Simultaneous measurement of proton and lepton kinematics in quasielastic-like ν_{μ} -hydrocarbon interactions from 2 to 20 GeV

D. Ruterbories,¹ S. Akhter,² Z. Ahmad Dar,^{3,2} F. Akbar,² V. Ansari,² M. V. Ascencio,^{4,*} M. Sajjad Athar,²

A. Bashyal,^{5,†} A. Bercellie,¹ M. Betancourt,⁶ A. Bodek,¹ J. L. Bonilla,⁷ A. Bravar,⁸ H. Budd,¹

G. Caceres,^{9,‡} T. Cai,¹ M.F. Carneiro,^{5,9,§} G.A. Díaz,¹ H. da Motta,⁹ J. Felix,⁷ L. Fields,¹⁰ A. Filkins,³

R. Fine,^{1,¶} A.M. Gago,⁴ H. Gallagher,¹¹ P.K.Gaur,² A. Ghosh,^{12,9} S.M. Gilligan,⁵ R. Gran,¹³ E. Haase,¹³

D.A. Harris,^{14,6} S. Henry,¹ K. Jacobsen,¹³ D. Jena,⁶ S. Jena,¹⁵ J. Kleykamp,^{1, **} A. Klustová,¹⁶

M. Kordosky,³ D. Last,¹⁷ A. Lozano,⁹ X.-G. Lu,^{18, 19} E. Maher,²⁰ S. Manly,¹ W.A. Mann,¹¹ C. Mauger,¹⁷

K.S. McFarland,¹ A.M. McGowan,¹ B. Messerly,^{21, ††} J. Miller,¹² O. Moreno,^{3,7} J.G. Morfín,⁶ D. Naples,²¹

J.K. Nelson,³ C. Nguyen,²² A. Olivier,¹ V. Paolone,²¹ G.N. Perdue,^{6, 1} K.-J. Plows,¹⁹ M.A. Ramírez,^{17, 7}

R.D. Ransome,²³ H. Ray,²² H. Schellman,⁵ C.J. Solano Salinas,²⁴ H. Su,²¹ M. Sultana,¹ V.S. Syrotenko,¹¹

E. Valencia,^{3,7} N.H. Vaughan,⁵ A.V. Waldron,¹⁶ M.O. Wascko,¹⁶ C. Wret,¹ B. Yaeggy,^{12, ‡‡} and L. Zazueta³

(The MINERvA Collaboration)

¹Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627 USA

²AMU Campus, Aligarh, Uttar Pradesh 202001, India

³Department of Physics, William & Mary, Williamsburg, Virginia 23187, USA

⁴Sección Física, Departamento de Ciencias, Pontificia Universidad Católica del Perú, Apartado 1761, Lima, Perú

⁵Department of Physics, Oregon State University, Corvallis, Oregon 97331, USA

⁶Fermi National Accelerator Laboratory, Batavia, Illinois 60510, USA

⁷Campus León y Campus Guanajuato, Universidad de Guanajuato, Lascurain

de Retana No. 5, Colonia Centro, Guanajuato 36000, Guanajuato México.

⁸University of Geneva, 1211 Geneva 4, Switzerland

⁹Centro Brasileiro de Pesquisas Físicas, Rua Dr. Xavier Sigaud 150, Urca, Rio de Janeiro, Rio de Janeiro, 22290-180, Brazil

¹⁰Department of Physics, University of Notre Dame, Notre Dame, Indiana 46556, USA

¹¹Physics Department, Tufts University, Medford, Massachusetts 02155, USA

¹²Departamento de Física, Universidad Técnica Federico Santa María, Avenida España 1680 Casilla 110-V, Valparaíso, Chile

¹³Department of Physics, University of Minnesota – Duluth, Duluth, Minnesota 55812, USA

¹⁴ York University, Department of Physics and Astronomy, Toronto, Ontario, M3J 1P3 Canada

¹⁵Department of Physical Sciences, IISER Mohali, Knowledge City, SAS Nagar, Mohali - 140306, Punjab, India

¹⁶ The Blackett Laboratory, Imperial College London, London SW7 2BW, United Kingdom

¹⁷Department of Physics and Astronomy, University of Pennsylvania, Philadelphia, PA 19104

¹⁸Department of Physics, University of Warwick, Coventry, CV4 7AL, UK

¹⁹Oxford University, Department of Physics, Oxford, OX1 3PJ United Kingdom

²⁰Massachusetts College of Liberal Arts, 375 Church Street, North Adams, MA 01247

²¹Department of Physics and Astronomy, University of Pittsburgh, Pittsburgh, Pennsylvania 15260, USA ²²University of Florida, Department of Physics, Gainesville, FL 32611

²³ Rutgers, The State University of New Jersey, Piscataway, New Jersey 08854, USA

