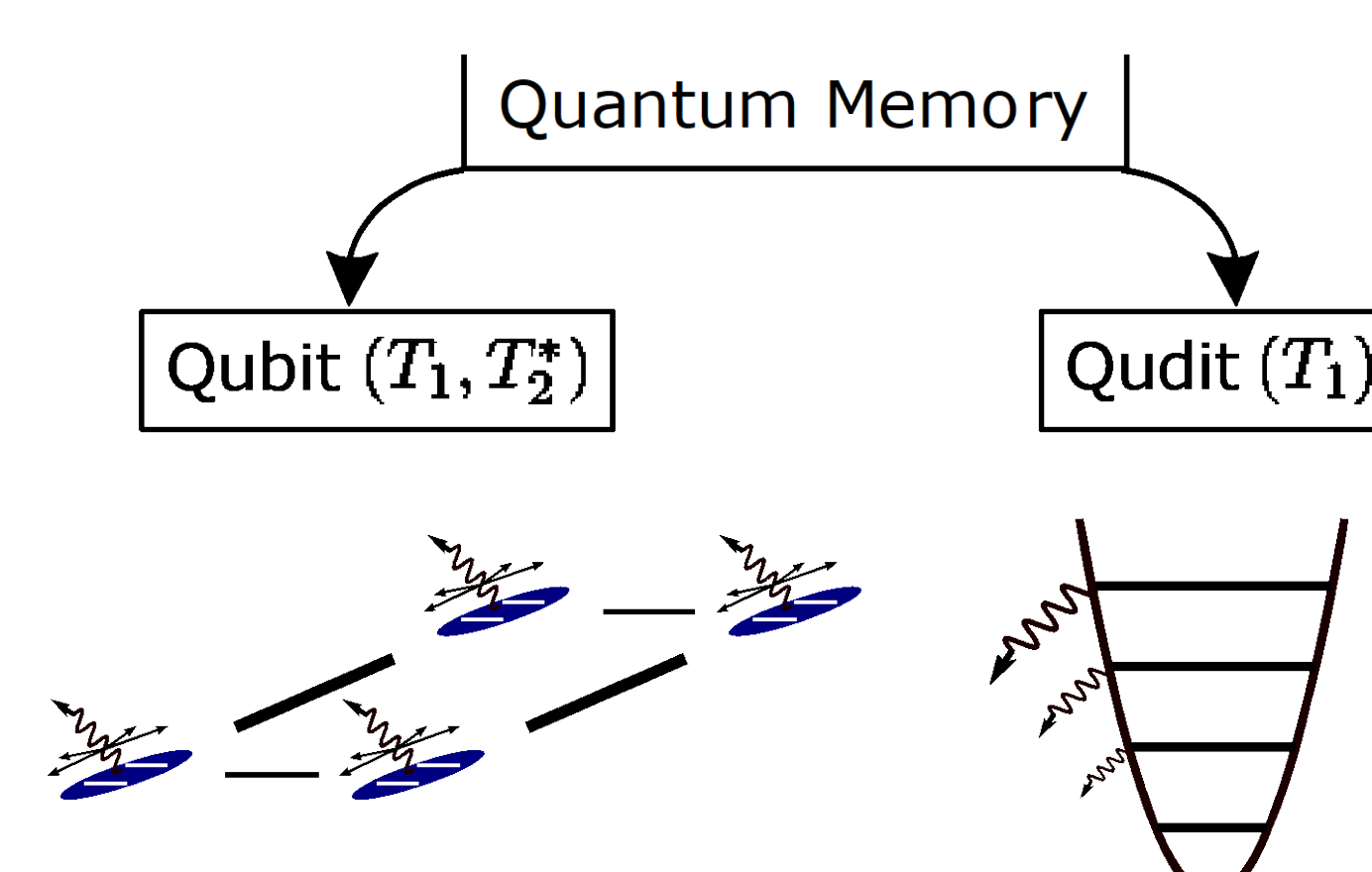


# Qudits as Quantum Memory (Phys. Rev. A 104, 012605 (2021))

