

Measurement of inclusive charged-current ν_μ cross sections as a function of muon kinematics at $\langle E_\nu \rangle \sim 6$ GeV on hydrocarbon

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MINERvA presents a new analysis of inclusive charged-current neutrino interactions on a hydrocarbon target. We report single and double-differential cross sections in muon transverse and longitudinal momentum. These measurements are compared to neutrino interaction generator predictions from GENIE, NuWro, GiBUU, and NEUT. In addition, comparisons against models with different treatments of multi-nucleon correlations, nuclear effects, resonant pion production, and deep inelastic scattering are presented. The data recorded corresponds to 10.61×10^{20} protons on target with a peak neutrino energy of approximately 6 GeV. The higher energy and larger statistics of these data extend the kinematic range for model testing beyond previous MINERvA inclusive charged-current measurements. The results are not well modeled by several generator predictions using a variety of input models.

I. INTRODUCTION

Neutrino oscillation experiments [1–4] depend on neutrino interaction models to correct for detector and nuclear effects. Oscillation experiments at a few GeV of mean neutrino energy plan to use an inclusive charged-current (CC) signal to maximize far detector statistical

precision. An important component of these measurements is the identification of the resulting lepton. Accurate prediction of the momentum and angular distributions of the lepton are required to correct the measured rate for efficiency and acceptance. Models of neutrino interactions are also used as input to neutrino energy reconstruction; mismodeling of lepton energy is *prima facie* evidence that neutrino energy reconstruction will be similarly flawed when using that neutrino interaction model as an input.

Inclusive cross section measurements have been made on a variety of nuclear targets in the past. Micro-

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BooNE [5] and T2K [6] have measured double-differential cross sections as a function of muon momentum and angle on argon and hydrocarbon, but at a lower mean neutrino energy than MINERvA [7]. NOMAD [8] as well as MINOS [9] and CCFR [10] made measurements as a function of neutrino energy on carbon and iron respectively. MINERvA has made measurements as a function of neutrino energy using the low- ν method on carbon for both neutrino and anti-neutrino beams [11, 12] and as a function of muon transverse and longitudinal momentum in the Low Energy (LE) NuMI beam with a neutrino flux peaked at 3 GeV [13]. The result presented here increases the phase space accepted into the multi-GeV regime and as a result expands the range of transverse momentum from 2.5 to 4.5 GeV and longitudinal momentum from 20 to 60 GeV with a ~ 12 times larger sample size and flux normalization uncertainty of approximately 1/2 the size of the previous result.

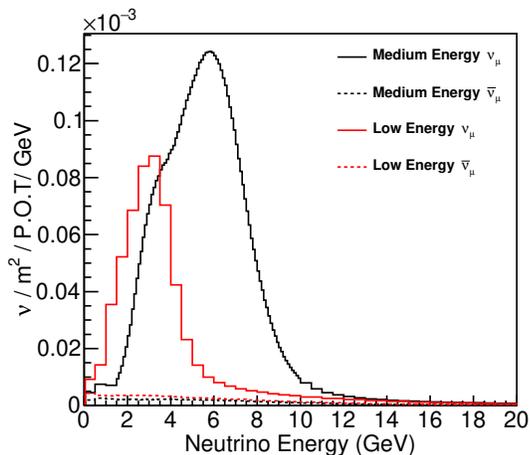


FIG. 1. Medium and Low Energy fluxes in the neutrino focused mode at MINERvA. In addition to the ν_μ flux, the $\bar{\nu}_\mu$ contamination is shown.

We present here the two-dimensional cross section for the inclusive neutrino scattering as a function of the muon transverse (p_t) and longitudinal momentum ($p_{||}$) in the Medium Energy (ME) NuMI beam, which has a neutrino flux peaked near 6 GeV. Figure 1 compares the Low and Medium Energy fluxes used by MINERvA. The muon momentum and angle can be precisely measured. These muon variables are suitable for comparison to exclusive channel measurements and provide a foundation to understand how model predictions combine to form an inclusive cross section prediction. In addition to the two-dimensional cross sections, one-dimensional projections, limited to the phase space of the double-differential cross section, are also provided.

Section II describes the experimental setup. Section III describes the simulation of the neutrino interactions, the modifications made to the interaction model, and the simulation of particle propagation through the detector.

The event selection and measurement methods used to extract the differential cross sections are described in Section IV. A description of the sources and determination of systematic uncertainties are presented in Section V. Section VI describes the cross section results while Section VII provides a set of comparisons to multiple neutrino generator predictions as well as modifications to these predictions. Finally, Section VIII provides conclusions that can be drawn from these comparisons.

II. EXPERIMENT

The MINERvA experiment employs a fine-grained tracking detector for recording neutrino interactions produced by the NuMI beamline at Fermilab [14, 15]. Neutrinos are created by directing 120 GeV protons from the Main Injector onto a graphite target. The resulting charged pions and kaons are focused by two magnetic horns. A neutrino-dominated or anti-neutrino-dominated beam is produced by switching the polarity of the horns. This analysis uses data from neutrino-dominated beam.

The MINERvA detector [7] consists of 120 hexagonal modules that create an active tracking volume preceded by a set of passive nuclear targets. This result includes only those interactions in the active tracking volume with a fiducial mass of 5.48 tons. The active target volume is surrounded by electromagnetic and hadronic calorimeters.

Each tracking module is made of two planes. Each plane is comprised of triangular polystyrene scintillator strips with a 1.7 cm strip-to-strip pitch. To allow for better three-dimensional reconstruction in a high-multiplicity environment, planes are oriented in three different directions, 0° and $\pm 60^\circ$ relative to the vertical axis of the detector. The downstream and side electromagnetic calorimeter consists of alternating layers of scintillator and 2 mm thick lead planes. The downstream and side hadronic calorimeters consists of alternating scintillator and 2.54 cm thick steel planes.

Multi-anode photomultiplier tubes read out the scintillator strips via wavelength-shifting fibers. The timing resolution measured by thoroughgoing muons is 3.0 ns and sufficient to separate multiple interactions within a single NuMI beam spill. Muons that originate in MINERvA are analyzed by the MINOS near detector [16], a magnetized spectrometer composed of scintillator and iron and located 2 m downstream of the MINERvA detector. The requirement that muons are analyzed in MINOS restricts this analysis to muons with $p_{||} > 1.5$ GeV/c and $\theta_\mu < 20^\circ$, which means a restricted acceptance for events with $Q^2 \lesssim \frac{p_{||}^2}{8}$.

This analysis uses data that correspond to 10.61×10^{20} protons on target (POT), received between September 2013 and February 2017 while the horn polarity was set to focus positively charged particles, creating a beam that

is predominantly muon neutrinos.

III. SIMULATION

A GEANT4-based simulation of the NuMI beamline is used to predict the neutrino flux. To improve the prediction, the simulation is reweighted as a function of pion kinematics to correct for differences between the GEANT4 [17]¹ prediction and hadron production measurements of 158 GeV protons on carbon from the NA49 experiment [18] and other relevant hadron production measurements. A description of this procedure is found in Ref. [15]. In addition, an *in situ* measurement of neutrino scattering off atomic electrons is used, as described in Ref. [19], to constrain the flux prediction.

Neutrino interactions are simulated using the GENIE neutrino event generator [20] version 2.12.6. Quasi-elastic (1p1h) interactions are simulated using the Llewellyn-Smith formalism [21] with the vector form factors modeled using the BBBA05 model [22]. The axial vector form factor uses the dipole form with an axial mass of $M_A = 0.99 \text{ GeV}/c^2$. Resonance production is simulated using the Rein-Sehgal model [23] with an axial mass of $M_A^{RES} = 1.12 \text{ GeV}/c^2$. Higher invariant mass interactions are simulated using a leading order model for deep inelastic scattering (DIS) with the Bodek-Yang prescription [24] for the modification at low square of the momentum transfer, Q^2 .

A relativistic Fermi gas model [25] is used with an additional Bodek-Ritchie high momentum tail [26] to account for nucleon-nucleon short range correlations. The maximum momentum for Fermi motion is assumed to be $k_F = 0.221 \text{ GeV}/c$. GENIE models intranuclear rescattering, or final state interactions (FSI), of the produced hadrons using the INTRANUKE-hA package [27].

To better describe MINERvA data, a variety of modifications to the interaction model are made. To better simulate quasielastic events, the cross section is modified as a function of energy and three momentum transfer based on the random phase approximation (RPA) part of the Valencia model [28, 29] appropriate for a Fermi gas [30, 31]. Multi-nucleon scattering (2p2h) is simulated by the same Valencia model [32–34], but the cross section is increased in specific regions of energy and three momentum transfer based on fits to MINERvA data [35] in a lower energy beam configuration. Integrated over all phase space, the rate of 2p2h is increased by 50% over the nominal prediction. Based on fits done in Ref. [36], we decrease the non-resonant pion production by 43% and reduce the uncertainty compared to the base GENIE model uncertainties. This modified version of the simulation is referred to as MINERvA Tune v1.

The response of the MINERvA detector is simulated using GEANT4 [17] version 4.9.3.p6 with the QGSP_BERT physics list. The optical and electronics performance is also simulated. Through-going muons are used to set the absolute energy scale of minimum ionizing energy depositions by requiring the average and RMS of energy deposits match between data and simulation as a function of time. A full description is found in Ref. [7]. Measurements using a charged particle test beam [37] and a scaled-down version of the MINERvA detector set the absolute energy response to charged hadrons. The effects of accidental activity are simulated by overlaying hits in both MINERvA and MINOS from data corresponding to random beam spills appropriate to the time periods in the simulation.