²⁴ Facultad de Ciencias, Universidad Nacional de Ingeniería, Apartado 31139, Lima, Perú

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Neutrino charged-current quasielastic-like scattering, a reaction category extensively used in neutrino oscillation measurements, probes nuclear effects that govern neutrino-nucleus interactions. This Letter reports the first measurement of the triple-differential cross section for ν_{μ} quasielasticlike reactions using the hydrocarbon medium of the MINERvA detector exposed to a wide-band beam spanning $2 \leq E_{\nu} \leq 20$ GeV. The measurement maps the correlations among transverse and longitudinal muon momenta and summed proton kinetic energies, and compares them to predictions from a state-of-art simulation. Discrepancies are observed that likely reflect shortfalls with modeling of pion and nucleon intranuclear scattering and/or spectator nucleon ejection from struck nuclei. The separate determination of leptonic and hadronic variables can inform experimental approaches to neutrino-energy estimation.

Current and future long baseline neutrino experiments [1–4] seek to delineate the neutrino mass ordering and to quantify the presence of CP violation in the neutrino sector. These experiments will utilize neutrinos of energies from 0.3 to 4 GeV and higher if tau-neutrino

appearance is explored [5]. Accurate models of neutrinonucleus interactions are required to relate the energies of visible final-state particles of events observed in the detectors to the initiating true neutrino energies that underwrite the oscillations of neutrino flavor. A leading contributor to charged-current (CC) neutrino interactions at these energies is the quasielastic-like channel:

$$\nu_{\mu} + \mathcal{A} \to \mu^{-} + \text{nucleons} + \mathcal{A}'.$$
 (1)

Charged-current neutrino interactions within nuclei, even those with apparent two-body quasielastic final states, are altered by a number of poorly understood effects: The struck nucleons of the initial state are bound and in motion [6, 7]; short-range multinucleon processes give rise to enhanced reaction rates relative to scattering on free nucleons [8–13], and hadrons produced in the parent ν_{μ} interactions with nucleons undergo intranuclear finalstate interactions (FSI) within the target nuclei. While the reaction $\nu_{\mu} + \mathcal{A} \rightarrow \mu^{-} + p + \mathcal{A}'$, wherein nearly all final-state energy is visible, is thought to be the main contributor to the quasielastic-like channel (Eqn. 1) a significant number of events may have energy deposited in undetected neutrons or light nuclear fragments. Final states of the latter kind complicate the task of inferring neutrino energy from samples of quasielastic-like events.

The reaction of Eqn. 1 has received repeated experimental scrutiny; however only single- or doubledifferential cross sections in muon kinematics have been reported, mostly carried out with ν_{μ} of incident energies, E_{ν} , of sub-GeV to few GeV [14–31]. This Letter reports a new measurement of the quasielastic-like channel in which the final-state muon transverse (p_t) and longitudinal ($p_{||}$) momenta are measured in each event simultaneously with the total "available" (calorimetrically visible) recoil energy ($E_{\text{available}}$) used in previous analyses of data from MINERvA [13, 32, 33]. Since the signal requires final state muon plus nucleons only, $E_{\text{available}}$ is the sum of the kinetic energies of all protons, denoted ΣT_p .

Under the assumption of a stationary target neutron in $\nu_{\mu} + n(bound) \rightarrow \mu^{-} + p$, energy transfer also can be inferred to be:

$$q_0^{(\text{QE})} \equiv \frac{m_p^2 - (m_n - E_B)^2 - m_\mu^2 + 2(E_\mu - p_\mu \cos\theta_\mu)E_\mu}{2(m_n - E_b) - E_\mu + p_\mu \cos\theta_\mu}.$$
(2)

Here, m_{μ} , m_p , and m_n are the masses of the muon, proton, and neutron, E_b is the average binding energy of 34 MeV [6, 34, 35], and E_{μ} , p_{μ} , and θ_{μ} are the muon energy, momentum and angle with respect to the neutrino beam. The $q_0^{(QE)}$ of Eq. (2) is the quantity added to the reconstructed muon energy by T2K to estimate neutrino energy of quasielastic-like events, while ΣT_p is the amount added to the muon energy by NOvA to form its neutrino energy estimator for events with quasielasticlike topologies. Combined T2K-NOvA analyses will be credible to the extent that interaction models correctly predict the relationship between these quantities. The measurements of this Letter elicit the correlations among the kinematic variables of quasielastic channels, thereby confronting the models with information of a kind that heretofore has not been available.

