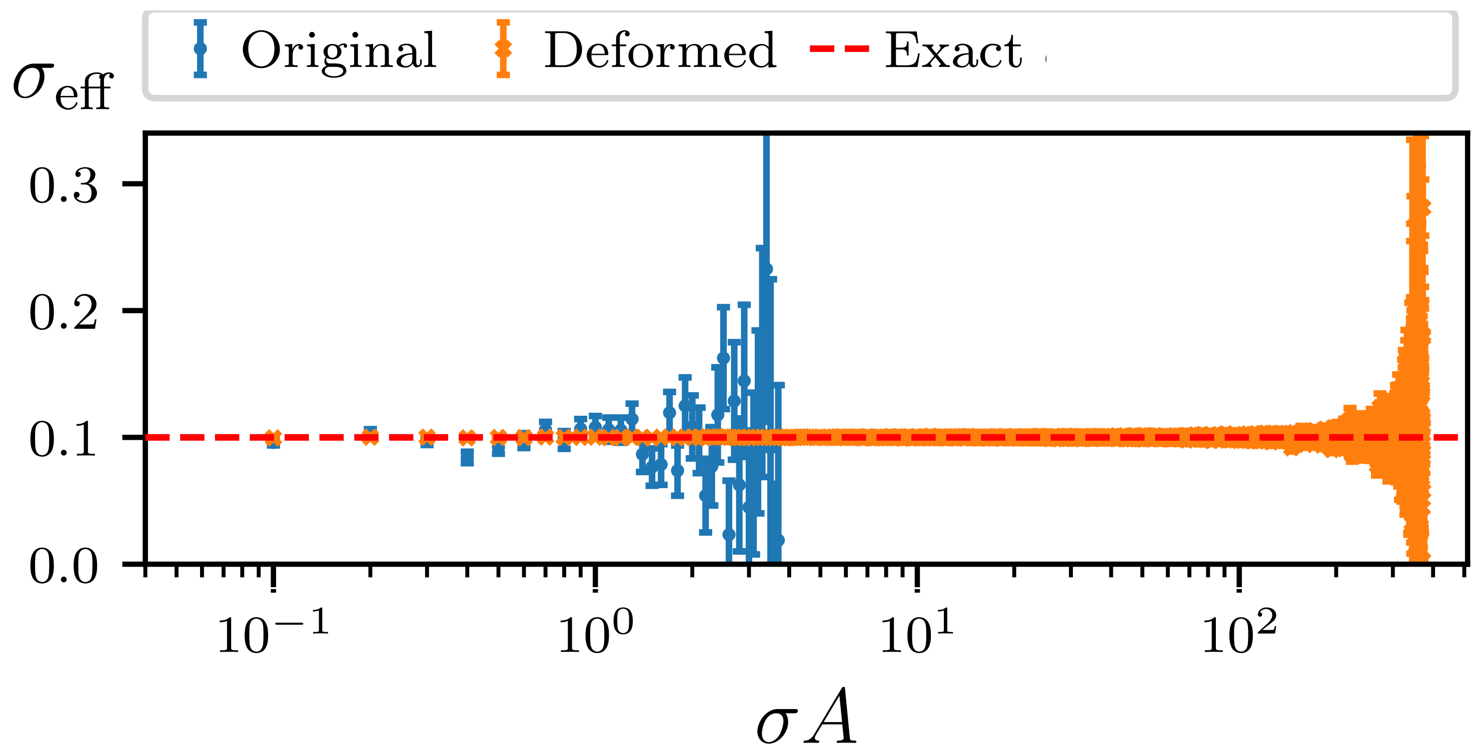
FERMILAB-SLIDES-21-032-T

Sign(al-to-noise) Problems in Lattice Gauge Theory

Michael Wagman



Fermilab

University of Iowa
Nuclear and Particle Physics Seminar

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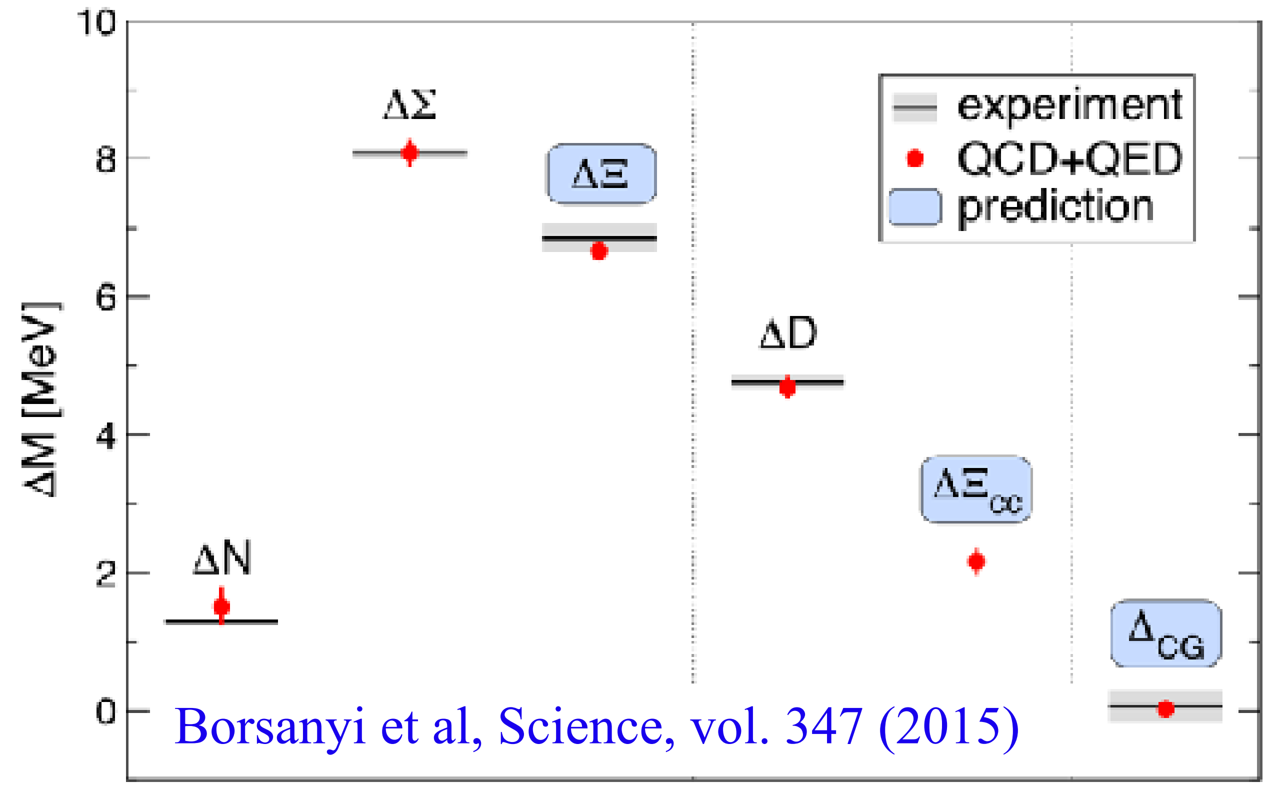
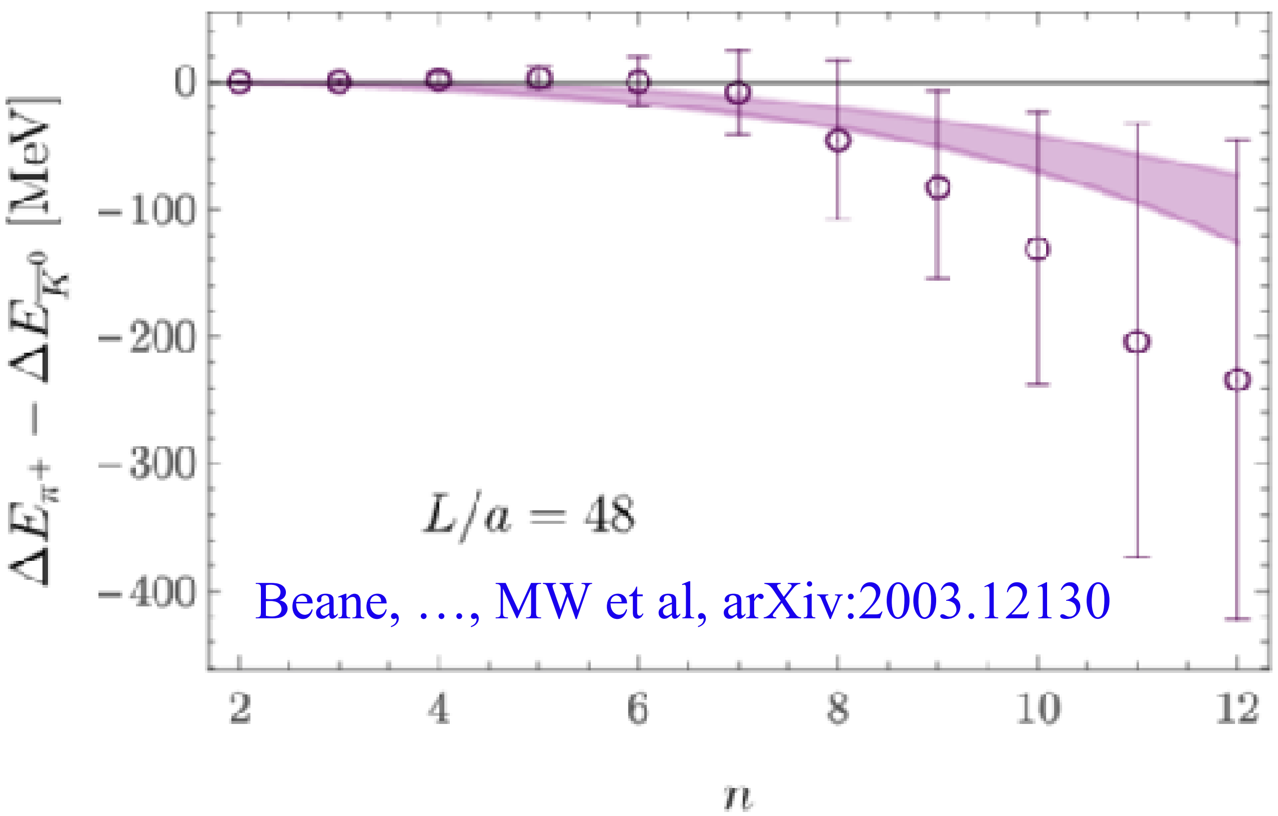
Solving the Standard Model

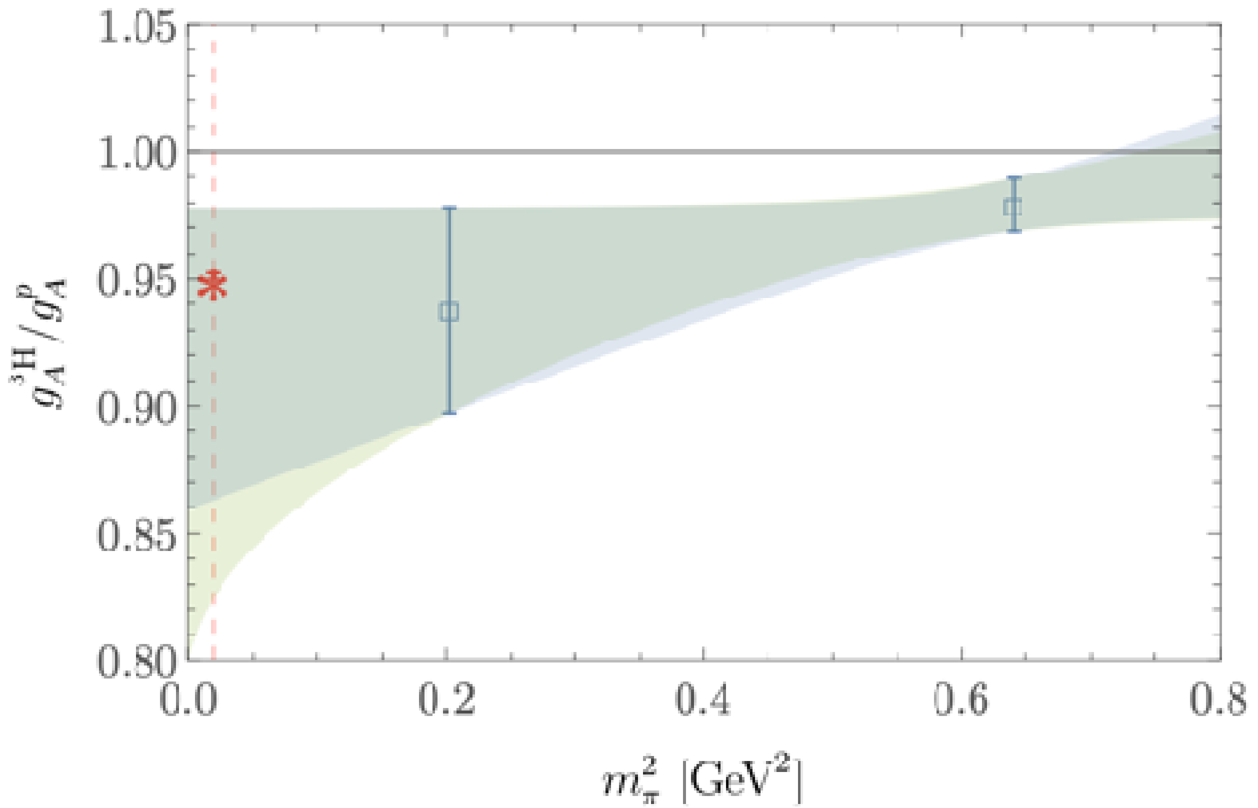
Simple quantities like the proton-neutron mass difference have been computed with percent-level precision in lattice QCD+QED





Increasingly complex effects such as QED effects on charged multi-meson systems are being explored

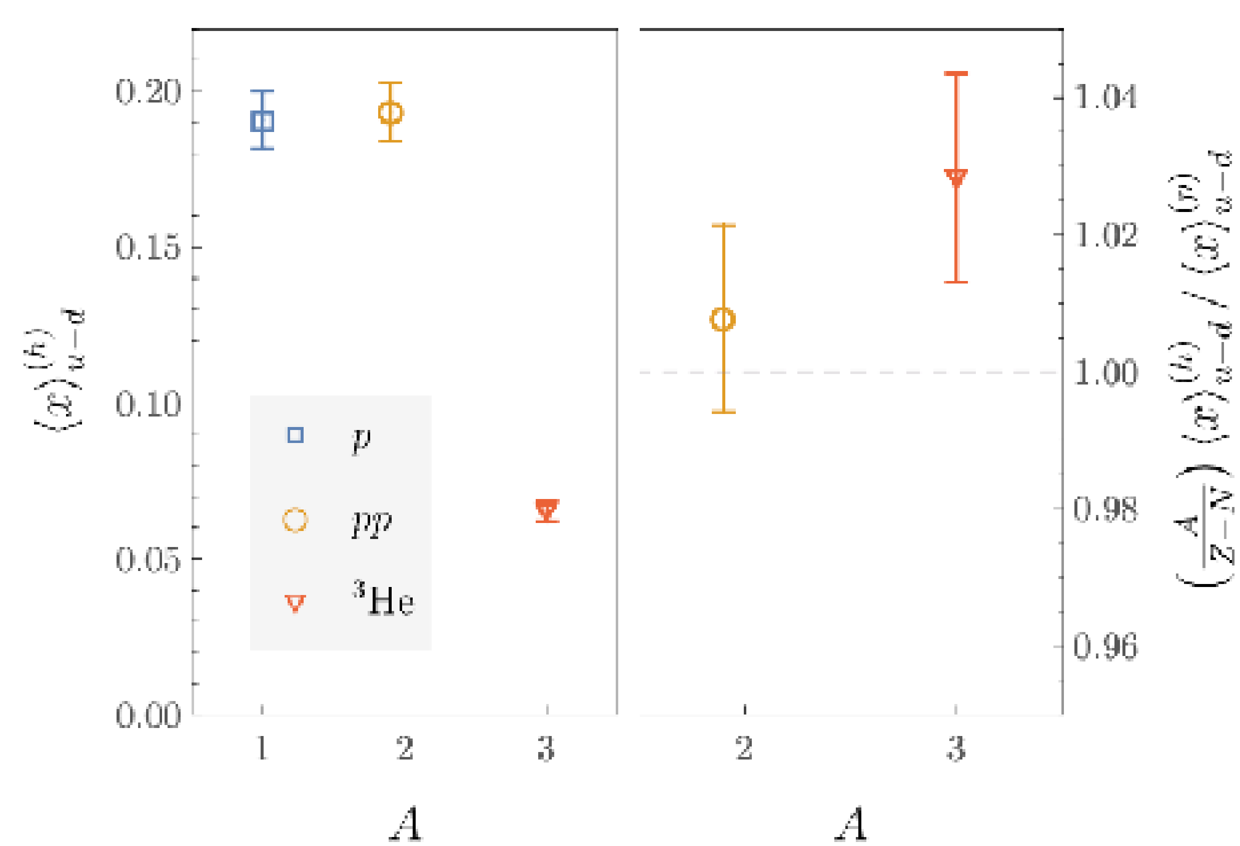
Nuclear matrix elements from LQCD



Parreño, …, MW, et al [NPLQCD] arXiv:2102.xx

Triton beta-decay rate computed from lattice QCD, effects of axial two-body currents visible

