General Approach to Physics Limits of Ultimate Colliders

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Abstract:

This paper presents an attempt to evaluate limits on energy, luminosity and social affordability of the ultimate future colliders - linear and circular, proton, electron positron and muon, based on traditional as well as on advanced accelerator technologies. .

Limits on E, L, C, P, size, etc

Limits on *Energy* (1)

- Linear vs Circular

$$\Delta U_{SR} = \frac{90\,\mathrm{keV}\cdot E_e^4(\mathrm{GeV})}{R[\mathrm{m}]}$$

$$\Delta U_{SR} < E_e$$

$$E_e < 500\,\mathrm{GeV}\cdot\left(\frac{R}{10\,\mathrm{km}}\right)^{1/3}$$

for muons: $\times\left(\frac{m_\mu}{m_e}\right)^{4/3}$

$$E_\mu < 600\,\mathrm{TeV}\cdot\left(\frac{R}{10\,\mathrm{km}}\right)^{1/3}$$

for protons: $\times\left(\frac{m_p}{m_e}\right)^{4/3}$

$$E_p < 10\,\mathrm{PeV}\cdot\left(\frac{R}{10\,\mathrm{km}}\right)^{1/3}$$

- Particles don't survive acceleration $dN/dt = -N/\gamma\tau_0$; $\frac{N}{N_0} \approx \left(\frac{m_\mu c^2}{E}\right)^{\kappa}$, $\kappa = (m_\mu c/\tau_0 G)$
  - Unstable particles: for muons $G \gg 3$ MeV m^{-1}; for τ-leptons $G \gg 0.3$ TeV m^{-1}
  - Lossy transport from cell to cell (loss in plasma material, c-t-c efficiency): $(1-\frac{\Delta N}{N})^M \leq 1$, $M = \frac{E}{\Delta E_{cell}} = \frac{5\,\mathrm{TeV}}{5\,\mathrm{GeV}} = 10^3$, $M\cdot\frac{\Delta N}{N} \leq 1 \Rightarrow \frac{\Delta N}{N} < 10^{-3}$

Circumference 100 km , B<16 T , E<50 TeV

Circumference 40,000 km, B=1 T, E<13 PeV

Length 50 km , G<0.1 GV/m, E<5 TeV

Length 10 km, G<1 TV/m, E<10 PeV

Limits on luminosity:

beam power

beam-beam limit… space-charge limit…beam loading…event pile-up…

particle production…beamstrahlung…synchrotron radiation… SR/m…

IR rad damage…ν-radiation dose… instabilities…jitter/emittance growth

Probability ≈ Cost^2/(1+Cost^4)

! WARNING !

The *αβγ* cost model:

$$\mathrm{Cost(TPC)} = \alpha\, L^{1/2} + \beta\, E^{1/2} + \gamma\, P^{1/2}$$

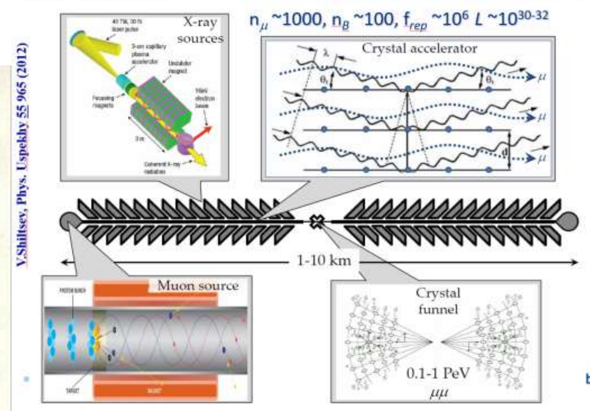
a) Is for a "green field" facility !
b) US-Accounting !
c) There is hidden correlation btw *E* and technology progress
d) Pay attention to units(10 km for *L*, 1 TeV for *E*, 100 MW for *P*)
- *α*≈ 2B$/sqrt(L/10 km)
- *β*≈ 10B$/sqrt(E/TeV) for SC/NC RF
- *β*≈ 2B$ /sqrt(E/TeV) for SC magnets
- *β*≈ 1B$ /sqrt(E/TeV) for NC magnets
- *γ*≈ 2B$/sqrt(P/100 MW)

USE AT YOUR OWN RISK!