The analysis utilizes high-statistics samples of ν_{μ} CC interactions recorded by the MINERvA detector [36] exposed to the wide-band, medium energy NuMI beam [37] at Fermi National Accelerator Laboratory. In the NuMI beam, 120 GeV protons impinging upon a carbon target produce pions and kaons that are subsequently chargeselected, focused by a magnetic horn system, and directed into a pipe where they decay. The resulting neutrino flux is calculated using a GEANT4 simulation of the beam optics with input from hadronic interaction data relevant to the beam and materials [38]. The neutrino flux is constrained by previous measurements of neutrino elastic scattering from atomic electrons, $\nu e^- \rightarrow \nu e^-$ [39]. This constraint reduces the normalization uncertainty from 7.8% to 3.9% for muon neutrinos of energies between 2 and 20 GeV. The neutrino interactions occur in the central scintillator tracker of the MINERvA spectrometer which has a mass fraction of 88.5% carbon, 8.2%hydrogen, 2.5% oxygen, and trace amounts of other elements. Primary vertices of selected events are restricted to a central 5.3 ton region. The spatial resolution of the tracker enables reconstruction of final-state protons and Michel electrons from the $\pi^+ \to \mu^+(\nu_\mu) \to e^+(\nu_e \overline{\nu}_\mu)$ decay chain, as well as the tracks of muons. The magnetized MINOS near detector, located downstream of MIN-ERvA, is used to determine the charge and momenta of exiting muons. The scintillator tracker and the surrounding sampling calorimeters enable calorimetric measurement of ΣT_p and of photon showers from $\pi^0 \to \gamma \gamma$. Occasionally, final-state neutrons leave a small amount of energy which is tagged as a photon or included in ΣT_p ; the reference simulation predicts and corrects for this effect. The average ΣT_p for protons is ≈ 250 MeV; neutrons contribute less than 10 MeV of energy in 74%of events and an average of 85 MeV for the rest.

The MINERvA detector response is simulated using GEANT4 [40] version 4.9.4p2 with the QGSP_BERT physics list. The optical and electronics performance is also simulated. Through-going muons are used to determine the absolute energy scale. Full descriptions of calibrations are given in Refs. [36, 41]. The absolute energy response to charged hadrons is set according to measurements using a charged particle test beam [42] and a scaled-down version of the MINERvA detector. The effects of accidental activity as a function of beam intensity are simulated by overlaying hits from data in both MINERvA and MINOS.

The reference signal and background models for this analysis are based on a modified version of the GE-NIE [43] v.2.12.6 event generator. Quasielastic interactions are modeled using the Llewellyn Smith formalism [44] with BBBA05 vector form factors [45] and an axial-vector form factor based on a z-expansion fit to deuterium data [46]. Resonance production is simulated using the Rein-Sehgal model [47] with a dipole axial mass of $M_A^{RES} = 1.12 \text{ GeV/c}^2$. The nuclear initial state is a rela-

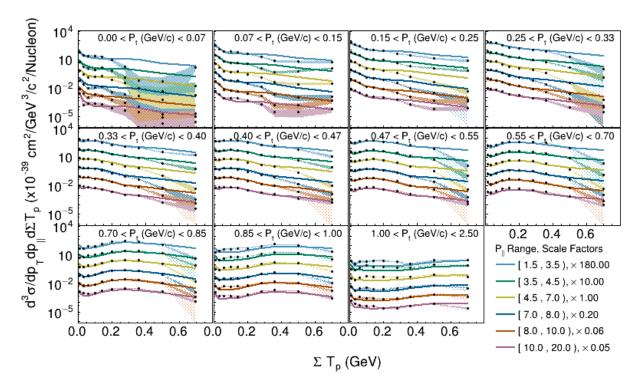


FIG. 1. The flux-averaged triple-differential cross section for quasielastic-like events, $d^3\sigma/dp_{||}dp_td\Sigma T_p$, shown as points with colored error bands for designated intervals of $p_{||}$ in panels of p_t . Note the use of scaling factors and log scale to elicit the trends and consistency across all $p_{||}$. The predictions of the reference model MINERvA tune v4.4.1 are shown as lines in each panel.

tivistic Fermi gas model [48] with $k_F = 0.221$ GeV/c and with a Bodek-Ritchie high momentum tail [49]. Multinucleon quasielastic-like interactions are simulated by the "Valencia model" described in Refs. [10, 11, 50]. Intranuclear final-state interactions of produced hadrons are modeled using the INTRANUKE-hA package [51].

To better describe MINERvA data, a number of modifications are made in the reference model which are collectedly denoted MINERvA tune v4.4.1. The quasielastic cross section is modified as a function of energy and threemomentum transfer based on the random phase approximation (RPA) of the Valencia model [52, 53] appropriate for a Fermi gas [54, 55] to account for long range correlations between nucleons. To account for an observed excess in specific regions of three-momentum transfer and ΣT_p , the multinucleon cross section is increased based on fits to MINERvA data [13] from a lower energy beam configuration. Additionally, based on fits to ν_{μ} -hydrogen data [56], the non-resonant CC pion production is decreased by 43%, the overall baryon-resonance pion production is increased by 15%, and $M_{A,Res}$ is set to 0.94 GeV.

