Charged hadron interactions in QCD+QED

Michael Wagman

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Lattice QCD+QED

High precision calculations of physical quantities require including dynamical QED effects nonperturbatively in lattice QCD calculations

Precise LQCD+QED determinations of meson and baryon masses have reproduced experimental results and made new predictions

Extending precise LQCD+QED results to multi-hadron systems including nuclei faces both numerical and theoretical challenges

Lattice QCD+QED

High precision calculations of physical quantities require including dynamical QED effects nonperturbatively in lattice QCD calculations

Extending precise LQCD+QED results to multi-hadron systems including nuclei faces both numerical and theoretical challenges

For more see:

Beane et al, arXiv:2003.12130

Charged multi-hadron systems in lattice QCD+QED

S. R. Beane,¹ W. Detmold,² R. Horsley,³ M. Illa,⁴ M. Jafry,¹ D. J. Murphy,²
Y. Nakamura,⁵ H. Perlt,⁶ P. E. L. Rakow,⁷ G. Schierholz,⁸ P. E. Shanahan,²
H. Stüben,⁹ M. L. Wagman,^{2,10} F. Winter,¹¹ R. D. Young,¹² and J. M. Zanotti¹²

QCD+QED Phenomenology

Charged hadron scattering

LQCD+QED can separate strong and electromagnetic isospin violation in nucleon-nucleon scattering lengths

Precise predictions of isospin-violating differences between pp , pn , and nn scattering lengths would improve nuclear EFTs

High charge-density systems

Boundaries of periodic table determined by competition between QCD and QED

LQCD+QED calculations in principle can disentangle nonperturbative QED and QCD effects in nuclei

QCD+QED Theory

LQCD+QED for $\alpha Z > 1$ nuclei faces several obstacles:

Charged systems in a box with periodic boundary conditions violate Gauss's law

- *Either boundary conditions or photon field must be modified*

Review: Patella, PoS LATTICE 2016 (2017)

- *QED_L removes spatial zero-mode of photon, equivalent to adding uniform density of opposite charge*

Duncan et al, PRL 76 (1996)

Blum et al, PRD 76 (2007)

Hayakawa and Uno, Prog. Theor. Phys. 120 (2008)

QCD+QED_L preserves symmetries of QCD+QED but violates locality

- *Subtleties of nonlocal field theory must be understood in order to match LQCD+QED_L results to hadronic EFTs*

Exponential signal-to-noise degradation with increasing baryon number

- *Not this talk. Start with LQCD+QED_L for charged mesons with unphysically large α to explore high charge density systems at lower cost*

NRQED

Relativistic formalism for FV effects on generic particle-number systems unknown

Non-Relativistic QED (NRQED) describes low-energy QED for electromagnetic bound states, charged particle scattering, hadronic reactions and decays, ...

Caswell and Lepage, Phys. Lett. 167B (1985)

Kong and Ravndal, Nucl. Phys. A 665 (2000)

Carrasco et al, PRD 91 (2015)

...

Dual expansion:

$$\alpha \quad p/M$$

Power counting (on-shell matter particles):

$$\text{---} \leftarrow \text{---} \quad \sim \frac{1}{E - (p + k)^2/M} \sim \frac{M}{p^2}$$

$$\text{~~~~~} \quad \sim \frac{1}{p^2}$$

$$\int dE \int d^3k \sim \frac{p^5}{M}$$

With multiple charged particles, loop expansion does not match EFT expansion, Coulomb ladder diagrams must be resummed for $\eta = \frac{\alpha M}{2p} > 1$

$$\text{---} \leftarrow \text{---} \quad \text{~~~~~} \quad \text{---} \leftarrow \text{---} \quad \sim \frac{\alpha}{p^2}$$

$$\text{---} \leftarrow \text{---} \quad \text{~~~~~} \quad \text{~~~~~} \quad \text{---} \leftarrow \text{---} \quad \sim \frac{\alpha}{p^2} \left(\frac{\alpha M}{p} \right)$$