IV. CROSS SECTION EXTRACTION

A sample of neutrino charged-current interactions is extracted by requiring the track identified as being from a muon to be matched between MINERvA and MINOS, and to be negatively charged. In addition, the reconstructed interaction vertex must be within a specified fiducial volume. To avoid model dependence introduced by correcting for kinematic regions without acceptance, we only report results for charged-current cross section where the muon angle with respect to the neutrino direction is less than 20° , the muon p_t is less than 4.5 GeV, and the muon $p_{||}$ is between 1.5 GeV and 60 GeV.

Using these criteria, a sample of 4,105,696 interactions was selected. The simulation predicts an average selection efficiency of 64% in the p_t - $p_{||}$ phase space, where the efficiency loss is due to the MINERvA-MINOS geometric acceptance. After all selection cuts, the sample in muon transverse and longitudinal momentum space is shown in Fig. 2, decomposed into predicted components. Events are labeled by categories within GENIE except for events given a DIS label. To explore how contributions from DIS events rely on the validity of the neutrino-quark scattering model with different “depth” of the inelasticity, DIS is divided into two categories. “True DIS” events are those events where the invariant mass of the hadronic system, W , is greater than $2.0 \text{ GeV}/c^2$ and Q^2 greater than $1.0 \text{ GeV}^2/c^4$. “Soft DIS” represents the remainder of the GENIE DIS events. While the cut defining the ‘True DIS’ regime is purely kinematic, and could therefore be used for model comparisons, the ‘Soft DIS’ definition is not. The modeling of inelastic events below the ‘True DIS’ region can vary widely across generators, in terms of the kinematic coverage of the resonance model and handling of non-resonant contributions in the resonance region [38]. No two generators handle these aspects in exactly the same way, so the ‘Soft DIS’ label here is relevant only to GENIE simulations. A final category is “other CC”, which contains CC events not belonging to the other categories, such as coherent charged pion production. The background category con-

¹ The MINERvA beam simulation uses GEANT4 version 4.9.2.p3 with the FTFP BERT physics list.

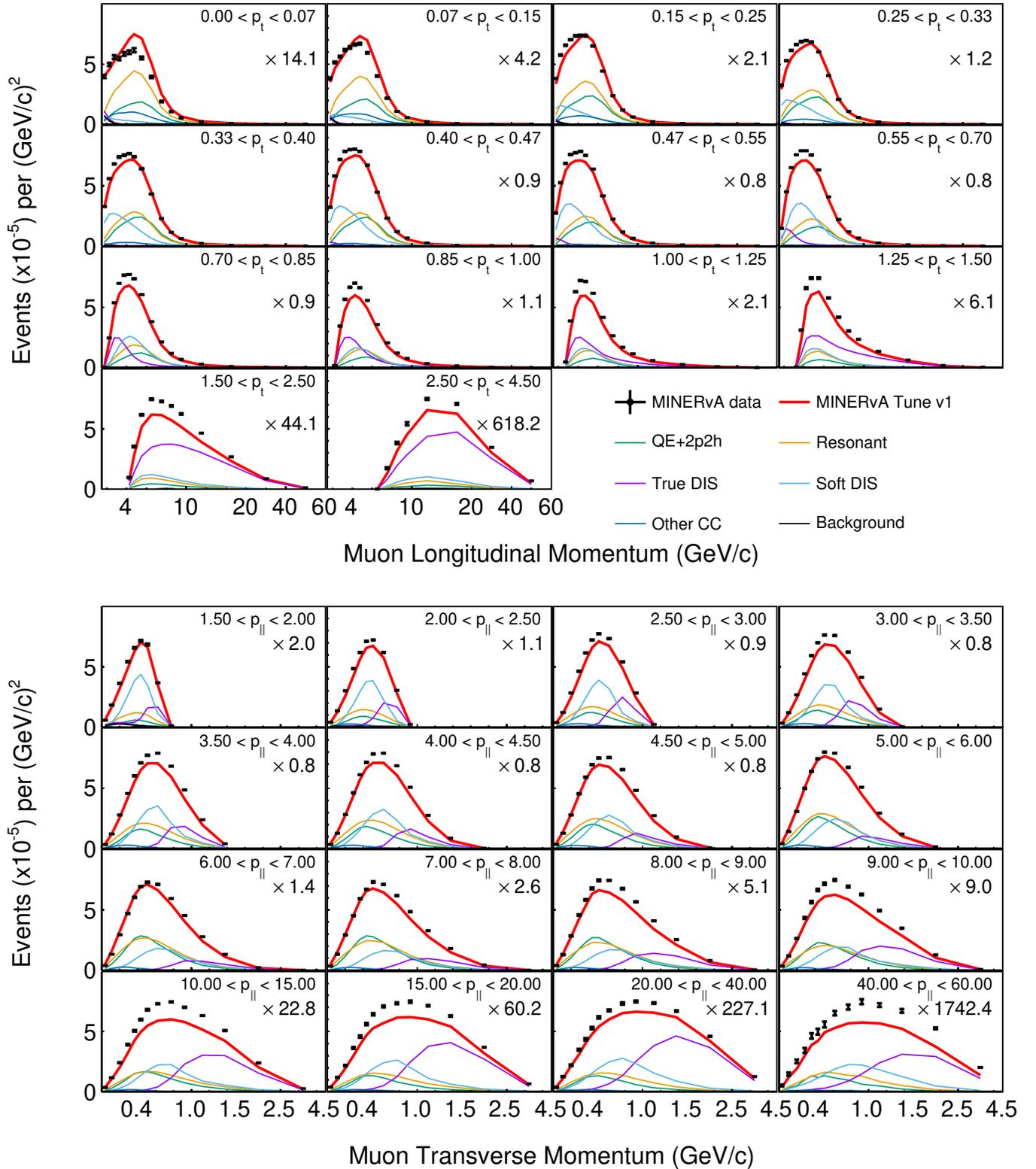


FIG. 2. Selected events passing all cuts in data (black points) and simulation (red line). Predictions from the simulation, MINERvA Tune v1, for various sample components (unstacked), in particular “Soft DIS”, are based on the GENIE generator and defined in Sec. IV. The indicated scale factors are applied to individual panel contents. The x-axis binning reduces the width of the largest p_t and p_{\parallel} bins for visual compactness. Only statistical uncertainty is shown.

tains charged-current events from other neutrino flavors and anti-muon type neutrinos as well as neutral-current interactions. A total of 8655 (0.2%) background events are predicted. Backgrounds at $p_{||}$ less than 2.5 GeV and p_t less than 0.4 GeV are primarily from neutral-current interactions where a pion was reconstructed as a muon in MINOS. These backgrounds have a maximum contribution of 10 percent of the predicted event rate at these low $p_{||}$ and p_t , and are typically much smaller. Backgrounds at high p_t are mostly anti-neutrino contamination due to muon charge misidentification which accounts for about one percent of the sample in the highest p_t bin.

The predicted background contributions are subtracted from the sample. Detector resolution effects (see Figs. 3 and 4) are then removed using the D'Agostini unfolding method [39, 40], via the implementation in RooUnfold [41]. To understand the necessary regularization strength, 10 different model predictions as pseudo-data were unfolded using the MINERvA Tune v1. The unfolded models were then compared to their true distributions via a χ^2 test taking full consideration of correlations. The optimal number of iterations was determined when the χ^2 approached one per degree of freedom and was not changing as a function of the number of iterations. Only statistical uncertainties were considered in determining the number of iterations. In all variations the required number of iterations was no more than 10. In addition, a fit to the data in reconstructed p_t - $p_{||}$ was performed, and the MINERvA Tune v1 prediction reweighted to the data as an additional fake data sample. Reweighting is done in true kinematic quantities and propagated through the Monte Carlo detector response prediction. Based on these studies, the data was unfolded using 10 iterations.

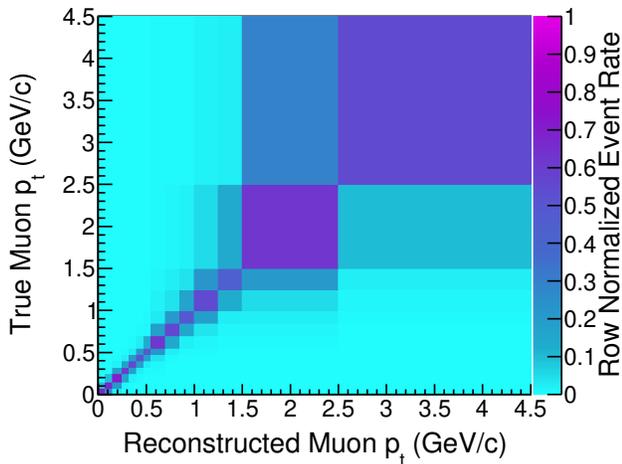


FIG. 3. Event migration between simulated and reconstructed p_t bins projected over all $p_{||}$.

