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# Simulating phi-4 scalar field on quantum computers

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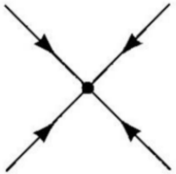
# Scalar field theory with $\phi^4$ interaction

- Lagrangian density $\mathcal{L} = \frac{1}{2}\left(\partial_0 \phi\right)^2 - \frac{1}{2}\left(\nabla \phi\right)^2 - \frac{1}{2} m_0^2 \phi^2 - \frac{\lambda}{4!}\phi^4$



- Discretized model in $(d+1)$-dimension with lattice spacing $a$

$$H = \sum_j \left[ \frac{1}{2}\Pi_j^2 + \frac{1}{2}\left(m_{0r}^2 + 2d\right)\Phi_j^2 - \sum_{e=1}^{d} \Phi_j \Phi_{j+e} + \frac{g_0}{4!}\Phi_j^4 \right] \quad m_{0r}^2 \equiv m_0^2 a^2 \text{ and } g_0 \equiv \lambda a^{3-d}$$

- Continuous model with Lorentz invariance recovers in the limit $a \to 0$

- Outline:
  - Binary encoding in position basis
  - State preparation: vacuum state

Quantum Algorithms for Quantum Field Theories

Stephen P. Jordan[1,*], Keith S. M. Lee[2], John Preskill[3]
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# Binary encoding in position basis

PHYSICAL REVIEW LETTERS **121,** 110504 (2018)

**Electron-Phonon Systems on a Universal Quantum Computer**

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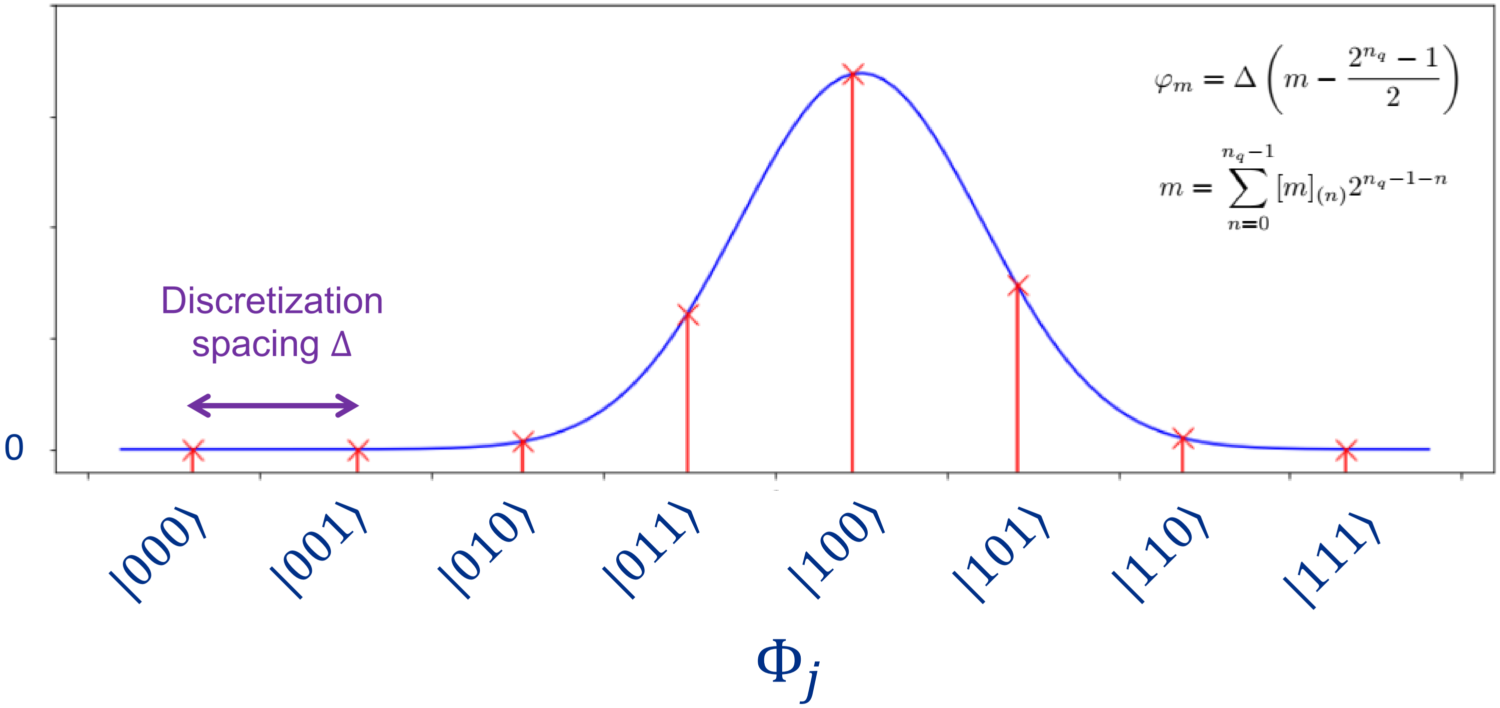
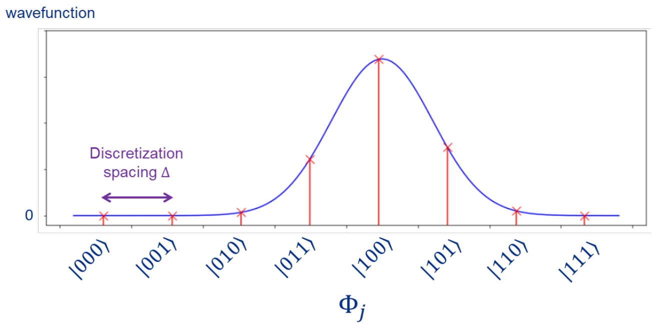
wavefunction