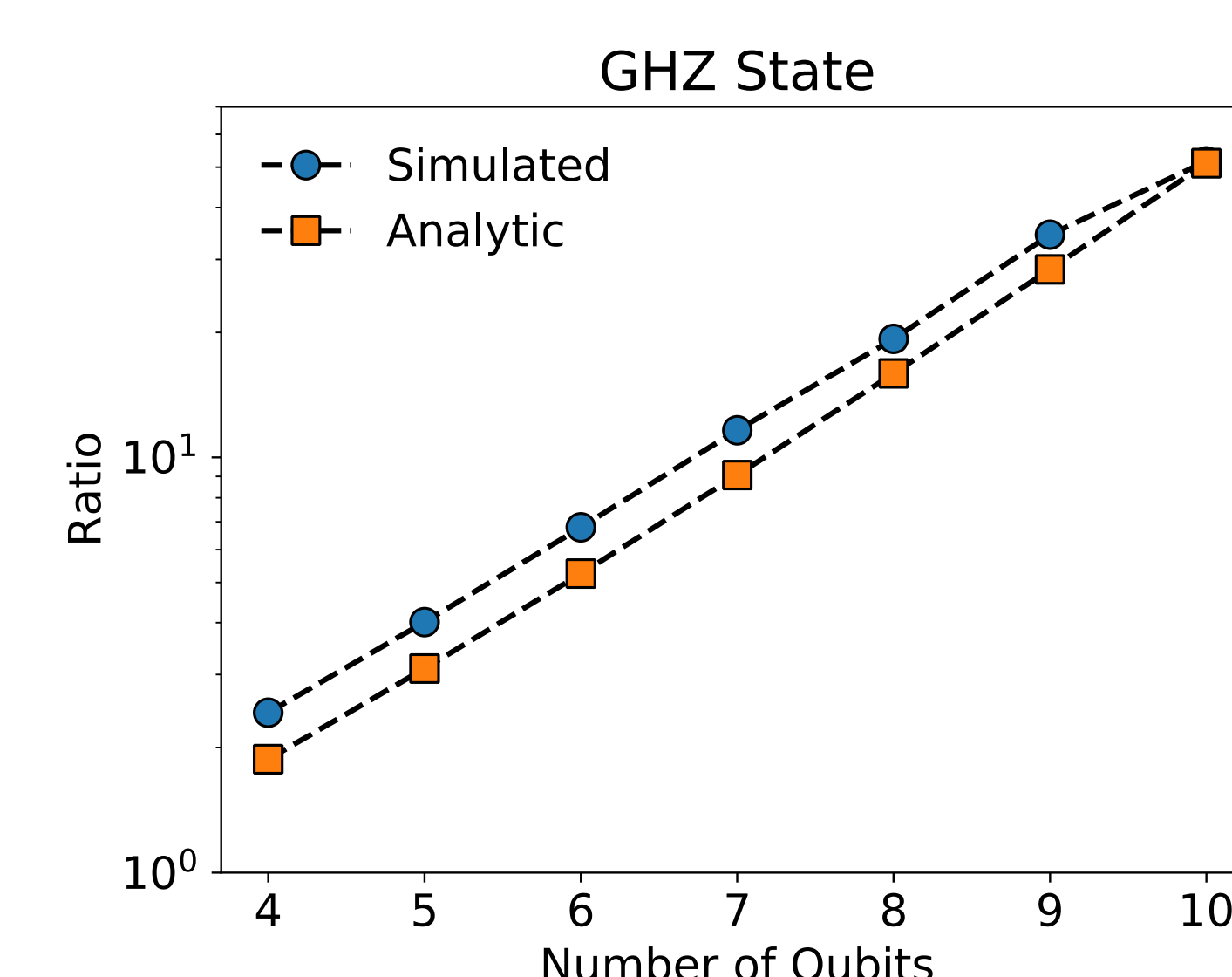
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## Introduction

