

Beam dynamics corrections to the Run-1 measurement of the muon anomaly a_μ at Fermilab

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The Fermi National Accelerator Laboratory (FNAL) Muon $g-2$ Experiment has measured the positive muon magnetic anomaly $a_\mu \equiv (g_\mu - 2)/2$ with a precision of 0.46 parts per million, with data collected during its first physics run in 2018. The experimental result combined with the measurement from the former experiment at Brookhaven National Laboratory increases the tension with the Standard Model expectation to 4.2σ , thus strengthening evidence for new physics. The magnetic anomaly is determined from the precision measurements of the muon spin precession frequency, relative to the muon momentum, and the average magnetic field seen by the beam. In an ideal case with muons orbiting in a horizontal plane with a uniform vertical magnetic field, the anomalous precession frequency ω_a is given by the difference between the spin (s) and cyclotron (c) frequencies, $\omega_a = \omega_s - \omega_c$. The observable ω_a is proportional to a_μ .

This proceeding presents the beam dynamics systematic corrections that are required to adjust the measured muon precession frequency ω_a^m to its true physical value ω_a .

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1. Introduction

The Fermilab Muon $g-2$ Experiment (E989) reports a new measurement of the positive muon magnetic anomaly $a_\mu \equiv (g_\mu - 2)/2$, where g_μ is the gyromagnetic ratio of the muon [1]. The Run-1 result is $a_\mu = 116\,592\,040(54) \times 10^{-11}$ (0.46 ppm), where the total uncertainty includes the dominant statistical uncertainty combined with the precession-rate systematic [2], magnetic systematic [3], and beam-dynamics systematic uncertainties [4].

This experiment measures a_μ by observing the anomalous spin precession frequency ω_a of a muon ensemble within a storage ring. In an ideal case with muons orbiting in a horizontal plane within a uniform vertical magnetic field, the anomalous precession frequency ω_a is given by the difference between the spin (s) and cyclotron (c) frequencies, $\omega_a = \omega_s - \omega_c$. Eq. 1 gives the rate of change of the component of spin \vec{S} parallel to the velocity $\vec{\beta} = \vec{v}/c$ in the presence of an electric field \vec{E} and a magnetic field \vec{B} .

$$\frac{d(\hat{\beta} \cdot \vec{S})}{dt} = -\frac{q}{m} \vec{S}_T \cdot \left[a_\mu \hat{\beta} \times \vec{B} + \beta \left(a_\mu - \frac{1}{\gamma^2 - 1} \right) \frac{\vec{E}}{c} \right], \quad (1)$$

where q , m and γ are the muon charge, mass, and Lorentz factor respectively, and \vec{S}_T is the component of \vec{S} perpendicular to $\hat{\beta}$. The electric field term in Eq. 1 vanishes for muons having the “magic” momentum $p_0 = 3.094 \text{ GeV}/c$ ($\gamma \sim 29.3$). Thus, the experiment is designed around injection and storage of muons centered on p_0 .

The Fermilab Muon Campus [5] delivers sixteen highly polarized positive muon (μ^+) beam bunches every 1.4 s into a 14.2-m-diameter superconducting storage ring (SR), which has a highly uniform $\sim 1.45 \text{ T}$ vertical magnetic field. After traversing one quarter of the SR, the beam is deflected by a fast pulsed-kicker magnet system [6] onto the intended storage orbit. Four electrostatic quadrupole (ESQ) sections [7] installed symmetrically around the ring confine the beam vertically. The Run-1 data, collected in 2018, are grouped into four datasets (a-d) corresponding to four different kicker magnet and ESQ voltage combinations. The ω_a frequency is encoded in the modulation of the decay-positron energy spectrum as muons circulate in the ring. Because of parity violation in the μ^+ weak decay, positrons are emitted with an energy and an angular distribution that are each correlated to the muon spin direction in its rest frame. Ignoring effects from beam dynamics, 24 calorimeter stations [8] distributed uniformly around the inner radius of the SR see a positron count rate versus time-in-fill t and positron energy E with the following equation:

$$N(t, E) = N_0(E) e^{-t/\gamma\tau_\mu} \{1 + A(E) \cos[\omega_a t + \varphi_0(E)]\}. \quad (2)$$