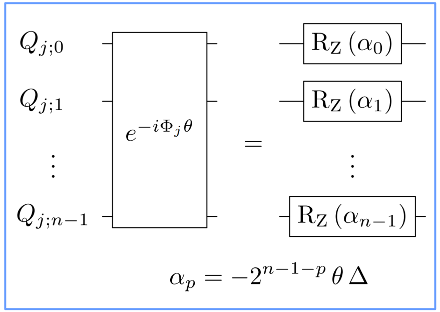


# Field operators in binary encoding




- Discretization and binary representation

$$\varphi_m = \Delta\left(m - \frac{2^{n_q}-1}{2}\right) \qquad m = \sum_{n=0}^{n_q-1} [m]_{(n)} 2^{n_q-1-n}$$




- Field operator $\Phi = \Delta\left(\sum_{n=0}^{n_q-1} \frac{-\sigma_n^z+1}{2} 2^{n_q-1-n} - \frac{2^{n_q}-1}{2}\right) = \Delta \sum_{n=0}^{n_q-1} \frac{-\sigma_n^z}{2} 2^{n_q-1-n}$

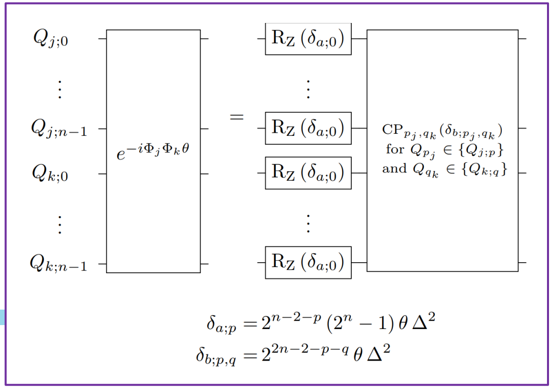
$$e^{-i\theta\Phi} = e^{-i\sum_{n=0}^{n_q-1} \frac{-\sigma_n^z}{2} 2^{n_q-1-n}\Delta\,\theta} = \prod_{n=0}^{n_q-1} \mathrm{R}_\mathrm{Z}^{(n)}(-2^{n_q-1-n}\Delta\,\theta)$$