Samples for measuring quasielastic-like interactions and their backgrounds require a muon track that starts in the fiducial volume and is identified in MINOS as negatively charged. All other tracked particles originating from the interaction vertex at the beginning of the muon track must have dE/dx consistent with a proton. Signal and background samples are formed by counting the number of Michel electron candidates within 600 mm long, 600 mm diameter cylinders centered on the neutrino vertex and on endpoints of tracked particles, and by counting isolated clusters constructed from two-dimensional clusters with at least 1 MeV visible energy. The former identify π^+ , and the latter identify photons from π^0 decays. Clusters with an energy less than 10 MeV per hit are assumed to be caused by neutrons producing low energy protons and are not used.

Events that contain either a π^+ (67%), a π^0 (19%), or both (14%), comprise the dominate backgrounds to the quasielastic-like signal. Four exclusive samples are assembled using the criteria of 0 or ≥ 1 Michel electrons, and ≤ 1 or ≥ 2 isolated clusters. Sample A with no Michel electrons and ≤ 1 isolated cluster is the signal sample. Sample B is the Michel-only sample; it is dominated by events with a single π^+ . Sample C is the isolated clusters-only sample comprised of events with having single π^0 . Sample D is with any Michel electrons and ≥ 2 isolated clusters; it is dominated by events with multiple pions. Details concerning these four samples and reconstruction methods that relate to them are given in Ref. [25].

For each bin of p_t and ΣT_p , a joint fit to the abovelisted four samples is used to determine scale factors applied to the signal sample (A) and to each of the backgrounds (single π^+ (B), single π^0 (C), and multiplon (D)). The fit minimizes a χ^2 over the four scale factors using a singular value decomposition (SVD) which drops singular values with condition number $< 10^{-3}$ to avoid numerical instability and forbids negative scale factors for any component. The background-subtracted event rate is unfolded using an iterative technique [57] from the RooUnfold framework [58] which is regularized by the number of iterations. A regularization of ten iterations was chosen by generating randomly fluctuated pseudodata samples with a number of different underlying physics models to ensure fidelity with different assumed data models. The statistical covariance matrix is scaled to account for the finite Monte Carlo statistics in the true-to-reconstructed migration matrix. The unfolded 3D distribution is then corrected for the predicted event loss from selection inefficiencies and detector effects. The average efficiency is between 40% and 75% range over all bins. The triple-differential cross section is obtained by normalizing the distribution according to the number of neutrinos incident on the detector and the number of scattering centers. The final result is shown in Fig. 1. To zeroth order, Fig. 1 shows that the reference simulation (solid lines of different colors in the panels) describes the general trends in the data points. Upon closer inspection, discrepancies are apparent. In the lower p_t range of the uppermost panels, for example, the prediction exceeds the data for all $p_{||}$ for $\Sigma T_p \ge 0.2 \,\text{GeV}$.

Figure 2 shows the average ΣT_p in each $p_t - p_{||}$ bin. The average recoil energy in data falls ~ 50 MeV below the reference model at low p_t , $\lesssim 0.5$ GeV/c, then rises to be comparable to the model ~ 0.9 GeV/c, and finally exceeds the model prediction in the highest p_t bin. The abrupt change in the highest p_t bins may be due to a cutoff in the Valencia multinucleon model that eliminates this process above three-momentum transfer of 1.2 GeV/c [10, 11].

Figures 1 and 2 show discrepancies between data and the reference simulation that persist as a function of $p_{||}$. Figure 3 displays the ratio of data to simulation at the peak $p_{||}$ bin. Here the discrepancies can be associated with three different kinematic regions of $p_t - \Sigma T_p$, enumerated below. Since the prominent processes in each region can be identified, each discrepancy suggests specific model developments.

Low p_t , high ΣT_p : For $0 < p_t < 0.4$ and $\Sigma T_p \geq 0.3$ -0.8 GeV, the reference model predicts significant cross section in a region where the data indicates very little cross section. The predicted events are approximately half multinucleon and half low W pion production where the pion is absorbed in FSI. Therefore it is likely that both processes are over-predicted. In this region, the multinucleon prediction is from the Valencia model with little effect from MINERvA tune v4.4.1. In pion production, such an effect may arise either from over-prediction

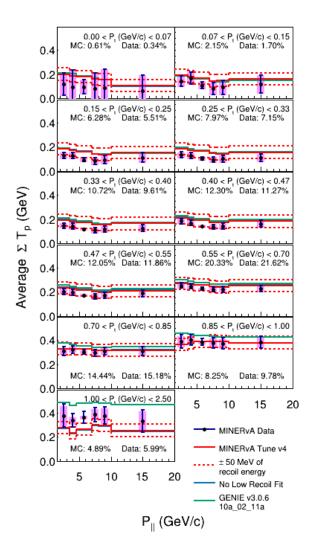


FIG. 2. Distribution of average ΣT_p in $p_t p_{||}$ bins. Legends within each panel give the percentage contribution of the p_t panel to the total integrated cross section for data and for the reference simulation. Statistical uncertainty is denoted by the colored box while the total uncertainty by the error bar.