Two Charged Particles in a Box

In finite-volume with PBCs, momenta are quantized $p = \frac{2\pi}{L}n, n \in \mathbb{Z}^3$

Coulomb ladder diagrams must be nonperturbatively resummed for large enough volumes that $\alpha M/p \sim \alpha M L \gg 1$

— technically challenging unsolved problem

For intermediate volumes $\frac{1}{m_\pi} \ll L \ll \frac{1}{\alpha M}$ Coulomb effects are perturbative

Beane and Savage derived generalized Lüscher quantization condition relating FV energy shifts and scattering phase shifts accurate to $\mathcal{O}(\alpha), \mathcal{O}(\alpha M L)$

$$C_{\eta(p)}^2 p \cot \delta + \alpha M h(\eta) = -\frac{1}{a_C} + \frac{1}{2} r_0 p^2 + \dots = \frac{1}{\pi L} \mathcal{S}^C \left(\frac{pL}{2\pi} \right) + \alpha M \left[\ln \left(\frac{4\pi}{\alpha M L} \right) - \gamma_E \right]$$

Beane and Savage, PRD 90 (2014)

Range of validity of Beane-Savage regime needs to be explored in LQCD+QED_L

Lattice QCD+QED_L

Dynamical QCD + QED_L ensembles generated by CSSM/QCDSF/UKQCD collaborations

Horsley et al, J. Phys. G 43 (2016)

Horsley et al, JHEP 1604 (2016)

Horsley et al, J. Phys G 46 (2019)

$$\alpha = 0.1 \quad a = 0.068(2) \text{ fm} \quad m_{\overline{K}^0} = 404(1)(12) \text{ MeV} \quad aL = 32, 48$$

$N_f = 1 + 2$ quark flavors with masses tuned to symmetric point where all neutral pseudoscalar mesons are degenerate, $m_{\overline{u}u} \approx m_{\overline{d}d} = m_{\overline{s}s}$

QED effects explored by comparing systems of 1-12 π^+ mesons with systems of 1-12 $\overline{K}^0$ mesons

Quark mass tuning removes strong isospin breaking between π^+ and $\overline{K}^0$ masses, remaining mass difference pure QED

Many Charged Mesons

Correlation functions for up to N=12 charged pions can be computed from one pair of point-to-all quark propagators, as in previous NPLQCD studies without QED

Beane et al, PRL 100 (2008)

Detmold et al, PRD 78 (2008)

Detmold and Savage, PRD 82 (2010)

Allows study of large charge densities: $Z\alpha \leq 1.2$ $\frac{Z\alpha}{L^3} \lesssim 0.12 \text{ fm}^{-3}$

Thermal effects become increasingly important for larger particle number

Fit results for $k < N$ particles used to determine thermal effects for N particles

$$G(N, t) = \sum_n Z_n \left(e^{-E_n(N)t} + e^{-E_n(N)(\beta-t)} \right) + \sum_{k=1}^{[n/2]} \tilde{Z}_0^k \left(e^{-E_0(k)t} e^{-E_0(N-k)(\beta-t)} + e^{-E_0(k)(\beta-t)} e^{-E_0(N-k)t} + \dots \right)$$

Many Fits

Wide range of possible fit ranges
sampled using automated procedure

Weighted average of all fits determines
final results

Charged Hadron Masses

FV effects on π^+ mass described by NRQED_L at NLO (1 fit parameter) for $L = 32, 48$

e^{-mL} FV effects clearly present at $L=24$ for π^+ and $\bar{K}^0$, volume excluded from analysis

[Horsley et al, J. Phys. G 43 \(2016\)](#)

