Nuclei from lattice QCD

Michael Wagman

APS April Meeting 2020: From Quarks to the Cosmos

April, 20 2020

Nuclear models

Nucleon-nucleon scattering can be *approximately* described by a non-relativistic two-body potential

Fundamentally, this description is incomplete

Nuclear models

Nuclear scattering and reaction rates can be ***approximately*** described using a two-nucleon potential and one-nucleon currents

Fundamentally, this description is incomplete

Protons and neutrons are composite objects made of quarks and gluons

Nuclear processes affected by correlations:

- *Three-nucleon forces*
- *Two-nucleon currents*

Nuclei, neutrinos, and new physics

Many experiments are searching for new physics using nuclear targets

— *Dark matter, fundamental symmetry violation, ...*

Accurate theoretical predictions are needed to reliably interpret new physics searches

Electroweak reaction cross-sections critical for neutrino oscillation experiments, $0\nu\beta\beta$ searches

QCD and DUNE

Neutrino-nucleus cross-section needed from theory (QCD + leptons) for DUNE to determine neutrino mass hierarchy and CP violation

Oscillation parameters determined by comparing reconstructed spectrum at far detector vs predictions from near detector + theory

Neutrino energy reconstruction with at least 100 MeV accuracy needed to observe CP violation

Mosel Ann. Rev. Nucl. Part. Sci. 66, (2016)

Abi et al (DUNE), arXiv 1807.10334

Acciarri et al (DUNE) arXiv 1512.06148

Double β - decay

Doubly-weak reactions important but less well-understood phenomenologically

Double-beta decay reactions include additional two-body currents not present in single-beta decay, e.g. isotensor axial polarizability for $2\nu\beta\beta$

Shanahan, MW et al [NPLQCD], PRL 119 (2017)

Tiburzi, MW et al [NPLQCD], PRD 96 (2017)

Two-body currents needed for $0\nu\beta\beta$
at leading order in ChEFT

Cirigliano et al, PRL 120 (2018)

LQCD studies of two-nucleon systems
can be used to fix unknown LEC

From quarks to nuclei

LQCD

Experiment

Nuclear many-body

Lattice QCD

Approximating spacetime as a discrete lattice with a finite size makes QCD path integrals finite

$$\langle \mathcal{O} \rangle = Z^{-1} \int \mathcal{D}U \mathcal{D}\bar{q} \mathcal{D}q e^{-S_{QCD}(U, q, \bar{q})} \mathcal{O}(U, q, \bar{q})$$

$$= Z^{-1} \int \mathcal{D}U e^{-S_g(U)} \det(\not{D}(U) + m_q) \mathcal{O}(U, [\not{D}(U) + m_q]^{-1})$$

Lattice action defines an EFT for QCD
valid at lengths larger than lattice scale

QCD properties of hadrons reproduced by
LQCD up to lattice cutoff artifacts
removed by continuum extrapolation

Hadron-hadron interactions

Volume scaling of energy levels allows bound and scattering states to be distinguished using finite-volume Euclidean information

**Infinite-volume
scattering state**

$$[E(L) - E(\infty)] \propto \frac{a}{ML^3}$$

Huang, Yang, Phys. Rev. 105 (1957)

**Infinite-volume
bound state**

$$[E(L) - E(\infty)] \propto \frac{e^{-\gamma L}}{L}$$

Beane et al, Phys. Lett. B 585 (2004)

Quantitative information
about interactions
encoded in volume
dependence

Lüscher, Commun. Math.
Phys. 105 (1986)

Recent review: Briceño, Dudek, Young, Rev. Mod. Phys. 90 (2018)

Neutrino-Nucleus Scattering

Cross-section governed by vector and axial form factors of nuclei in QCD

Vector form factors can be measured in electron scattering experiments

Axial form factors, flavor dependence less well-known

Single-nucleon response dominates for small nuclei

Multi-nucleon effects more important for larger nuclei

g_A

L_{1A}

Single-Nucleon QCD Input

Nucleon axial charge computed in LQCD by several groups

Results (with controlled systematics) consistent with neutron decay experiments

Aoki et al [FLAG], arXiv:1902.08191

Nucleon axial form factors computed by multiple LQCD groups, ongoing effort to fully control systematics