The *u-d* quark momentum fractions of light nuclei have been computed in exploratory calculations at heavier-than-physical quark masses



Detmold, …, MW, et al [NPLQCD], arXiv:2101.12668

Excited-state effects

Euclidean correlation functions have spectral representation

$$G(\tau) = \left\langle N(\tau)\overline{N}(0)\right\rangle = \sum_n \langle 0|N|n\rangle\, e^{-E_n\tau}\, \langle n|\overline{N}|0\rangle$$
$$\equiv \sum_n Z_n e^{-E_n\tau}$$

Precise studies of multi-nucleon systems face challenges from unbound finite-volume excited states

$$\vec{p} = \frac{2\pi\vec{n}}{L} \qquad\qquad E_{NN} \approx 2\sqrt{M_N^2 + \left(\frac{2\pi n}{L}\right)^2}$$

Analogous issues with $N\pi$ excited-states may impact axial form factor calculations

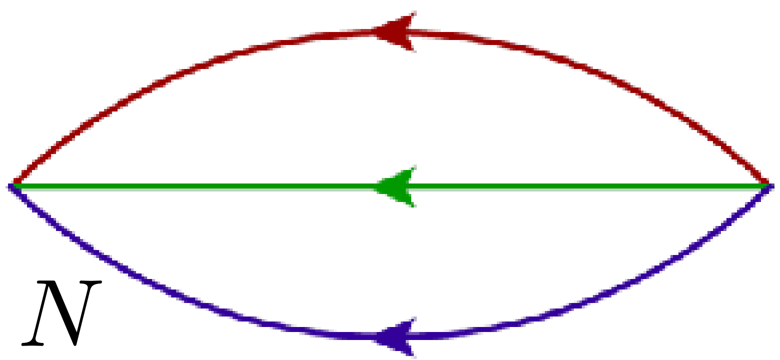
Jang, Gupta, Yoon, Bhattacharya, PRL 124 (2020) Alexandrou et al [ETMC], arXiv:2011.13342

Better tools for needed for dealing with multi-hadron excited states

The signal-to-noise problem

Monte Carlo noise in nucleon/nuclear correlation functions grows exponentially with Euclidean time separation

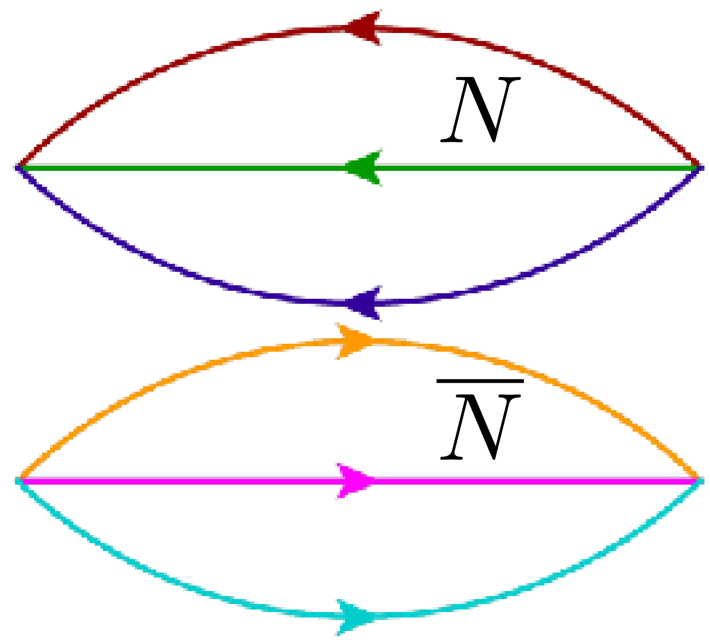
Correlation function variance can be analyzed as a correlation function

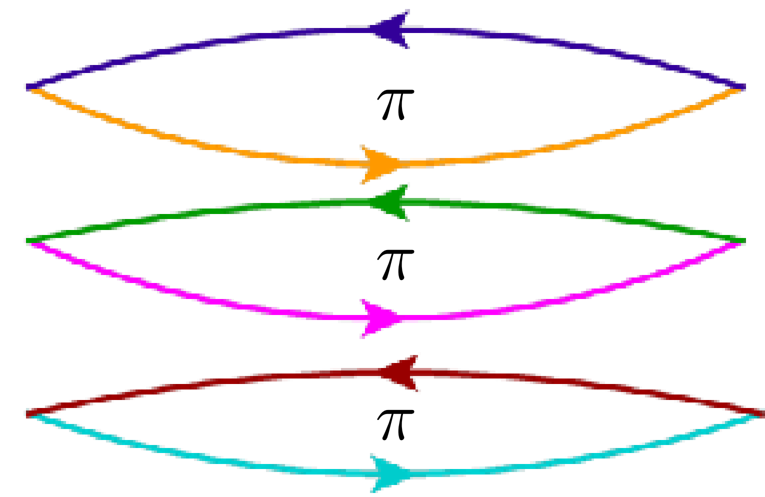


$$G_N \sim e^{-M_N t}$$

$$\mathrm{Var}(G_N) \sim \left\langle \left| N(t) N(0)^\dagger \right| \right\rangle$$

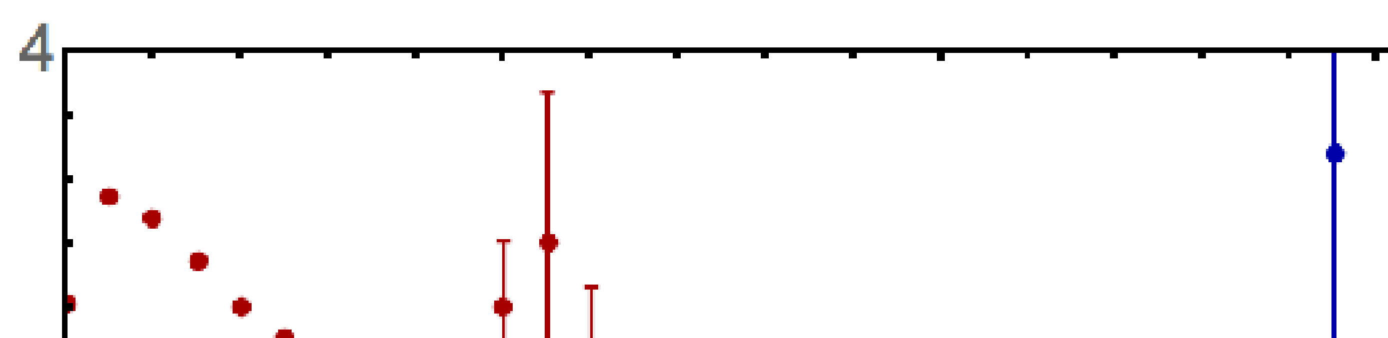
$$\sim e^{-3 m_\pi t}$$





$$\frac{G_N}{\sqrt{\mathrm{Var}(G_N)}} \sim e^{-\left(M_N - \frac{3}{2} m_\pi\right) t}$$

Multi-nucleon: $\sim e^{-A\left(M_N - \frac{3}{2} m_\pi\right) t}$



The sign(al-to-noise) problem

Nucleon correlation functions are real on average but complex in generic gauge field backgrounds

$$G_N(t) = \left\langle e^{R(t)+i\theta(t)} \right\rangle$$

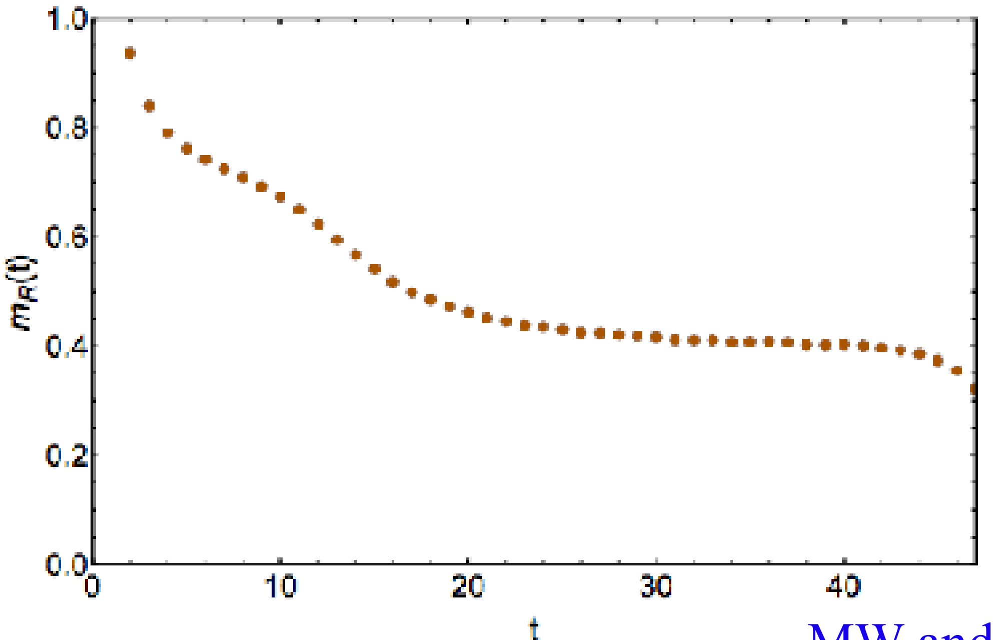
Contributions to the energy from phase fluctuations lead to StN problems

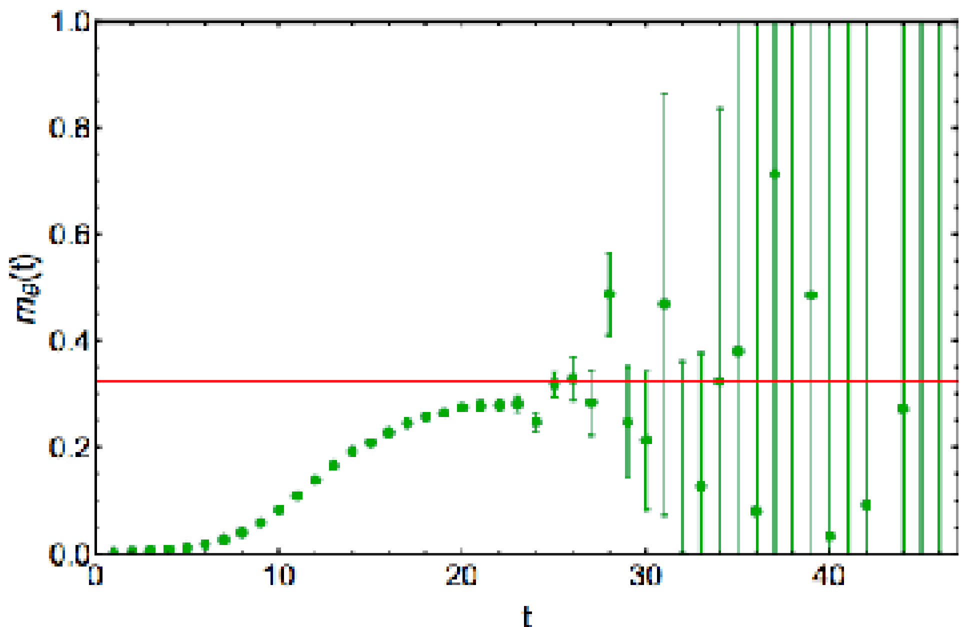
$$\mathrm{StN}(e^{i\theta(t)}) \sim \frac{\left\langle e^{i\theta(t)} \right\rangle}{\left\langle |e^{i\theta(t)}|^2 \right\rangle} = \left\langle e^{i\theta(t)} \right\rangle \sim e^{-m_\theta t}$$

Empirically, phase fluctuations ("sign problem") reproduce the Parisi-Lepage signal-to-noise problem

$$m_R(t) = \ln\left(\frac{\left\langle e^{R(t)} \right\rangle}{\left\langle e^{R(t+1)} \right\rangle}\right) \sim \frac{3}{2} m_\pi$$

$$m_\theta(t) = \ln\left(\frac{\left\langle e^{i\theta(t)} \right\rangle}{\left\langle e^{i\theta(t+1)} \right\rangle}\right) \sim M_N - \frac{3}{2} m_\pi$$





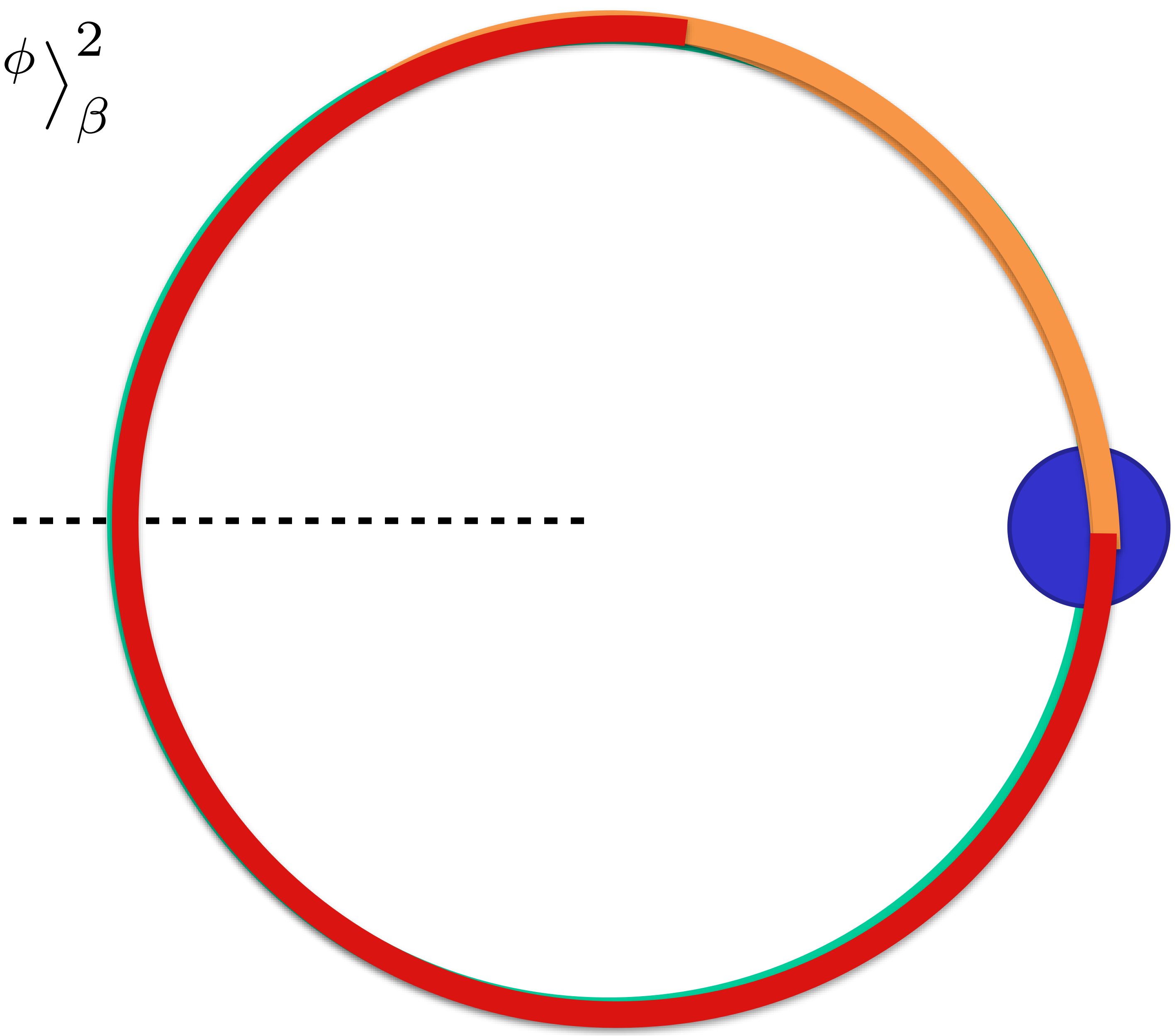
MW and Savage, PRD 96 (2017)

A nice toy sign problem

$$\int_{-\pi}^{\pi} \frac{d\phi}{2\pi}\, e^{i\phi}\, e^{\beta\cos(\phi)} = I_1(\beta) = \left\langle e^{i\phi}\right\rangle_\beta I_0(\beta)$$

$$\mathrm{Var}_\beta[\mathrm{Re}\; e^{i\phi}] = \left\langle \cos^2(\phi)\right\rangle_\beta - \left\langle e^{i\phi}\right\rangle_\beta^2$$

$$= \frac{1}{2}\left[1 + \frac{I_2(\beta)}{I_0(\beta)}\right] - \left[\frac{I_1(\beta)}{I_0(\beta)}\right]^2$$



