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Higgs alignment and the top quark

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Abstract

There is a surprising connection between the top quark and Higgs alignment in Gildener-Weinberg multi-Higgs doublet models. Were it not for the top quark and its large mass, the coupling of the 125 GeV Higgs boson H to gauge bosons and fermions would be indistinguishable from those of the Standard Model Higgs. The top quark's coupling to a single Higgs doublet breaks this perfect alignment in higher orders of the Coleman-Weinberg loop expansion of the effective potential. But the effect is still small, $\lesssim \mathcal{O}(1\%)$, and probably experimentally inaccessible.

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The 125 GeV Higgs boson H discovered at the LHC in 2012 [1, 2] is consistent in all measurements with the single Higgs boson of the Standard Model (SM).[1] This is puzzling, because many well-motivated attempts to cure the problems of the SM — most famously, naturalness — require two or more Higgs multiplets. Why, then, does H have SM couplings? The usual answer is "Higgs alignment" [3, 4, 5, 6]. However, with a few exceptions that rely on elaborate global symmetries or supersymmetry [7, 8, 9, 10], implementations of alignment suffer large radiative corrections.

In Gildener-Weinberg (GW) multi-Higgs models of electroweak (EW) symmetry breaking [11], H is a Goldstone boson of spontaneously broken scale symmetry. In tree approximation, H *naturally* has the same structure structure as the Goldstone bosons eaten by $W^\pm$ and Z^0. In N-Higgs-doublet models (NHDMs),

$$\Phi_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}\phi_i^+ \\ \rho_i + i a_i \end{pmatrix}, \quad i = 1, 2, \dots N, \tag{1}$$

these Goldstone bosons are

$$w^\pm = \sum_{i=1}^N v_i \phi_i^\pm / v, \quad z = \sum_{i=1}^N v_i a_i / v, \tag{2}$$

where v_i is the vacuum expectation value (VEV) of the CP-even scalar ρ_i and $v = \sqrt{\sum_{i=1}^N v_i^2}$. The Higgs boson is

$$H = \sum_{i=1}^N v_i \rho_i / v. \tag{3}$$

Thus, at tree level, H has exactly the same couplings to EW gauge bosons and to fermions (and, hence, to the gluon and the photon) as the single Higgs boson of the Standard Model; i.e., H is aligned.

In this Letter, we show that, but for the top quark, this alignment would be perfect through second order in the Coleman-Weinberg loop expansion of

[1] `https://atlas.web.cern.ch/Atlas/GROUPS/PHYSICS/CombinedSummaryPlots/HIGGS/`, `https://cms-results.web.cern.ch/cms-results/public-results/publications/HIG/index.html`, `https://pdg.lbl.gov/2020/reviews/rpp2020-rev-higgs-boson.pdf`.

the effective potential [12]. The top quark's presence upsets perfect alignment, but only by a small amount, at most $\mathcal{O}(1\%)$. The consequence of this is that experimental searches for new Higgs boson, such as H', A and $H^\pm$, via fusion of and decay to weak boson pairs and via Drell-Yan production in association with H — will remain fruitless. Promising discovery modes at the LHC still are gg-fusion of A, H' and gb-production of $H^\pm$ in association with a top quark. The new Higgs scalars in GW models must be lighter than about 400 GeV, so they are well within reach of the LHC [13, 14].

We discuss the top quark's role in Higgs alignment in the context of an $N = 2$ Higgs doublet model introduced by Lee and Pilaftsis in 2012 [15]. However, by the Glashow-Weinberg criterion that all quarks of a given electric charge must couple to only one scalar doublet to avoid flavor-changing neutral current (FCNC) interactions mediated by neutral Higgs exchange [16], our conclusion is true in any GW-NHDM. This GW-2HDM was updated in 2018 [13] to make it consistent with LHC data. The modification required the following $\mathbb{Z}_2$ symmetry on the Higgs doublets and fermions:

$$\Phi_1 \to -\Phi_1,\ \Phi_2 \to \Phi_2, \quad \psi_L \to -\psi_L,\ \psi_{uR} \to \psi_{uR},\ \psi_{dR} \to \psi_{dR}. \tag{4}$$

This is the usual type-I 2HDM [17], but with Φ_1 and Φ_2 interchanged. (The effect of this is that experimental limits on $\tan\beta = v_2/v_1$ in type-I models limit $\cot\beta$ in this model.) The operative experimental constraint came from CMS [18] and ATLAS [19] searches for charged Higgs decay into $t\bar{b}$. Consistency with those experiments required $\tan\beta \lesssim 0.5$ for $M_{H^\pm} \lesssim 500\,\mathrm{GeV}$. Because t and b-backgrounds at low masses are quite large, this is still the limit on $\tan\beta$. We urge much greater effort to suppress these backgrounds in the 200–500 GeV mass range.