FV effects on proton mass described by NRQED_L, suppressed compared to pion

$$\alpha/(M_p L) \ll \alpha/(m_{\pi^+} L)$$

e^{-mL} FV effects on neutron mass appear negligible

Charged Hadron Interactions

Coulomb effects on $\pi^+\pi^+$ FV energy shift might be expected to be large since

$$\alpha m_{\pi^+} L \sim 0.50, 0.74$$

QED effects < 10% of total shift, even at largest volume

$$\Delta E_{\pi^+\pi^+} = 7.4(1.7) \text{ MeV}$$

$$\Delta E_{\pi^+\pi^+} - \Delta E_{K^0 K^0} = 0.26(58) \text{ MeV}$$

Nucleon-nucleon systems have stronger Coulomb effects, still not resolved despite

$$\alpha M_p L \sim 1.3, 1.9$$

Is this consistent with NRQED_L?

Nonlocality

Details of NRQED_L must be understood to interpret $\text{LQCD} + \text{QED}_L$ results in EFT

Significant recent progress in understanding NRQED_L subtleties for single hadrons

Davoudi and Savage, PRD 90 (2014) Borsanyi et al, Science, vol. 347 (2015) Fodor et al, Phys. Lett. B 755 (2016)

Lee and Tiburzi, PRD 93 (2016) Matzelle and Tiburzi, PRD 95 (2017)

Davoudi, Harrison, Jüttner, Portelli, and Savage, PRD 99 (2019)

- locality violation from zero-mode subtraction allows IR scales such as L to appear in coefficients of UV divergences and renormalized couplings
- careful matching between QED_L and NRQED_L allows nonlocal counterterms to be determined

$$\left(\text{Diagram with wavy line} \right)_{\text{QED}_L} = \left(\text{Diagram with wavy line} + \text{Diagram with cross in circle} \right)_{\text{NRQED}_L}$$

$\delta m \sim \frac{1}{mL^3}$

Remaining questions for multi-particle systems:

How do non-local counterterms modify 2-body interactions?

Are non-local 2-body effects enhanced by Coulomb ladder diagrams?

Matching NRQED_L and QED_L

Tree-level QED_L 2 → 2 amplitude reproduced by NRQED_L with $\psi = \sqrt{2M}e^{iMt}\varphi$

$$\mathcal{L}^{\text{QED}_L} = -(D_\mu \varphi)^\dagger D^\mu \varphi - M^2 \varphi^\dagger \varphi - 8\pi a M (\varphi^\dagger \varphi)^2 + \mathcal{L}_\gamma^\xi$$

$$\mathcal{L}^{\text{NRQED}_L} = \psi^\dagger \left(iD_0 - \frac{D_i D^i}{2M} \right) \psi - \frac{2\pi a}{M} (\psi^\dagger \psi)^2 + \mathcal{L}_\gamma^\xi$$

One-level QED_L 2 → 2 amplitude includes antiparticle pole contributions not present in NRQED_L

Explicit calculation shows antiparticle pole contribution suppressed by $\frac{\alpha}{(ML)^3}$

Antiparticle poles in Coulomb ladder diagrams further suppressed, nonlocal quartic interactions in NRQED_L only arise as high-order relativistic corrections

Matching pNRQED_L and NRQED_L

Dominant contributions arise from diagrams with Coulomb photon exchange between space-like separated particles

All nonlocal effects suppressed by powers of $1/(ML)$

Contributions from radiation photon diagrams suppressed by $1/(ML)$

Coulomb photon contributions reproduced in Rayleigh-Schrödinger perturbation theory with potential

$$\langle p'_1, p'_2 | V | p_1, p_2 \rangle = \left[\left(\frac{4\pi a}{ML^3} \right) + \frac{\alpha}{\pi L} \sum'_{i \in \mathbb{Z}^3} \frac{1}{|i|^2} \right] \delta_{p_1 + p_2 - p'_1 - p'_2}$$

Defines leading-order potential NRQED_L (pNRQED_L) accurate up to $1/(ML)$ effects, higher-order potential computable as in infinite-volume pNRQED/pNRQCD