For uniform phase fluctuations, average phase vanishes but variance is always O(1)

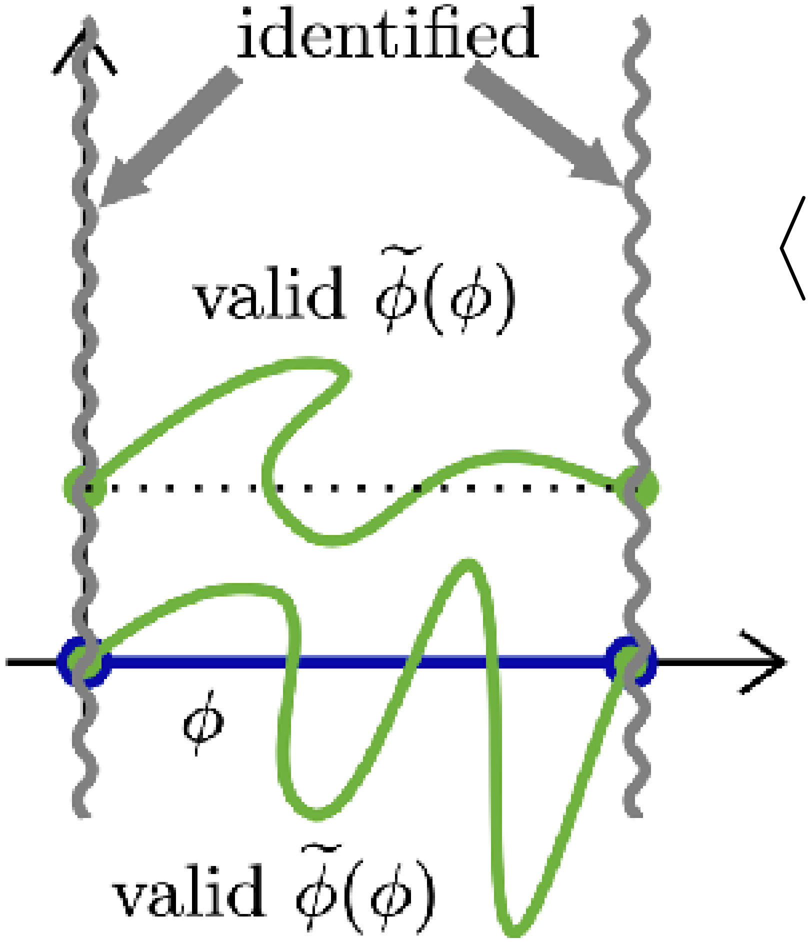
Detmold, Kanwar, MW, Warrington, PRD 102 (2020)

Detmold, Kanwar, Lamm, MW, Warrington, arXiv:2101.12668

Contour deformations

$$\int_{-\pi}^{\pi} \frac{d\phi}{2\pi}\, e^{i\phi}\, e^{\beta\cos(\phi)} = I_1(\beta) = \left\langle e^{i\phi}\right\rangle_\beta I_0(\beta)$$

Stokes' theorem + holomorphic integrand ⇒ integral result unaffected by contour deormation



Constant vertical deformation:

$$\left\langle e^{i\phi}\right\rangle_\beta = \int_{-\pi+if}^{\pi+if} \frac{d\phi}{2\pi I_0(\beta)}\, e^{i\phi}\, e^{\beta\cos(\phi)}$$

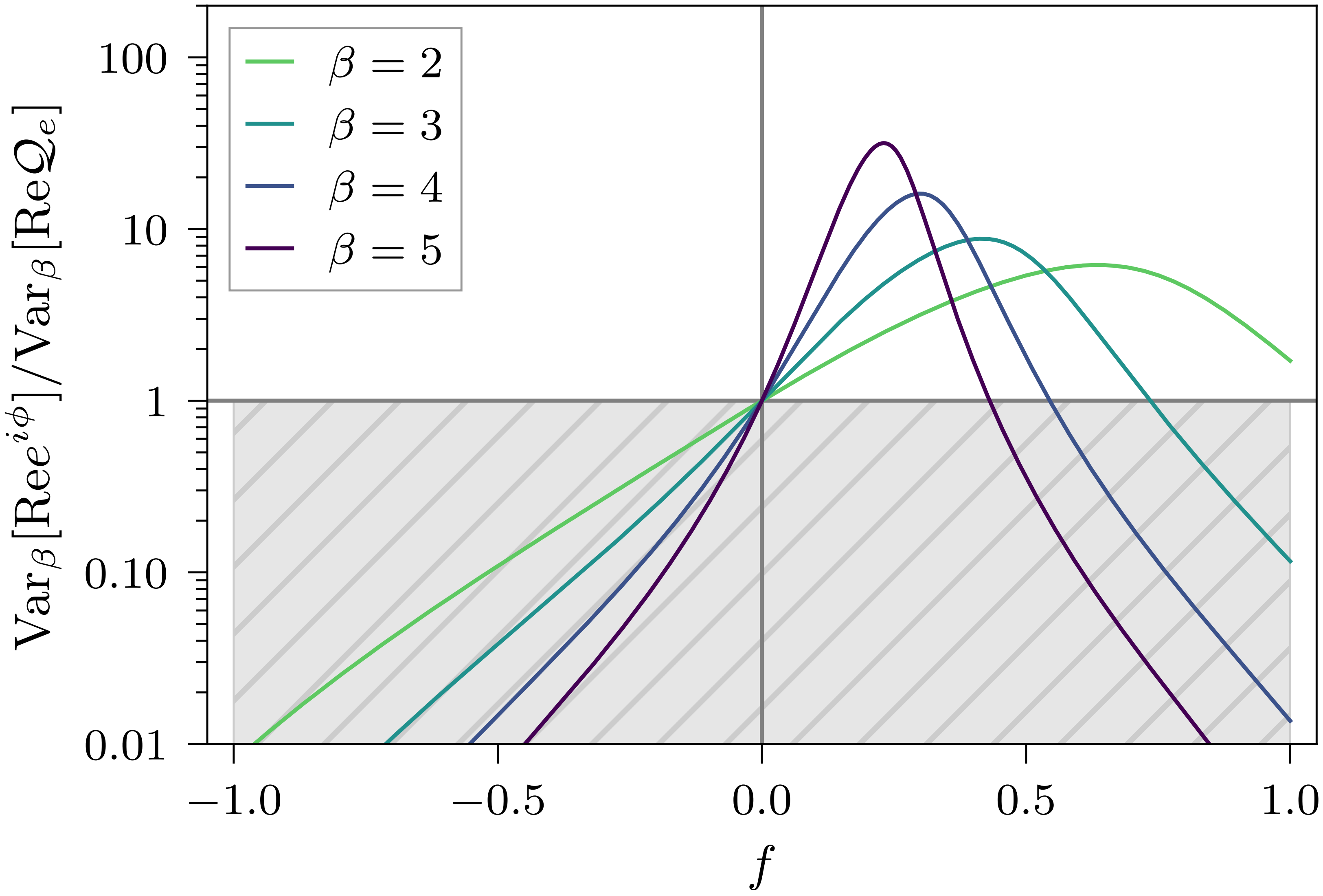
$$= \int_{-\pi}^{\pi} \frac{d\phi}{2\pi}\, e^{i\phi-f}\, e^{\beta\cos(\phi+if)}$$

$$= \left\langle e^{i\phi-f} e^{\beta\cos(\phi+if)-\beta\cos(\phi)}\right\rangle_\beta \equiv \left\langle \mathcal{Q}_e\right\rangle_\beta$$

Variance reduction

The variance involves non-holomorphic integrands

$$\mathrm{Var}_\beta[\mathrm{Re}\ \mathcal{Q}_e] = \left\langle (\mathrm{Re}\ \mathcal{Q}_e)^2 \right\rangle_\beta - \left\langle e^{i\phi} \right\rangle_\beta^2 \neq \mathrm{Var}_\beta[\mathrm{Re}\ e^{i\phi}]$$



$$\mathcal{Q}_e = e^{-f} e^{i\phi} e^{\Delta S}$$

Detmold, Kanwar, Lamm, MW, Warrington, arXiv:2101.12668

Path integral contour deformations

Consider a path integral with a sign problem

$$\langle \mathcal{O} \rangle_M = \frac{1}{Z_M} \int_{\mathcal{M}} \mathcal{D}U \; e^{iS_M(U)} \; \mathcal{O}(U)$$

Deform the integration contour

$$= \frac{1}{Z_M} \int_{\widetilde{\mathcal{M}}} \mathcal{D}U \; e^{iS_M(\widetilde{U})} \; \mathcal{O}(\widetilde{U})$$

$$= \frac{1}{Z_M} \int_{\mathcal{M}} \mathcal{D}U \; J(U) \; e^{iS_M(\widetilde{U}(U))} \; \mathcal{O}(\widetilde{U}(U))$$

Deformed integrand can have less severe sign problem

$$= \frac{1}{Z_M} \int_{\mathcal{M}} \mathcal{D}U \; |J(U)| \; e^{-\mathrm{Im}[S_M(\widetilde{U}(U))]} \; \mathcal{O}(\widetilde{U}(U))$$

$$\times e^{i\mathrm{Re}[S_M(\widetilde{U}(U))] + i\arg[J(U)]}$$

Witten, AMS/IP Stud.Adv.Math. 50 (2011)

Cristoforetti, Di Renzo, Scorzato, PRD 86 (2012)

…

Review:

Alexandru, Basar, Bedaque, Warrington, arXiv:2007.05436

Observifolds

Deformed observables method: contour deformations without modifying Monte Carlo sampling

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int_{\mathcal{M}} \mathcal{D}U \; e^{-S(U)} \; \mathcal{O}(U)$$

$$= \frac{1}{Z} \int_{\widetilde{\mathcal{M}}} \mathcal{D}\widetilde{U} \; e^{-S(\widetilde{U})} \; \mathcal{O}(\widetilde{U})$$

$$= \frac{1}{Z} \int_{\mathcal{M}} \mathcal{D}U \; e^{-S(U)} \; \det\left(\frac{\partial \widetilde{U}}{\partial U}\right) \; e^{-S(\widetilde{U}(U))+S(U)} \; \mathcal{O}(\widetilde{U}(U))$$

$$\equiv \frac{1}{Z} \int_{\mathcal{M}} \mathcal{D}U \; e^{-S(U)} \; \boxed{\mathcal{Q}(U)}$$

$$\langle \mathcal{O} \rangle = \langle \mathcal{Q} \rangle$$

$$\mathrm{Var}[\mathcal{O}] \neq \mathrm{Var}[\mathcal{Q}]$$

Detmold, Kanwar, MW, Warrington, PRD 102 (2020)

Lattice gauge theory

Gauge transformations act on matter fields as

$$\psi_x^a \to \Omega_x^{ab}\psi_x^b$$

$$\Omega_x \in SU(N),\ U(1)$$

Gauge field acts as parallel transporter in color space

$$U_{x,\mu} \in SU(N),\ U(1)$$

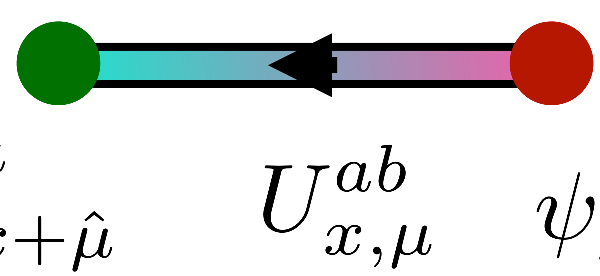
$$U_{x,\mu} = e^{i\int_x^{x+\hat{\mu}} dy A_\mu(y)}$$

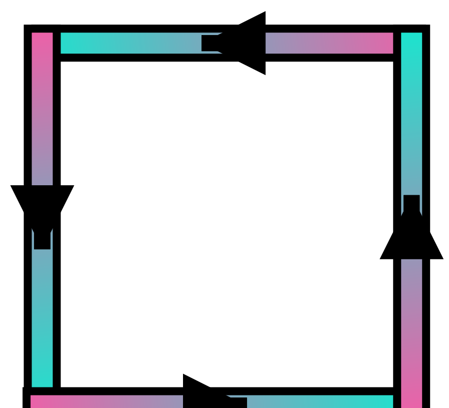
$$U_{x,\mu} \to \Omega_x U_{x,\mu} \Omega_{x+\hat{\mu}}^\dagger$$

$$D_\mu^{ab}\psi_x^b = U_{x,\mu}^{ab}\psi_{x+\hat{\mu}}^b - \psi_x^a$$

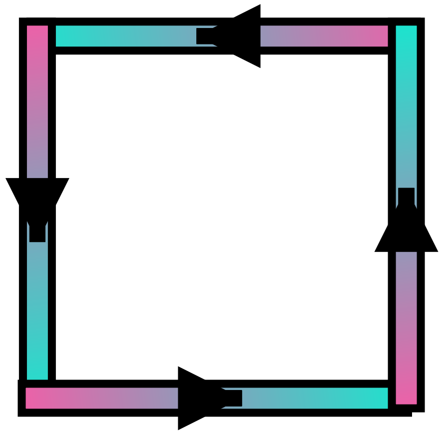
$$D_\mu\psi_x^a \to \Omega_x^{ab} D_\mu \psi_x^b$$

Gauge invariant building blocks:

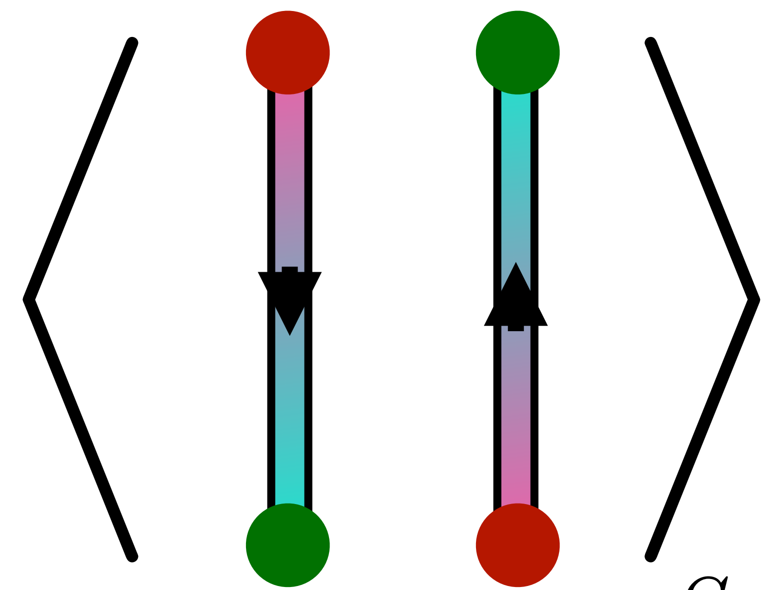


Tr ()

Wilson loops



$$W_{\mathcal{A}} = \prod_{x,\mu \in \partial\mathcal{A}} U_{x,\mu} \quad =$$



$S_{\psi,\mathrm{static}}$

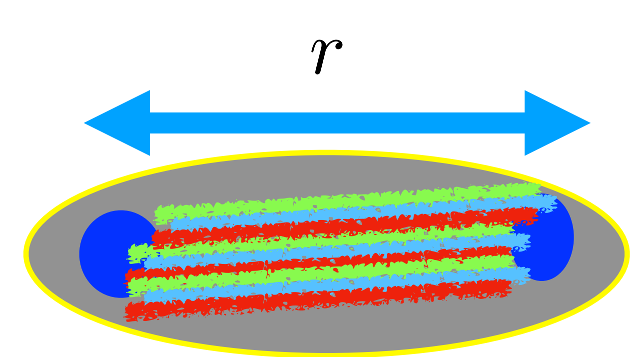
Wilson loops are equivalent to static quark propagators

$$S_{\psi,\mathrm{static}} = \sum_x \overline{\psi}_x D_4 \psi_x = \sum_x \overline{\psi}_x \left(U_{x,4} \psi_{x+\hat{4}} - \psi_x \right)$$

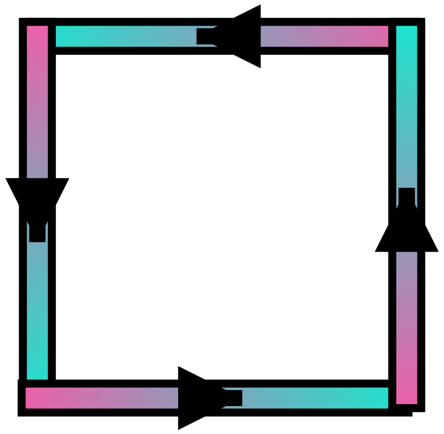
Since by equations of motion $\psi_{(\vec{x},\tau)} = \prod_{\tau'=0}^{\tau} U^{-1}_{(\vec{x},\tau'),4} \psi_{(\vec{x},0)}$