Finally, the sample is corrected for efficiency and acceptance. The selection efficiency is shown in Fig. 5. The large efficiency in the 6-7 GeV $p_{||}$ and highest p_t bin is

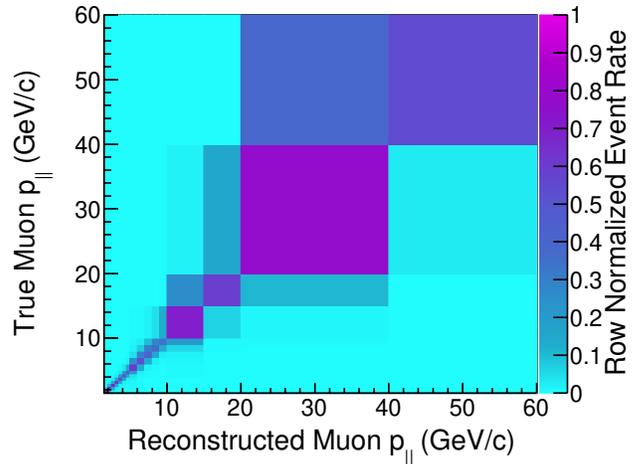


FIG. 4. Event migration between simulated and reconstructed $p_{||}$ bins projected over all p_t .

due to a fractionally large sample of muons with generated angle greater than 20 degrees passing event selection and appearing in this bin.

The efficiency corrected distribution is then divided by the integral of the flux with neutrino energies between 0 and 100 GeV averaged over the fiducial volume, which is $6.32 \times 10^{-8} \pm 3.9\%$ per cm^2 per proton on target, and the number of nucleons in the fiducial volume, $3.23 \times 10^{30} \pm 1.4\%$, with a mass fraction of 88.51% carbon, 8.18% hydrogen, 2.5% oxygen, 0.47% titanium, 0.2% chlorine, 0.07% aluminum, and 0.07% silicon.

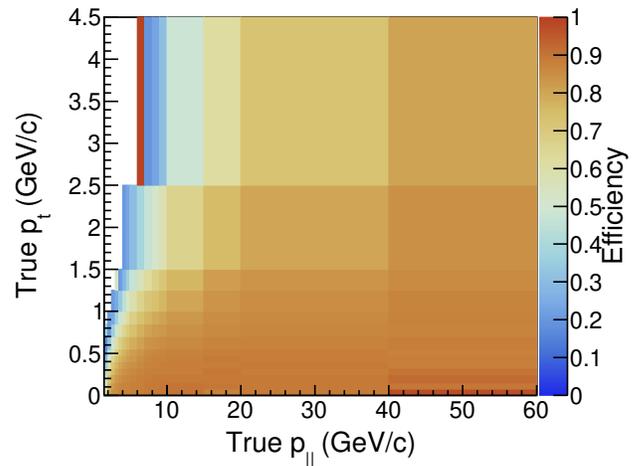


FIG. 5. Selection efficiency as a function of p_t and $p_{||}$.

V. SYSTEMATIC UNCERTAINTIES

Systematic uncertainties in this analysis fall under three different categories: flux, detector response, and neutrino interaction model uncertainties. The uncertainties from individual sources are evaluated by re-extracting the cross section using modified simulations. The size of each modification is related to the uncertainty in each source. Flux uncertainty, a typical leading uncertainty in neutrino cross section measurements, is below 4% for almost all the phase space because of the flux constraint that comes from a measurement of neutrino-electron scattering in the same beam [19]. The normalization uncertainty of 1.4% corresponds to the uncertainty in the number of target nucleons and is based on material assays and weight measurements of production-quality scintillator planes.

Uncertainty in the detector response to hadrons is evaluated using shifts determined by *in situ* measurements of a smaller version of the detector in a test beam [37]. Uncertainties in inelastic interaction cross sections for particles in the detector material are independently varied based on data-Monte Carlo differences between GEANT particle cross sections and world data on neutrons [42–45], pions [46–49], and protons [50–52]. Muon reconstruction uncertainty is dominated by the muon energy scale uncertainty, which is constrained by a fit to the visible neutrino energy distribution for low recoil neutrino charged-current events, whose cross section is known to be flat as a function of neutrino energy. As described in more detail in Ref. [53], the resulting uncertainty in the muon energy scale is 1%. Uncertainty in the matching efficiency is from imperfect modeling of the efficiency loss from accidental activity in the MINOS near detector when matching muon tracks from MINERvA to MINOS. This last efficiency is also determined by a data-simulation comparison as a function of instantaneous neutrino beam intensity.

Interaction model uncertainties are evaluated using the standard GENIE reweighting infrastructure. Because this is an inclusive analysis with very low backgrounds and few selection cuts, model uncertainties are never the dominant uncertainty in any p_t - $p_{||}$ bin. These uncertainties are most significant at the highest p_t bins where the geometric acceptance changes dramatically and at low p_t bins where the backgrounds are the largest.

The fractional uncertainties in the one-dimensional projections are shown in Figs. 6 and 7. The fractional uncertainties in the two-dimensional result are shown in Fig. 8. The dominant uncertainties are the muon momentum scale uncertainty and the flux normalization.

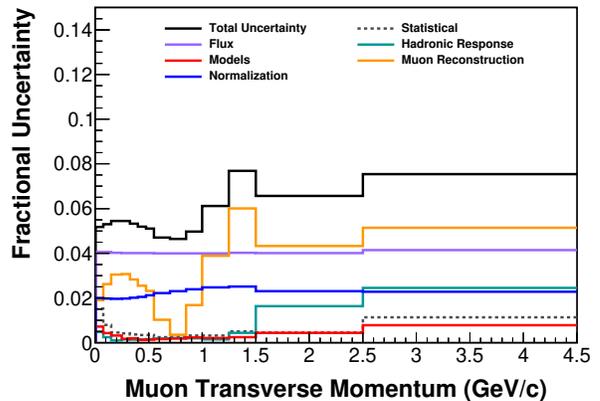


FIG. 6. Fractional uncertainty of the cross section when projected on the p_t axis.

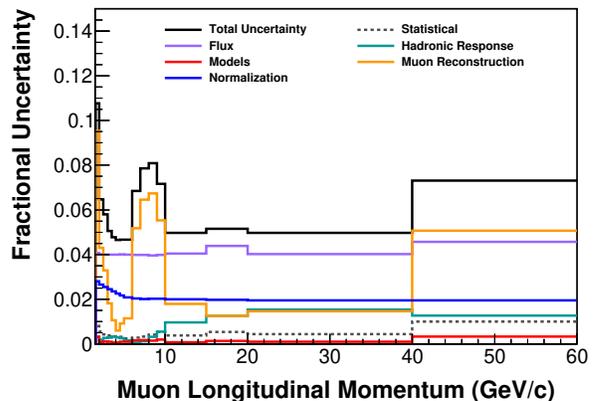


FIG. 7. Fractional uncertainty of the cross section when projected on the $p_{||}$ axis.

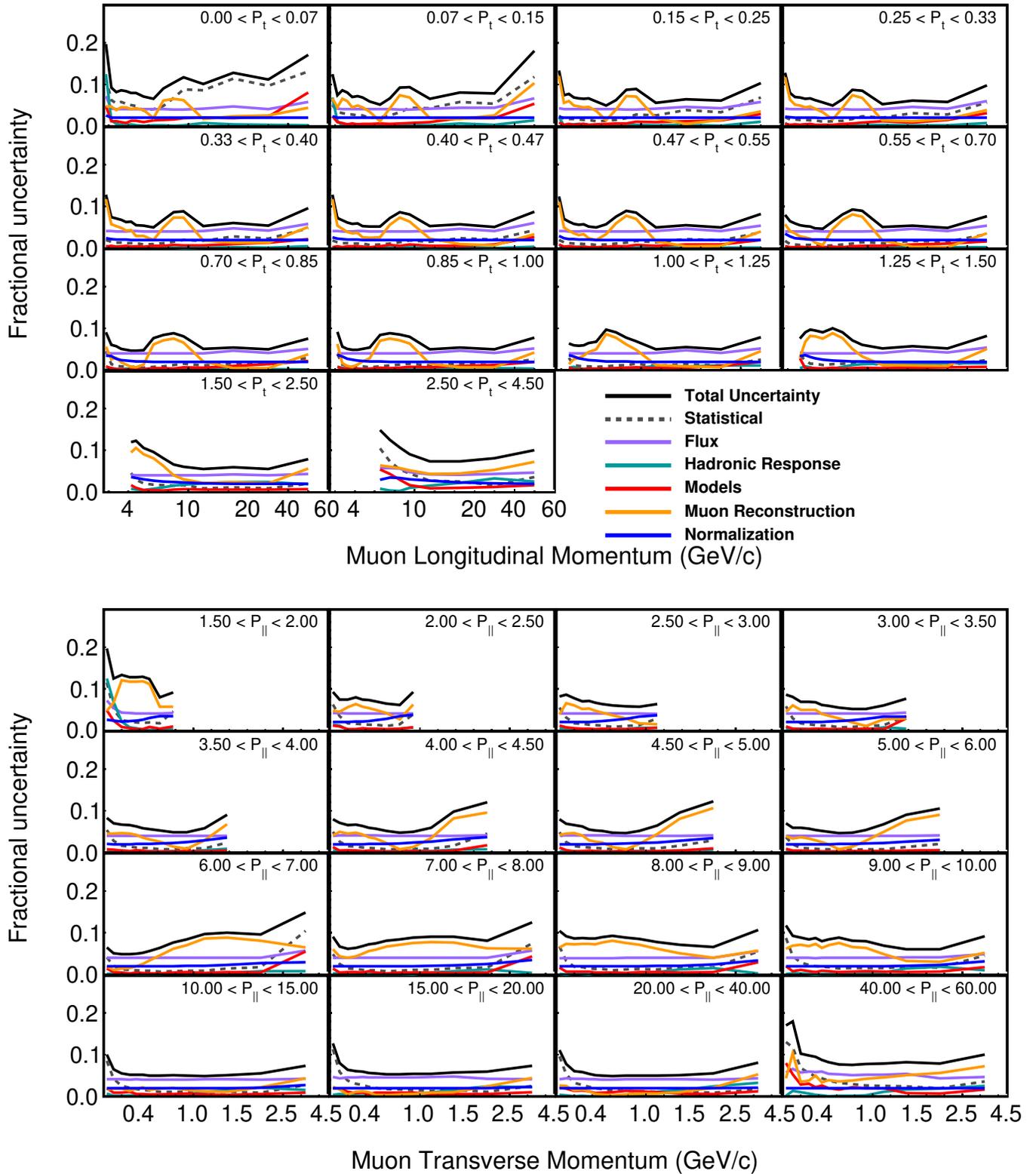


FIG. 8. Fractional uncertainties of the two-dimensional cross section as a function of p_t and $p_{||}$.