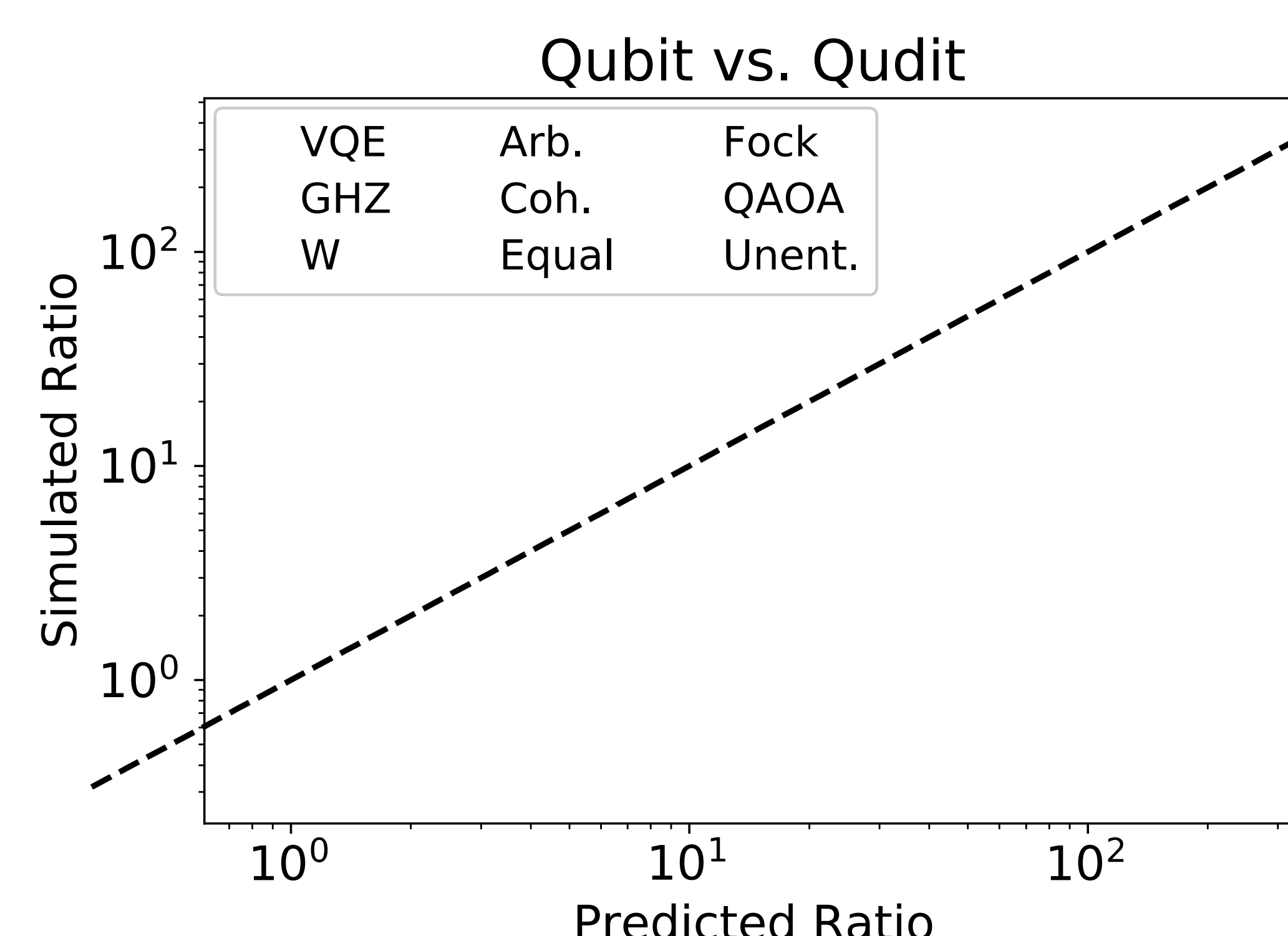
The need for high-quality quantum memories to store quantum states is becoming ever more pressing. Here, we study the correlation of the structure of quantum information with physical noise models of various possible quantum memory implementations. Through numerical simulation of different noise models and approximate analytical formulas applied to a variety of quantum states, we provide comparisons which point to simple, experimentally relevant formulas for the relative lifetimes of quantum information in different quantum memories.