Xtal Collider

n~10^22 cm^-3, 10 TeV/m → 1 PeV = 1000 TeV

n_μ~1000, n_B~100, f_{rep}~10^6 L~10^30-32



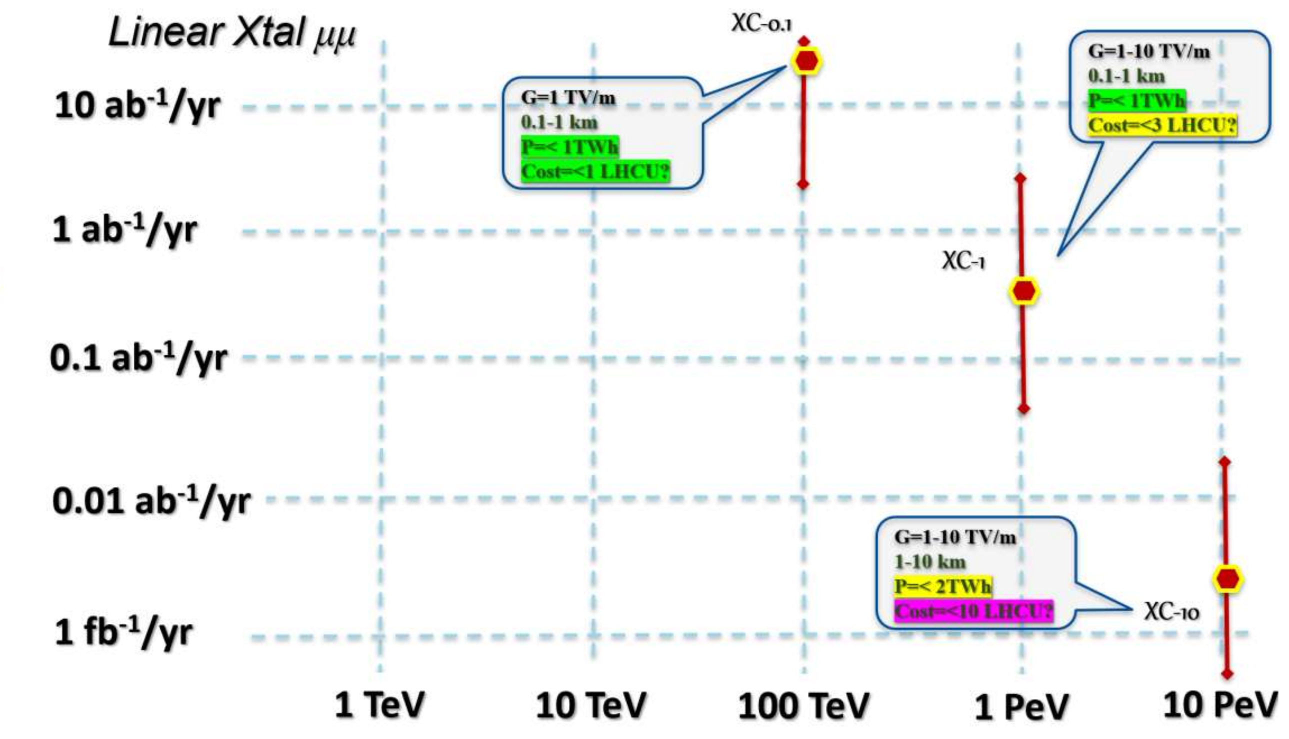
XC Luminosity

$$L = fN^2/A$$

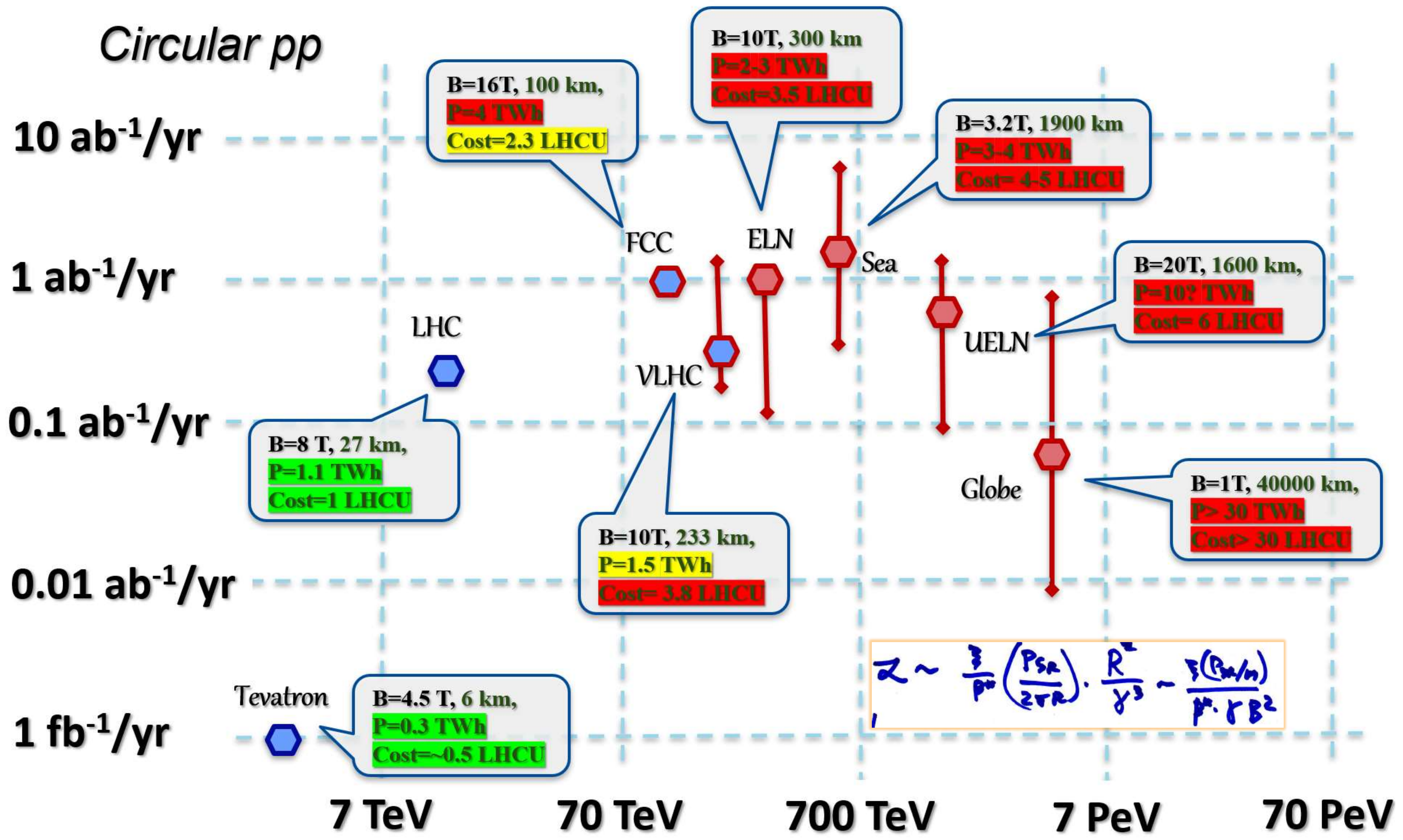
- Considerations :
  - Muons/bunch < Xtal electrons excited: $100\lambda_p \times \lambda_p^2$, $N_0 \sim 10^3$
  - Employ many channels: $n_{ch} \sim 100$, gain a factor of n_{ch}
  - Limit beam power O(10MW): $P = f n_{ch} N E$, $f = 10^6$ Hz
  - Combine *n_channels* to gain ***L*** via *crystal funnel* (? Is it possible)

$$L\,[\mathrm{sm}^{-2}\,\mathrm{s}^{-1}] \approx 4\times 10^{33-35}\,\frac{P^2\,[\mathrm{MW}]}{E^2\,[\mathrm{TeV}]\, f n_{ch}[10^8\,\mathrm{Hz}]}$$