- Field operator square

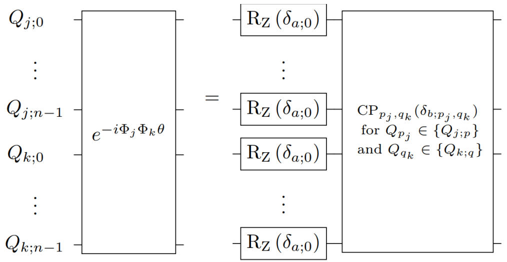
$$\Phi^2 = \Delta^2\left[\sum_{m<n} \frac{-\sigma_m^z+1}{2}\frac{-\sigma_n^z+1}{2} 2^{2n_q-1-m-n} + \sum_{n=0}^{n_q-1} \frac{-\sigma_n^z}{2} 2^{n_q-1-n}\left(2^{n_q-1-n} - 2^{n_q}+1\right)\right] + \phi_2$$

Control-phase gates

Z-rotation gates

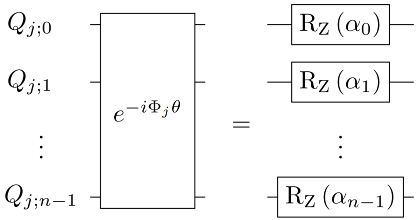


# Field operators: gate counts



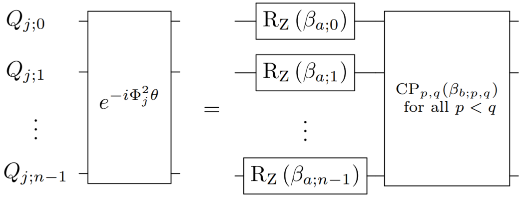

$$\delta_{a;p} = 2^{n-2-p}\,(2^n - 1)\,\theta\,\Delta^2$$
$$\delta_{b;p,q} = 2^{2n-2-p-q}\,\theta\,\Delta^2$$

| | CPHASE | CNOT |
|---|---|---|
| $e^{-i\Phi\theta}$ | $0$ | $0$ |
| $e^{-i\Phi^2\theta}$ | $n(n-1)/2$ | $0$ |
| $e^{-i\Phi^4\theta}$ | $n(n-1)/2$ | $n(n^3-6n^2+11n-6)/4$ |
| $e^{-i\Phi_j\Phi_k\theta}$ | $n^2$ | $0$ |

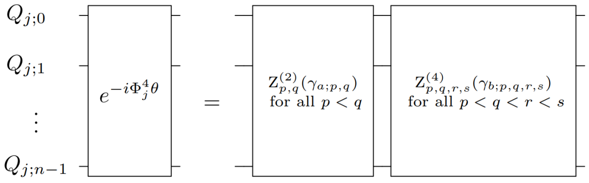


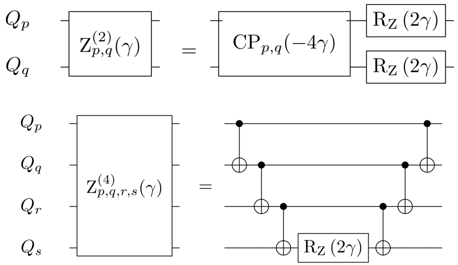

$$\alpha_p = -2^{n-1-p}\,\theta\,\Delta$$

$$H = \sum_j \left[\frac{1}{2}\Pi_j^2 + \frac{1}{2}\left(m_{0r}^2 + 2d\right)\Phi_j^2 - \sum_{e=1}^{d}\Phi_j\Phi_{j+e} + \frac{g_0}{4!}\Phi_j^4 + f\Phi_j\right]$$