Left: A quantum memory comprising qubits subject to amplitude damping and pure dephasing. Right: A quantum memory comprising a single qudit subject to amplitude damping. The noise grows as the number of levels increases.



Scaling ratio for the qudit-based quantum memory to perform as well as the qubit-based quantum memory versus the number of qubits for the GHZ state. The ratio grows exponentially as the number of qubits grows, which we can see by following the non-Hermitian treatment is expressed as  $\frac{t_b}{t_d} = \frac{2^{n_q}-1}{2^{n_q}}$  where  $n_q$  is the number of qubits needed to express the state.



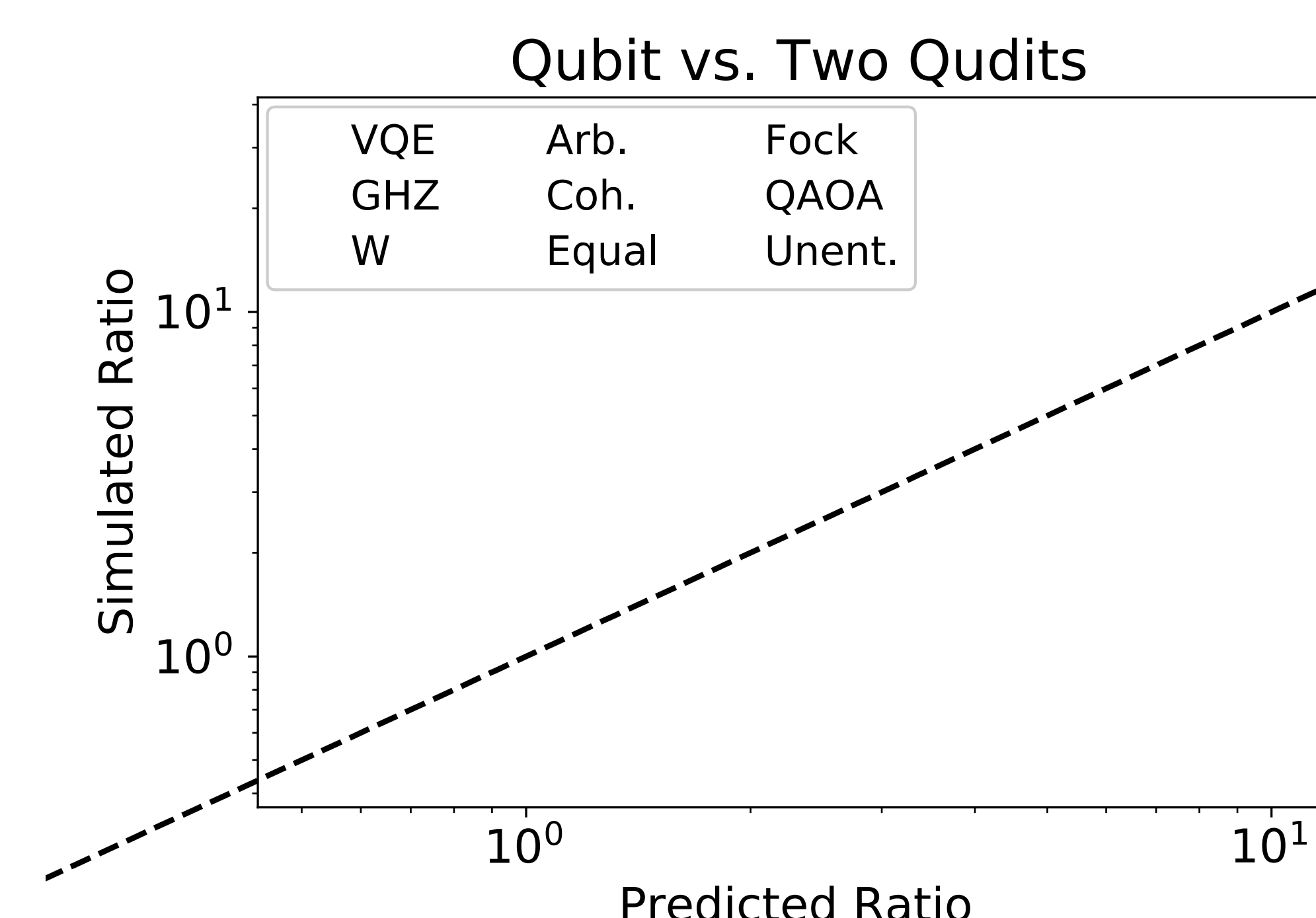
Comparison of the numerically simulated and analytically predicted scaling ratios between a qubit-based quantum memory with both amplitude damping and dephasing and a qudit-based quantum memory with only amplitude damping, for a wide of variety of interesting quantum states. The dashed black line is  $y=x$ ; the closer the points are to this line, the better the prediction. The prediction is derived from a Taylor expansion of the non-Hermitian analysis, giving an expression for the ratio as:  $\frac{t_b}{t_d} \approx \frac{\langle n_d \rangle}{2\langle n_b \rangle}$  where  $\langle n_d \rangle$  is the expectation value of the number operator for the qudit and  $\langle n_b \rangle$  is the Hamming weight of the qubits.