of the baryon-resonance pion production cross sections or from too-small suppression of primary pions due to FSI. A low Q^2 suppression of resonant pion production[18, 59] or a reduction in the visible energy from these events[33], as Fig. 2 also suggests, would improve agreement with this data. Shifts in energy transfers have been observed in (e,e') data for regions of low energy transfer [34, 60– 68]. An over-prediction of pion FSI could arise from finite hadronic formation time [69], an effect not included in the reference simulation. However this background arises mostly from absorption of slow pions ($p_{\pi} < 0.3 \,\text{GeV/c}$), hence pion formation time is unlikely to account for the entire effect.

Moderate p_t and ΣT_p just above the quasielastic peak: For ΣT_p of 0.2 GeV and 0.15 < p_t < 0.55 GeV/c, where

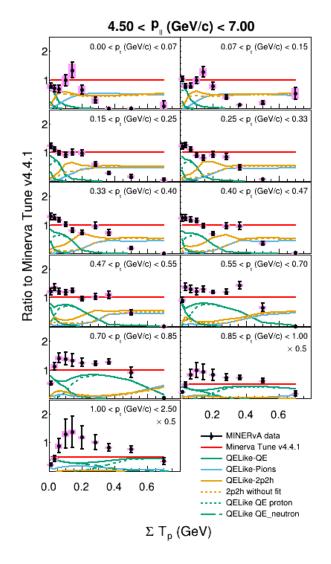


FIG. 3. Ratio of the differential cross-section in panels of p_t for the peak bin 4.5 $< p_{||} < 7.0$ GeV/c. The predicted cross section in the reference model is broken down into different contributions. Note that in the highest p_t panels, above 0.85 GeV/c, the ratio is shown scaled by 0.5 relative to the other panels.

the modifications of MINERvA tune v4.4.1 to multinucleon processes are large, the data and reference model would be in strong disagreement without these modifications. Figure 3 shows that the ratio of the data to the reference model dips near the peak of the tune, suggesting that the shape of the MINERvA tune v4.4.1 enhancement may not be accurate, either in rate or in fraction of events with a neutron in the final state. However, at $p_t > 0.55$ GeV/c, where the model predicts a smaller multinucleon contribution, the data mostly exceeds the reference prediction, suggesting that a significant enhancement to multinucleon processes at higher p_t than in MINERvA tune v4.4.1 may be needed.

High p_t and low ΣT_p : At $p_t > 0.55$ GeV/c and $\Sigma T_p < 50$

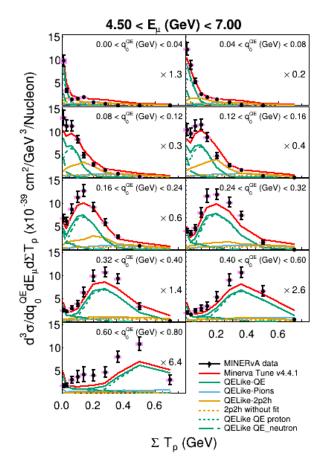


FIG. 4. Flux-averaged triple differential cross section, $d^3\sigma/dE_{\mu}dq_0^{(\text{QE})}d\Sigma T_p$. Note that scale factors are applied to each of the $q_0^{(\text{QE})}$ panels.

MeV, there is a significant over-prediction relative to data. This region is dominated by true quasielastic events where the final-state proton undergoes FSI and leaves the nucleus as one or more energetic neutrons; this suggests that too much strength is given to FSI in this kinematic region.

Figure 4 presents the flux-averaged triple-differential cross section $d^3\sigma/dE_{\mu}dq_0^{(\text{QE})}d\Sigma T_p$. Here as well, significant data versus reference model discrepancies are seen at low $q_0^{(\text{QE})}$ for ΣT_p beyond the peak of the quasielastic contribution. The previously noted discrepancy at low ΣT_p and high p_t corresponds to a predicted peak near zero ΣT_p at high $q_0^{(\text{QE})}$ which is absent from the data. The cross section in the quasielastic peak is underestimated, especially at higher $q_0^{(\text{QE})}$, and modified form factors could improve this agreement[46, 70–72]. This measurement directly probes the relationship between energy estimators used in oscillation experiments, and discrepancies with models suggest deficiencies in modeling those estimators.