Many Charged Particles in a Box

3+ particle systems achieve higher charge density in fixed volume, eventually probe nonperturbative relativistic QED effects

— comparison of LQCD+QED_L and NRQED_L for multi-pion systems provides strong tests of validity of NRQED_L

Coulomb photon contributions accessible to pNRQED_L dominant

$$H_{\text{int}} = - \sum_{\mathbf{k}} \tilde{\psi}_{\mathbf{k}}^{\dagger} \left(\frac{\mathbf{k}^4}{8M^3} \right) \tilde{\psi}_{\mathbf{k}} + \frac{1}{2L^3} \sum_{\mathbf{p}', \mathbf{p}, \mathbf{Q}} V(\mathbf{p}, \mathbf{p}') \tilde{\psi}_{\mathbf{Q}-\mathbf{p}'}^{\dagger} \tilde{\psi}_{\mathbf{Q}+\mathbf{p}'}^{\dagger} \tilde{\psi}_{\mathbf{Q}-\mathbf{p}} \tilde{\psi}_{\mathbf{Q}+\mathbf{p}} \\ + \frac{\eta_3(\mu)}{(3!)L^6} \sum_{\mathbf{Q}, \mathbf{p}, \mathbf{p}', \mathbf{q}, \mathbf{q}'} \tilde{\psi}_{\mathbf{Q}+\mathbf{p}'}^{\dagger} \tilde{\psi}_{\mathbf{Q}+\mathbf{q}'}^{\dagger} \tilde{\psi}_{\mathbf{Q}-\mathbf{p}'-\mathbf{q}'}^{\dagger} \tilde{\psi}_{\mathbf{Q}+\mathbf{p}} \tilde{\psi}_{\mathbf{Q}+\mathbf{q}} \tilde{\psi}_{\mathbf{Q}-\mathbf{p}-\mathbf{q}},$$

Radiation photon diagrams again suppressed by $1/(ML)$

pNRQED_L suitable for calculations of many-pion energy shifts up to $1/(ML)$ effects 16

Relativistic Corrections

Relativistic corrections to NRQED also arise at higher order

$$\mathcal{L} = \psi^\dagger \left(iD_0 - \frac{\mathbf{D}^2}{2M} \right) \psi - \frac{1}{2} \left(\frac{4\pi a}{M} \right) (\psi^\dagger \psi)^2 - \frac{\eta_3(\mu)}{3!} (\psi^\dagger \psi)^3 + \mathcal{L}_\gamma^\xi + \mathcal{L}_r.$$

LECs for relativistic corrections can be obtained by matching threshold expansion of quantization condition (without QED)

[Hansen and Sharpe, PRD 93 \(2016\)](#)

$$\mathcal{L}_r = \psi^\dagger \left(\frac{\mathbf{D}^4}{8M^3} \right) \psi - \frac{1}{2} \left(\frac{\pi a}{M} \right) \left(ar - \frac{1}{M^2} \right) (\psi^\dagger \psi)(\psi^\dagger \mathbf{D}^2 \psi + \psi \mathbf{D}^2 \psi^\dagger)$$

Matching on to potential NRQED_L gives

$$V(\mathbf{p}', \mathbf{p}) = \frac{4\pi}{M} \left(a + \frac{a}{4} \left(ar - \frac{1}{M^2} \right) (\mathbf{p}^2 + \mathbf{p}'^2) + \dots \right) + \frac{4\pi\alpha}{|\mathbf{p}' - \mathbf{p}|^2} (1 - \delta_{\mathbf{p}, \mathbf{p}'})$$