$$\mathrm{CP}_{p,q}(\beta) = e^{i\beta|11\rangle\langle 11|} = \text{Control-Phase gate on } Q_{j;p} \text{ and } Q_{j;q}$$
$$\beta_{a;p} = -\,2^{n-1-p}\left(2^{n-1-p} - 2^n + 1\right)\,\theta\,\Delta^2$$
$$\beta_{b;p,q} = -\,2^{2n-1-p-q}\,\theta\,\Delta^2$$




$$\mathrm{Z}^{(2)}_{p,q}(\gamma) = e^{-i\gamma \mathrm{Z}_p \mathrm{Z}_q}$$
$$\mathrm{Z}^{(4)}_{p,q,r,s}(\gamma) = e^{-i\mathrm{Z}_p\gamma \mathrm{Z}_q \mathrm{Z}_r \mathrm{Z}_s}$$
$$\gamma_{a;p,q} = 2^{4n-4-p-q}\left(1 - 2^{-2n} - 2^{-1-2p} - 2^{-1-2q}\right)\,\theta\,\Delta^4$$
$$\gamma_{b;p,q,r,s} = 3 \times 2^{4n-5-p-q-r-s}\,\theta\,\Delta^4$$

# Time evolution in binary encoding

$$H = \sum_j \left[ \frac{1}{2}\Pi_j^2 + \frac{1}{2}\left(m_{0r}^2 + 2d\right)\Phi_j^2 - \sum_{e=1}^{d} \Phi_j \Phi_{j+e} + \frac{g_0}{4!}\Phi_j^4 + f\Phi_j \right]$$

- $f(\Pi_j) = \mathrm{FT}^{-1} f(\Phi_j)\,\mathrm{FT}$
- $\mathrm{FT} = \prod_p R_Z(\pi \frac{n-1}{2n} p)\ \mathrm{QFT} \prod_p R_Z(\pi \frac{n-1}{2n} p)$
- Cost of $\mathrm{QFT} = \frac{\mathrm{n(n-1)}}{2}$ CPHASE gates
- 1 first-order Trotter step with 32 states per site ($n = 5$) and $M$ sites on a 1D chain: $75M$ CPHASE, $30M$ CNOT

| | CPHASE | CNOT |
|---|---|---|
| $e^{-i\Phi\theta}$ | 0 | 0 |
| $e^{-i\Phi^2\theta}$ | $n(n-1)/2$ | 0 |
| $e^{-i\Phi^4\theta}$ | $n(n-1)/2$ | $n(n^3 - 6n^2 + 11n - 6)/4$ |
| $e^{-i\Phi_j\Phi_k\theta}$ | $n^2$ | 0 |
| $e^{-i\Pi^2\theta}$ | $3n(n-1)/2$ | 0 |

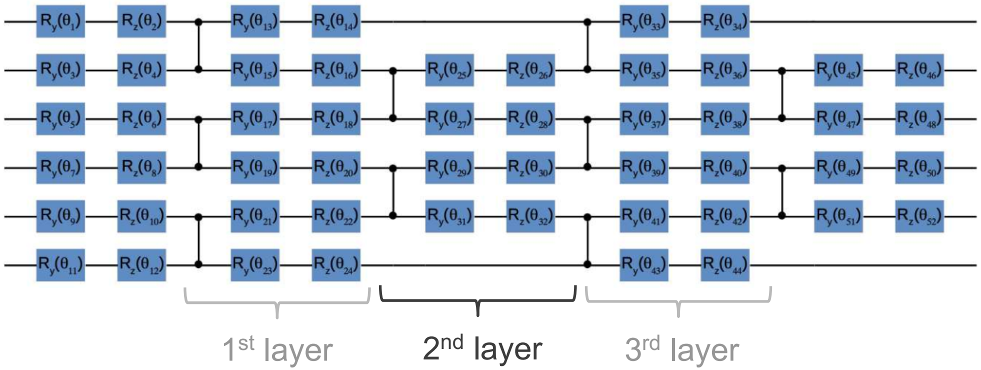
# Vacuum state preparation: local variational + adiabatic transfer

$$H = \sum_j \left[ \frac{1}{2}\Pi_j^2 + \frac{1}{2}\left(m_{0r}^2 + 2d\right)\Phi_j^2 + \frac{g_0}{4!}\Phi_j^4 - \sum_{e=1}^{d} \Phi_j \Phi_{j+e} \right]$$