Static quark potential accessible from Wilson loops

$$\langle W_{r\times\tau} \rangle = \sum_n Z_n \, e^{-E_n(r)\tau} = e^{-V(r)\tau} + \ldots$$



Plaquettes



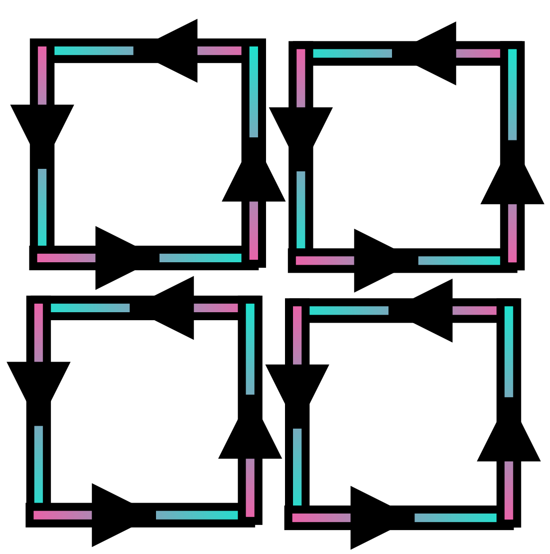
“Plaquettes” are 1x1 Wilson loops

$$P_{x,\mu\nu} = U_{x,\mu} U_{x+\hat{\mu},\nu} U^{-1}_{x+\hat{\mu}+\widehat{nu},\mu} U^{-1}_{x+\hat{\nu},\nu}$$

Wilson action provides simple, gauge-invariant action with correct naive continuum limit

$$S_W(U) = \frac{1}{g^2} \sum_x \sum_{\mu<\nu} \mathrm{Tr}\left[2 - P_{x,\mu\nu} - P^{-1}_{x,\mu\nu}\right]$$

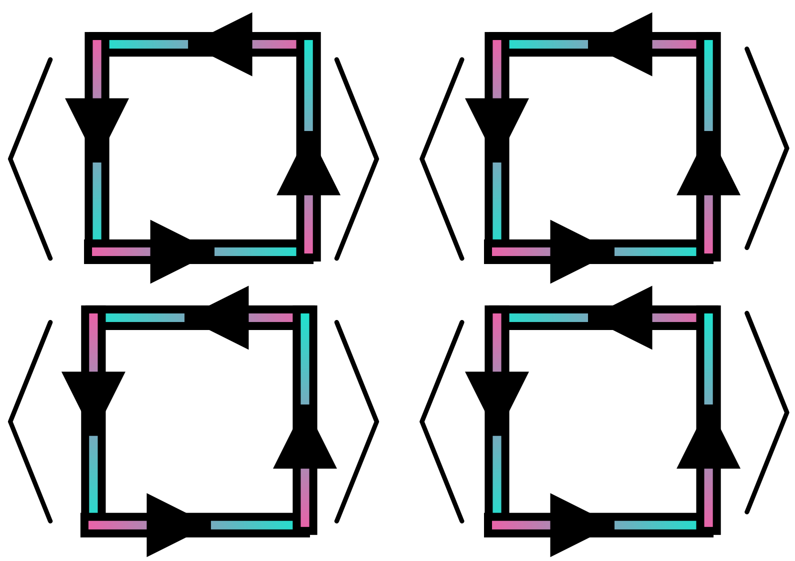
Wilson loops can be expressed using plaquettes, most simply taking open boundary conditions and gauge-fixing $U_{x,\nu} = 1$



$$W_{\mathcal{A}} = \prod_{x,\mu \in \partial\mathcal{A}} U_{x,\mu} = \prod_{x \in \mathcal{A}} P_{x,\mu\nu}$$

2D Confinement

In 2D, Wilson loop expectation values further factorize into products of single-plaquette expectation values



$$\langle \mathrm{Tr}(W_{\mathcal{A}}) \rangle = \prod_{x \in \mathcal{A}} \langle \mathrm{Tr}(P_x) \rangle = \langle \mathrm{Tr}(P) \rangle^{A}$$

Implies confinement, static quark potential

$$V(r) = \sigma r$$

String tension: $$\sigma = -\ln \left\langle \frac{1}{N} \mathrm{Tr}(P) \right\rangle$$

Confining potential arises for any gauge group in 2D from factorization

$$\sigma_{U(1)} = \ln \left(\frac{I_0(1/e^2)}{I_1(1/e^2)} \right)$$

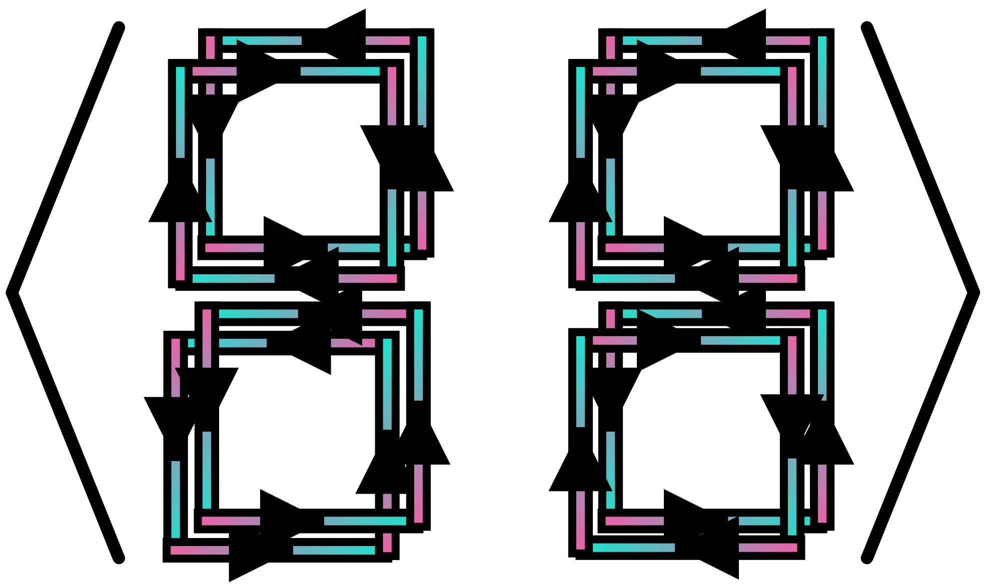
$$\sigma_{SU(2)} = \ln \left(\frac{I_1(4/g^2)}{I_2(4/g^2)} \right)$$

Gross and Witten, PRD 21 (1980)

Wadia, arXiv:1212.2906 (1979)

2D Wilson loop noise

Squared traces also factorize, but differently



$$\left\langle \left| \mathrm{Tr}(W_{\mathcal{A}})^2 \right| \right\rangle = 1 + (N^2 - 1) \left\langle \chi_{1,-1} \right\rangle^A$$

$$\left\langle \mathrm{Tr}(W_{\mathcal{A}})^2 \right\rangle = \frac{N(N+1)}{2} \left\langle \chi_2 \right\rangle^A + \frac{N(N-1)}{2} \left\langle \chi_{1,-1} \right\rangle^A$$

Detmold, Kanwar, Lamm, MW, Warrington, arXiv:2101.12668

Phase (sign in N=2) fluctuations of Wilson loop traces lead to exponential StN problem

$$\mathrm{Var}\left[\frac{1}{N} \mathrm{Re}\ \mathrm{Tr}(W_{\mathcal{A}}) \right] = \frac{1}{2N^2} + \dots$$

$$\mathrm{StN}\left[\frac{1}{N} \mathrm{Re}\ \mathrm{Tr}(W_{\mathcal{A}}) \right] = \sqrt{2} N n e^{-\sigma A} + \dots$$

2D U(1) contour deformations

Using the parameterization

$$P = e^{i\phi} \in U(1)$$

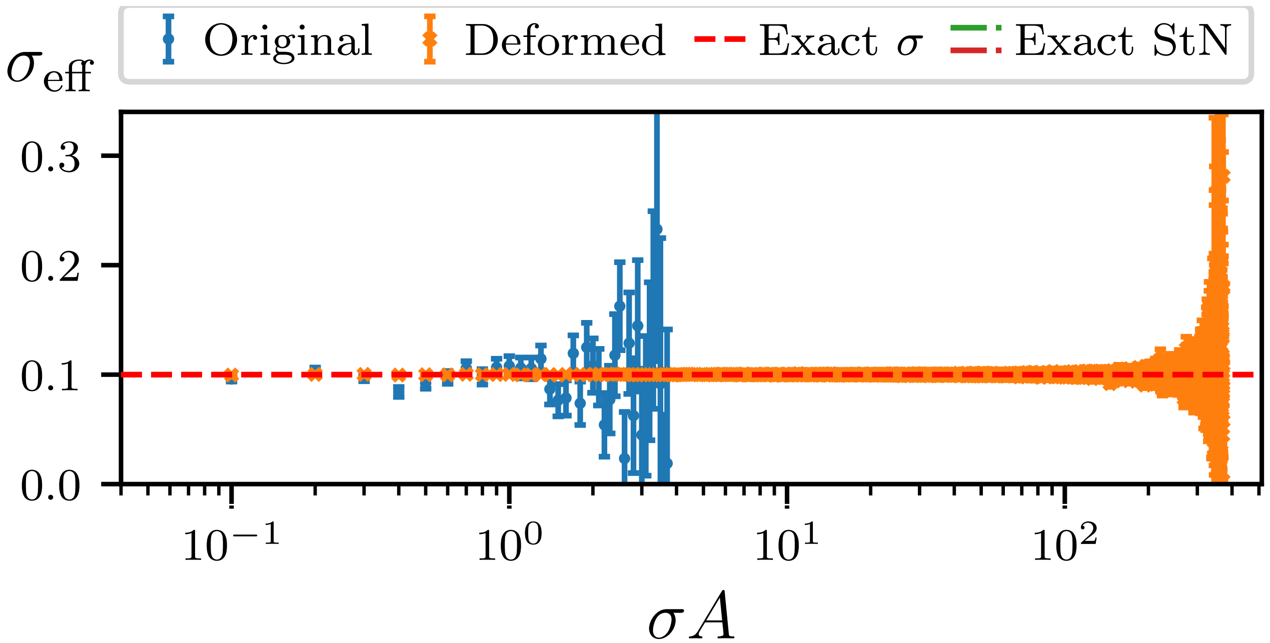
$$\langle W_{\mathcal{A}} \rangle = \left(\int \frac{dP}{2\pi I_0(1/e^2)} \; P \; e^{\frac{1}{2e^2}(P+P^{-1})} \right)^A$$

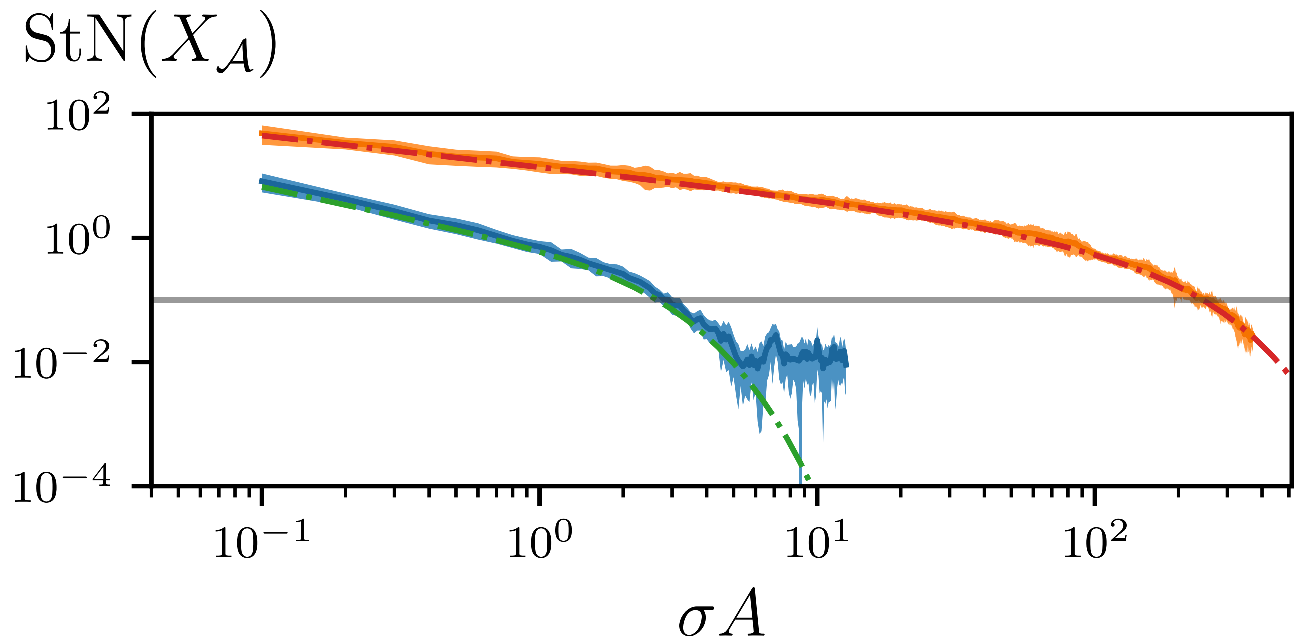
U(1) Wilson loops are products of toy sign problem integrals

$$= \left(\int_{-\pi}^{\pi} \frac{d\phi}{2\pi I_0(1/e^2)} \; e^{i\phi} \; e^{\frac{1}{e^2}\cos(\phi)} \right)^A$$

Contour deformation analogous to toy problem for U(1) Wilson loops

$$e^{i\phi} \to e^{i(\phi + if)}$$





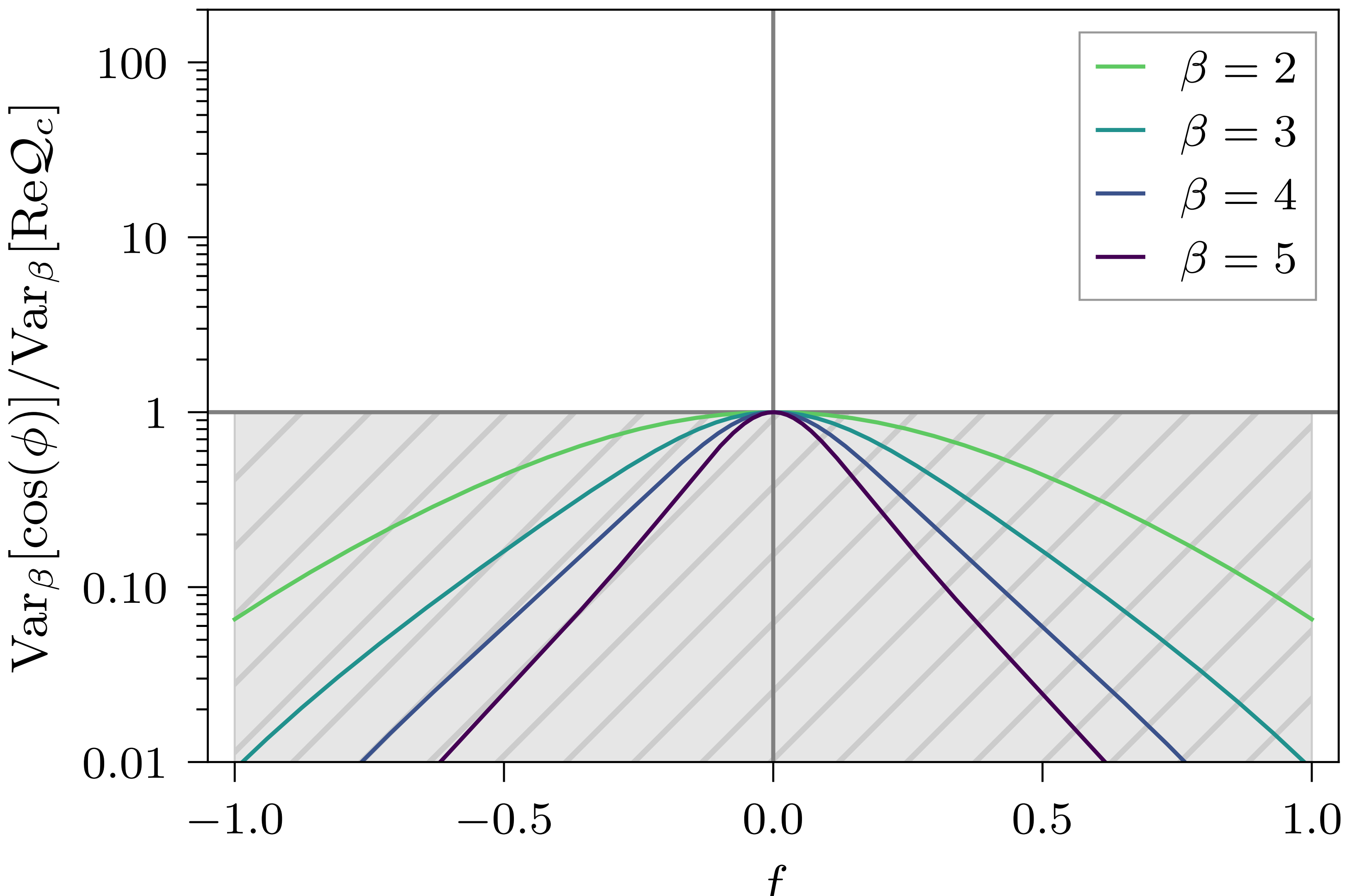
Detmold, Kanwar, MW, Warrington, PRD 102 (2020)