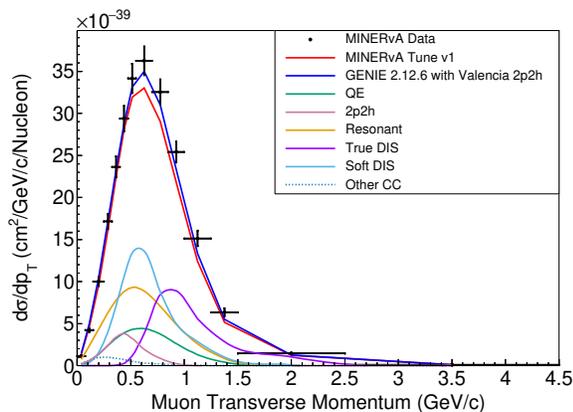


FIG. 9. Cross section projected onto the p_t axis showing contributions as predicted by MINERvA Tune v1 where the different contributions, in particular "Soft DIS", are based on the GENIE generator and are defined in Section IV. The GENIE 2.12.6 with Valencia 2p2h prediction is also shown. Total uncertainty is shown.

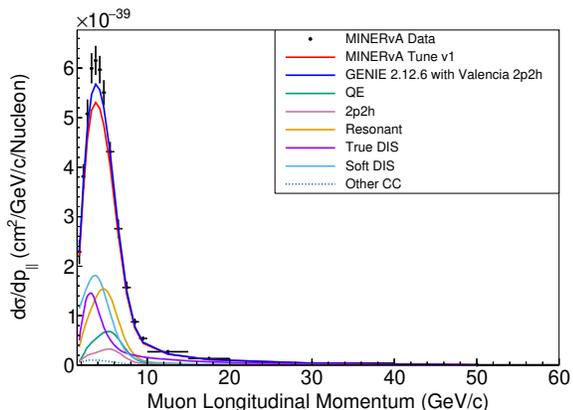


FIG. 10. Cross section projected onto the $p_{||}$ axis showing as predicted by MINERvA Tune v1 where the different contributions, in particular "Soft DIS", are based on the GENIE generator and are defined in Section IV. The GENIE 2.12.6 with Valencia 2p2h prediction is also shown. Total uncertainty is shown.

VI. RESULTS

Three results are presented: two single-differential cross sections and a double-differential cross section using the p_t and $p_{||}$ of the muon. The single-differential cross sections are shown in Figs. 9 and 10. The ratio of data to MINERvA Tune v1 for the single-differential results is shown in Figs. 11 and 12. The double-differential cross section as a function of p_t and $p_{||}$ is shown in Fig. 13.

The single-differential cross sections are derived from the two-dimensional result which means the additional phase space restrictions of the two-dimensional spaces are incorporated. The cross section versus p_t includes a

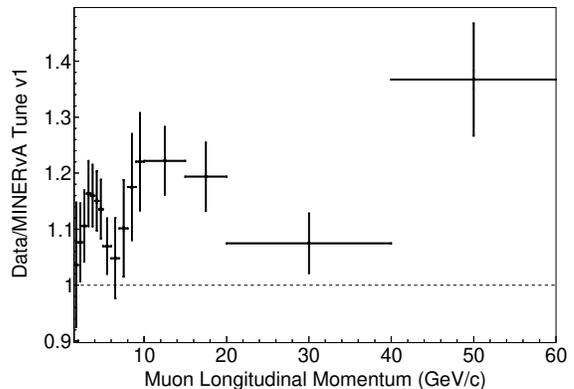


FIG. 11. Ratio of measured to MINERvA Tune v1 as projected onto the $p_{||}$ axis. Total uncertainty is shown.

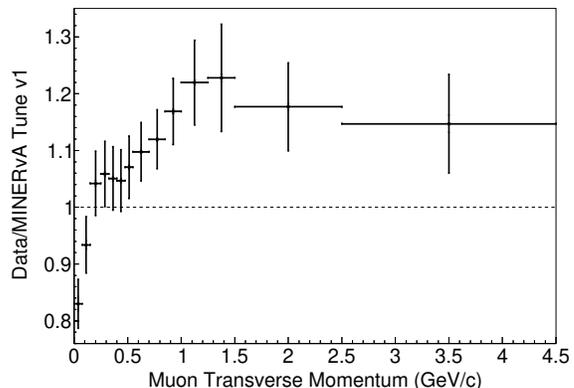


FIG. 12. Ratio of measured to MINERvA Tune v1 as projected onto the p_t axis. Total uncertainty is shown.

restriction of $1.5 \leq p_{||} \leq 60$ GeV/c, while the cross section versus $p_{||}$ includes a restriction of $p_t \leq 4.5$ GeV/c.

The difference between data and MINERvA Tune v1 is a convolution of the angular acceptance of the sample, detector resolutions, and mismodeling of the cross section as a function of p_t - $p_{||}$. Figure 11 has a shape which is due to these effects. A study to separate some of these effects was performed by correcting the prediction to match the data in the p_t projection the overall normalization and angular acceptance discrepancies are partially removed. The $p_{||}$ prediction after this correction was compared to the data and found to be consistent within the uncertainty of the muon energy scale.

Figure 14 shows the ratio of data to simulation. For $p_{||}$ between 3 and 15 GeV and low values of p_t , the cross section is overpredicted. In this region the dominant process is resonant pion production which has been previously measured by MINERvA [54–59], MiniBooNE [60], and T2K [61]. Many of these measurements, including a MINOS measurement [62], indicate the need to reduce the predicted cross section at low Q^2 which corresponds

here to regions of low p_t . A set of comparisons against a variety of resonant pion production model modifications is shown in Sec. VII. At $p_t > 0.85$ GeV/c the Monte Carlo prediction consistently underpredicts the data by 10-25%. The high p_t regions is dominated by the “True DIS” process for $p_{||} > 6$ GeV/c. Poorly understood neutrino DIS nuclear effects could contribute to the underprediction in this region of kinematics.

VII. COMPARISONS

In this section the extracted cross sections are compared to a variety of predictions. Four different groups of models and the data are presented as a ratio with respect to MINERvA Tune v1. Each group investigates a different aspect of the MINERvA Tune v1. No model combination completely describes the data. We also compare to a group of predictions from different neutrino event generators.

The first of these groups of comparisons, see Fig. 15, considers alterations to MINERvA Tune v1 by adding or subtracting selected changes that were made to the original default GENIE prediction. The cases plotted are:

- (a) GENIE v2.12.6 with Valencia 2p2h [33, 34, 63, 64], (GENIE 2.12.6);
- (b) Case a) but with a reduction in the non-resonant pion production of 43% [36], (NonResPionTune Only);
- (c) Case b) but with RPA from the Valencia model applied to quasielastic events, (QE RPA);
- (d) Case b) but with the application of the empirical enhancement of 2p2h production as described in Sec. III, (Low Recoil Enhancement);
- (e) The full set of corrections, (MINERvA Tune v1).

The second group of comparisons, see Fig. 16, use the MINERvA Tune v1 as the baseline prediction with modifications to the DIS model for $W \geq 2.0$ GeV/ c^2 and $Q^2 \geq 1.0$ GeV²/ c^4 . The modifications plotted are:

- (a) MINERvA Tune v1, which uses the LHAPDF5 parton distributions functions (PDFs), (MINERvA Tune v1);
- (b) Case a) but with the nCTEQ15 PDFs, which are determined from charged lepton-nucleus scattering [65], rather than the LHAPDF5 PDFs, (nCTEQ15 DIS);
- (c) Case a) but with the nCTEQ15 ν pdfs, which are determined from neutrino-nucleus scattering [66], rather than the LHAPDF5 PDFs, (nCTEQ ν DIS);
- (d) Case a) but employing the microscopic model developed at Aligarh Muslim University [67], (AMU DIS).

The third set of comparisons, see Fig. 17, use MINERvA Tune v1 as the baseline with modifications of various resonant pion production channels. These include:

- (a) MINERvA Tune v1 with the nominal GENIE resonant pion model, (MINERvA Tune v1);
- (b) Case a) but with the non-resonant pion reduction removed and a reweighting of resonant pion production to the MK model [68], (MK Model);
- (c) Case a) but with a reweighting to reduce the resonant pion cross section at low Q^2 according to a MINOS parameterization [62], (Pion LowQ2-MINOS);
- (d) Case a) but with a reweighting to reduce the resonant pion cross section at low Q^2 according to a MINERvA parameterization [69], (MINERvA Tune v2).