Kronfeld et al [USQCD], arXiv:1904.09931

Axial currents in nuclei

Axial current matrix elements are smaller than predicted by shell models, experiment reproduced by rescaling $g_A \rightarrow q g_A$ with $q \sim 0.75$

Modern calculations including multi-nucleon correlations can reproduce experiment without “quenching” g_A

Gysbers et al, Nature Phys. 15 (2019)

Pastore et al, PRC 97 (2018)

Can the emergence of nuclear effects on axial currents be predicted directly from the Standard Model?

Nuclei from LQCD

LQCD can calculate spectra and matrix elements of QCD with IR cutoff (finite volume) and UV cutoff (lattice spacing) in Euclidean spacetime

Volume dependence encodes scattering phase shifts, binding energies determined by QCD spectrum in large volumes

NPLQCD collaboration finds nuclei and hypernuclei with 2-5 baryons are bound at $m_\pi \sim 806$ MeV more deeply than in nature

Beane et al [NPLQCD], PRD 87 (2013)

$L = 2.9 \text{ fm} \rightarrow 5.8 \text{ fm}$

$a = 0.145 \text{ fm}$

PACS collaboration finds similar results at $m_\pi \sim 510$ MeV

Yamazaki et al, PRD 86 (2012)

$L = 3.5 \text{ fm} \rightarrow 7.0 \text{ fm}$

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Nuclear matrix elements

LQCD spectrum determined from 2-point correlation functions

$$C_A(t) = \langle A(t) A^\dagger(0) \rangle = \sum_n |Z_n|^2 e^{-E_n t}$$

Axial matrix elements determined from 3-point correlation functions including an axial current operator insertion

Fixed-order background fields

Basic input: linear combinations of quark propagators with propagators that contain a current inserted everywhere (“background field”)

The diagram shows a quark propagator with a background field current insertion (represented by a blue circle with a cross) equal to the sum of the free quark propagator and a term proportional to λ times the propagator with a current insertion. The current insertion is represented by a blue circle with a cross and a wavy line.

$$\text{Quark propagator with current insertion} = \text{Free quark propagator} + \lambda \text{Quark propagator with current insertion}$$

Correlation functions encode linear (quadratic, ...) response to background field in linear (quadratic, ...) terms of polynomial

The diagram shows a two-point correlation function with a background field current insertion (represented by a blue circle with a cross) equal to the sum of the free two-point function and terms proportional to λ and λ^2 times the two-point function with one or two current insertions, respectively. The current insertions are represented by blue circles with crosses and wavy lines.

$$\text{Two-point function with current insertion} = \text{Free two-point function} + \lambda \text{Two-point function with one current insertion} + \lambda^2 \text{Two-point function with two current insertions} + \dots$$

Nuclear correlation functions containing current insertions can be formed as straightforwardly as two-point functions

The diagram shows a nuclear correlation function with a background field current insertion (represented by a blue circle with a cross) equal to the sum of the free nuclear correlation function and terms proportional to λ and higher powers of λ times the nuclear correlation function with one or more current insertions, respectively. The current insertions are represented by blue circles with crosses and wavy lines.

$$\text{Nuclear correlation function with current insertion} = \text{Free nuclear correlation function} + \lambda \text{Nuclear correlation function with one current insertion} + \dots$$

Extracting matrix elements

Correlation functions exact
polynomials in background field
strength

Current matrix elements
obtained from linear term

$$C_A^{(\lambda)}(t) = \langle A(t)A^\dagger(0) \rangle + \lambda \sum_y \langle A(t)J(y)A^\dagger(0) \rangle + O(\lambda^2)$$