A mean toy sign problem

An alternative expression of the same expectation value

$$\langle \cos(\phi) \rangle_\beta = \int_{-\pi}^{\pi} \frac{d\phi}{2\pi I_0(\beta)} \; \cos(\phi)\; e^{\beta \cos(\phi)} = \frac{I_1(\beta)}{I_0(\beta)} = \left\langle e^{i\phi} \right\rangle_\beta$$

has identical variance but different contour deformation possibilities



$$\mathcal{Q}_c = \cos(\phi + if)e^{\Delta S}$$

Detmold, Kanwar, Lamm, MW, Warrington, arXiv:2101.12668

Parameterizing SU(2)

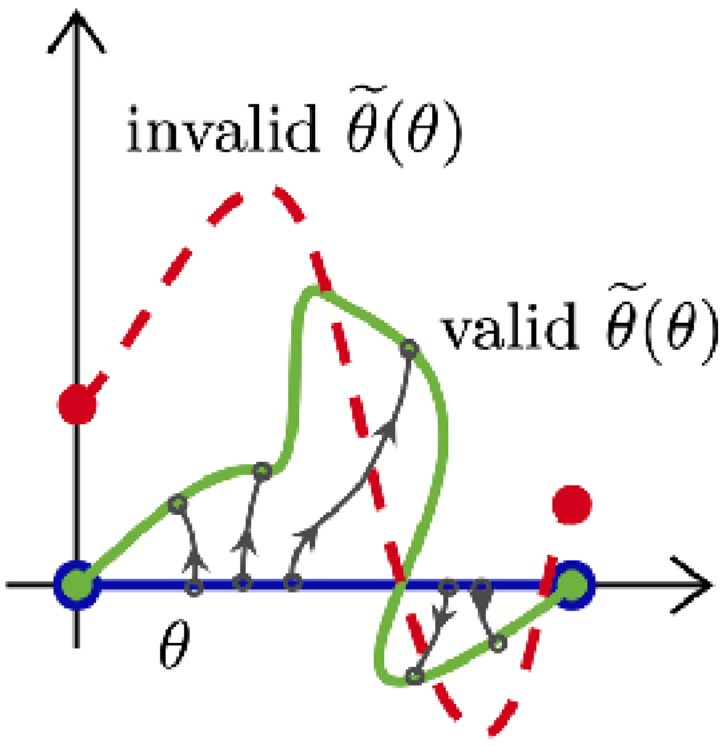
Mean toy sign problem — SU(2) Wilson loops parameterized as

$$P = \cos(\alpha/2) + i\sin(\alpha/2)\hat{n}\cdot\sigma$$

$$\frac{1}{2}\mathrm{Tr}(P) = \cos(\alpha/2)$$

Useful to instead use angular parameterization

$$P = \begin{pmatrix} \sin(\theta)e^{i\phi_1} & \cos(\theta)e^{i\phi_2} \\ -\cos(\theta)e^{-i\phi_2} & \sin(\theta)e^{i\phi_1} \end{pmatrix}$$



Nice toy sign problem — (1,1) component only

$$\left\langle \frac{1}{2}\mathrm{Tr}(P)\right\rangle = \langle P_{11}\rangle = \left\langle \sin(\widetilde{\theta})e^{i\widetilde{\phi}}e^{\Delta S}J(\theta,\phi)\right\rangle$$

Detmold, Kanwar, Lamm, MW, Warrington, arXiv:2101.12668

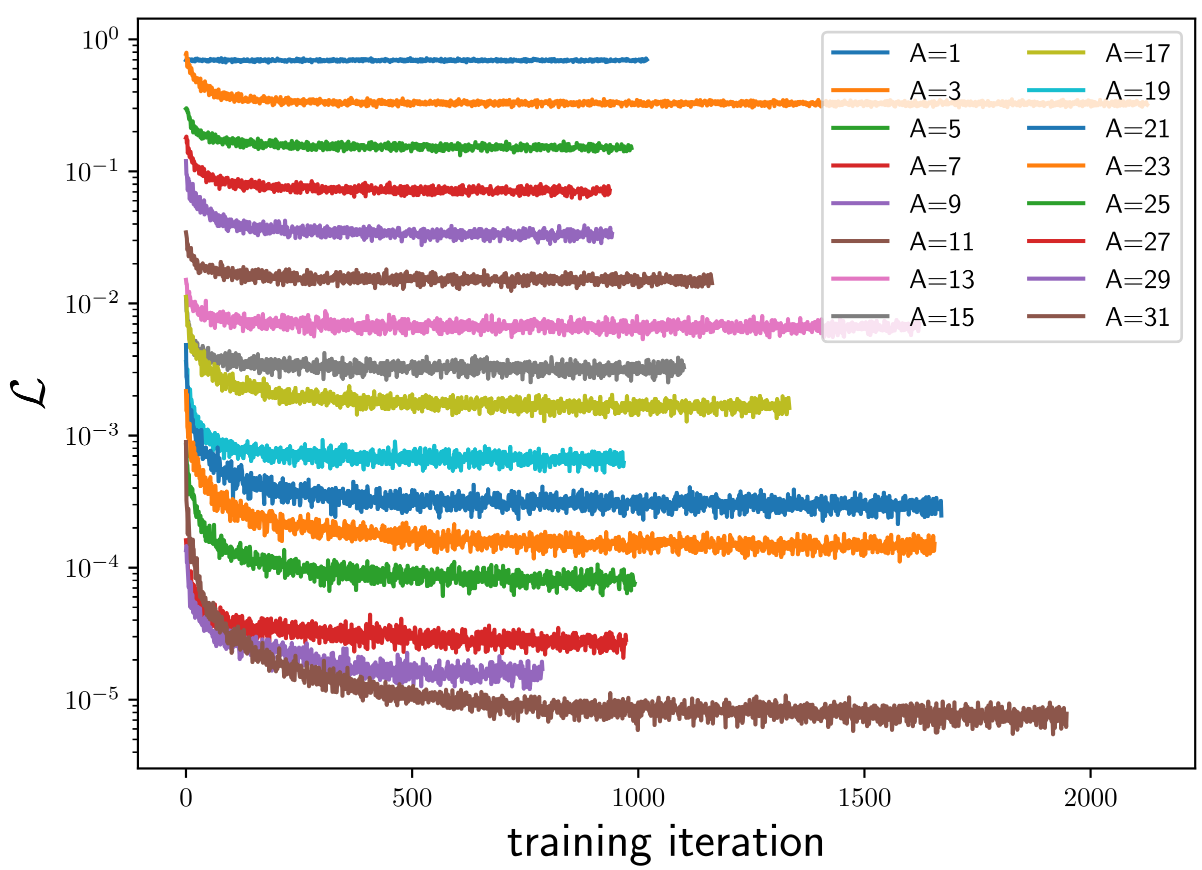
Parameterizing SU(2) contours

More general manifolds can be parameterized using gauge-field-dependent vertical deformations (ordered in spacetime)

$$\widetilde{\Omega} = \Omega + i \sum_{\{n_i\}=0}^{\Lambda} \sum_{\{m_j\}=1}^{\Lambda} \lambda_{n_1 \ldots m_K} \prod_{i=1}^{J} \sin(\phi_i n_i + \chi^i) \prod_{j=1}^{K} \sin(2\theta_j m_j)$$

Variance minimization is a well-posed optimization problem suitable for machine learning techniques

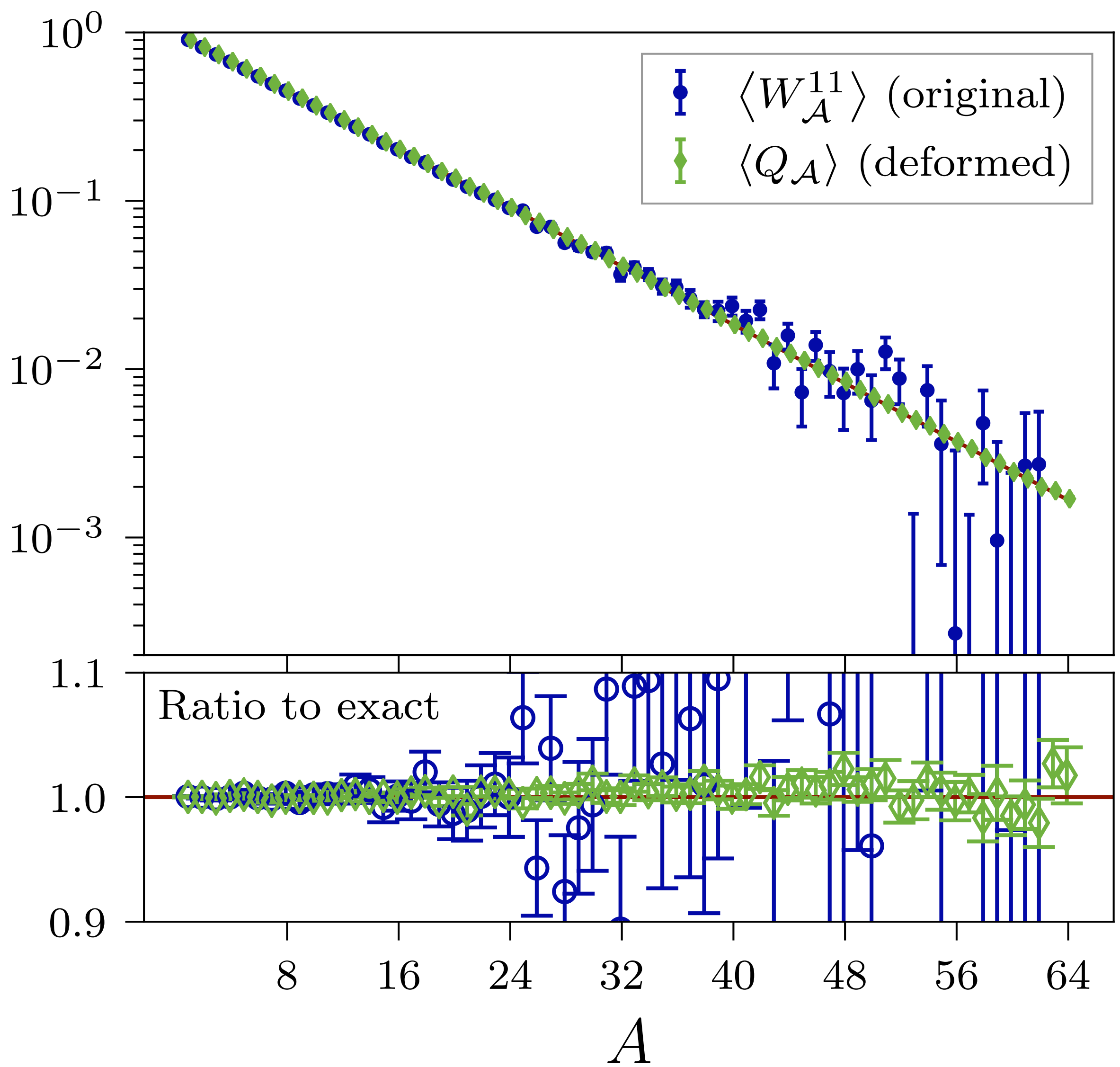
Overtraining on sample variance avoided using resampling (mini-batches) and using a separate test set for train rate scheduling

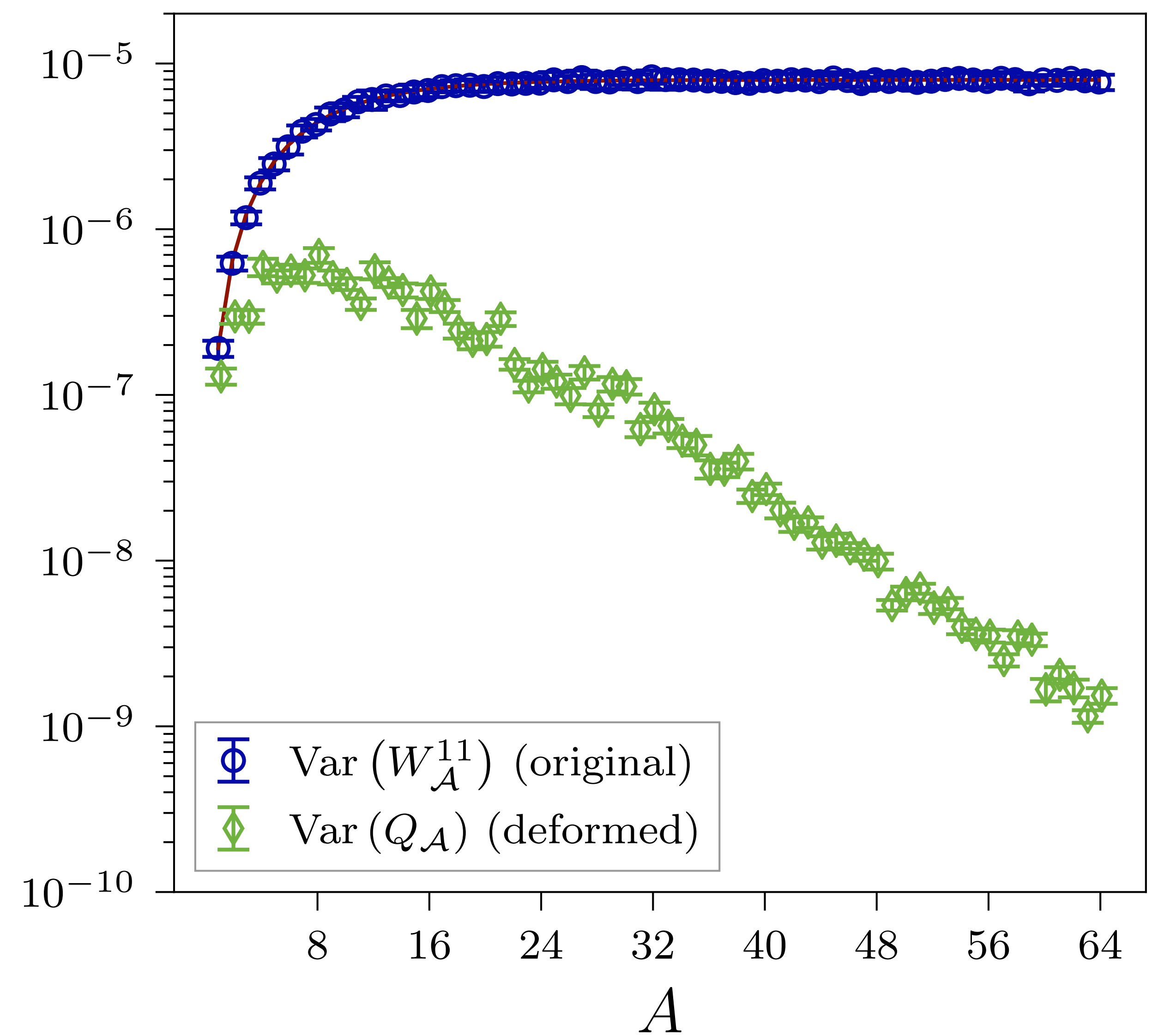


Detmold, Kanwar, Lamm, MW, Warrington, arXiv:2101.12668

SU(2) deformation results

Optimized manifolds lead to exponentially increasing variance reduction for large area Wilson loops



