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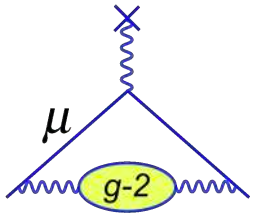
The muon g-2 and $\Delta\alpha$ connection

Keshavarzi, Marciano, Passera and Sirlin
Phys. Rev. D 102 (2020) 033002

Alex Keshavarzi

New Perspectives 2020
24th August 2020



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This manuscript has been authored by Fermi Research Alliance, LLC under Contract No. DE-AC02-07CH11359 with the U.S. Department of Energy, Office of Science, Office of High Energy Physics.

The muon $g-2$

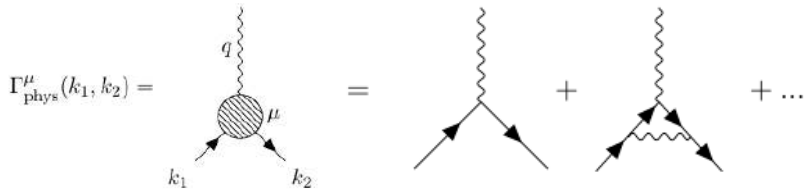
Magnetic moment: $\vec{\mu} = \frac{-e}{2m} g_\mu \vec{S}$: $g = 2$ (Dirac) $\rightarrow$ $g = 2 + 2a_\mu$ (+ radiative corrections)

- a_μ arises due to quantum corrections inherent from RQFTs (QED, SM)
- These effects manifest differently for the theoretical/experimental determination.

Theory, a_μ^{SM}

$\rightarrow$ 1948, Schwinger: $a_e = \frac{g_e - 2}{2} = \frac{\alpha}{2\pi} = 0.001162$

$\rightarrow$ Corrections to the QED vertex function:

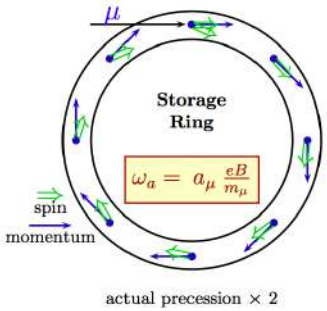


$\rightarrow$ Must determine all SM contributions to sufficient loop order

Experiment, a_μ^{exp}

$\rightarrow$ 1948, Kusch & Foley: $g_e = 2.00238$

$\rightarrow$ Anomalous precession frequency, ω_a



$$\overrightarrow{\omega_a} = \overrightarrow{\omega_s} - \overrightarrow{\omega_c}.$$
$$= -a_\mu \frac{Qe}{m} \vec{B}$$

Experimental measurements naturally contains all higher order effects

$\rightarrow$ Compare a_μ^{SM} and a_μ^{exp} to rigorously test SM

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Theory, a_μ^{SM}

$\rightarrow$ Today, The Muon g-2 Theory Initiative

Experiment, a_μ^{exp}

$\rightarrow$ 2003, BNL measurement

4-year long international collaborative effort:

arXiv.org > hep-ph > arXiv:2006.04822

High Energy Physics - Phenomenology

(Submitted on 8 Jun 2020)

The anomalous magnetic moment of the muon in the Standard Model

T. Aoyama, N. Asmussen, M. Benayoun, J. Bijnens, T. Blum, M. Bruno, I. Caprini, C. M. Carloni Calame, M. Cè, G. Colangelo, F. Curciarello, H. Czyż, I. Danilkin, M. Davier, C. T. H. Davies, M. Della Morte, S. I. Eidelman, A. X. El-Khadra, A. Gérardin, D. Giusti, M. Golterman, Steven Gottlieb, V. Gülpers, F. Hagelstein, M. Hayakawa, G. Herdoiza, D. W. Hertzog, A. Hoecker, M. Hoferichter, B.-L. Hoid, R. J. Hudspith, F. Ignatov, T. Izubuchi, F. Jegerlehner, L. Jin, A. Keshavarzi, T. Kinoshita, B. Kubis, A. Kupich, A. Kupść, L. Laub, C. Lehner, L. Lellouch, I. Logashenko, B. Malaescu, K. Maltman, M. K. Marinković, P. Masjuan, A. S. Meyer, H. B. Meyer, T. Mibe, K. Miura, S. E. Müller, M. Nio, D. Nomura, A. Nyffeler, V. Pascalutsa, M. Passera, E. Perez del Rio, S. Peris, A. Portelli, M. Procura, C. F. Redmer, B. L. Roberts, P. Sánchez-Puertas, S. Serednyakov, B. Shwartz, S. Simula, D. Stöckinger, H. Stöckinger-Kim, P. Stoffer, T. Teubner, R. Van de Water, M. Vanderhaeghen, G. Venanzoni, G. von Hippel, H. Wittig, Z. Zhang, M. N. Achasov, A. Bashir, N. Cardoso, B. Chakraborty, E.-H. Chao, J. Charles, A. Crivellin, O. Deineka, A. Denig, C. DeTar, C. A. Dominguez, A. E. Dorokhov, V. P. Druzhinin, G. Eichmann, M. Fael, C. S. Fischer, E. Gámiz, Z. Gelzer, J. R. Green, S. Guellati-Khelifa, D. Hatton, N. Hermansson-Truedsson et al. (32 additional authors not shown)