The GW tree-level potential for this model is purely quartic [11] so that, since all masses in the model arise from Higgs VEVs, the Lagrangian is scale-invariant at this level:

$$\begin{aligned} V_0(\Phi_1,\Phi_2) &= \lambda_1(\Phi_1^\dagger\Phi_1)^2 + \lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) \\ &+ \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + \tfrac{1}{2}\lambda_5\left((\Phi_1^\dagger\Phi_2)^2 + (\Phi_2^\dagger\Phi_1)^2\right). \end{aligned} \tag{5}$$

The five quartic couplings λ_i in Eq. (5) are real and, so, V_0 is CP-invariant. The couplings $\lambda_{1,2} > 0$ for positivity of the potential.

The trivial minimum of V_0 occurs at $\Phi_1 = \Phi_2 = 0$. But a nontrivial flat minimum of V_0 can occur on the ray

$$\Phi_{1\beta} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \phi\, c_\beta \end{pmatrix}, \quad \Phi_{2\beta} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \phi\, s_\beta \end{pmatrix}, \tag{6}$$

where $c_\beta = \cos\beta$, $s_\beta = \sin\beta$ with $\beta \neq 0, \pi/2$ a fixed angle and $0 < \phi < \infty$ a real mass scale.[2] The nontrivial extremal conditions are

$$\lambda_1 + \tfrac{1}{2}\lambda_{345}\tan^2\beta = \lambda_2 + \tfrac{1}{2}\lambda_{345}\cot^2\beta = 0, \tag{7}$$

where $\lambda_{345} = \lambda_3 + \lambda_4 + \lambda_5 < 0$ for positivity of V_0. Eqs. (7) hold in all orders of the loop expansion of the effective potential [11]; this is important in our subsequent development (see Eqs. (12,13,14)). This extremum spontaneously (but not explicitly) breaks scale invariance as well as the EW gauge symmetry and H is the corresponding Goldstone boson.

Following Ref. [14], we use the "aligned basis" of the Higgs fields because the scalars' mass matrices will remain very nearly diagonal in that basis beyond the tree approximation (also see Ref. [6]). That is the essence of Higgs alignment in GW models and, in this and similar models, it is broken, but only slightly, by the top quark. This basis is:

$$\begin{aligned} \Phi &= \Phi_1 c_\beta + \Phi_2 s_\beta = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2} w^+ \\ H + iz \end{pmatrix}, \\ \Phi' &= -\Phi_1 s_\beta + \Phi_2 c_\beta = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2} H^+ \\ H' + iA \end{pmatrix}. \end{aligned} \tag{8}$$

On the ray Eq. (6) on which V_0 has nontrivial extrema, these fields are $\Phi = (0, \phi)/\sqrt{2}$ and $\Phi' = 0$. In this basis, $H, z, w^\pm$ are massless and unmixed with the $H', A, H^\pm$ whose "masses" are

$$M_{H'}^2 = -\lambda_{345}\phi^2, \;\; M_A^2 = -\lambda_5 \phi^2, \;\; M_{H^\pm}^2 = -\tfrac{1}{2}\lambda_{45}\phi^2. \tag{9}$$

Thus, the flat potential is indeed a minimum on the ray (6) (albeit degenerate with the trivial one) if, like λ_{345}, λ_5 and $\lambda_{45} = \lambda_4 + \lambda_5$ are negative.

[2] It is easily proved that any such purely quartic potential as well as its first derivatives vanish at *any* extremum so that $V_0(\Phi_{i\beta}) = 0$ [14].