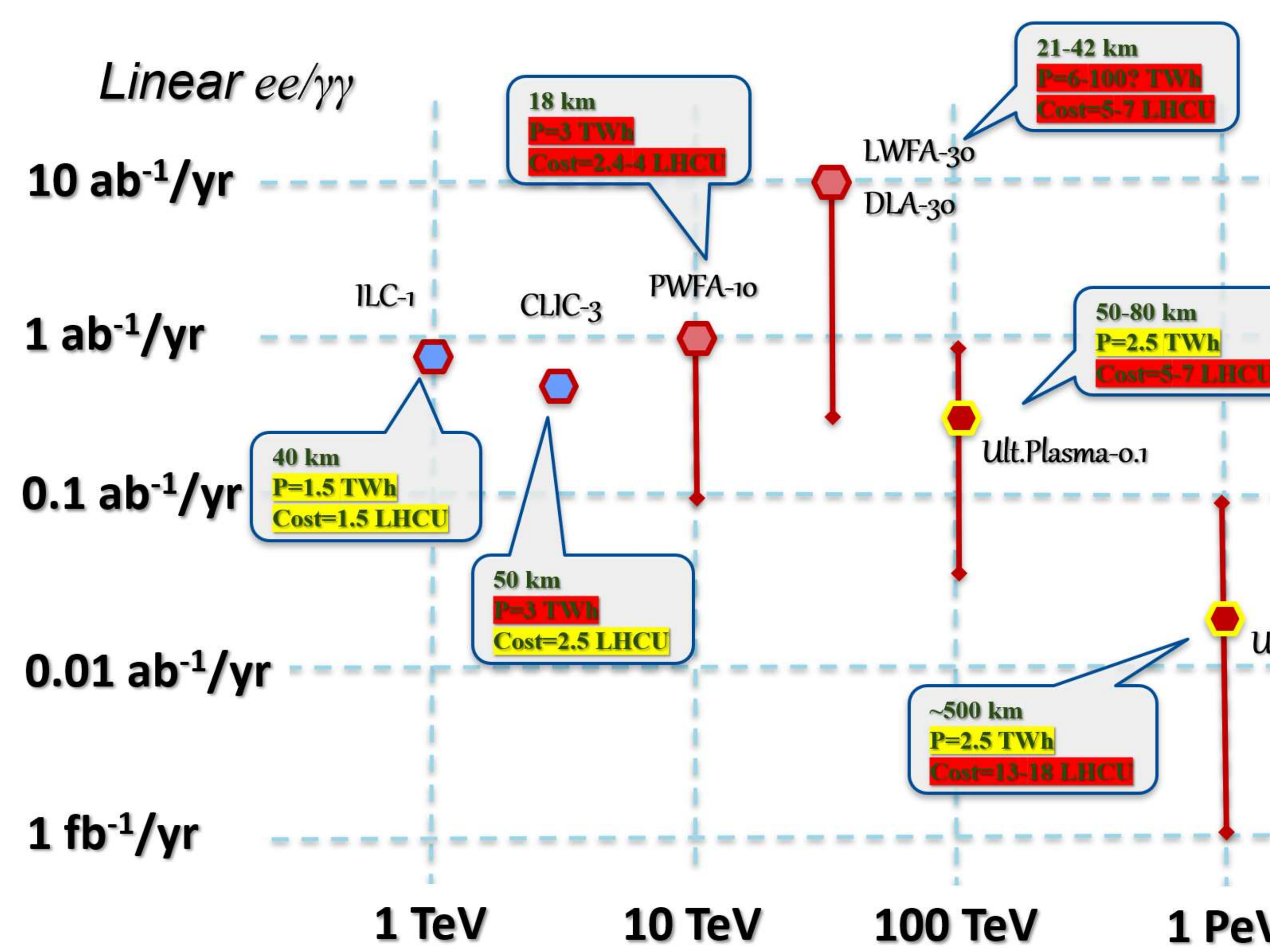
Xtal Colliders: *Lumi* and *Cost* vs *Energy*