Theory Initiative Website:
https://muon-gm2-theory.illinois.edu/white-paper/

Community approved a_μ^{SM} vs. BNL a_μ^{exp} yields $\Delta a_\mu = a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{SM}} = (27.9 \pm 7.6) \times 10^{-10}$

$\rightarrow 3.7\sigma$ discrepancy hints at new physics beyond the SM.

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Theory Initiative Website:
https://muon-gm2-theory.illinois.edu/white-paper/

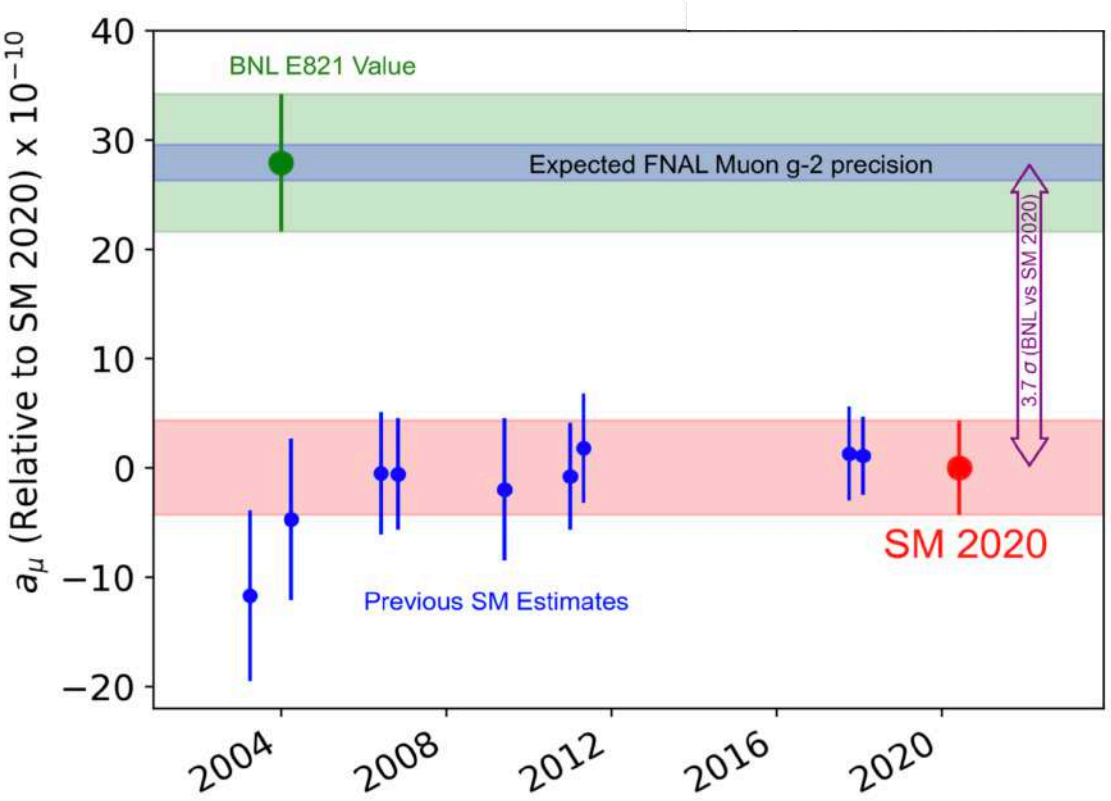
Experiment, a_μ^{exp}

$\rightarrow$ Today, Muon g-2 Experiment at Fermilab

Aiming for x4 improvement in total uncertainty.

Analysis of Run-1 data being finalised for publication.
$\rightarrow$ Will confirm validity of Muon g-2 discrepancy and potential for BSM discovery.

Theory $g-2$ and hadronic data

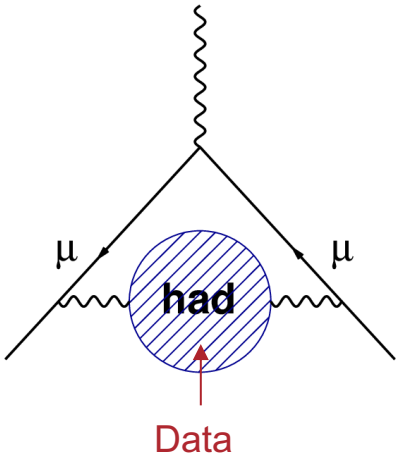


Without theory improvements:

$\Delta a_\mu \sim 6\sigma$

With theory improvements:

$\Delta a_\mu > 10\sigma$

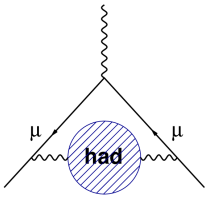


- Theory uncertainty is entirely dominated by hadronic contributions.
- These hadronic contributions rely on experimentally measured data.
- What if these data are the source of the muon g-2 discrepancy?
- If the data are adjusted to fix g-2, what impact does this have on other areas of physics?