Figure 18 shows a comparison between various neutrino generator predictions including GiBUU [70, 71] for two different versions, two different nuclear models from NuWro [72, 73], and NEUT [74].²

Table I gives the χ^2 statistics comparing data to the various model predictions listed above. The standard χ^2 calculation assumes the underlying uncertainty is normally distributed. This assumption is not correct as some sources of uncertainty are log-normal. An example is the flux normalization which introduces uncertainty in the measurement by division. To understand the span of these differences we report both the standard χ^2 and a log-normal version. The model with the lowest standard and log-normal χ^2 is the NuWro prediction with a local Fermi gas nuclear model. The effect of Peele’s Pertinent Puzzle [76–78] is clearly shown by the effect of the GiBUU v2019 prediction (or the difference in the peak region between MINERvA Tune v1 and GENIE v2.12.6 with Valencia 2p2h in Figs. 9 and 10) which, by eye, has a different normalization than the data. This effect occurs when the dominant uncertainty of a result is a highly correlated normalization uncertainty. The log-normal χ^2 increases significantly for models where the dominant difference with data is the normalization. Because of the flux is a dominant uncertainty in this analysis the log-normal χ^2 is a better estimation, but the ordering of models within each group of model modifications is the same using either estimator.

The MINERvA Tune v1 agrees better with the data than GENIE v2.12.6 with Valencia 2p2h. The inclusion or removal of components can either improve or degrade the data Monte Carlo agreement. The inclusion of the non-resonant pion reduction results in poorer agreement than the base model. GENIE v2.12.6 with Valencia 2p2h and QE RPA and a non-resonant pion production reduction is the best combination to predict the data while the model with the low recoil enhancement has the largest χ^2 . The MINERvA Tune v1 is supported by a variety of exclusive measurements in a 3 GeV neutrino focused beam [36, 79, 80] indicate a need for the all modifications but data using the 6 GeV beam [53] indicates a preference

² Predictions for NuWro and NEUT were produced using NUISANCE [75]

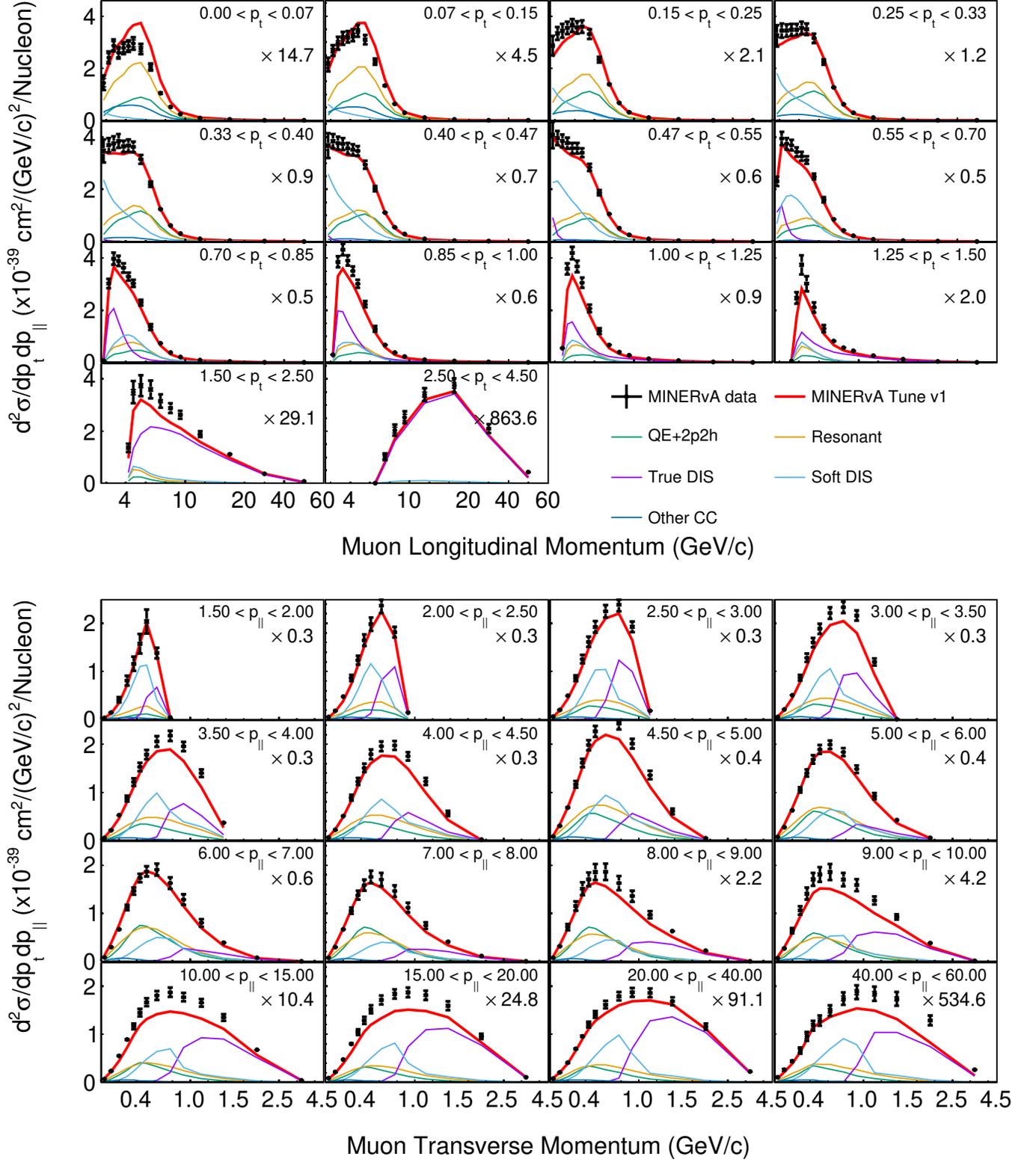


FIG. 13. Extracted cross section compared to MINERvA Tune v1. Predictions represent MINERvA Tune v1, with various sample components (unstacked), in particular “Soft DIS”, based on the GENIE generator and defined in Sec. IV. The indicated scale factors are applied to individual panel contents. The x-axis binning reduces the width of the largest p_t and p_{\parallel} bins for visual compactness. Inner (outer) ticks denote statistical (total) uncertainty.

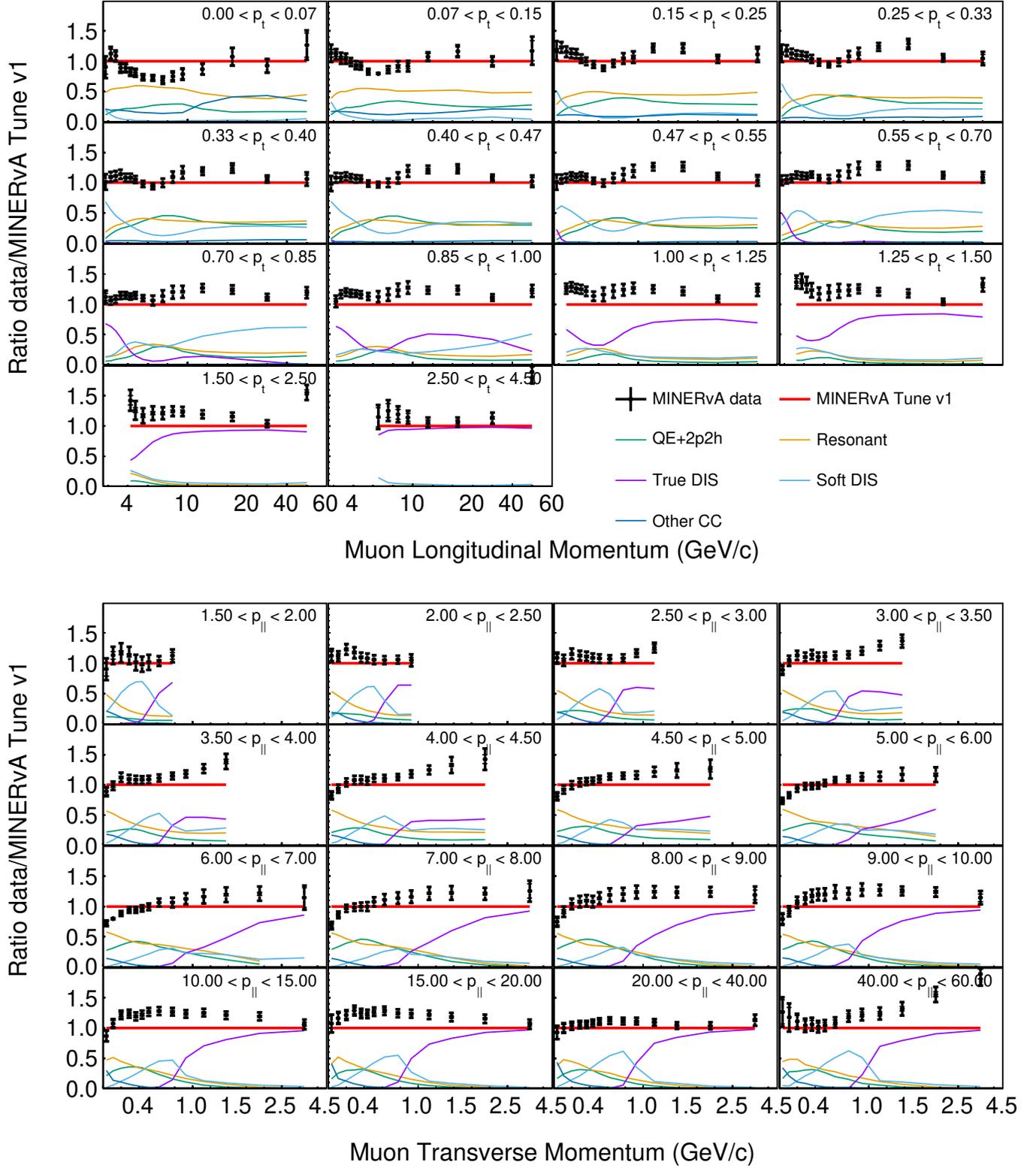


FIG. 14. Ratio of the extracted cross section to MINERvA Tune v1. The various sample components, in particular “Soft DIS”, are based on the GENIE generator and defined in Sec. IV. Inner (outer) ticks denote statistical (total) uncertainty.

for GENIE v2.12.6 with Valencia 2p2h and QE RPA and a non-resonant pion production reduction. The modifications to DIS have mild modifications to the prediction. Models which further modify the resonant pion cross section improve the prediction. The data prefer a low Q^2 type suppression for resonant pions. To understand the effect of the modifications in detail the χ^2 is broken down into contributions for each kinematic bin.