In summary, a number of modelling shortfalls for neutrino-nucleus quasielastic-like scattering are identified by this measurement. These imply that relationships between the true neutrino energy of quasielastic-like events and experimental estimators, such as $E_{\text{available}} + E_{\mu}$ and $q_0^{(\text{QE})} + E_{\mu}$, differ from those predicted by current neutrino generators. The triple-differential cross section presented in Fig. 4 can serve as a benchmark for neutrinonucleus interaction simulations employed in ongoing and future neutrino oscillation experiments.

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- * Now at Iowa State University, Ames, IA 50011, USA
- [†] Now at High Energy Physics/Center for Computational Excellence Department, Argonne National Lab, 9700 S Cass Ave, Lemont, IL 60439
- [‡] now at Department of Physics and Astronomy, University of California at Davis, Davis, CA 95616, USA
- [§] Now at Brookhaven National Laboratory, Upton, New York 11973-5000, USA
- Now at Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA
- ** Now at Department of Physics and Astronomy, University of Mississippi, Oxford, MS 38677
- ^{††} Now at University of Minnesota, Minneapolis, Minnesota

55455, USA

- ^{‡‡} Now at Department of Physics, University of Cincinnati, Cincinnati, Ohio 45221, USA
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PRESENTATION OF ALL SLICES OF THE TRIPLE DIFFERENTIAL CROSS SECTIONS

Figures S1, S2, S3, S4, S5, and S6 show the ratio of the triple differential cross section to the reference model in the $p_t p_{||} - \Sigma T_p$, grouped in bins of $p_{||}$. Figures S7, S8, S9, S10, S11, and S12 present the triple differential cross section in $q_0^{(QE)} - E_\mu - \Sigma T_p$, grouped in bins of E_μ .

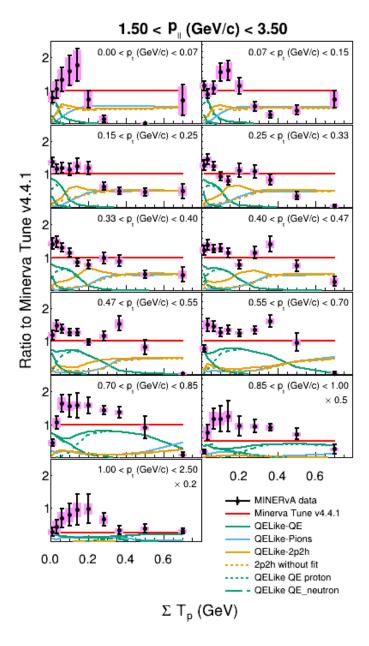


FIG. S1. Ratio of the differential cross-section in panels of p_t for the peak bin $1.5 < p_{||} < 3.5$ GeV/c. The predicted cross section in the reference model is broken down into different contributions. Note that in the highest p_t panels, above 0.85 GeV/c, the ratio is shown scaled by 0.5 and 0.2 relative to the other panels.

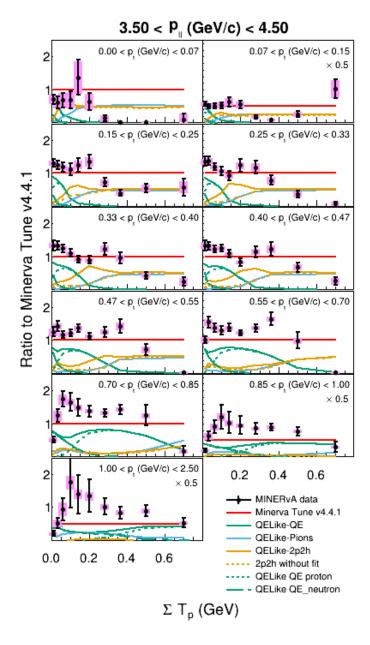


FIG. S2. Ratio of the differential cross-section in panels of p_t for the peak bin $3.5 < p_{||} < 4.5$ GeV/c. The predicted cross section in the reference model is broken down into different contributions. Note that in the highest p_t panels, above 0.85 GeV/c, the ratio is shown scaled by 0.5 relative to the other panels.

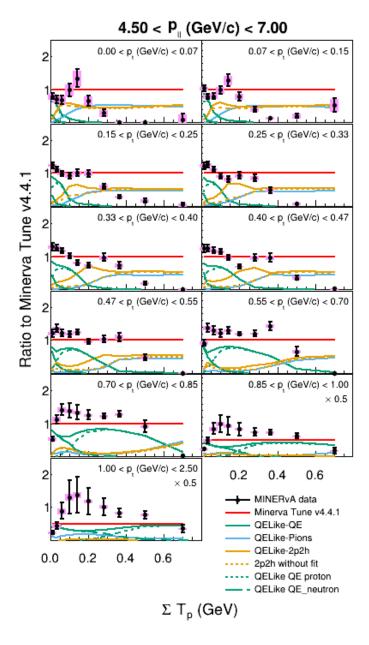


FIG. S3. Ratio of the differential cross-section in panels of p_t for the peak bin 4.5 $< p_{||} < 7.0$ GeV/c. The predicted cross section in the reference model is broken down into different contributions. Note that in the highest p_t panels, above 0.85 GeV/c, the ratio is shown scaled by 0.5 relative to the other panels.