Hadronic cross section data

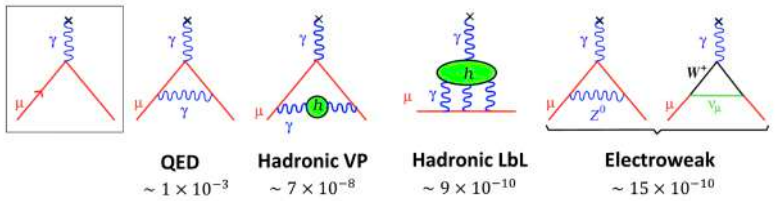
Experimentally measured hadronic cross section: σ_{had}

Muon g-2:
hadronic vacuum polarisation contribution



$$a_\mu^{\text{had, VP}} = \frac{1}{4\pi^3}\int_{m_\pi}^{\infty} ds\ \sigma_{\text{had}}(s)\, K(s)$$

… sum with other SM contributions…



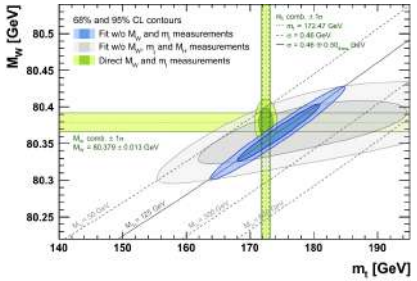
→ Determines a_μ^{SM} and $\Delta a_\mu = 3.7\sigma$

Increase cross section so that $\Delta a_\mu = 0$?
→ Solves muon g-2 discrepancy

Running QED coupling:
hadronic contribution to running

$$\Delta\alpha^{(5)}_{\text{had}}(q^2) = \frac{q^2}{4\pi\alpha^2}\int_{m_\pi}^{\infty} ds\ \sigma_{\text{had}}(s)\,\frac{q^2}{(q^2 - s)}$$

… evaluate at $q^2 = M_Z^2$ and input into **global EW fit**…



→ Predicts M_W, M_H, $\sin^2\theta_{eff}^{lep}$ and more…

Increase cross section so that $\Delta a_\mu = 0$?
→ What happens to precision EW parameters?

Muon g-2 and $\Delta\alpha$

Slide content by Massimo Passera.

- Can Δa_μ be due to hypothetical mistakes in the hadronic $\sigma(s)$?
- An upward shift of $\sigma(s)$ also induces an increase of $\Delta\alpha_{had}^{(5)}(M_Z)$.
- Consider:

$$a_\mu^{HLO} \rightarrow a = \int_{4m_\pi^2}^{s_u} ds\, f(s)\,\sigma(s), \qquad f(s) = \frac{K(s)}{4\pi^3},\ s_u < M_Z^2,$$

$$\Delta\alpha_{had}^{(5)} \rightarrow b = \int_{4m_\pi^2}^{s_u} ds\, g(s)\,\sigma(s), \qquad g(s) = \frac{M_Z^2}{(M_Z^2 - s)(4\alpha\pi^2)},$$

and the increase

$$\Delta\sigma(s) = \epsilon\sigma(s)$$

ϵ>0, in the range:

$$\sqrt{s} \in [\sqrt{s_0} - \delta/2, \sqrt{s_0} + \delta/2]$$

Note the very different energy-dependent weighting of the integrands…

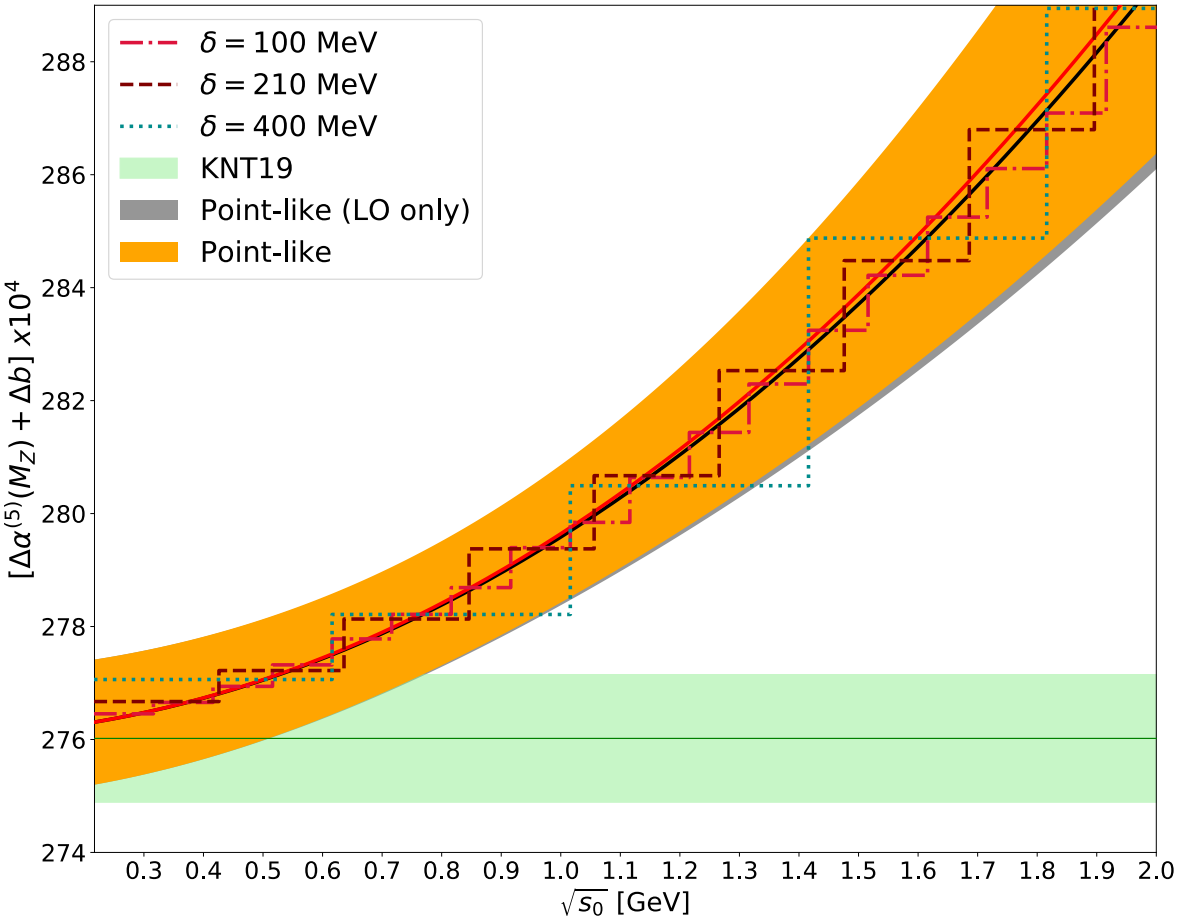
Use precise and up-to-date compilation of total hadronic cross section from KNT

Keshavarzi, Nomura and Teubner, Phys.Rev.D 101 (2020) 014029, arXiv:1911.00367

The muon g-2 and $\Delta\alpha$ connection

Keshavarzi, Marciano, Passera and Sirlin, Phys. Rev. D 102 (2020) 033002

- Shift KNT hadronic cross section in fully energy-dependent (point-like and binned) analysis to account for Δa_μ.



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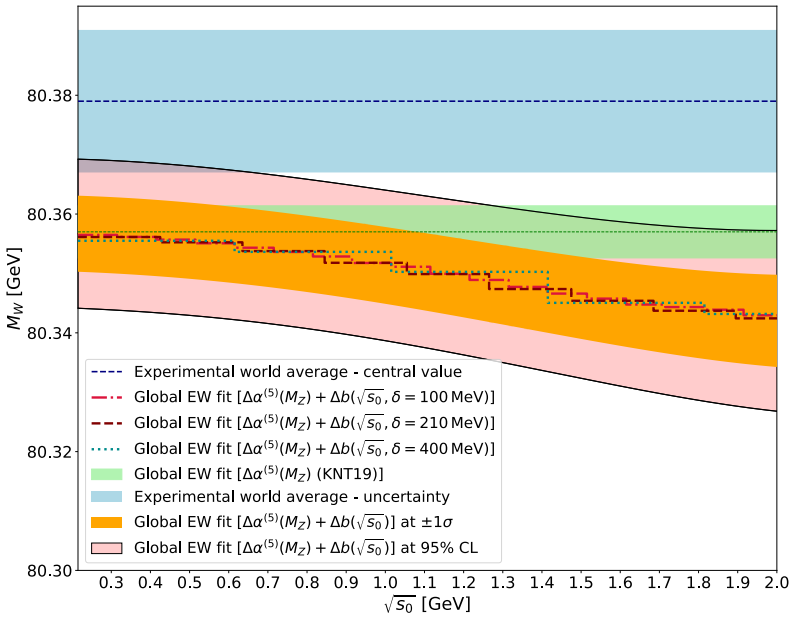
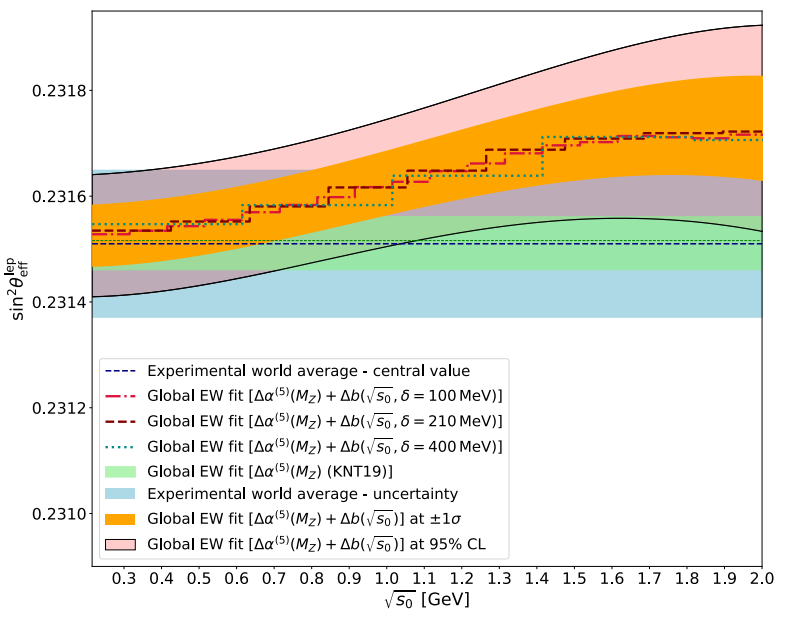
Keshavarzi, Marciano, Passera and Sirlin, Phys. Rev. D 102 (2020) 033002