## Lindblad Master Equation

The Lindblad Master Equation provides time evolution of a quantum state in an open quantum system. We utilize this to see the effects of noise on time evolution of a quantum state in a quantum memory:  $\frac{d\rho(t)}{dt} = \sum_i \gamma_i L(C_i)[\rho(t)]$  where

$$L(C_i)[\rho(t)] = \left( C_i \rho(t) C_i^\dagger - \frac{1}{2} \{ C_i^\dagger C_i, \rho(t) \} \right)$$

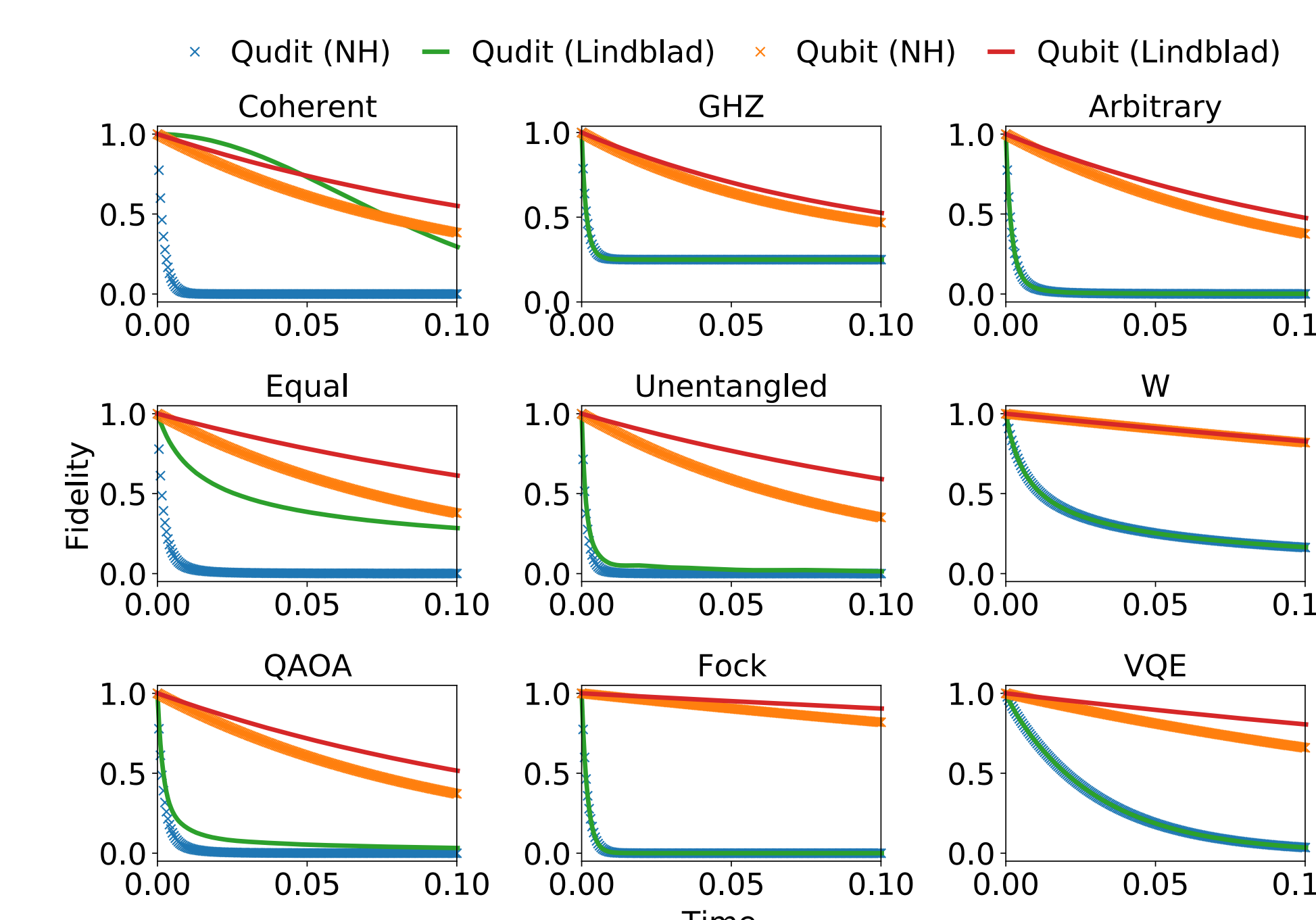
We utilize QuaC\* to provide the time evolution in this way. From here we can compare performance of noise models.