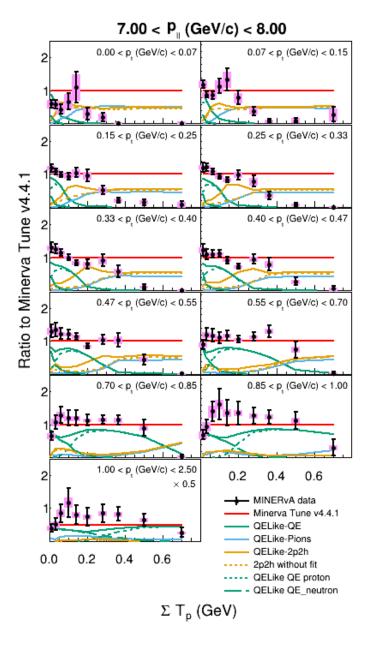


FIG. S4. Ratio of the differential cross-section in panels of p_t for the peak bin $7.0 < p_{||} < 8.0$ GeV/c. The predicted cross section in the reference model is broken down into different contributions. Note that in the highest p_t panels, above 1.0 GeV/c, the ratio is shown scaled by 0.5 relative to the other panels.

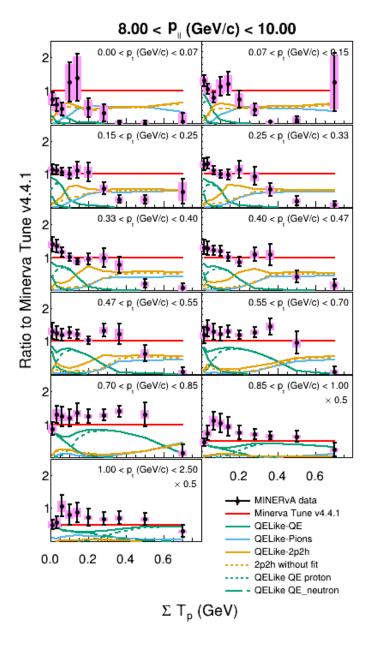


FIG. S5. Ratio of the differential cross-section in panels of p_t for the peak bin $8.0 < p_{||} < 10.0$ GeV/c. The predicted cross section in the reference model is broken down into different contributions. Note that in the highest p_t panels, above 0.85 GeV/c, the ratio is shown scaled by 0.5 relative to the other panels.

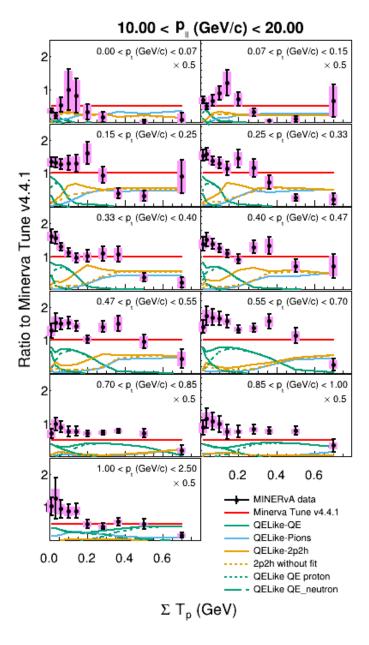


FIG. S6. Ratio of the differential cross-section in panels of p_t for the peak bin $10.0 < p_{||} < 20.0$ GeV/c. The predicted cross section in the reference model is broken down into different contributions. Note that in the highest p_t panels, above 0.70 GeV/c, the ratio is shown scaled by 0.5 relative to the other panels.

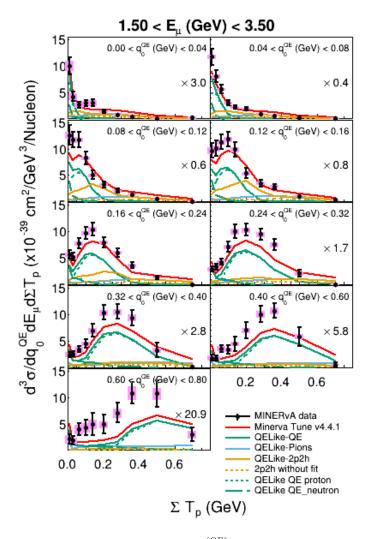


FIG. S7. Flux-averaged triple differential cross section, $d^3\sigma/dE_{\mu}dq_0^{(\text{QE})}d\Sigma T_p$. Note that scale factors are applied to each of the $q_0^{(\text{QE})}$ panels.