Inserting complete sets of states gives Euclidean spectral representation

$$C_A^{(\lambda)}(t) \Big|_{O(\lambda)} = |Z_0|^2 e^{-E_0 t} [c + t \langle A|J|A \rangle + O(e^{-\delta t})] \quad \leftarrow \text{Fit to LQCD results}$$

Fixed-order background field formalism also helps organize EFT matching

Neutron β -decay

Neutron decay rate governed by axial charge

$$g_A = \langle p | A_3^+ | n \rangle = \langle p | A_3^3 | p \rangle$$

← Isospin limit

$$A_\mu^a = A_\mu^a(\mathbf{q} = 0) = \int_{\mathbf{y}} \bar{q}(\mathbf{y}) \gamma_\mu \gamma_5 \tau^a q(\mathbf{y})$$

g_A obtained from nucleon background field correlation function

$$\partial_t \left(\frac{C_p^{(\lambda_u - \lambda_d)}(t)}{C_p(t)} \Big|_{O(\lambda)} \right) = \langle p | A_3^3 | p \rangle + O(e^{-\delta t})$$

Axial current renormalization Z_A computed separately

Accurate LQCD calculations at physical quark masses performed by several groups, reviewed by FLAG: [Aoki et al \[FLAG\], arXiv:1902.08191](#)

Proton-proton fusion

Combining lattice QCD and EFT allows more complex reactions to be studied

LQCD/EFT matching for two nucleon correlation functions in a box with a background axial field:

$$iC_{pp \rightarrow np(^3S_1)} = \text{diagram 1} + \text{diagram 2} \quad \mathcal{M}_{pp \rightarrow np(^3S_1)} = gA(1 + S) - L_{1A}$$

Short-distance QCD physics

Used for LQCD determination of two-body currents in proton-proton fusion

$$L_{1A} = 3.9(0.2)(1.0)(0.4)(0.9) \text{ fm}^3$$

Savage, MW et al [NPLQCD], PRL 119 (2017)

method		$L_{1A} \text{ (fm}^3\text{)}$
two-body		
reactor $\bar{\nu} + d$		3.6 ± 5.5 [11]
ES, CC, NC in SNO		4.0 ± 6.3 [41]
MuSun proposal		± 1.25
three-body		
tritium beta decay		4.2 ± 3.7 [11], 4.2 ± 0.1 [41]
other		
helioseismology		4.8 ± 6.7 [42]

Andreev et al (MuSun) arXiv 1004.1754

Tritium β -decay

Triton decay rate governed by Gamow-Teller matrix element

$$g_A(^3\text{H}) = | \langle ^3\text{He} | A_3^+ | ^3\text{H} \rangle | = | \langle ^3\text{H} | A_3^+ | ^3\text{H} \rangle |$$

Computed in ChEFT

Baroni et al, PRC 98 (2018)

After fitting LECs to experimental triton β decay rate predicts

$$\frac{| \langle ^3\text{He} | A_3^+ | ^3\text{H} \rangle |}{g_A} = 0.951(13)$$

Deviations from 1 arise from two-body currents and multi-nucleon interactions

NLO calculations in pionless EFT relate nuclear effects to the two-body axial current coupling L_{1A} appearing in proton-proton fusion

De-Leon, Platter, Gazit (2016)

Tritium β -decay, $m_\pi \sim 806$ MeV

With fixed-order axial background fields, triton GT matrix element calculation analogous to proton axial charge

$$\partial_t \left(\frac{C_{^3\text{He}}^{(\lambda_u - \lambda_d)}(t)}{C_{^3\text{He}}(t)} \right) \Big|_{O(\lambda)} = \langle ^3\text{He} | A_3^3 | ^3\text{He} \rangle + O(e^{-\delta t})$$

Renormalization factors cancel from isovector matrix element ratios, precise results obtained for ratio

$$\frac{|\langle ^3\text{He} | A_3^+ | ^3\text{H} \rangle|}{g_A} = 0.987(4)$$

$N_f = 3, m_\pi = 806(9)$ MeV, $a = 0.145(2)$ fm

Savage et al [NPLQCD], PRL 119 (2017)