- Shift KNT hadronic cross section in fully energy-dependent (point-like and binned) analysis to account for Δa_μ.
- Input new values of $\Delta\alpha$ into Gfitter to predict EW observables.
- Analysis greatly constrained from precise EW observables measurements and comprehensive hadronic cross section data.

M_W



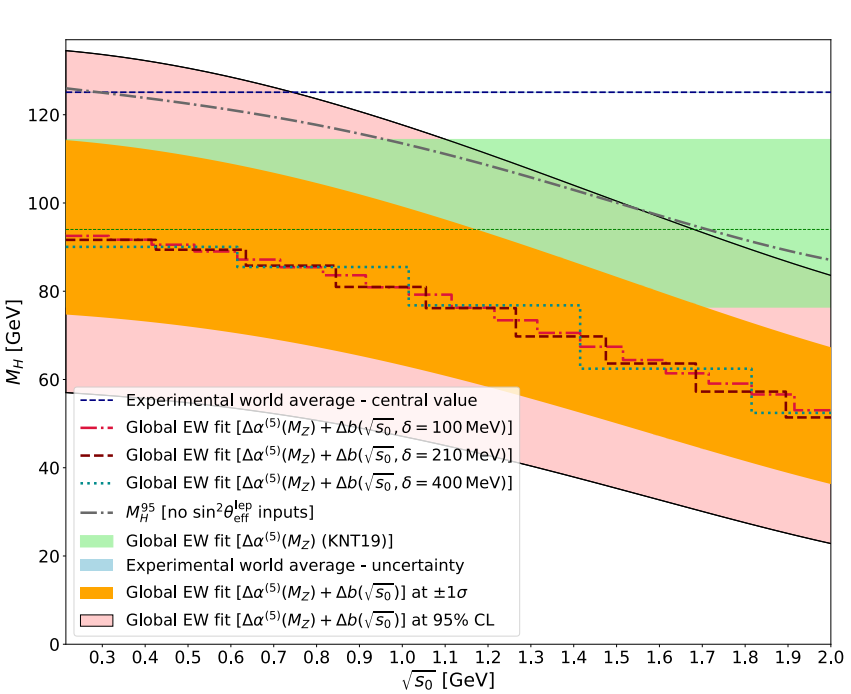
$\sin^2\theta_{eff}^{lep}$