To establish the top quark's role in Higgs alignment of GW-NHDMs, it suffices to consider this model in one-loop order of the effective potential, $V_0 + V_1$. This potential provides a lower minimum than $V_0 = 0$ by picking out a particular value v of ϕ, explicitly breaking the scale symmetry of V_0, and giving H a nonzero mass. The one-loop potential is [20]

$$V_1 = \frac{1}{64\pi^2} \sum_n \alpha_n \overline{M}_n^4 \left(\ln \frac{\overline{M}_n^2}{\Lambda_{\rm GW}^2} - k_n \right). \tag{10}$$

Only very massive particles contribute to V_1. They are $n = (W^\pm, Z, t, H', A, H^\pm)$ in this model. The constants are $\alpha_n = (6, 3, -12, 1, 1, 2)$; $k_n = 5/6$ for the weak gauge bosons and $3/2$ for scalars and the top-quark.[3] The background-field-dependent masses $\overline{M}_n^2$ in Eq. (10) are [21, 15]

$$\overline{M}_n^2 = \begin{cases} M_n^2 \left(2\left(\Phi^\dagger\Phi + \Phi'^\dagger\Phi'\right)/\phi^2\right) = M_n^2 \left(H^2 + H'^2 + \cdots/\phi^2\right), & n \neq t \\ M_t^2 \left(2\Phi_1^\dagger\Phi_1/(\phi c_\beta)^2\right) = M_t^2 \left((Hc_\beta - H's_\beta)^2 + \cdots\right)/(\phi c_\beta)^2, & \end{cases} \tag{11}$$

where $M_n^2 \propto \phi^2$ is the actual squared "mass" of particle n. The form of $\overline{M}_t^2$ is dictated by the type-I coupling of fermions to the Φ_1 doublet in Eq.(4). This difference controls the breaking of Higgs alignment through second order in the loop expansion. The renormalization scale $\Lambda_{\rm GW}$ will be fixed relative to the Higgs VEV $v = 246\,\text{GeV}$ in Eq. (15) below.

The one-loop extremal conditions are [11]

$$\left.\frac{\partial(V_0 + V_1)}{\partial H}\right|_{\langle\rangle + \delta_1 H + \delta_1 H'} = \left.\frac{\partial(V_0 + V_1)}{\partial H'}\right|_{\langle\rangle + \delta_1 H + \delta_1 H'} = 0. \tag{12}$$

Here, we follow GW's analysis by expanding around the tree-level VEVs $\langle H\rangle = \phi$, $\langle H'\rangle = 0$ while allowing for $\mathcal{O}(V_1)$ shifts $\delta_1 H$ and $\delta_1 H'$ in those VEVs — and from perfect Higgs alignment. Recall that the tree-level extremal conditions $(\partial V_0/\partial H)_{\langle\rangle} = (\partial V_0/\partial H')_{\langle\rangle} = 0$ remain in force. To $\mathcal{O}(V_1)$ this expansion results in

$$\left.\frac{\partial V_1}{\partial H}\right|_{\langle\rangle} = \frac{1}{16\pi^2 v} \sum_n \alpha_n M_n^4 \left(\ln \frac{M_n^2}{\Lambda_{\rm GW}^2} + \frac{1}{2} - k_n \right) = 0, \tag{13}$$

$$\left.\frac{\partial^2 V_0}{\partial H'^2}\right|_{\langle\rangle} \delta_1 H' + \left.\frac{\partial V_1}{\partial H'}\right|_{\langle\rangle} = M_{H'}^2\, \delta_1 H' - \frac{\alpha_t M_t^4 \tan\beta}{16\pi^2 v} \left(\ln \frac{M_t^2}{\Lambda_{\rm GW}^2} + \frac{1}{2} - k_t \right) = 0, \tag{14}$$

[3] V_1 was calculated in the Landau gauge using the $\overline{\rm MS}$ renormalization scheme.

where, by Eq. (11), the first derivative with respect to H' of the $n \neq t$ terms in V_1 vanish because they are quadratic in H'. Eq. (13) provides a definition of the renormalization scale $\Lambda_{\rm GW}$ in terms of the VEV $\phi = v$ at which the minimum of V_1 occurs. It can be rewritten as [11]

$$0 = \sum_n \alpha_n M_n^4 \left(\ln \frac{M_n^2}{\Lambda_{\rm GW}^2} + \frac{1}{2} - k_n\right) = A + \tfrac{1}{2}B + B \ln\left(\frac{v^2}{\Lambda_{\rm GW}^2}\right), \qquad (15)$$

where $A = \sum_n \alpha_n M_n^4(\ln(M_n^2/v^2) - k_n)$ and $B = \sum_n \alpha_n M_n^4$, so that $\ln(\Lambda_{\rm GW}^2/v^2) = A/B + \frac{1}{2}$.[4] Note that $M_n^2 \propto v^2$ so that $\Lambda_{\rm GW}/v$ is a function of coupling constants only.