pp Colliders: *Lumi* and *Cost* vs *Energy*

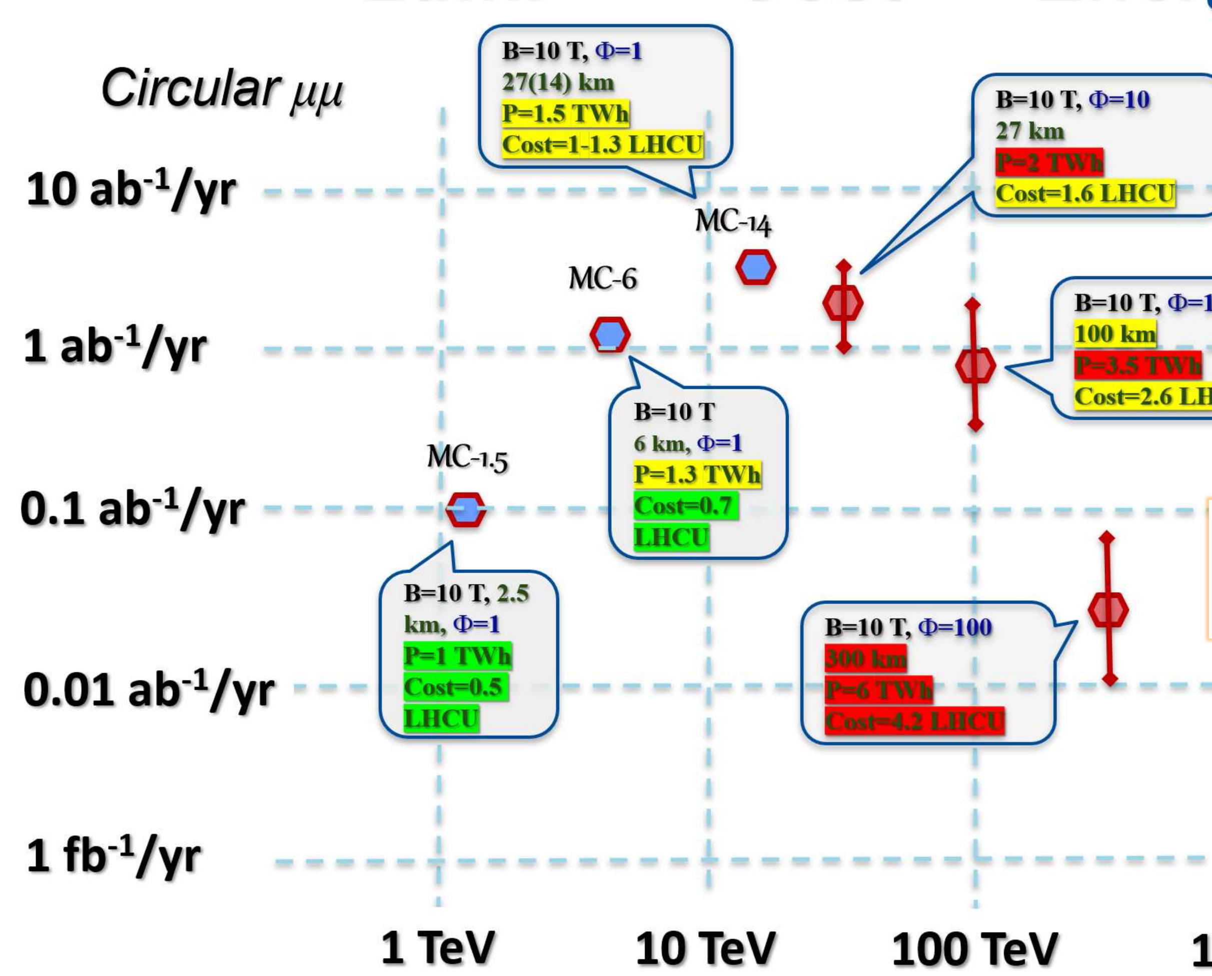


Linear RF and Plasma: *Lumi* and *Cost* vs *Energy*



$$L \propto \frac{\eta_{linac} P_{wall}}{E}\frac{N_\gamma}{\sigma_y}$$

MC: *Lumi* and *Cost* vs *Energy*



$$\mathcal{L} \sim \frac{B}{f_{rep}}\cdot\frac{(D\Phi)^2}{\gamma^4}$$

Main Conclusions:

- **For ultimate high energy colliders:**
  - **Major thrust is *Energy***
  - **Major concern/limit *is Cost***
  - **Main focus is *Luminosity* and *Power***
- **Cost:**
  - **Critically dependent on core acceleration technology**
  - **Existing injectors and infrastructure greatly help**
- **High *E*nergy means low *L*uminosity :**
  - **Don't expect more than 0.1-1 ab⁻¹/yr at 30TeV-1 PeV**
  - **Assume *Power* limited to 1-3 TWh/yr**
- **For considered collider types:**
  - **Circular *pp* – limit is close or below 100 TeV (14 TeV cm)**
  - **Circular ee – limit is ~0.4-0.5 TeV**
  - **Circular *μμ* – limit is between 30 and 100 TeV**
  - **Linear RF *ee/γγ*, Plasma *ee/γγ* – limit is between 3 and 10 TeV**
  - **Exotic crystal *μμ* – promise of 0.1-1 PeV, low Luminosity**
- **Muons are particles of the future**