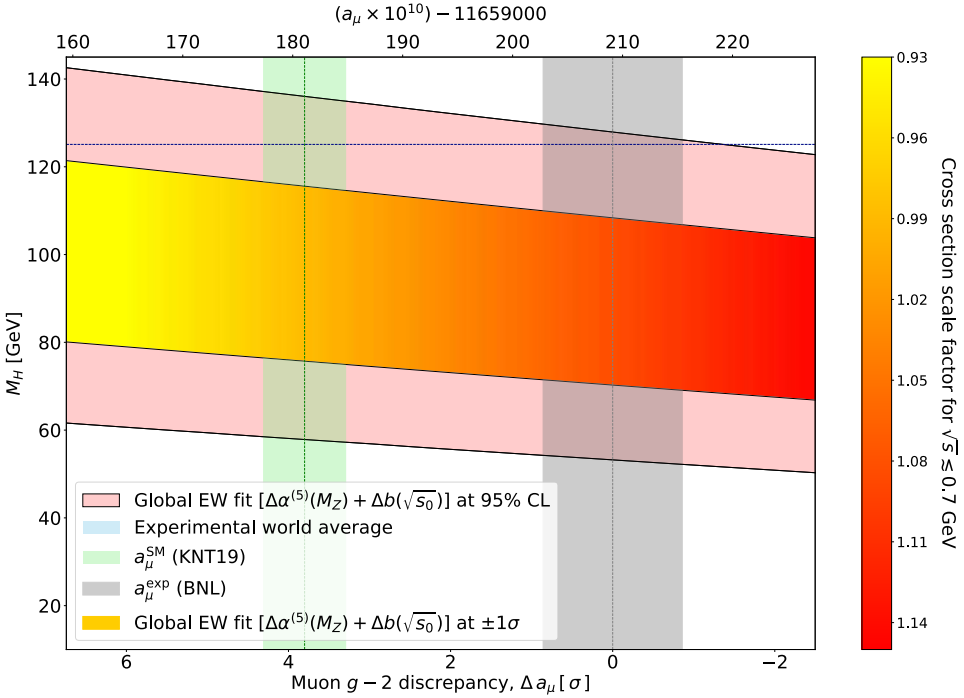


Bounds from the Higgs mass

Keshavarzi, Marciano, Passera and Sirlin, Phys. Rev. D 102 (2020) 033002

Shifts in σ(s) needed to bridge Δa_μ are found to be excluded above $\sqrt{s} > 0.7$ GeV at the 95%CL.



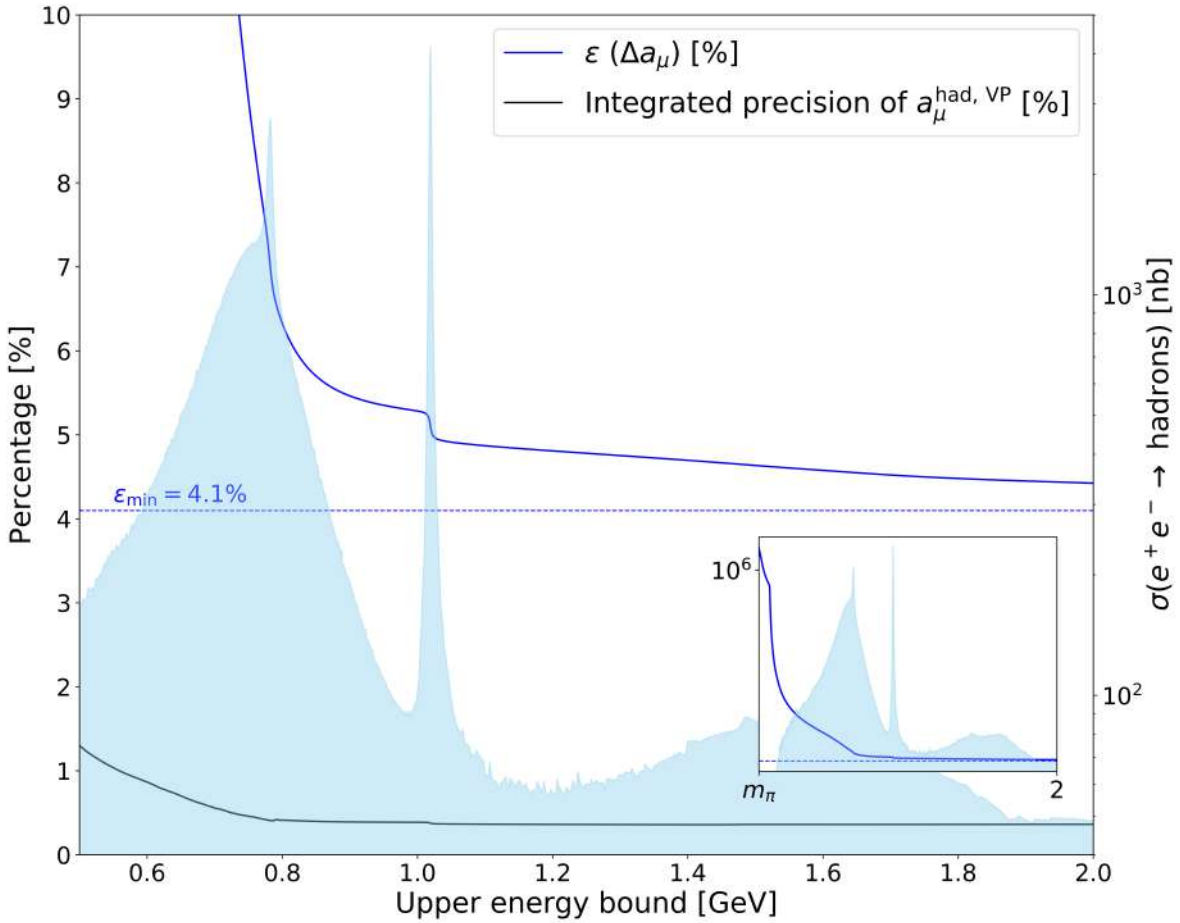


So, from EW sector, shifts to σ(s) to bridge g-2 discrepancy and BMW are allowed below 0.7 GeV…?