The normalization, time-dilated muon lifetime, asymmetry, anomalous precession frequency, and ensemble average phase at the time of injection are represented in Eq. 2 by $N_0(E)$, $\gamma\tau_\mu$, $A(E)$, ω_a and $\varphi_0(E)$, respectively. The frequency extracted by fitting the data is the measured quantity ω_a^m , which must be corrected for perturbations from beam dynamics effects. Two straw-tracker stations [9] are critical to the beam dynamics study since they provide detailed time-dependent stored-muon spatial profiles in two areas of the SR. The beam dynamics corrections are identified as electric field C_e , pitch C_p , muon loss C_{ml} , and phase acceptance C_{pa} . They are applied in a linear combination as $\omega_a \approx \omega_a^m \cdot (1 + C_e + C_p + C_{ml} + C_{pa})$ and are further discussed in Sec. 2 and in Sec. 3.

2. Electric Field Correction C_e and Pitch Correction C_p

The electric field term in Eq. 1 produces a rest frame magnetic field that affects the measured anomalous precession frequency ω_a^m . For the simple case, where we neglect the vertical betatron motion, Eq. 1 leads to

$$\omega_a^m = \frac{|q|}{m} a_\mu B_y \left[1 - \beta \frac{E_r}{c B_y} \left(1 - \frac{m^2 c^2}{a_\mu p^2} \right) \right], \quad (3)$$

where the subscripts r and y denote the radial and vertical components, respectively¹. The electric field term vanishes at the magic momentum, or when $E_r = 0$, which is the case of the central orbit radius R_0 as a result of the design of the ESQ system. In practice, the stored muon distribution has a finite momentum spread and is not well-centered at R_0 . A fast rotation analysis using the Fourier method yields the distribution of cyclotron frequencies f , which we convert to equilibrium radii R through the relation $R(2\pi f) = v$. We assume a fixed muon velocity v . Fig. 1a shows the distributions of equilibrium radial offsets $x_e := R - R_0$. The electric field correction depends on the mean and width of these distributions via $C_e \approx 2n(1-n)\beta_0^2 \frac{\langle x_e^2 \rangle}{R_0^2}$, where $\langle x_e^2 \rangle = \sigma_{x_e}^2 + \langle x_e \rangle^2$. The field index n is defined by $n = \frac{R_0}{v B_0} \frac{\partial E_y}{\partial y}$, with the field gradient determined by the quadrupole voltages and geometry. Finally, the electric field correction is $C_e = 489 \pm 53$ ppb.

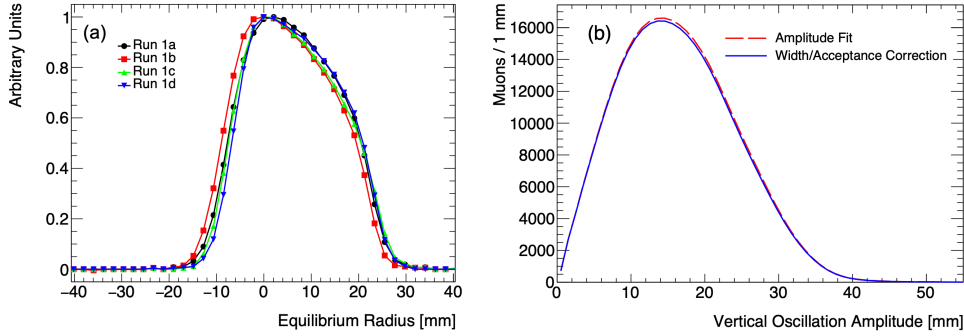


Figure 1: (a) The radial distribution extracted by the Fourier method for the four Run-1a datasets. The equilibrium radius is defined such that 0 mm corresponds to a magic-momentum muon [4]. (b) The fitted distribution of vertical oscillation amplitudes before (dashed red line) and after (solid blue line) the azimuthal averaging and calorimeter acceptance corrections [4].