Going to lighter quark masses

Much higher statistics needed at lighter quark masses

$$m_\pi \sim 806 \text{ MeV}$$

$$m_\pi \sim 450 \text{ MeV}$$

$$N_{src} = 10,592$$

$$N_{src} = 281,472$$

25x statistics increase mitigates 40x variance increase for $A=3$ at $t=1$ fm

$$\text{StN} \approx \frac{1}{\sqrt{N_{src}}} e^{-A(M_N - \frac{3}{2}m_\pi)t}$$

Faster noise growth makes identifying “plateau” challenging

Automated sampling of fitting choices essential for reliable results

“Autofitter” algorithm

The triton at $m_\pi \sim 450$ MeV

New calculations of ${}^3\text{H}$ performed with $m_\pi \sim 450$ MeV

[NPLQCD], arXiv:2004.XXXX

All possible fit ranges sampled, multi-state fit model selected for each, weighted average taken of all fits passing various reliability checks

Triton binding vs quark mass

Volume dependence of triton energy shift consistent with bound state

Pion mass dependence appears consistent between different LQCD results

Simple extrapolations to physical point imprecise but consistent with experimental results

Axial currents at $m_\pi \sim 450$ MeV

Same fitting procedure applied to correlation functions with background axial fields in order to extract matrix elements

Axial charge vs quark mass

Simple extrapolations of proton and ^3H axial charges similarly consistent with physical results

(Note many available LQCD results for the proton are not shown)

Nuclear effects on axial structure

Ratios of ${}^3\text{H}$ and proton axial charges determined more precisely than individual charges (renormalization cancels)

$$m_\pi \sim 450 \text{ MeV}$$

$$\frac{g_A^{{}^3\text{H}}}{g_A^p} = 0.952(9)$$

Results at $m_\pi \sim 450 \text{ MeV}$ and simple extrapolations consistent with physical results

Nuclear effects on axial structure more significant at lighter pion mass
— *are chiral dynamics important for emergence of nuclear effects on axial structure?*

Flavor dependence

Beyond validation, LQCD can compute properties of electroweak reactions that are difficult to access experimentally, e.g. flavor dependence

Nuclear modifications of u and d quark axial couplings $O(1\%)$ for $A=2-3$ nuclei, $m_\pi \sim 806$ MeV

$N_f = 3$, $m_\pi = 806(9)$ MeV, $a = 0.145(2)$ fm

Flavor-singlet matrix elements needed for neutral weak interactions, understanding quark/gluon spin structure of nuclei

$$g_{A,h}^{(u+d+s)} = \langle h, \mathbf{p}, \lambda | \bar{q} \gamma_\mu \gamma_5 q | h, \mathbf{p}, \lambda \rangle = s_\mu^h \int_0^1 dx \Delta q_h^i(x)$$

Extensions to include momentum transfer and/or derivative operators will constrain axial form factors and structure functions

Beyond the Standard Model currents

LQCD can also compute nuclear reactions induced by BSM currents, needed to interpret e.g. dark matter direct detection experiments

Larger nuclear modifications of scalar than axial structure at $m_\pi \sim 806$ MeV

Extensions to physical quark masses will provide inputs to EFTs for dark matter direct detection, e.g. [Fieguth et al, PRD 97 \(2018\)](#) [Hoferichter et al, PRD 99 \(2019\)](#)

Towards larger nuclei

LQCD results in the few-nucleon sector can be used to tune EFTs that can efficiently predict properties of larger nuclei

Provides near-term strategy for QCD-based predictions of the properties of larger nuclei

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Techniques to mitigate the signal-to-noise problem could eventually allow exponentially more efficient LQCD calculations of nuclei

Recent progress on signal-to-noise problems in 2D U(1) gauge theory achieved by path integral contour deformations

From quarks to nuclei

LQCD

Experiment

Nuclear many-body