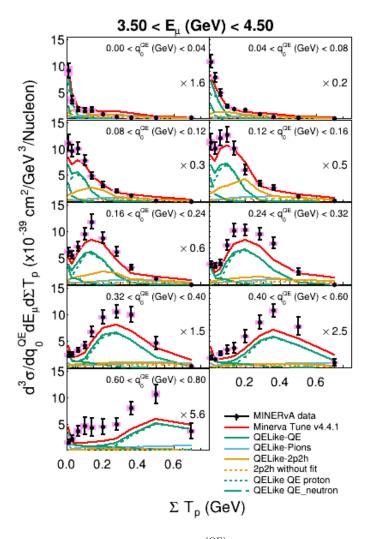


FIG. S8. Flux-averaged triple differential cross section, $d^3\sigma/dE_{\mu}dq_0^{(\text{QE})}d\Sigma T_p$. Note that scale factors are applied to each of the $q_0^{(\text{QE})}$ panels.

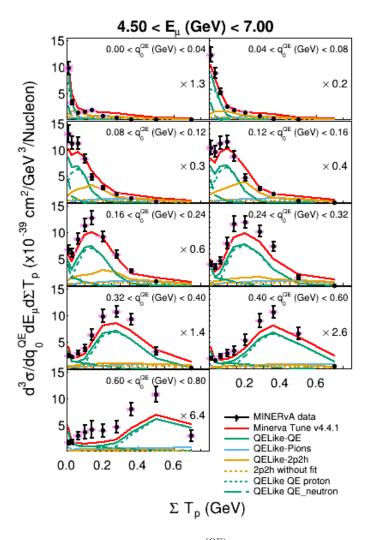


FIG. S9. Flux-averaged triple differential cross section, $d^3\sigma/dE_{\mu}dq_0^{(QE)}d\Sigma T_p$. Note that scale factors are applied to each of the $q_0^{(QE)}$ panels.

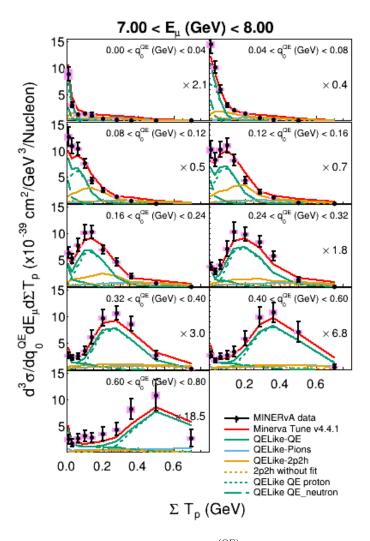


FIG. S10. Flux-averaged triple differential cross section, $d^3\sigma/dE_{\mu}dq_0^{(\text{QE})}d\Sigma T_p$. Note that scale factors are applied to each of the $q_0^{(\text{QE})}$ panels.

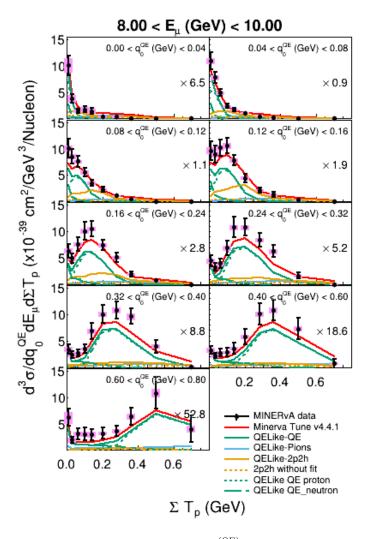


FIG. S11. Flux-averaged triple differential cross section, $d^3\sigma/dE_{\mu}dq_0^{(\text{QE})}d\Sigma T_p$. Note that scale factors are applied to each of the $q_0^{(\text{QE})}$ panels.

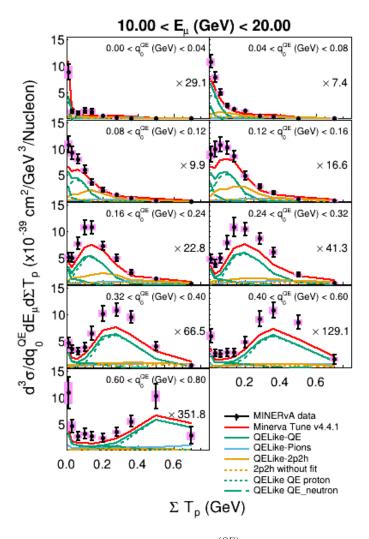


FIG. S12. Flux-averaged triple differential cross section, $d^3\sigma/dE_{\mu}dq_0^{(\text{QE})}d\Sigma T_p$. Note that scale factors are applied to each of the $q_0^{(\text{QE})}$ panels.