Driven by the vertical focusing provided by the ESQ system, the pitch correction is derived as $C_p \approx \frac{n}{2} \frac{\langle y^2 \rangle}{R_0^2} = \frac{n}{4} \frac{\langle A^2 \rangle}{R_0^2}$. The vertical oscillation amplitude A is extracted from tracker measurements. Numerical simulations are used to model the azimuthal variations around the SR from the discrete ESQ sections. Moreover, not all decay positrons hit a calorimeter and enter into the determination of ω_a^m . To account for this, each calorimeter acceptance has been modeled as a function of transverse and azimuthal decay position. Fig. 1b shows the fitted distribution of vertical oscillation amplitudes before and after these corrections. The resulting pitch correction is $C_p = 180 \pm 13$ ppb.

¹Our coordinate system is with respect to the center of the storage volume at radius $R_0 = 7.112$ m, with x or r radially outward, y vertically up, and ϕ increasing clockwise when viewed from above.

3. Muon Loss Correction C_{ml} and Phase Acceptance Correction C_{pa}

The variable φ_0 in Eq. 2 depends on the stored muon momentum p_μ , the decay positron energy E , and the transverse decay coordinates (x, y) inside the SR. If the stored muon average transverse distribution and the detector gains are stable throughout a fill, that average phase remains constant. However, two resistors in the ESQ system were faulty, consequently causing slower-than-designed charging times during the first $\sim 100 \mu\text{s}$ of the measurement period (see Fig. 2a). This led to a time-dependent phase shift from the correlations of decay position to average phase owing to the detector acceptance. An extensive study of this effect involved the determination of the time-dependent muon spatial distributions for each tracker station and for each dataset, and the evolution of these distributions using beam dynamics models and simulations to produce $M^c(x, y, t)$ for all azimuthal locations where calorimeters (c) are placed. To extract the time-dependent phase for each calorimeter $\varphi_{pa}^c(t)$, the $M^c(x, y, t)$ distributions are then combined with acceptance ε^c , asymmetry A^c , and phase maps φ_{pa}^c using the following weighted sum over all spatial bins:

$$\varphi_{pa}^c(t) = \arctan \frac{\sum_{ij} M^c(x_i, y_j, t) \cdot \varepsilon^c(x_i, y_j) \cdot A^c(x_i, y_j) \cdot \sin(\varphi_{pa}^c(x_i, y_j))}{\sum_{ij} M^c(x_i, y_j, t) \cdot \varepsilon^c(x_i, y_j) \cdot A^c(x_i, y_j) \cdot \cos(\varphi_{pa}^c(x_i, y_j))}. \quad (4)$$

An example evaluation of $\varphi_{pa}^c(t)$ for the Run-1d dataset is shown in Fig. 2b. Simulated data are generated to evaluate the size of the mismeasurement of ω_a^m as a result of the time-dependent phase $\varphi_{pa}^c(t)$ by comparing the ω_a^m obtained with the fit function used to extract ω_a^m . This difference yields the correction factor for each calorimeter. The phase-acceptance correction value is: $C_{pa} = 158 \pm 75$ ppb.