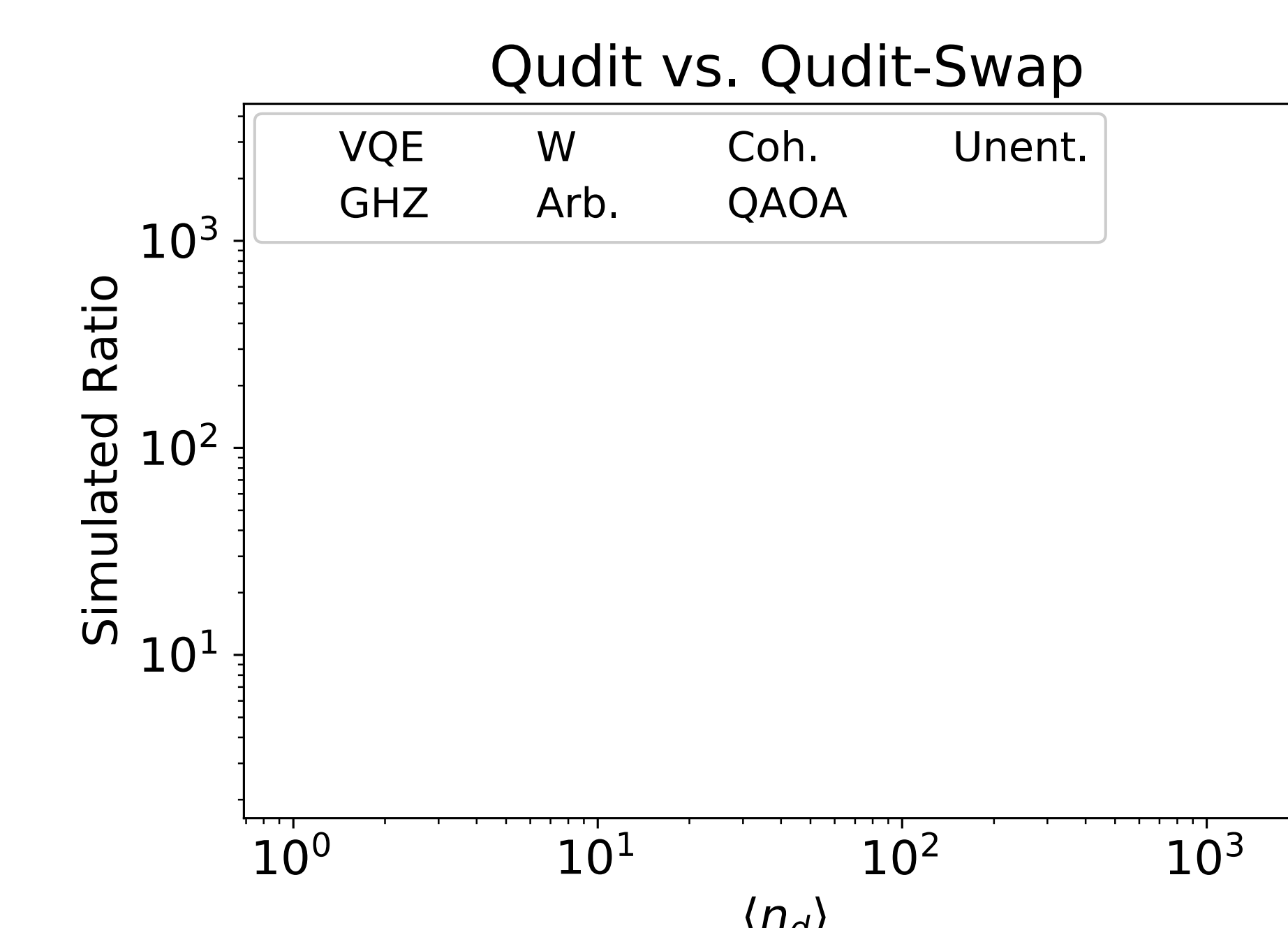
Comparison of the numerically simulated and analytically predicted scaling ratios between a qubit-based quantum memory with both amplitude damping and dephasing and an array of qudits with only amplitude damping, for a wide of variety of interesting quantum states. The dashed black line is  $y=x$ ; the closer the points are to this line, the better the prediction. The prediction is derived from a Taylor expansion of the non-Hermitian analysis, giving an expression for the ratio as:  $\frac{t_b}{t_{int}} \approx \frac{\langle n_{int} \rangle}{2\langle n_b \rangle}$  where  $\langle n_{int} \rangle$  is the expectation value of the number operator for the qudits and  $\langle n_b \rangle$  is the Hamming weight of the qubits.