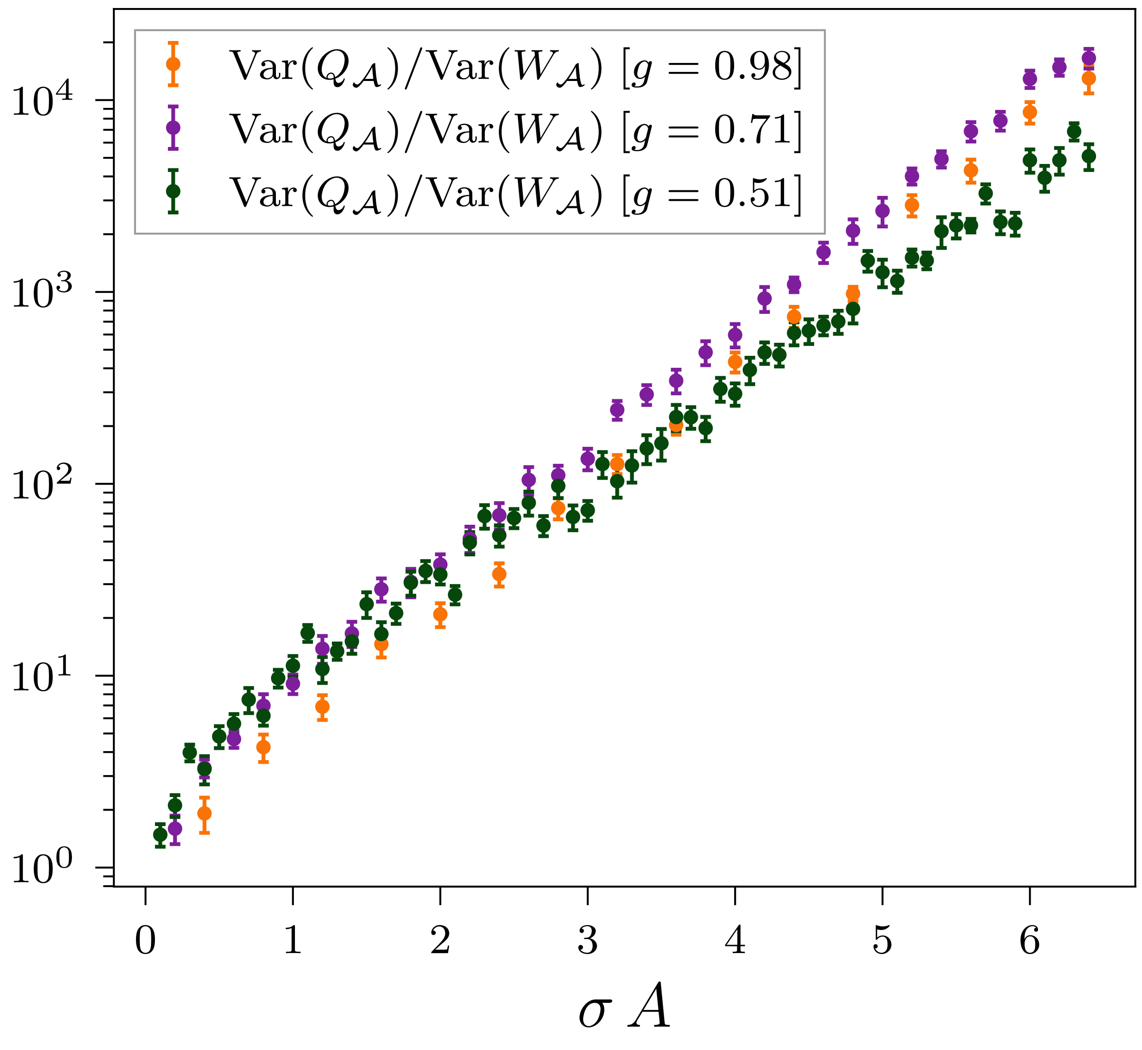


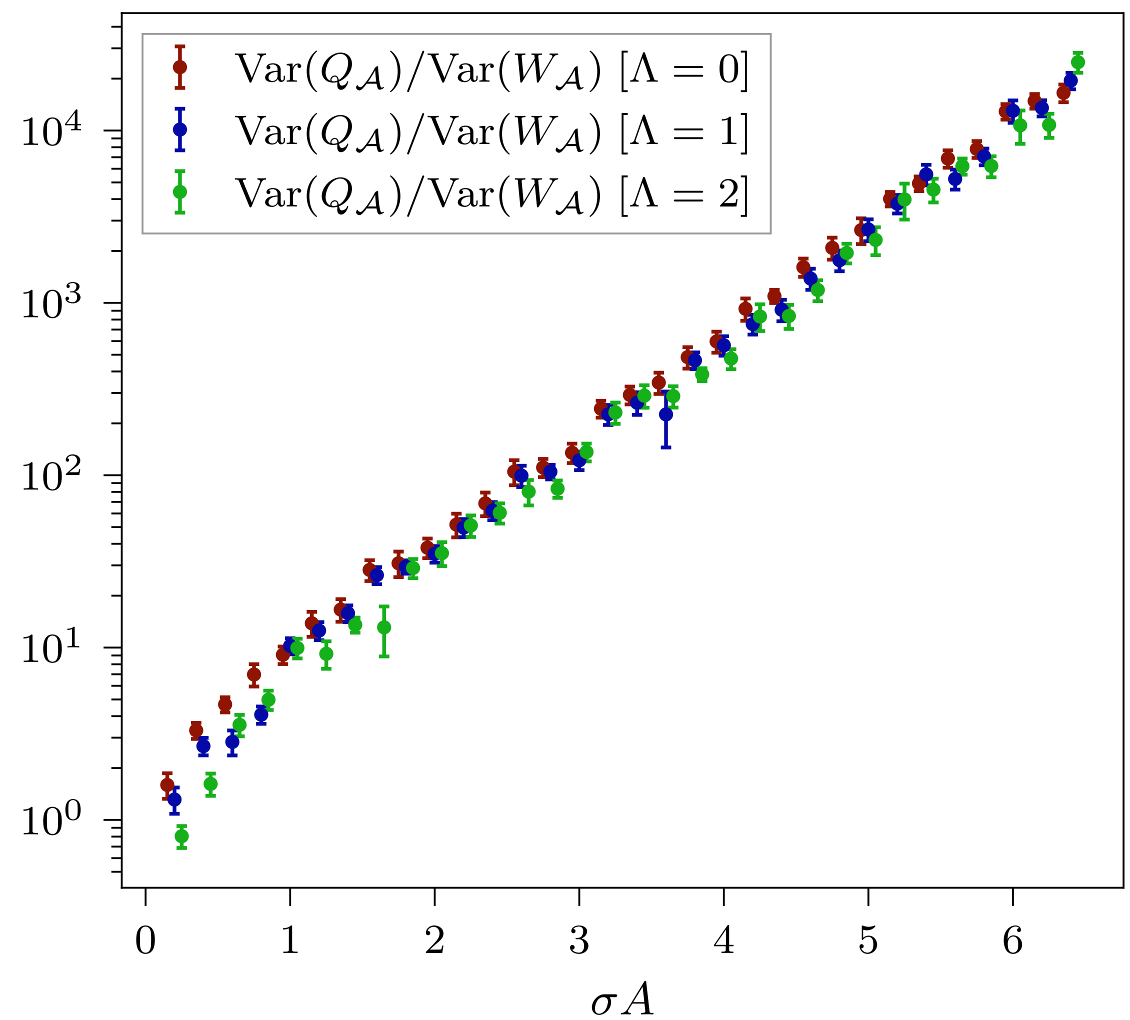
Detmold, Kanwar, Lamm, MW, Warrington, arXiv:2101.12668

SU(2) deformation results

Variance reduction scales in physical units as gauge coupling is decreased towards continuum limit

Gains predominantly coming from zeroth order Fourier cutoff (constant vertical deformations)

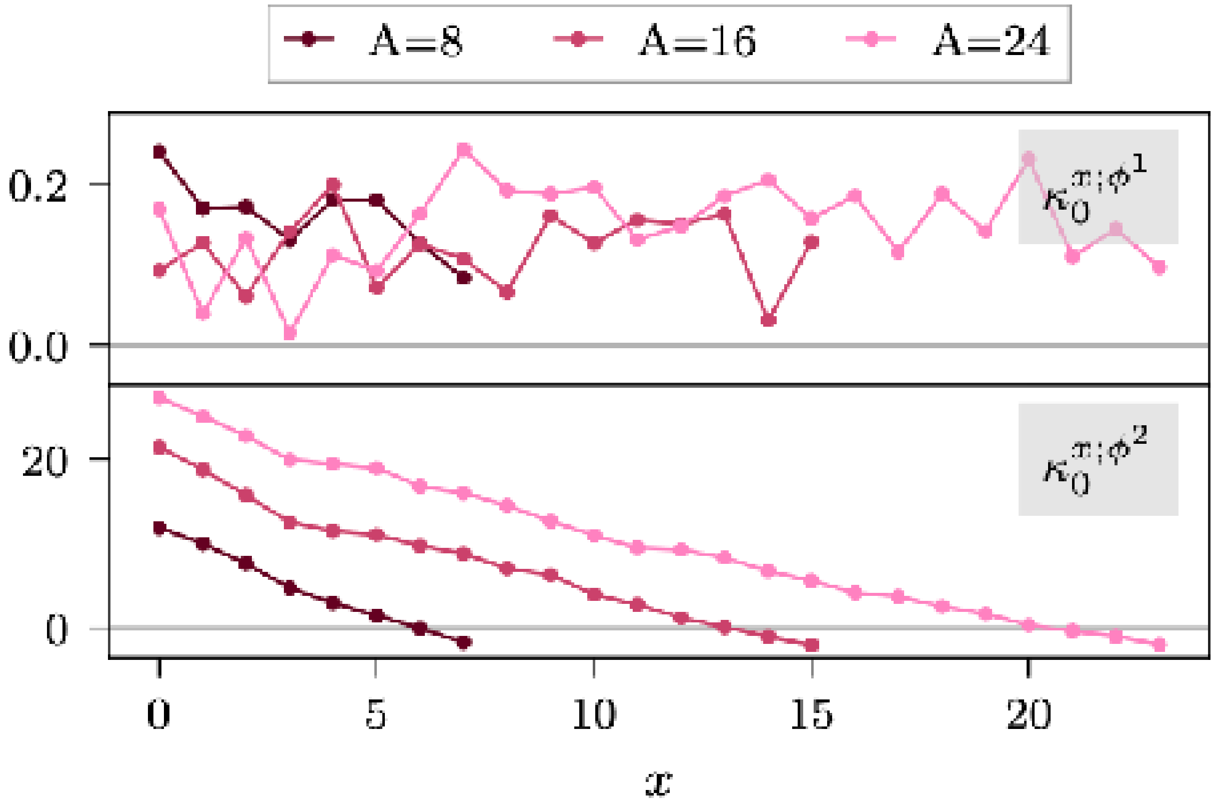




Detmold, Kanwar, Lamm, MW, Warrington, arXiv:2101.12668

SU(2) deformation results

Optimal manifold resembles a constant magnitude deformation of diagonal phase throughout Wilson loop, linearly decreasing deformation of off-diagonal phase



Qualitative features follow from minimizing magnitude of product of two plaquettes

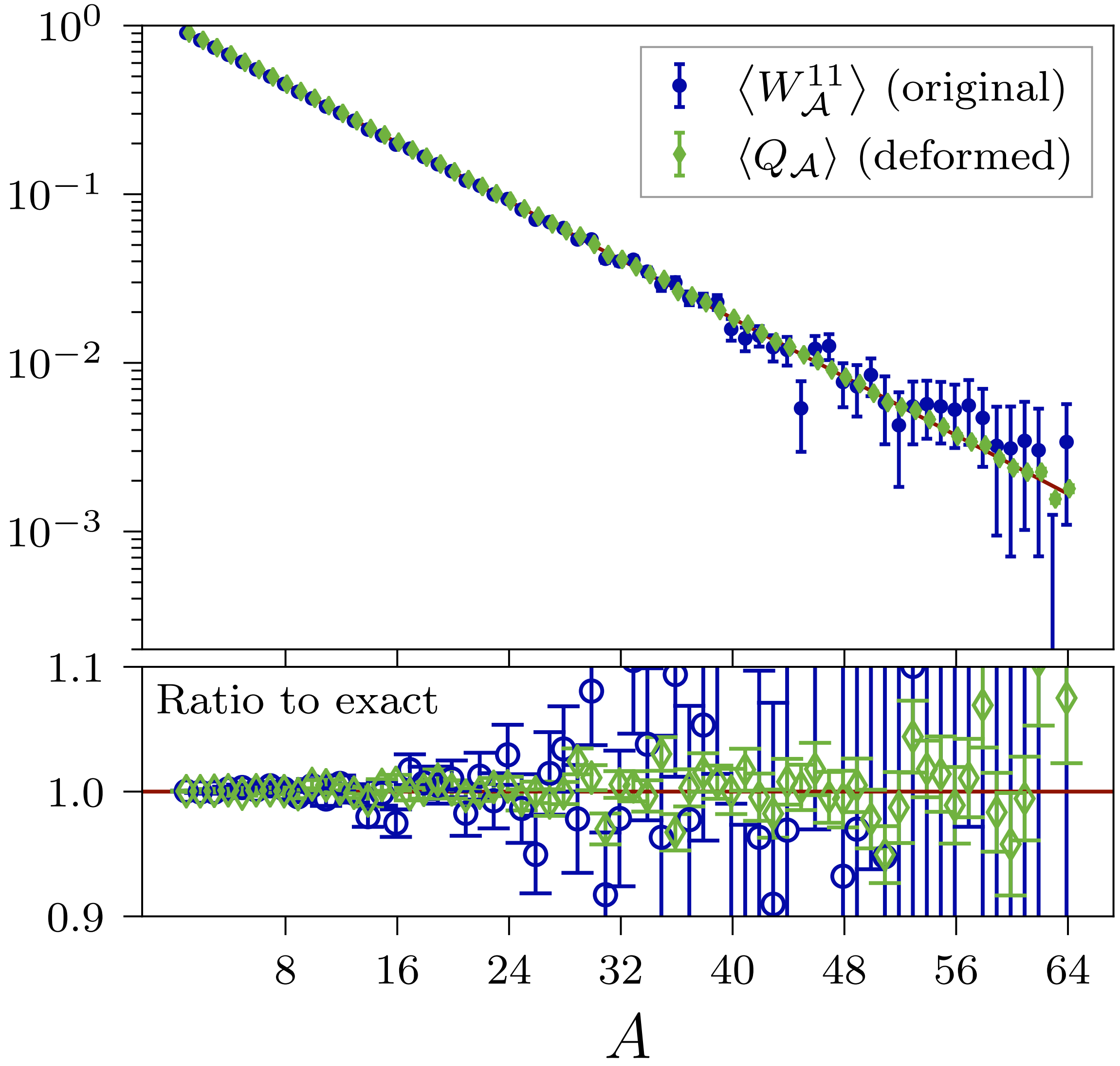
$$(P_x P_{x'})^{11} = \sin\theta_x \sin\theta_{x'} e^{i\phi_x^1 + i\phi_{x'}^1} + \cos\theta_x \cos\theta_{x'} e^{i\phi_x^2 - i\phi_{x'}^2}$$

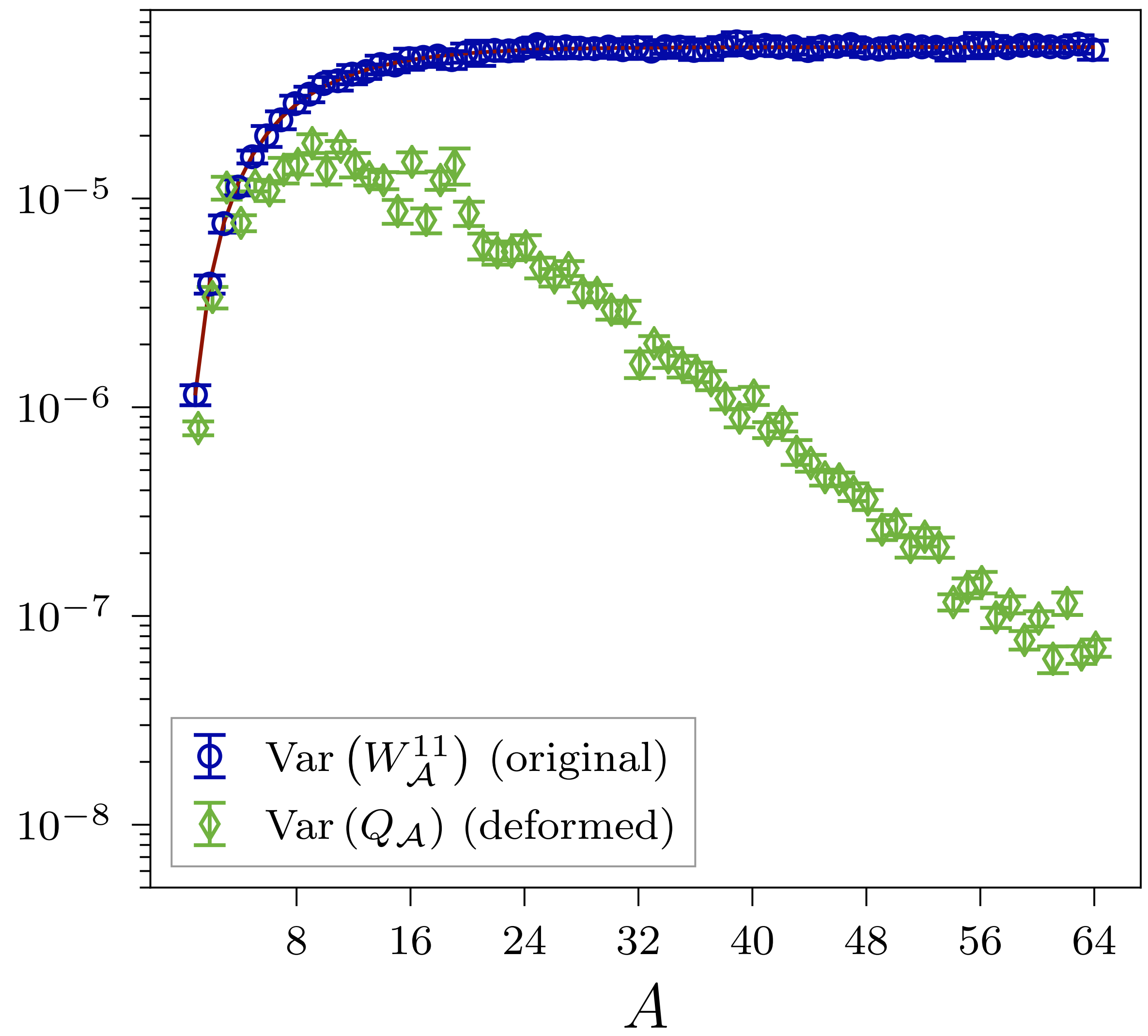
Detmold, Kanwar, Lamm, MW, Warrington, arXiv:2101.12668