Local Hamiltonian $H_0$ Coupling between sites $H_1$

Adiabatically switch on $H_1$

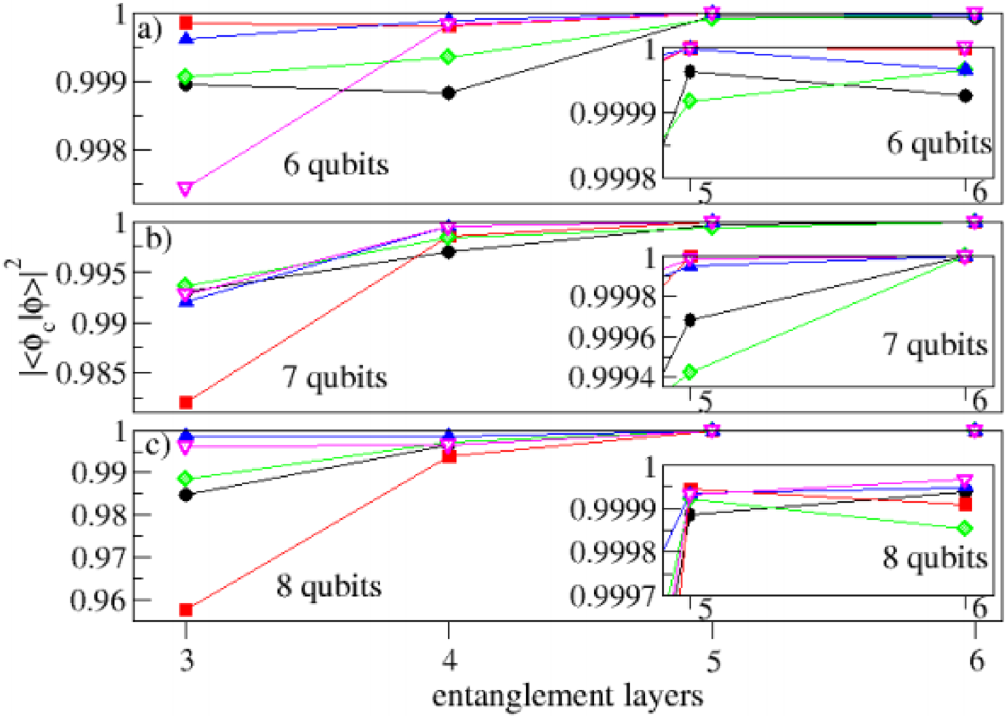
Variational preparation of ground state $H_0$ $|g\rangle$ → $|0\rangle$ $H_0 + \mathrm{H}_1$