→ But, how realistic are the required shifts in σ(s)?

How realistic are the required shifts in σ(s)?

Keshavarzi, Marciano, Passera and Sirlin, Phys. Rev. D 102 (2020) 033002

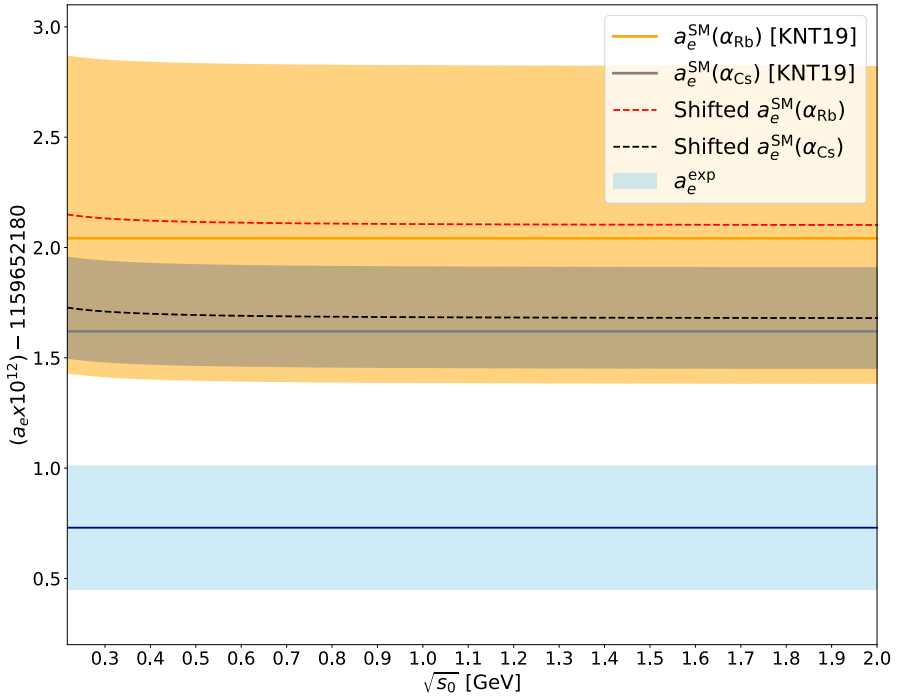


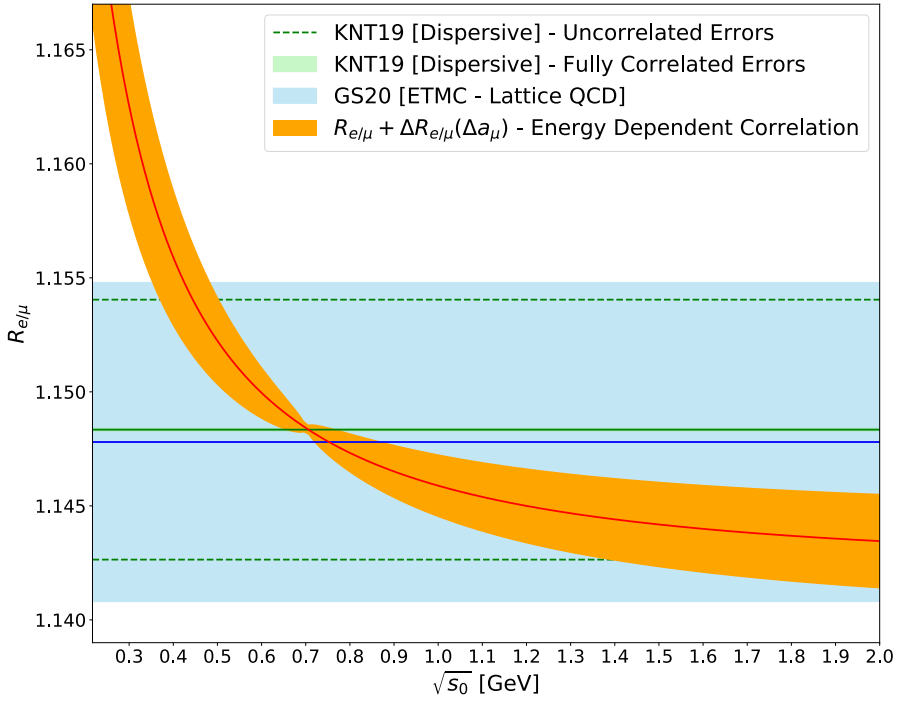
Size of missed contributions would need to be implausibly large given the robust status of the hadronic cross section measurements.