Power Counting

With $m_{\pi^+} = 449(1)(13)$ MeV and volumes $L = 2.2$ fm, 3.2 fm

$$\frac{\alpha m_{\pi} L}{4\pi} = 0.04, 0.06$$

$$\frac{a_{\pi\pi}}{L} = 0.10, 0.07$$

Hierarchy between QCD and QED effects volume dependent, two reasonable power counting for this range of volumes

$$\mathbf{PC1} \quad \frac{\alpha m_{\pi} L}{4\pi} \sim \frac{a_{\pi\pi}}{L}$$

$$\mathbf{PC2} \quad \frac{\alpha m_{\pi} L}{4\pi} \sim \left(\frac{a_{\pi\pi}}{L} \right)^2$$

Previous result ordered by loop expansion appropriate for PC1 at NNLO

Known higher-order results for pure QCD allow PC2 to be extend to N³LO

Beane, Detmold, and Savage, PRD 76 (2007)

Hansen and Sharpe, PRD 93 (2016)

FV Energy Shifts in pNRQED_L

n -boson energy shifts straightforwardly computable in pNRQED_L

$$|\mathbf{p}_1, \dots, \mathbf{p}_n\rangle = \frac{1}{\sqrt{n}} \tilde{\psi}_{\mathbf{p}_1}^\dagger \times \dots \times \tilde{\psi}_{\mathbf{p}_n}^\dagger |0\rangle \quad \langle 0, \dots, 0 | H_{\text{int}} | 0, \dots, 0 \rangle = \frac{1}{L^3} \binom{n}{2} V(0, 0) + \frac{1}{L^6} \binom{n}{3} \eta_3(\mu).$$

Higher-orders, e.g. N³LO in PC2, given by Rayleigh-Schrödinger perturbation theory

$$\begin{aligned} \Delta E_n^{\text{N}^3\text{LO,PC2}} = & \frac{4\pi\bar{a}_C}{ML^3} \binom{n}{2} \left\{ 1 - \left(\frac{\bar{a}_C}{\pi L} \right) \mathcal{I} + \left(\frac{\bar{a}_C}{\pi L} \right)^2 [\mathcal{I}^2 + (2n-5)\mathcal{J}] \right. \\ & \left. - \left(\frac{\bar{a}_C}{\pi L} \right)^3 [\mathcal{I}^3 + (2n-7)\mathcal{I}\mathcal{J} + (5n^2-41n+63)\mathcal{K}] \right\} \\ & + \frac{4\eta_L}{ML^2} \binom{n}{2} \left\{ - \left(\frac{\bar{a}_C}{\pi L} \right) 2\mathcal{J} - \left(\frac{\eta_L}{\pi^2} \right) \mathcal{K} \right. \\ & \left. + \left(\frac{\bar{a}_C}{\pi L} \right)^2 [2\mathcal{I}\mathcal{J} + \mathcal{R}_{22} + (4n-10)\mathcal{K} - 4\pi^4 [\ln(\eta_L) + \gamma_E]] \right\} \\ & + \binom{n}{3} \frac{1}{L^6} \left[\eta_3(\mu) + \frac{64\pi\bar{a}_C^4}{M} (3\sqrt{3} - 4\pi) \ln(\mu L) - \frac{96\bar{a}_C^4}{\pi^2 M} \mathcal{S}_{\text{MS}} \right]. \end{aligned}$$

Effective range and relativistic corrections act as modification to scattering length

$$a_C = \bar{a}_C(L) - \frac{2\pi\bar{a}_C(L)^2}{L^3} \left(\bar{a}_C(L)r - \frac{1}{2M^2} \right)$$

Beane, Detmold, and Savage, PRD 76 (2007)

Beane and Savage, PRD 90 (2014)

Extends: Huang and Yang, Phys. Rev. 105 (1957)

Hansen and Sharpe, PRD 93 (2016)