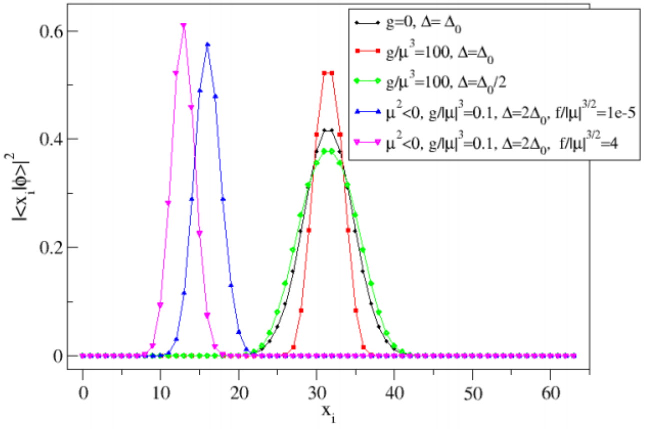


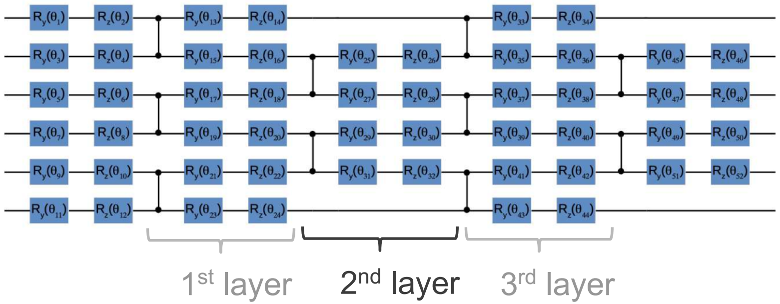

Advantage:

1. Efficient variational preparation compared to Kitaev-Webb method [arXiv:0801.0342 (2008)]
2. Prepare the vacuum state for any $\lambda_0$ and $m_0^2$ (including $m_0^2 < 0$)

# High fidelity local state preparation by hardware efficient ansatz



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# Adiabatic state transfer in the broken-symmetry phase

$$H = \sum_j \left[ \frac{1}{2}\Pi_j^2 + \frac{1}{2}\left(m_{0r}^2 + 2d\right)\Phi_j^2 + \frac{g_0}{4!}\Phi_j^4 - \sum_{e=1}^{d} \Phi_j \Phi_{j+e} \right]$$

PHYSICAL REVIEW D VOLUME 13, NUMBER 10 15 MAY 1976

**Existence of a second-order phase transition in a two-dimensional $\phi^4$ field theory***

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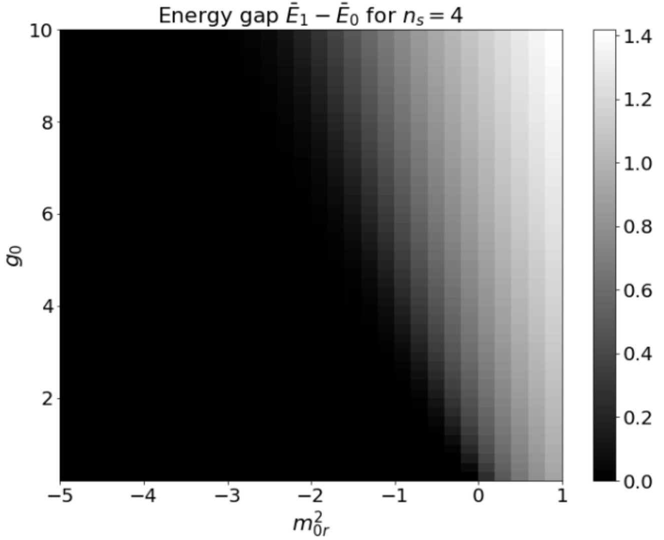
*Department of Physics, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801*

(Received 9 February 1976)

- Renormalization
  large $\lambda \rightarrow m_{0r}^2 < 0$

- Broken symmetry phase $\langle \Phi_j \rangle \neq 0$
  double-well potential
  degenerate ground states




Adiabatically switch off

$$H = \sum_j \left[ \frac{1}{2}\Pi_j^2 + \frac{1}{2}\left(m_{0r}^2 + 2d\right)\Phi_j^2 + \frac{g_0}{4!}\Phi_j^4 + f\Phi_j - \sum_{e=1}^{d} \Phi_j \Phi_{j+e} \right]$$

