than a percent difference between the central α_s^2 and α_s^3 predictions. Nevertheless, the uncertainties we obtain are noticeably larger than the fixed-order uncertainties. We further observe that going from N^{3}LL/α_s^2 to N^{4}LL/α_s^3 does not reduce uncertainties as substantially as when going from α_s to α_s^2. This is because the resummation uncertainties around the Sudakov peak region at small $q_T \sim 5\,\text{GeV}$ do not improve dramatically.

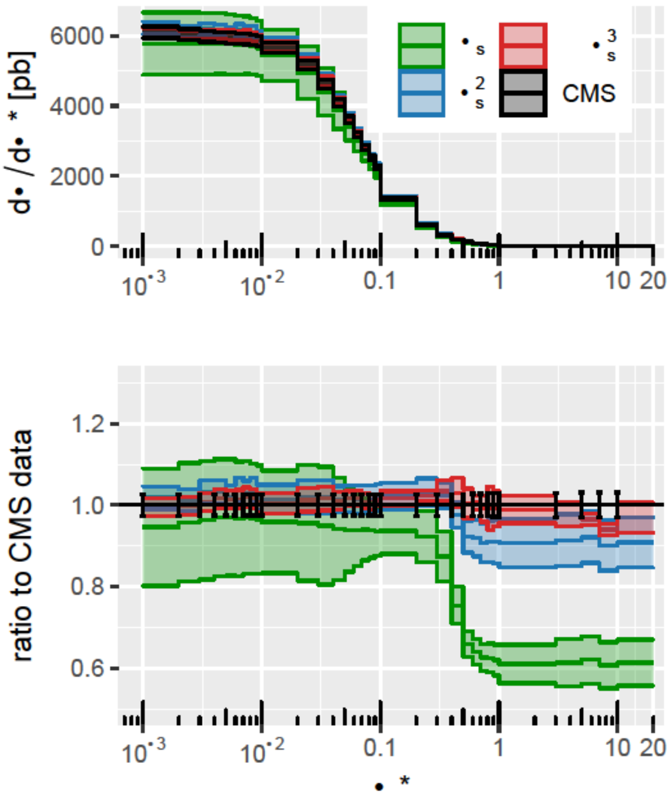

Figure 2.: Differential transverse-momentum resummation improved predictions for the Φ^* distribution at order α_s, α_s^2 and α_s^3.

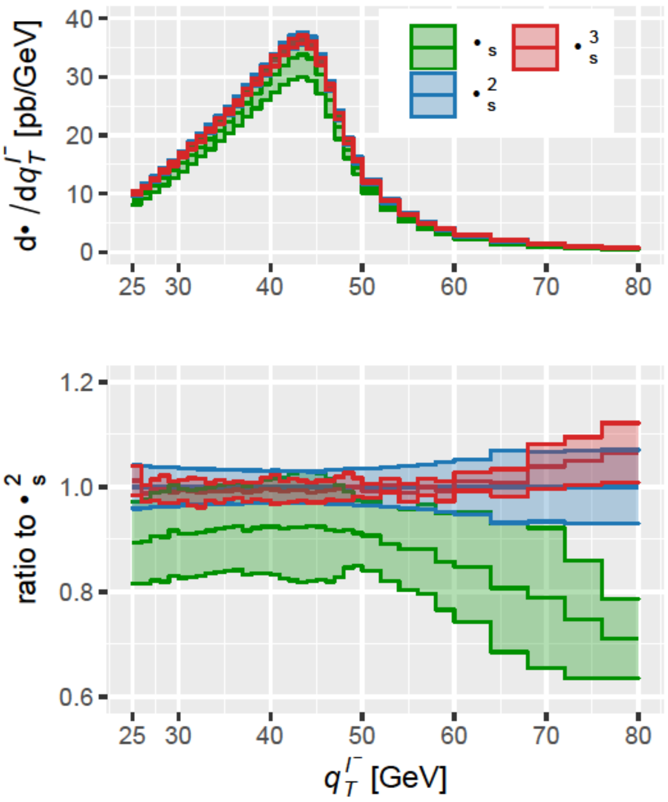

Figure 3.: Differential transverse-momentum resummation improved predictions for the lepton transverse momentum distribution at order α_s, α_s^2 and α_s^3.

While this behavior, of only moderately decreasing uncertainties going from α_s^2 to α_s^3, is consistent with the findings of ref. [15] using RadISH resummation, our uncertainties of the resummation improved fiducial cross-section are larger than the uncertainties presented there. Our α_s^3 prediction has uncertainties of about 2.5%, while using RadISH for the resummation results in uncertainties of about 1%. Given that differentially in fig. 1 we see still some variation in the low q_T region between the central α_s^2 and α_s^3 results, we are confident in our more conservative uncertainty estimate.

Indeed, theory uncertainties have become an important topic within recent years [73]. First, they cannot be interpreted statistically and second, perturbative predictions are limited to the level presented here for the foreseeable future. It is therefore important to study them with as much scrutiny as possible. An approach followed in ref. [13] has been to take half the difference between the two highest order results as an uncertainty. This would bring our uncertainties closer in line with the uncertainties presented in ref. [15], less than one percent.

4. Conclusions & Outlook.

Z-boson production is the most precisely measured process at the LHC and meanwhile solely limited in precision by the beam luminosity uncertainty. At the same time it is one of the most important standard candles and enters many precision prediction ingredients like PDFs and SM input parameters. It is crucial that theory predictions are available at the same level of precision to make best use of the available measurements.