FV Energy Shifts in pNRQED_L

$$\begin{aligned}\Delta E_n^{\text{N}^3\text{LO,PC2}} = & \frac{4\pi\bar{a}_C}{ML^3} \binom{n}{2} \left\{ 1 - \left(\frac{\bar{a}_C}{\pi L}\right) \mathcal{I} + \left(\frac{\bar{a}_C}{\pi L}\right)^2 [\mathcal{I}^2 + (2n-5)\mathcal{J}] - \left(\frac{\bar{a}_C}{\pi L}\right)^3 [\mathcal{I}^3 + (2n-7)\mathcal{I}\mathcal{J} + (5n^2 - 41n + 63)\mathcal{K}] \right\} \\ & + \frac{4\eta_L}{ML^2} \binom{n}{2} \left\{ - \left(\frac{\bar{a}_C}{\pi L}\right) 2\mathcal{J} - \left(\frac{\eta_L}{\pi^2}\right) \mathcal{K} + \left(\frac{\bar{a}_C}{\pi L}\right)^2 [2\mathcal{I}\mathcal{J} + \mathcal{R}_{22} + (4n-10)\mathcal{K} - 4\pi^4 [\ln(\eta_L) + \gamma_E]] \right\} \\ & + \binom{n}{3} \frac{1}{L^6} \left[\eta_3(\mu) + \frac{64\pi\bar{a}_C^4}{M} (3\sqrt{3} - 4\pi) \ln(\mu L) - \frac{96\bar{a}_C^4}{\pi^2 M} \mathcal{S}_{\text{MS}} \right].\end{aligned}$$

Important features:

2-body log involves FV analog of Coulomb expansion parameter $\eta_L = \frac{\alpha M}{2p} = \frac{\alpha M L}{4\pi}$

Coulomb photon effects enter as power series in η_L with O(1) coefficients

FV Coulomb perturbative when $\frac{\alpha M L}{4\pi} \ll 1$, including $\alpha M L \sim 1$

Physical quark mass
protons with $\alpha = 0.1$

$$\alpha M L \ll 1$$

$$L \ll 2.1 \text{ fm}$$

$$\frac{\alpha M L}{4\pi} \ll 1$$

$$L \ll 26 \text{ fm}$$

EFT Fits

EFT parameters $a^{K^0 K^0}$, $a_C^{\pi^+ \pi^+}$ and three-body couplings $\eta^{\overline{K}^0 \overline{K}^0 \overline{K}^0}$, $\eta^{\pi^+ \pi^+ \pi^+}$ determined from global fit to all FV shifts with 2-12 mesons and correlated differences $\Delta E_{N\pi^+} - \Delta E_{NK^0}$

NNLO fits with both power counting give $\chi^2/\text{dof} \sim 1.3$

N³LO fits (pc 2) add three-body forces, better agreement $\chi^2/\text{dof} \sim 0.8$

EFT Fits

QED effects resolved at $1 - 2\sigma$ for systems of $3 - 12$ mesons

Negative sign arises from zero-mode subtraction, full QED effects reproduced by Coulomb effects in NRQED_L

EFT Results

Coulomb scattering length for $\pi^+\pi^+$ differs from K^0K^0 scattering length

$$a^{\overline{K}^0\overline{K}^0} m_{\overline{K}^0} = 0.337(19)$$

$$a_C^{\pi^+\pi^+} m_{\overline{K}^0} = 0.464(29)$$

Three-body forces clearly resolved, no QED effects visible on strength of three-body forces

(all results valid for unphysical quark masses used in calculation)

Summary

Nonlocal counterterms in NRQED_L are not enhanced by Coulomb ladders (at least at 2-loop order), appear only as relativistic corrections

FV effects on interacting charged hadron systems in LQCD+QED_L consistent with NRQED_L up to $Z\alpha \leq 1.2$, $Z\alpha/L^3 \lesssim 0.12 \text{ fm}^{-3}$ at unphysically large quark mass

Size of QED effects consistent with Coulomb expansion parameter

$$\eta_L = \frac{\alpha M}{2p} = \frac{\alpha M L}{4\pi} \ll \alpha M L$$

NRQED_L with perturbative Coulomb should converge for proton-proton systems at the physical point with $L \ll 20 \text{ fm}$