Figure 19 shows the bin-by-bin contributions to the overall χ^2 . The best prediction, based on overall χ^2 , from each model group from Sec. VII is compared against the MINERvA Tune v1. The metric used is the difference in χ^2 on a bin-by-bin basis. The value for the i^{th} bin is the result of the calculation shown in Eqs. 1 - 2

$$\chi_{i,j_{\text{model}}}^2 = (x_{i,\text{measured}} - x_{i,\text{expected}_{\text{model}}}) \times V_{ij}^{-1} \times (x_{j,\text{measured}} - x_{j,\text{expected}_{\text{model}}}), \quad (1)$$

$$\Delta\chi_i^2 = \sum_j (\chi_{i,j_{\text{model}}}^2 - \chi_{i,j_{\text{MINERvA Tune v1}}}^2), \quad (2)$$

where x is the cross section and V is the measurement covariance matrix. Due to the anti-correlations introduced by the unfolding procedure some large bin-to-bin anti-correlations will appear in this metric. Negative values indicate an improvement due to the model with respect to MINERvA Tune v1.

GENIE 2.12.6 with Valencia 2p2h and QE RPA and a non-resonant pion reduction, MINERvA Tune v1 with low Q^2 pion suppression models, and NuWro with a local Fermi gas roughly improve in the same regions of phase space. The low Q^2 pion suppression modification appears to be an overcorrection for the lowest p_t - $p_{||}$ bins, which is evident in Fig. 17. The modification to the DIS model is different than the other three modifications. The model improves agreement in regions of increasing p_t as a function of $p_{||}$. This is a kinematic boundary between the “Soft DIS” and “True DIS”.

VIII. CONCLUSIONS

This paper presents the double-differential and single-differential cross sections as a function of the muon transverse and longitudinal momenta using data recorded by the MINERvA detector in the NuMI beamline. The high statistics and high neutrino energy of these new data, combined with a low flux uncertainty compared to previous measurements [13] mean that new regions of kinematic phase space can be examined to unprecedented precision. The data are compared to model predictions that reweight different components in the GENIE prediction or to external generator predictions. Some modifications to the GENIE prediction are inspired by measurements of previous exclusive or restricted phase space measurements on earlier data taken by MINERvA. Other

Process Variant	Standard χ^2	Log-normal χ^2
MINERvA Tune v1	6786	7494
GENIE 2.12.6	8241	7892
GENIE 2.12.6 and NonResPionTune Only	9764	9910
GENIE 2.12.6 and QE RPA	5661	6544
GENIE 2.12.6 and Low Recoil Enhancement	12345	12074
MINERvA Tune v1 with nCTEQ15	6803	7530
MINERvA Tune v1 with nCTEQ ν	6954	7762
MINERvA Tune v1 with AMU	7652	8793
MINERvA Tune v1 using MK	6224	7049
MINERvA Tune v1 with Low Q^2 Pion - MINOS	4553	6388
MINERvA GENIE tune v2	5022	7833
GiBUU v2019	4149	9394
GiBUU v2021	4872	9842
NuWro with Spectral Function	5151	6394
NuWro with Local Fermi Gas	3789	4944
NEUT with Spectral Function	9151	10020
NEUT with Local Fermi Gas	6251	7452

TABLE I. χ^2 of various model variants compared to data using the standard and log-normal χ^2 where there are 205 degrees of freedom.

modifications to the GENIE prediction (MK, AMU, nCTEQ15, and nCTEQ ν) represent replacements of a particular set of interaction channels. Finally, generators other than GENIE provide a different set of nuclear models, particle transport, and interaction channel models. None of these predictions describe the data well based on χ^2 tests. Similar model preferences were measured by the inclusive double-differential cross section measured [13] in the 3 GeV neutrino focused beam.

The single- and double- differential measurements provide indication of the need for a low Q^2 suppression (low p_t) for the resonant pion production channel. In addition, most of the components in MINERvA Tune v1 are favored, while the low recoil enhancement from increased 2p2h production is disfavored. In Fig. 19 the region separating “Soft DIS” and “True DIS” (see Fig. 14, lower panel, blue and purple lines) shows up when applying the slightly preferred nCTEQ15 PDF. Overall, NuWro with a local Fermi gas nuclear model best describes the data. The result cannot differentiate the specific source of mis-modeling in regions where all the underlying processes contribute to the prediction. Other methods, either via exclusive, semi-inclusive, or inclusive measurements using other kinematic variables are needed to investigate these complex regions. This work represents an important benchmark that can be used to validate future ensembles of models tuned to agree with exclusive results.

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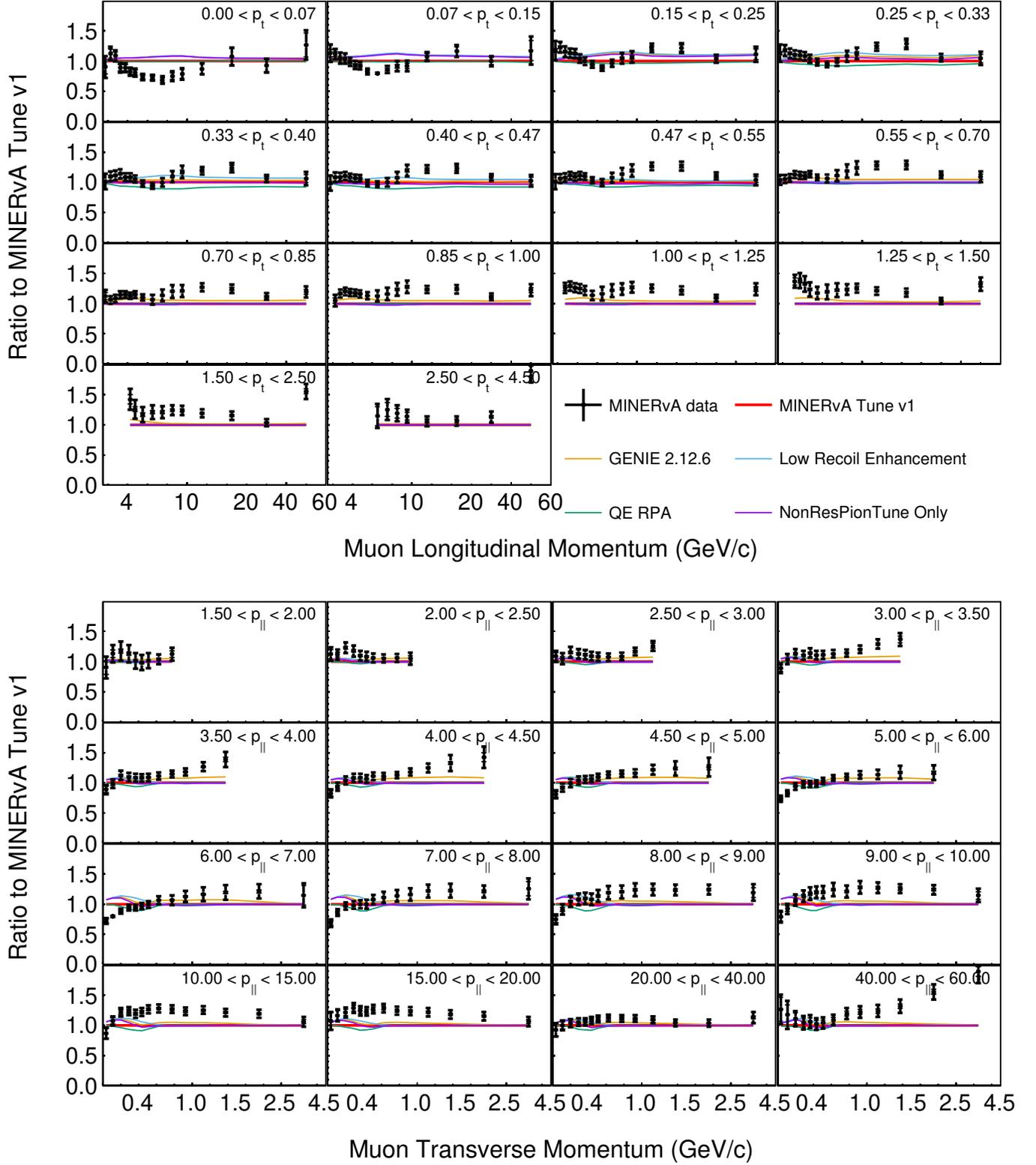


FIG. 15. Extracted cross section and four predictions based on specific modifications to components of the MINERvA Tune v1 displayed as a ratio to MINERvA Tune v1. Inner (outer) ticks denote statistical (total) uncertainty.