Variational state preparation

Adiabatically switch on



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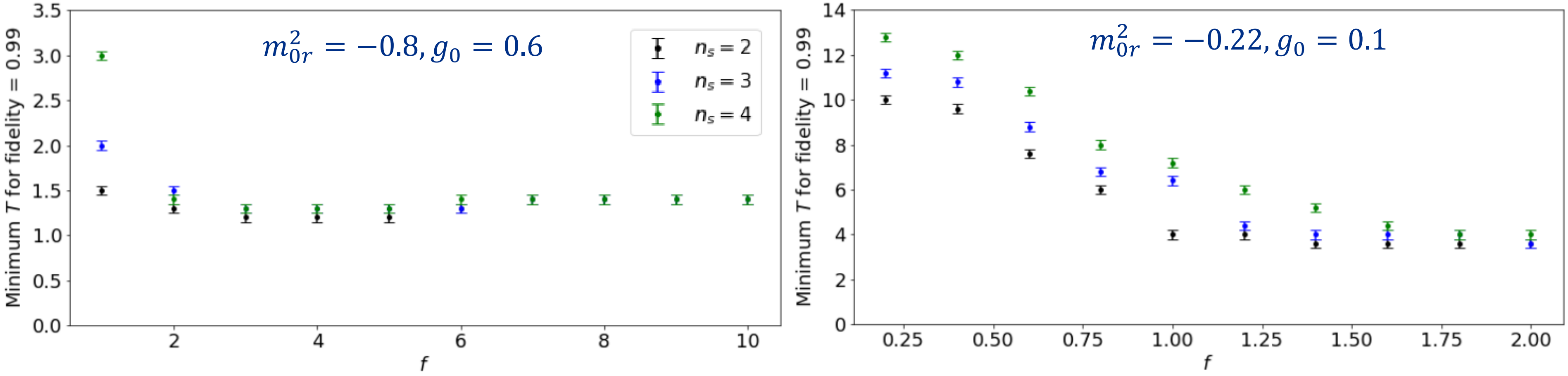
# Adiabatic time required to prepare broken-symmetry states

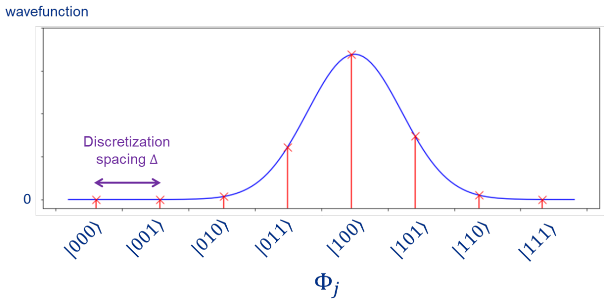


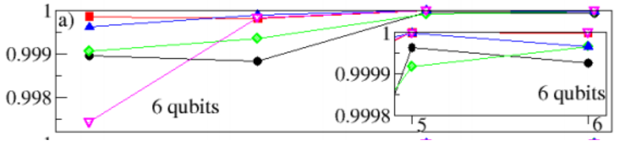

- Continuous limit: $a \to 0$, $g_0 = \lambda a^{3-d} \to 0$ for 1D and 2D systems
  – State preparation becomes more difficult due to stronger effective site-site coupling

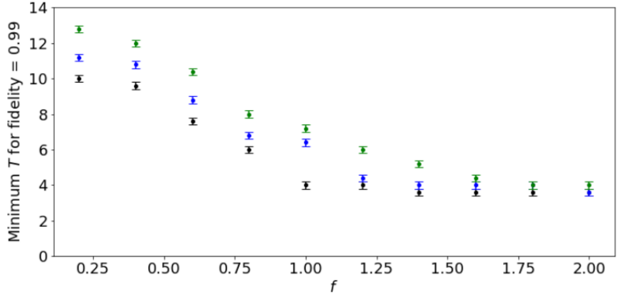
- System size dependence: require further study
  – Appears to be less sensitive with a sufficiently large $f$

# Summary

- Quantum simulation of $\phi^4$ scaler field theory using binary encoding in the position basis
- Variational hardware-efficient ansatz: flexible tool to prepare the local ground state with very high fidelity
  - Flexibility: design local Hamiltonian to further optimize the algorithm
- Adiabatic state transfer: connect the engineered local Hamiltonian to the full Hamiltonian
- Gate count: require better QPU to be implemeneted



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