In this paper we presented the first transverse-momentum (q_T) resummation improved calculation at the level of N^{4}LL+N^{3}LO, which broadly reduces theory uncertainties to the few percent level. Our results show excellent agreement with the 13 TeV CMS measurements

Table 2.: Fiducial cross-sections in pb for the cuts in table 1 and input parameters as in the text. Uncertainties for the resummation-improved results include matching to fixed-order (mat.) and by scale variation (sc.). We have not computed fixed-order N^{3}LO results. For comparison, the final row shows the CMS measurement (for electron and muon channels combined) [3].

Order k	fixed-order α_s^k	res. improved α_s^k
0	694^{+85}_{-92}	—
1	732^{+19}_{-30}	$637 \pm 8_{\text{mat.}} \pm 70_{\text{sc.}}$
2	720^{+4}_{-3}	$707 \pm 3_{\text{mat.}} \pm 29_{\text{sc.}}$
3	—	$702 \pm 1_{\text{mat.}} \pm 17_{\text{sc.}}$

$699 \pm 5\,(\text{syst.}) \pm 17\,(\text{lumi.})$ (e, μ combined) [3]

within a few percent both at the differential level from $q_T^Z = 1\,\text{GeV}$ to $\sim 500\,\text{GeV}$ and for Φ^* over the whole spectrum, as well as for the total fiducial cross-section. As a consequence of the resummation (and inclusion of linear power corrections), our calculation can provide reliable predictions also for past experimental analyses that would induce factorially divergent contributions at fixed order due to cuts, e.g. symmetric lepton cuts [35].

All previous calculations of order N^{3}LL+N^{3}LO rely on a single Z+jet NNLO calculation [27]. Further, uncertainties (via scale variation) for resummation improved results were only estimated by using the RadISH resummation framework [14, 38]. Due to the utmost importance of this process, it is crucial to provide an independent calculation using completely different methods to reliably estimate uncertainties. It allows future (experimental) studies to assess the validity of their input theory predictions through independent results. This becomes increasingly important with the advent of very precise collider measurements that might indicate tension with the SM [11]. The public availability of our calculation as part of the upcoming `CuTe-MCFM` release allows for a much larger audience to make use of this state-of-the-art precision, to implement modification of cuts and input parameters, and also to re-use parts and to validate other calculations [41].

Previously it was found that fiducial cross-section uncertainties at the level of α_s^3 are similar to those at α_s^2, about 1% using RadISH resummation [15]. With resummation, this uncertainty is dominated by the uncertainties around the Sudakov peak at small q_T, i.e. mostly within the pure resummation region. We find more conservative uncertainties of about 2.5% using `CuTe-MCFM` resummation.

Although the theoretical precision of the calculation discussed in this paper is now at an impressive level, there are two important aspects that require further work. Statistical PDF uncertainties have reached the level of one percent [70, 74] and systematic effects can no longer be neglected. Since these uncertainties are at the same level as perturbative truncation uncertainties, a careful study of PDF effects at this order will be an important future direction. Indeed, while finalizing this manuscript, approximate N^{3}LO PDFs have been introduced by the MSHT group [75]. They take into account approximations for the four loop splitting functions through known information on small and large x and available Mellin moments. Such theory approximations of missing higher-order effects are included in their Hessian procedure as nuisance parameters.[3]

In addition, in order to better match with data at very small q_T, it is possible to include a parametrization of non-perturbative effects, see e.g. refs. [76, 77]. This can then inform the modeling of the related process of W-boson production and thus have implications for the extraction of the W-boson mass. Extending W-boson production in `CuTe-MCFM` to α_s^3 accuracy will thus be a valuable extension that allows for very precise W/Z boson ratio predictions [78].

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[3] A preliminary study of the potential impact of this PDF set on the results shown in this paper is presented in appendix A.

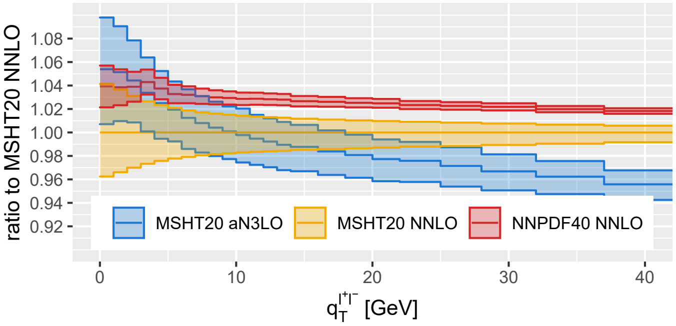

Figure 4.: PDF uncertainties of the purely resummed q_T spectrum as the ratio to the MSHT20 NNLO central value.

research used resources of the National Energy Research Scientific Computing Center, which is supported by the Office of Science of the U.S. Department of Energy under Contract No. DE-AC02-05CH11231. Some of the numerical calculations entering the results in this paper were performed using the Wilson High-Performance Computing Facility at Fermilab.

A. Impact of N^{3}LO PDFs.

Here we give a first impression of the impact of the approximate N^{3}LO PDFs of Ref. [75] by comparing the PDF uncertainties of this set compared to our default set NNPDF40 NNLO [70] and to MSHT20 NNLO [74]. Figure 4 shows the purely resummed spectrum up to 40 GeV, where matching corrections of about 5% are neglected at 20 GeV (less than 2% below 10 GeV). We do not expect that the matching corrections change the relative PDF results and uncertainties substantially. About two-thirds of the total fiducial cross-section originates from the integrated purely resummed spectrum up to 20 GeV. The results demonstrate that systematic differences between PDF sets are still dominant, comparable to the effect of N^{3}LO corrections in the PDFs. Uncertainties for the MSHT20 aN^{3}LO PDF set are larger since it includes missing higher-order effects with the PDF uncertainties. Overall, combined statistical and systematic PDF uncertainties are comparable to the residual truncation uncertainties found in our paper.

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