SU(3) contour deformations

Analogous angular parameterization of SU(3) matrices

$$\phi_x^1, \ldots, \phi_x^5, \theta_x^1, \ldots, \theta_x^3$$



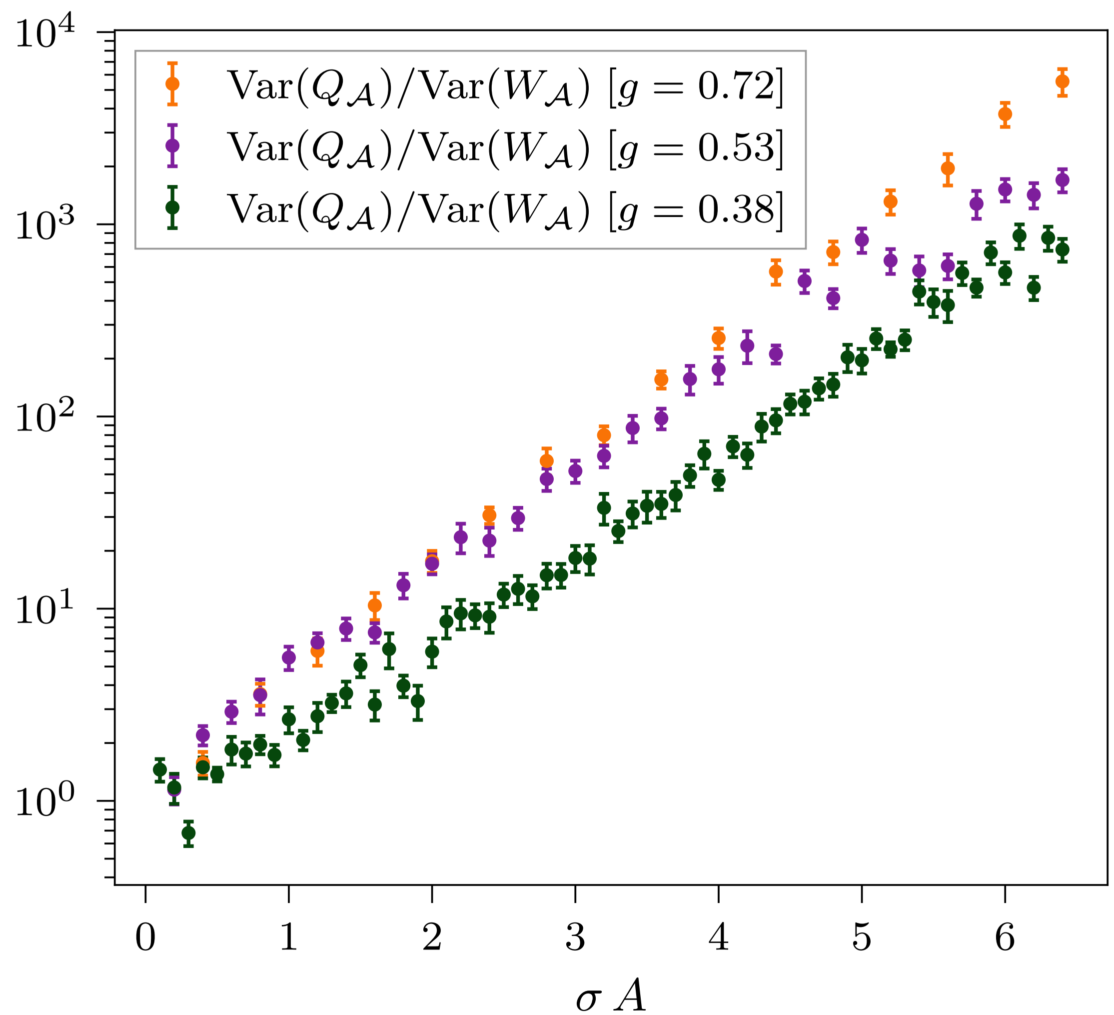


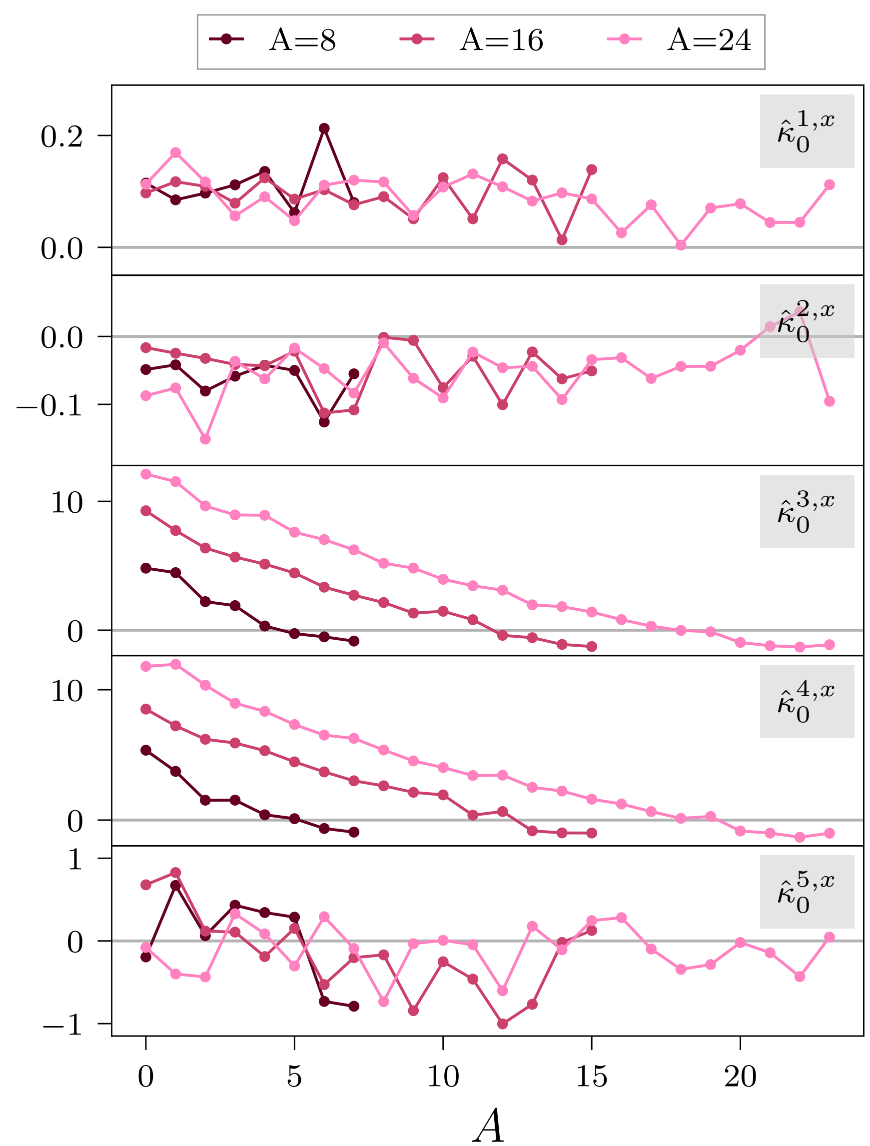
Detmold, Kanwar, Lamm, MW, Warrington, arXiv:2101.12668

SU(3) deformation results

Variance reduction roughly scales in physical units as gauge coupling is decreased towards continuum limit (training?)

Gains predominantly coming from zeroth order Fourier cutoff, qualitatively consistent with loop magnitude minimization





Detmold, Kanwar, Lamm, MW, Warrington, arXiv:2101.12668

Sign problem time

Real-time evolution faces a notorious sign problem

$$\langle \mathcal{O} \rangle_M = \frac{1}{Z_M} \int \mathcal{D}U \; e^{iS_M(U)} \; \mathcal{O}(U)$$

Can trade sign for StN problem (compact gauge group) $p(U)dU = dU$

$$\langle \mathcal{O} \rangle_M = \frac{\int \mathcal{D}U p(U) \; \mathcal{O}(U) \; e^{iS_M(U)}}{\int \mathcal{D}U p(U) \; e^{iS_M(U)}}$$

Still a problem, e.g. 2D U(1) $\mathrm{StN}(e^{iS_{M,U(1)}}) \approx \sqrt{n}\; e^{-2.3Vt}$

Improve with contour deformations $p(U)dU = e^{-\mathrm{Im}[S_M(\widetilde{U}(U))]}|J(U)|dU$

$$\langle \mathcal{O} \rangle_M = \frac{\int \mathcal{D}U p(U) \; \mathcal{O}(U) \; e^{i\mathrm{Re}[S_M(\widetilde{U}(U))] + i\arg[J(U)]}}{\int \mathcal{D}U p(U) \; e^{i\mathrm{Re}[S_M(\widetilde{U}(U))] + i\arg[J(U)]}}$$

Real-time scalar field theory: Alexandru, Basar, Bedaque, Vartak, and Warrington, PRL 117 (2016)

Real-time lattice gauge theory

A more basic question: what's the action?

$$S_{E,W}(U) = \frac{2}{g^2} \sum_x \sum_{\mu<\nu} \mathrm{Tr}(2 - P_{x,\mu\nu} - P_{x,\mu\nu}^{-1})$$

Identifying time-space (space-space) plaquette terms as kinetic and potential energies and performing usual Wick rotation gives

$$S_{M,W}(U) = \frac{2}{g^2} \sum_x \sum_k \mathrm{Tr}(2 - P_{x,0k} - P_{x,0k}^{-1})$$

$$- \frac{2}{g^2} \sum_x \sum_{i<j} \mathrm{Tr}(2 - P_{x,ij} - P_{x,ij}^{-1})$$

(1+1)D Minkowski Wilson loops

Wick rotate action

$$g^2 \to i g^2$$

$$\left\langle \frac{1}{2}\mathrm{Tr}(W_{\mathcal{A}}) \right\rangle_{M,W,SU(2)} = \left[\frac{I_1(4i/g^2)}{I_2(4i/g^2)} \right]^{-Lt}$$

Non-unitary time evolution

No continuum limit

$$\left\langle \frac{1}{2}\mathrm{Tr}(W_{\mathcal{A}}) \right\rangle_{E,W,SU(2)} = \left[\frac{I_1(4/g^2)}{I_2(4/g^2)} \right]^{-L\tau}$$

Analytically continue time

$$\tau \to it$$

$$\left\langle \frac{1}{2}\mathrm{Tr}(W_{\mathcal{A}}) \right\rangle_{M,HFK,SU(2)} = \left[\frac{I_1(4/g^2)}{I_2(4/g^2)} \right]^{-iLt}$$

The HFK action

A unitarity time-evolution operator exists for the Wilson action

$$\hat{T} = e^{-\hat{H}} \qquad\qquad \hat{U} = e^{-i\hat{H}} = \hat{T}^i$$

However, $\hat{U}$ is not the lattice time-evolution operator

An alternative action proposed by Hoshina, Fujii, and Kikukawa using character expansion leads to a unitary lattice time-evolution operator

$$e^{iS_{M,HKF}(U)} = e^{-\frac{i}{e^2}\sum_x \sum_{i<j}(1-\cos(\phi_{x,ij}))} \prod_{x,k} \left[\sum_{r=-\infty}^{\infty} [I_r(-i/e^2)]^i e^{i/e^2} e^{ir\phi_{x,0k}} \right]$$

$$P_{x,\mu\nu} = e^{i\phi_{x,\mu\nu}}$$

Hoshina, Fujii, Kikukawa, PoS LATTICE2019, 190 (2020)

The heat-kernel action

An alternative Euclidean action starts from "heat-kernel" equation

$$\partial_\tau \mathcal{K}_E(U,\tau) = \Delta \mathcal{K}_E(U,\tau)$$

Laplace-Beltrami operator for gauge group

Solution for U(1): $\mathcal{K}_{E,U(1)}(e^{i\phi}, -e^2) = \sum_{n=-\infty}^{\infty} \exp\left[-\frac{1}{2e^2}(\phi + 2\pi n)^2\right]$

Generalization to SU(N) gives heat-kernel action

Analytic continuation gives Schrödinger equation

$$i\partial_t \mathcal{K}_M(U,t) = -\Delta \mathcal{K}_M(U,t)$$

Solution for U(1): $\mathcal{K}_{M,U(1)}(e^{i\phi}, -e^2) = \sum_{n=-\infty}^{\infty} \exp\left[\frac{i}{2e^2}(\phi + 2\pi n)^2\right]$

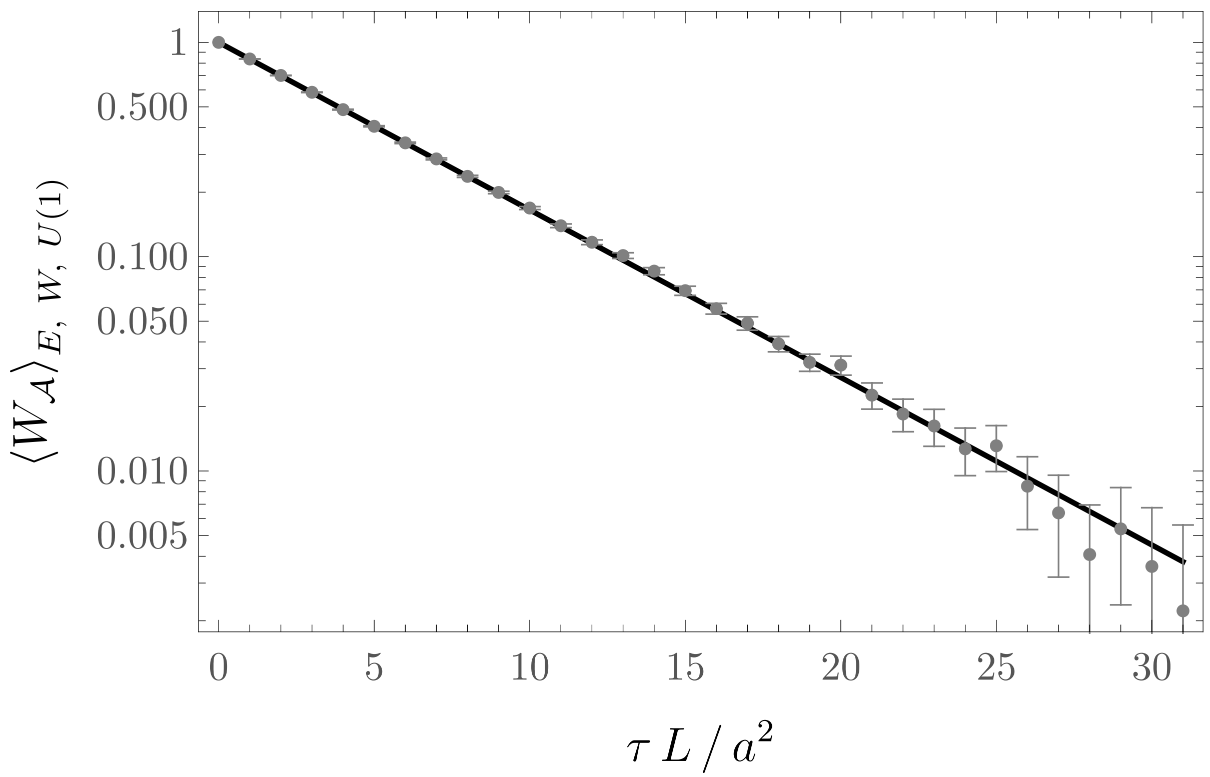
Kanwar and MW, arXiv:2102.xx

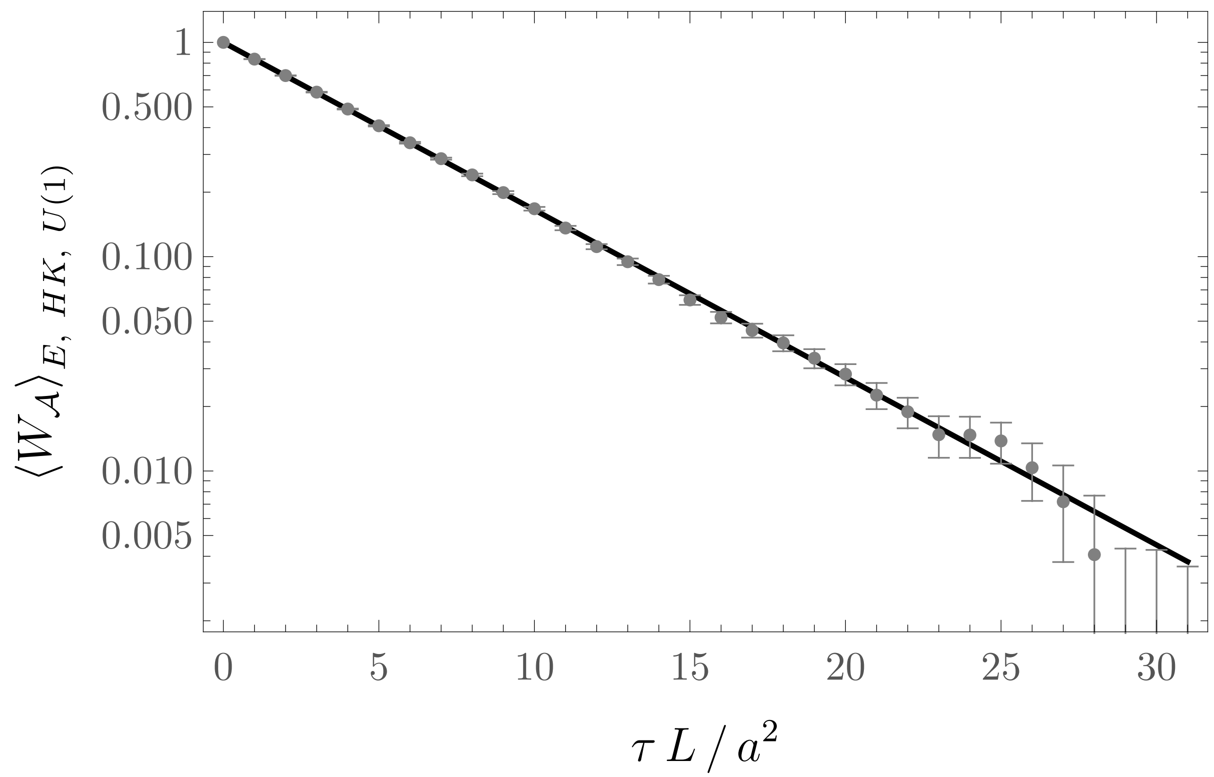
Euclidean U(1) heat-kernel results

Infinite sum in Euclidean heat-kernel action can be performed stochastically by introducing integer-valued auxiliary field