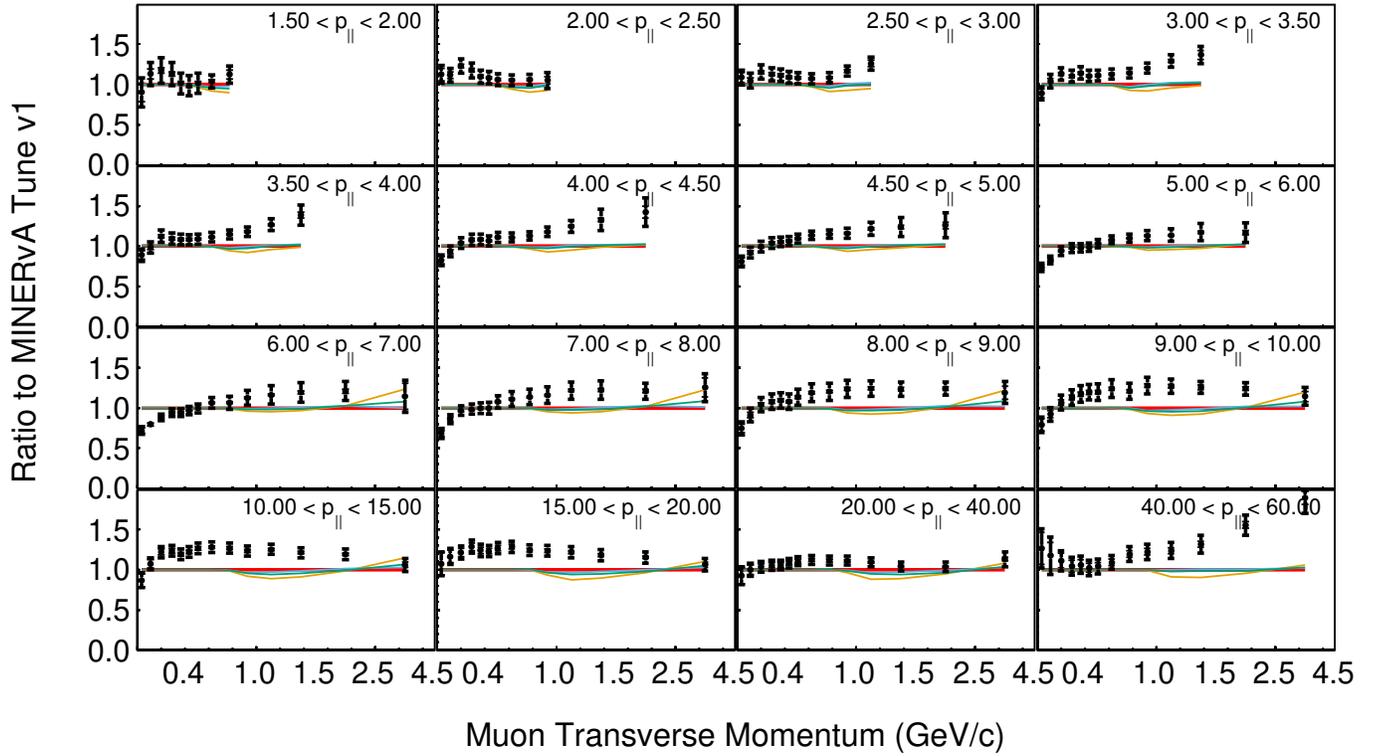
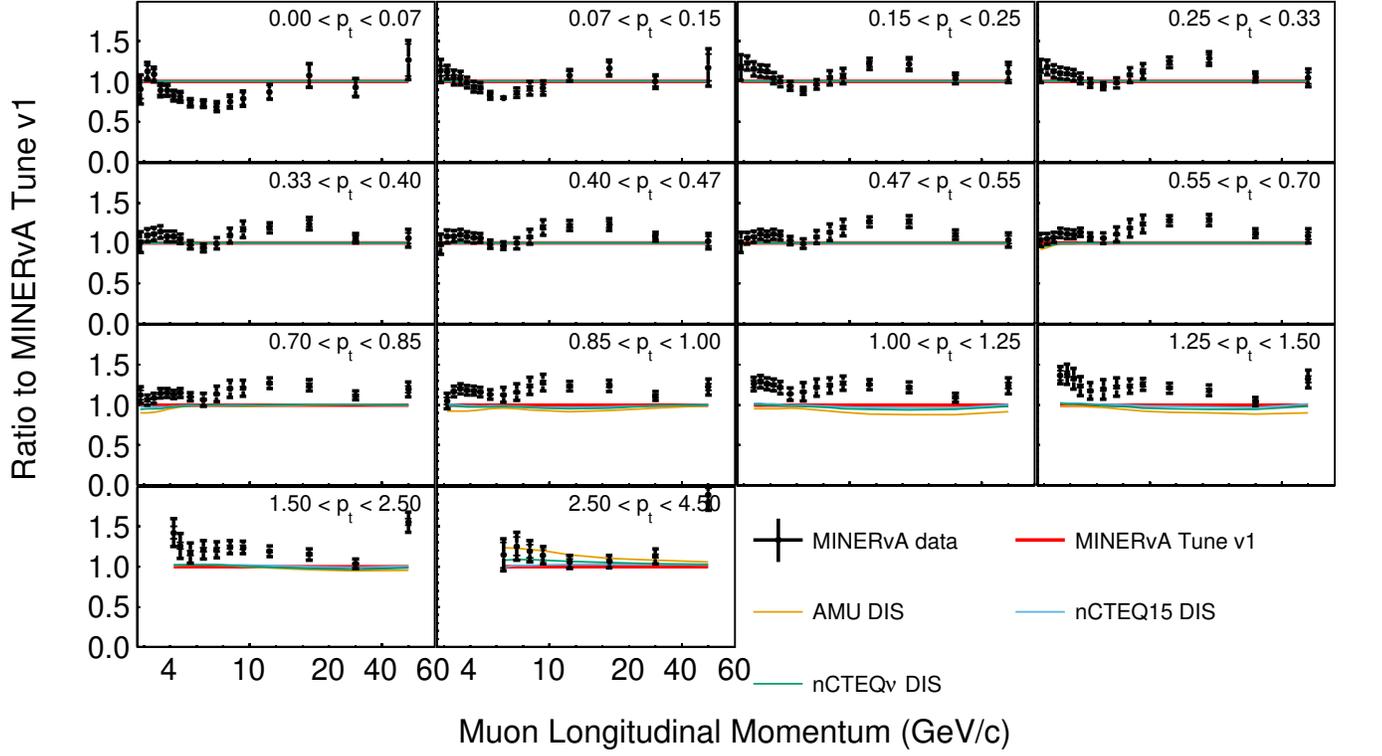


FIG. 16. The extracted cross section and predictions that modify the DIS models displayed as a ratio to MINERvA Tune v1. Inner (outer) ticks denote statistical (total) uncertainty.

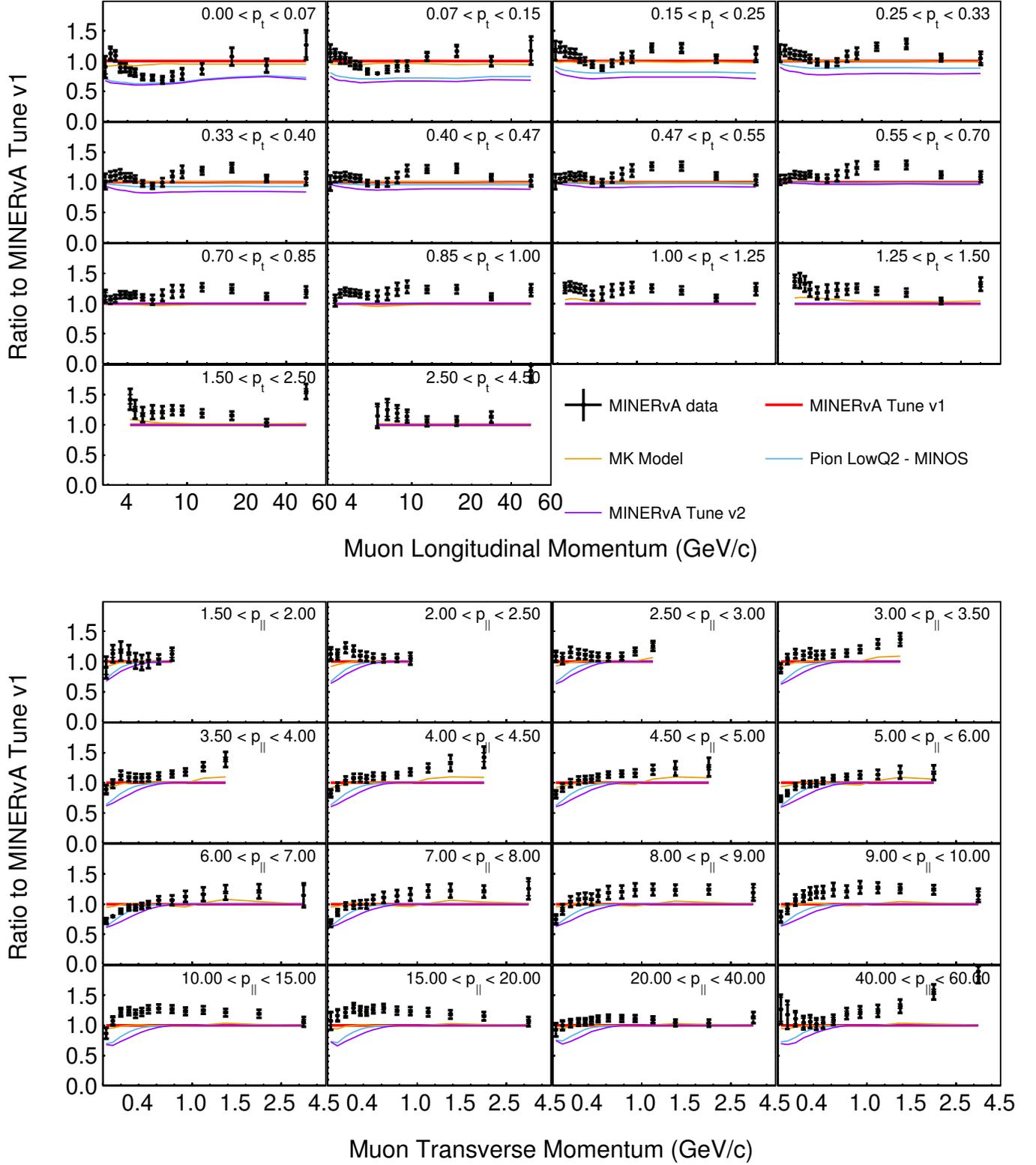


FIG. 17. The extracted cross section and predictions modifying the resonant pion prediction taken as a ratio to MINERvA Tune v1. Inner (outer) ticks denote statistical (total) uncertainty.