From Eq. (14), the shift $\delta_1 H'$ in $\langle H' \rangle$ is given by the tadpole formula:

$$\delta_1 H' = -\frac{1}{M_{H'}^2} \frac{\partial V_1}{\partial H'}\bigg|_{\langle\rangle} = \frac{\alpha_t M_t^4 \tan\beta}{16\pi^2 M_{H'}^2 v}\left(\ln \frac{M_t^2}{\Lambda_{\rm GW}^2} + \frac{1}{2} - k_t\right). \qquad (16)$$

As an example of its magnitude, we take $M_{H'} = 400\,\text{GeV}$, $\Lambda_{\rm GW} = 260\,\text{GeV}$ and $\tan\beta = 0.5$. Then $\delta_1 H' = 1.57\,\text{GeV}$ which, when added in quadrature with $v = 246\,\text{GeV}$, amounts to an increase of $0.002\,\%$.[5]

Eq. (16) establishes the connection of the top quark to Higgs alignment: The large mass of the top quark ensures its appearance in the effective potential V_1 while the Glashow-Weinberg criterion [16] implies $(\partial\overline{M}_t^2/\partial H')_{\langle\rangle} \neq 0$; hence the small $\mathcal{O}(V_1)$ shift away from perfect alignment. The elements of

[4] As discussed in Ref. [11], Eq. (13) does not lead to a minimum of V_1 unless $B > 0$. With the known masses of $W^\pm, Z, t$ and H, $B > 0$ and, importantly, it leads to a sum rule for the heavier Higgs masses that requires them to be $\lesssim \mathcal{O}(400\,\text{GeV})$ [15, 22, 13, 14].

[5] Because $\delta_1 H$ is not determined in $\mathcal{O}(V_1)$, we can set it to zero here. This is consistent with our expectation that $\delta H = \mathcal{O}(\delta^2)$ where $\delta = \mathcal{O}(V_1)$ is the H–H' mixing angle.

the CP-even mass matrix $\mathcal{M}^2_{0^+}$ in $CO(V_1)$ further emphasize this connection:

$$\mathcal{M}^2_{HH} = \left.\frac{\partial^2 V_1}{\partial H^2}\right|_{\langle\rangle} = \frac{1}{8\pi^2 v^2}\sum_n \alpha_n M_n^4, \tag{17}$$

$$\begin{aligned}\mathcal{M}^2_{HH'} &= \left.\frac{\partial^3 V_0}{\partial H \partial H'^2}\right|_{\langle\rangle} \delta_1 H' + \left.\frac{\partial^2 V_1}{\partial H \partial H'}\right|_{\langle\rangle} \\ &= -\frac{\alpha_t M_t^4 \tan\beta}{16\pi^2 v^2}\left(\ln\frac{M_t^2}{\Lambda^2_{\rm GW}} + \frac{5}{2} - k_t\right),\end{aligned} \tag{18}$$

$$\begin{aligned}\mathcal{M}^2_{H'H'} &= \left.\frac{\partial^2 V_0}{\partial H'^2}\right|_{\langle\rangle} + \left.\frac{\partial^3 V_0}{\partial H'^3}\right|_{\langle\rangle} \delta_1 H' + \left.\frac{\partial^2 V_1}{\partial H'^2}\right|_{\langle\rangle} \\ &= M^2_{H'} + \frac{\alpha_t M_t^4}{8\pi^2 v^2}\left(\ln\frac{M_t^2}{\Lambda^2_{\rm GW}} + \frac{1}{2} - k_t + \tan^2\beta\right).\end{aligned} \tag{19}$$

At this level, only the top quark prevents $\mathcal{M}^2_{0^+}$ being diagonal and the Higgs boson being completely aligned.

To repeat: Because the Glashow-Weinberg criterion applies to *any* EW model in which quarks of a given charge acquire all their mass from the scalars, the top quark's role in Higgs alignment holds in any GW-NHDM. The additional complications of the two-loop effective potential do not alter this conclusion.[6]

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