$$\int \mathcal{D}U \; e^{-S(U)} = \left(\int \mathcal{D}U \sum_{n=-\infty}^{\infty} \right) e^{-S(U,n)}$$

Monte Carlo results for 2D Wilson loops with heat-kernel action with appropriately tuned coupling identical to Wilson action





Kanwar and MW, arXiv:2102.xx

Real-time U(1) lattice gauge theory

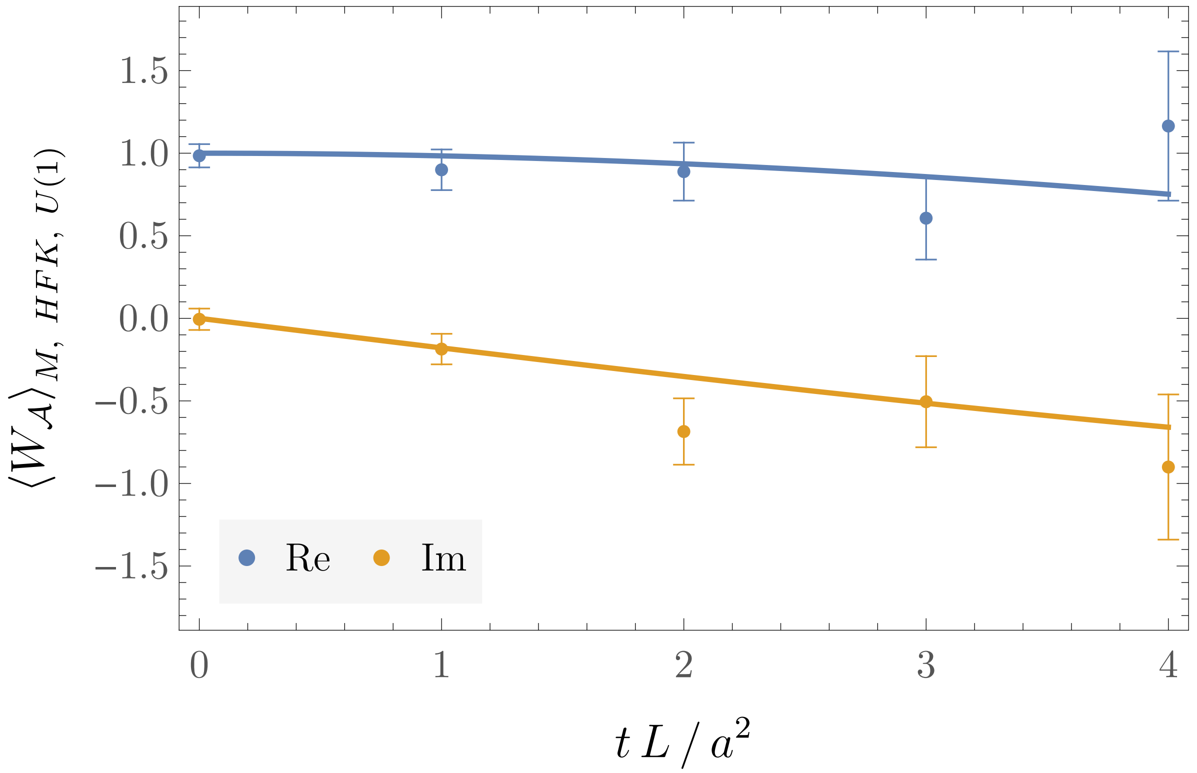
In Minkowski spacetime, the sums defining both the HFK and heat-kernel actions diverge

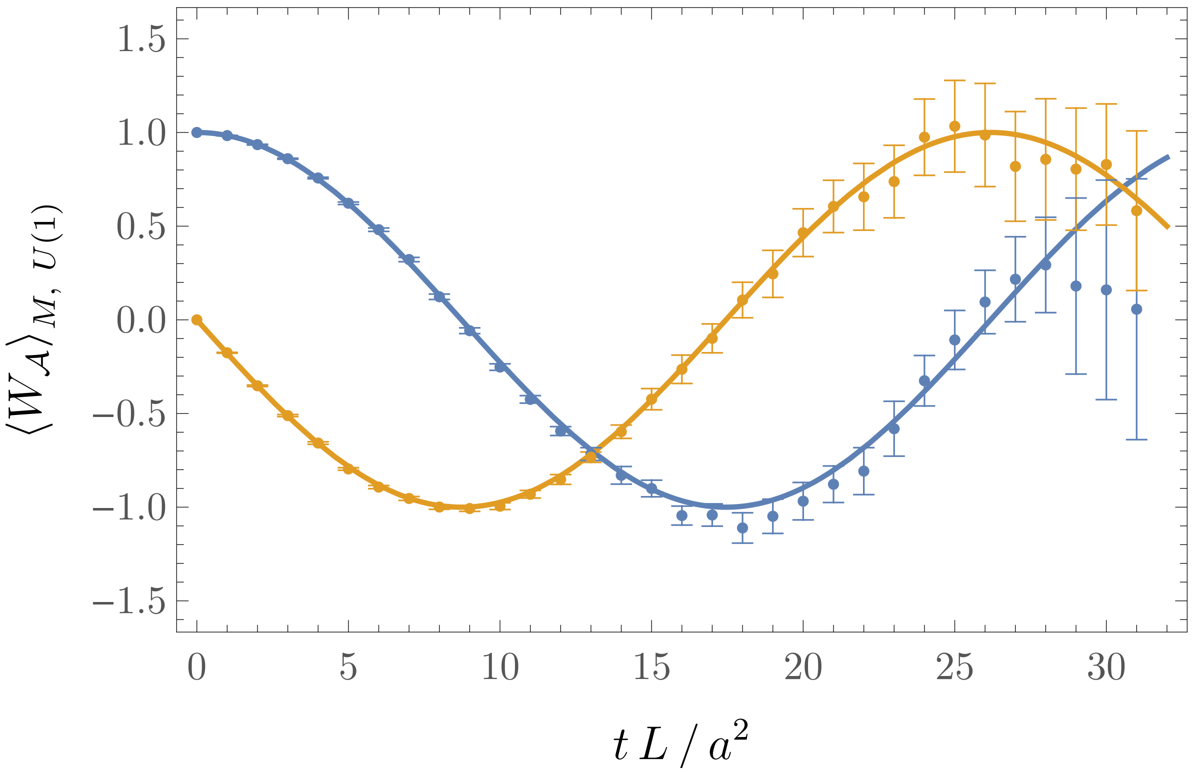
Path integral contour deformations can improve convergence

$$U \to \widetilde{U}(U, n)$$

With the heat-kernel action, they also “solve” the sign problem

$$e^{\frac{i}{2e^2}(\phi+2\pi n)^2} \to e^{-\frac{1}{2e^2}(\phi+2\pi n)^2}$$

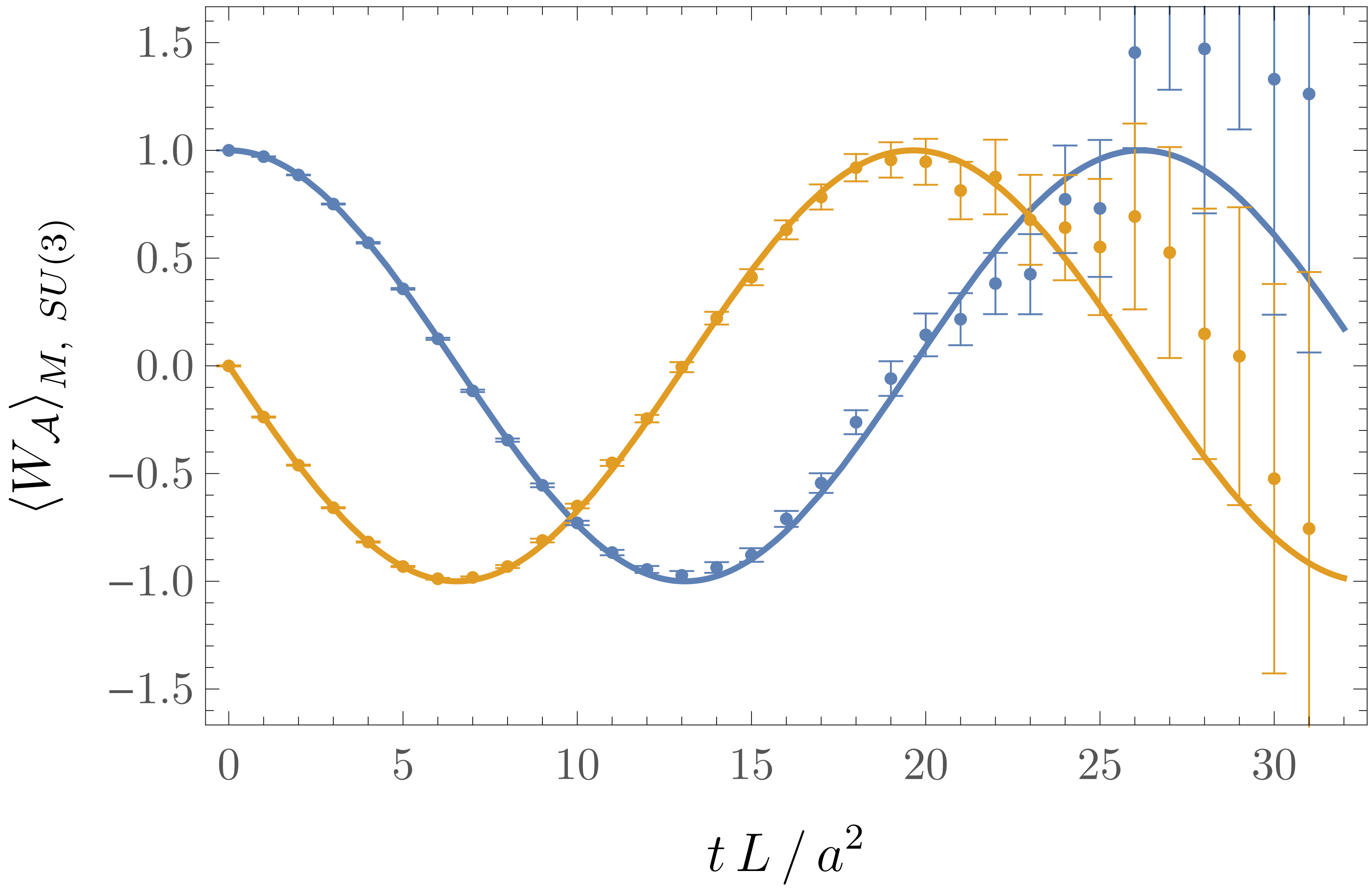




Kanwar and MW, arXiv:2102.xx

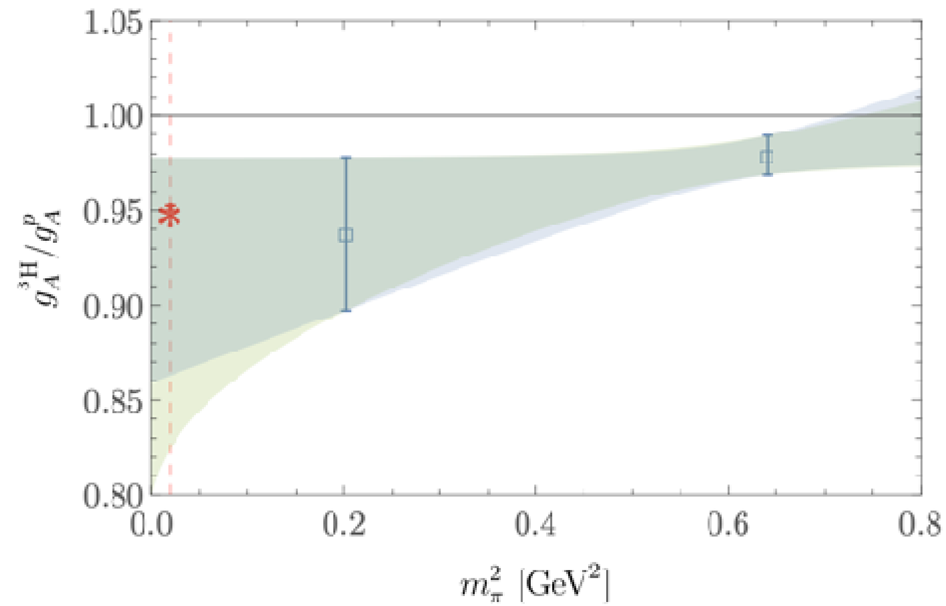
Real-time SU(3)

Works in (1+1)D

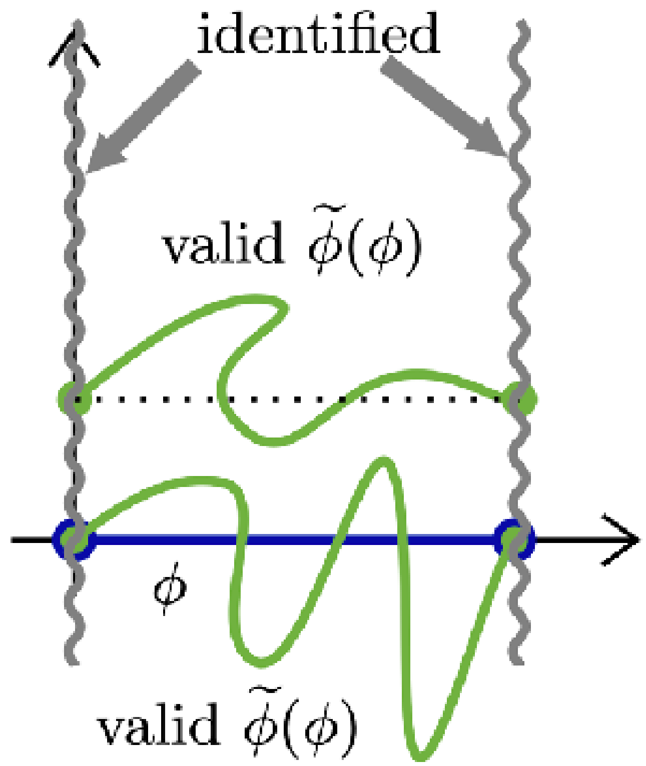


Kanwar and MW, arXiv:2102.xx

Conclusions



Lattice QCD is increasingly important for HEP/NP problems; sign(al-to-noise) problems present challenges



Path integral contour deformations have been shown to tame sign(al-to-noise) problems in 2D lattice gauge theory

Monte Carlo simulations of real-time lattice gauge theory are possible with the right action and contour deformation