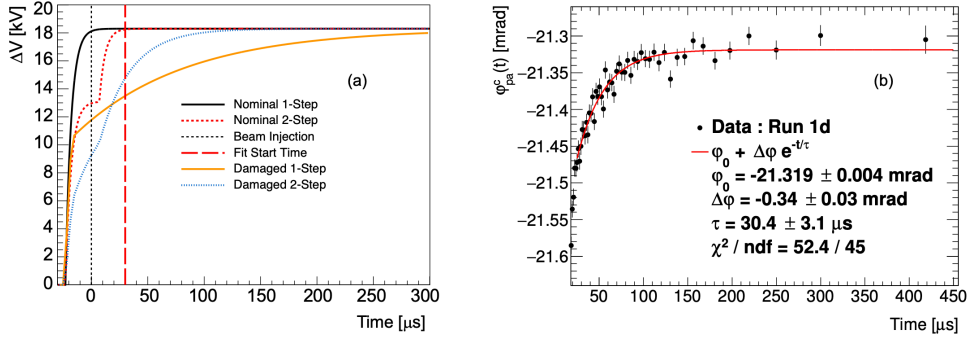


Figure 2: (a) Charging profiles for the nominal ESQ plates (black line and dotted red line). The two damaged resistors (solid orange and dotted blue lines) exhibit markedly different charging profiles during the data-fitting period [4]. (b) Calculation of $\varphi_{pa}^c(t)$ for Calorimeter 19 in Run-1d using Eq. 4 [4].

The muon loss correction takes into account the effect from muons that permanently escape from the storage volume during data taking and can potentially bias ω_a^m by inducing low drifts in the phase. A careful analysis demonstrated that lost muons do not significantly alter the ensemble-averaged spin phase, but we apply a small data-driven correction with $C_{ml} = -11 \pm 5$ ppb.

The damaged resistors were replaced after Run-1, which will significantly reduce the dominant contribution to C_{pa} and the overall magnitude of muon losses in the next runs.

4. Conclusion

The key findings discussed in this proceeding are tabulated in Table 1. We described four beam dynamics systematic corrections that are required to adjust the measured muon precession frequency ω_a^m to its true physical value ω_a . The pitch correction requires knowledge of the vertical stored muon distribution from the *in situ* tracker system. The electric field correction requires knowledge of the stored muon radial distribution, which is deduced from studying the time evolution of the incoming muon bunch. A small correction is applied for the muon loss-induced phase change. Finally, owing to two damaged high-voltage resistors in the ESQ system, the mean and rms of the stored muon distribution in Run-1 evolved throughout the first $\sim 100 \mu\text{s}$ of the measurement period. The sum of the corrections to ω_a^m is 0.50 ± 0.09 ppm; the uncertainty is small compared to the 0.43 ppm statistical precision of ω_a^m .

	C_e	C_p	C_{ml}	C_{pa}	C_{total}
Correction (ppb)	489	180	-11	-158	499
Uncertainty (ppb)	53	13	5	75	93

Table 1: The Run-1 combined beam dynamics corrections to ω_a^m from the four Run-1 datasets.

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References

- [1] B. Abi *et al.* (Muon $g-2$ Collaboration), *Measurement of the Positive Muon Anomalous Magnetic Moment to 0.46 ppm*, *Phys. Rev. Lett.* **126**, 141801 (2021).
- [2] T. Albahri *et al.* (Muon $g-2$ Collaboration), *Measurement of the anomalous precession frequency of the muon in the Fermilab Muon $g-2$ Experiment*, *Phys. Rev. D* **103**, 072002 (2021).
- [3] T. Albahri *et al.* (Muon $g-2$ Collaboration), *Magnetic-field measurement and analysis for the Muon $g-2$ Experiment at Fermilab*, *Phys. Rev. A* **103**, 042208 (2021).
- [4] T. Albahri *et al.* (Muon $g-2$ Collaboration), *Beam dynamics corrections to the Run-1 measurement of the muon anomalous magnetic moment at Fermilab*, *Phys. Rev. Accel. Beams* **24** 044002 (2021).
- [5] D. Stratakis *et al.*, *Accelerator performance analysis of the Fermilab Muon Campus*, *Phys. Rev. Accel. Beams* **20** 111003 (2017).
- [6] A. P. Schreckenberger *et al.*, *The fast non-ferric kicker system for the Muon $g-2$ Experiment at Fermilab*, *Nucl. Instrum. Meth. A* **1011** 165597 (2021).
- [7] J. Crnkovic *et al.*, *FERMILAB-CONF-18-466-PPD*
- [8] J. Kaspar *et al.*, *J. Instrum.* **12** P01009 (2017).
- [9] B.T. King *et al.*, *J. Instrum.* **17** P02035 (2022).
- [10] J. D. Jackson, *Classical Electrodynamics*, 3rd ed. (Wiley, New York, 1999).