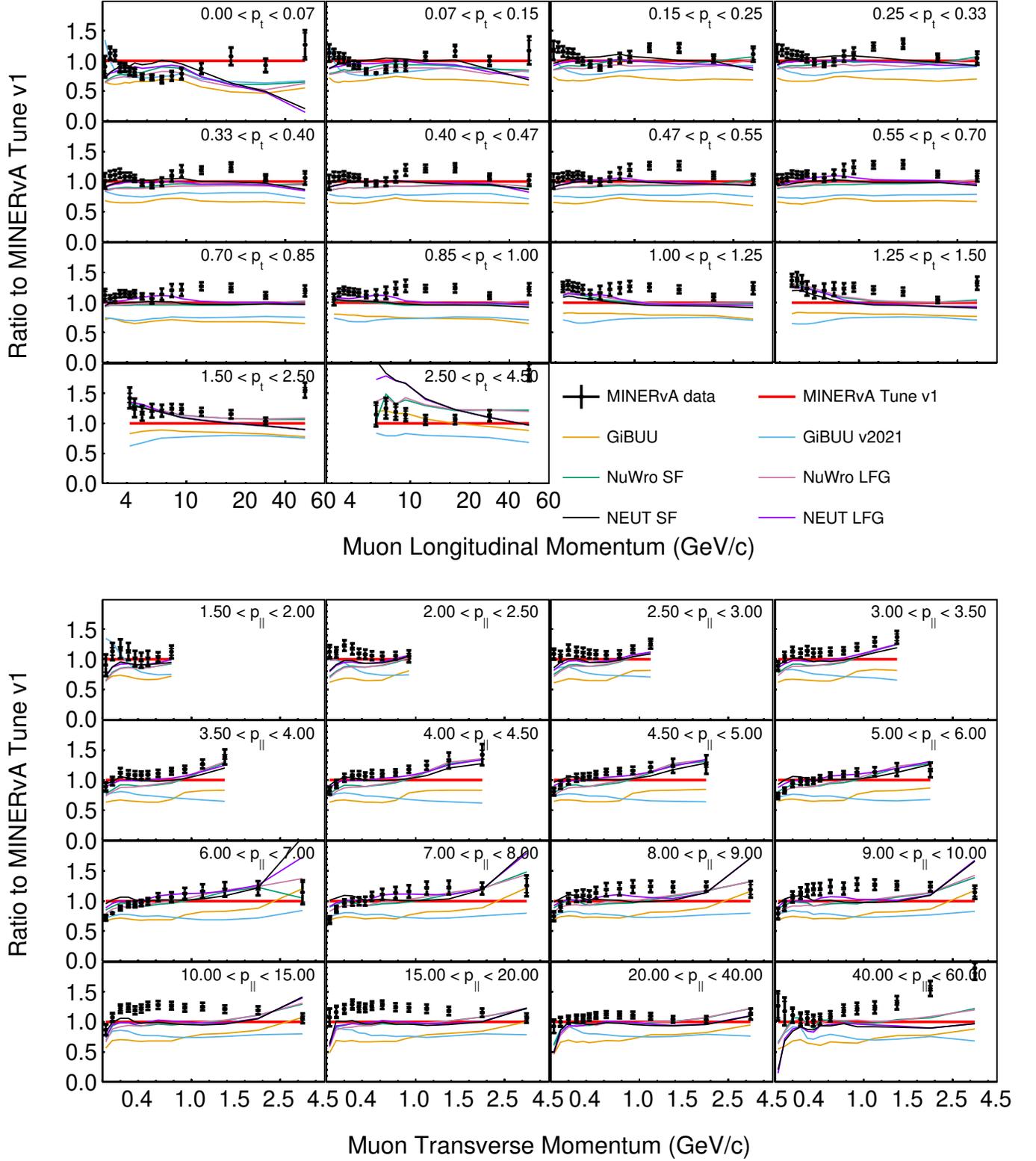


FIG. 18. The extracted cross section and external generator predictions displayed as a ratio to MINERvA Tune v1. Inner (outer) ticks denote statistical (total) uncertainty.

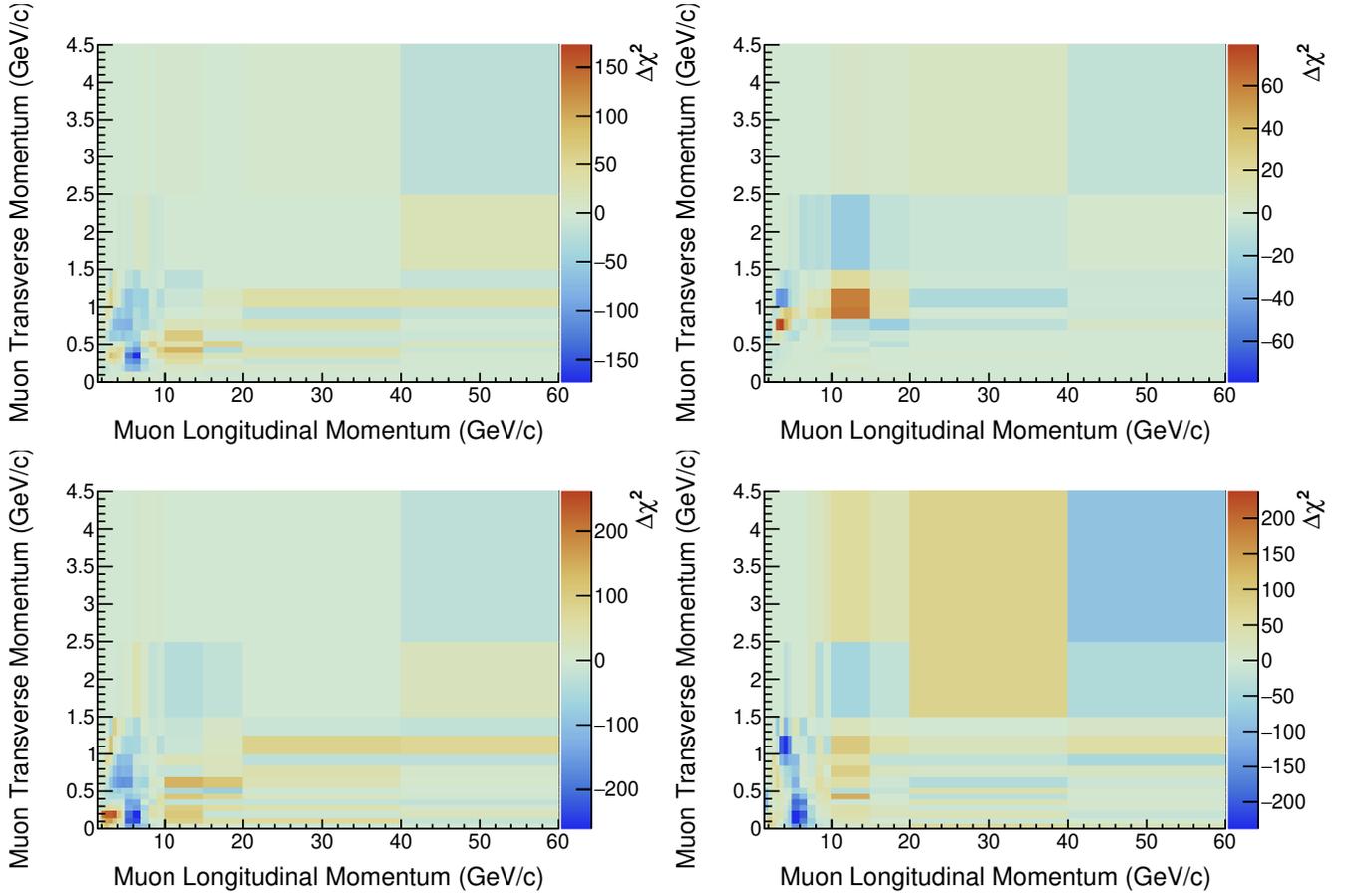


FIG. 19. Difference between MINERvA Tune v1 and the leading model from each group. Top left is GENIE 2.12.6 and QE RPA and a non-resonant pion reduction; top right is MINERvA Tune v1 with nCTEQ ν ; bottom left is MINERvA Tune v1 with low Q^2 pion - MINOS; bottom right is NuWro with local Fermi gas.

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