## Non-Hermitian Analysis

Using a non-Hermitian analysis, we see that the time evolution of a state can be given as:  $\frac{d|\psi(t)\rangle}{dt} = \sum_i \frac{-\gamma_i}{2} C_i^\dagger C_i |\psi(t)\rangle$  where each  $C_i$  is equivalent to the dissipation operators in the Lindblad Master Equation. We can now get the fidelity as a function of time by exponentiating the non-Hermitian Hamiltonian described above. For example, a qubit noise model would be described by:  $\sqrt{F(t)} = \sum_j |\alpha_j|^2 e^{-\gamma w(j)t}$  where the original state is given as  $|\psi(0)\rangle = \sum_j \alpha_j |j\rangle$  where  $|j\rangle$  are eigenstates and  $w(j)$  is the Hamming weight of each eigenstate. Similarly, for the qudit we can get the following expression:  $\sqrt{F(t)} = \sum_j |\alpha_j|^2 e^{-\frac{\gamma}{2} j t}$ . We analyze when each system reaches a given fidelity to characterize the performance of the qudit.



Comparison of the full Lindblad dynamics and the non-Hermitian (NH) approximation for specific 10 qubit instances of the various states studied. Generally accurate, notice the discrepancies in the coherent and equal superposition states. For example the coherent state suffers in the non-Hermitian treatment due to it not capturing properties of the state including the fact that the state is an eigenstate of the destruction operator.



Performance enhancement from reordering the quantum information in a qudit-based quantum memory with amplitude damping. The simulated performance gain grows with increasing average number of excitations in the original state.

\*M. Otten, QuaC: Open quantum systems in C, a time-dependent open quantum systems solver, <https://github.com/Ott3r/QuaC> (2017)