What do this shifts do the electron g-2?

Keshavarzi, Marciano, Passera and Sirlin, Phys. Rev. D 102 (2020) 033002

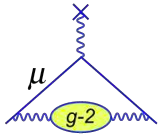
a_e^{SM} vs. a_e^{exp} yields $\Delta a_e = a_e^{\mathrm{exp}} - a_e^{\mathrm{SM}} = (-0.89 \pm 0.36)\times 10^{-12}$ $[2.5\sigma]$





Shifts at low-energy that are “allowed” in muon g-2 invoke additional tension for electron g-2.

Conclusions



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- The muon g-2 provides a robust test of the SM.
- Community-approved theory result from the Muon g-2 theory initiative is now released.
- The difference between theory and experiment is significant at 3.7σ.
- Both the hadronic contributions to the muon g-2 and the running QED coupling depend on the measured hadronic cross section.
- This connection between g-2 and $\Delta\alpha$ allows the impact of the muon g-2 discrepancy on the EW fit to be explored.
- Increases to the hadronic cross section to solve muon g-2 discrepancy affect the predictions of EW precision observables.
- This study excludes shifts to hadronic cross section above 0.7 GeV to bridge muon g-2 discrepancy.
- However, the required shifts to the hadronic cross section below 0.7 GeV are implausibly large.
- And, these low energy shifts also worsen tension in electron g-2.

Thank you for listening.

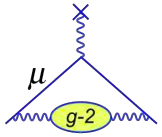
Backups

Theory initiative result released

Contribution	Value $\times 10^{11}$
Experiment (E821)	116 592 089(63)
HVP LO (e^+e^-)	6931(40)
HVP NLO (e^+e^-)	−98.3(7)
HVP NNLO (e^+e^-)	12.4(1)
HVP LO (lattice, *udsc*)	7116(184)
HLbL (phenomenology)	92(19)
HLbL NLO (phenomenology)	2(1)
HLbL (lattice, *uds*)	79(35)
HLbL (phenomenology + lattice)	90(17)
QED	116 584 718.931(104)
Electroweak	153.6(1.0)
HVP (e^+e^-, LO + NLO + NNLO)	6845(40)
HLbL (phenomenology + lattice + NLO)	92(18)
Total SM Value	116 591 810(43)
Difference: $\Delta a_\mu := a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$	279(76)

QED fully cross checked

Slide content by Thomas Teubner.



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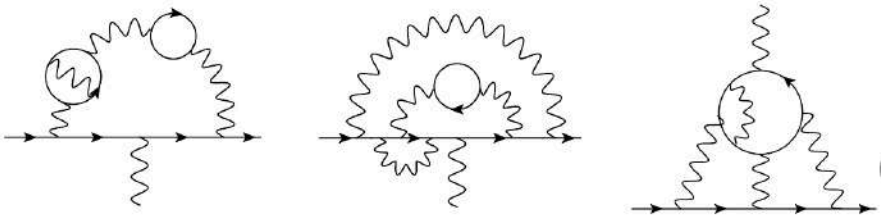
Kinoshita et al. 2012: 5-loop completed numerically (12672 diagrams):

... but 4-loop and 5-loop rely heavily on numerical integrations

Recently several independent checks of 4-loop and 5-loop diagrams:

Baikov, Maier, Marquard [NPB 877 (2013) 647], Kurz, Liu, Marquard, Smirnov AV+VA, Steinhauser [NPB 879 (2014) 1, PRD 92 (2015) 073019, 93 (2016) 053017]:

all 4-loop graphs with internal lepton loops now calculated independently, e.g.



(from Steinhauser et al., PRD 93 (2016) 053017)

4-loop universal (massless) term calculated semi-analytically to 1100 digits (!) by Laporta, arXiv:1704.06996, also new numerical results by Volkov, 1705.05800

all agree with Kinoshita et al.'s results, so QED is on safe ground ✓

$$a_\mu^{\mathrm{QED}}(\alpha(\mathrm{Cs})) = 116\,584\,718.931\,(104) \times 10^{-11}$$

[T. Aoyma et al, 2012, 2019, Laporta 2017,...]

uncertainty dominated by $\mathcal{O}(\alpha^6)$ contributions

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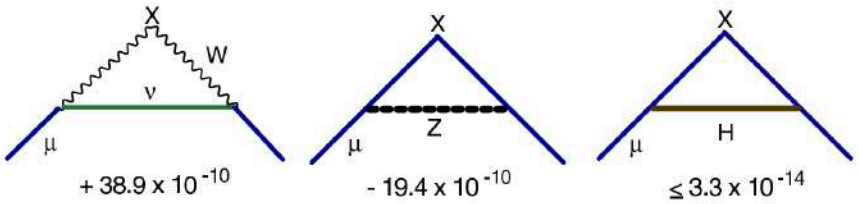
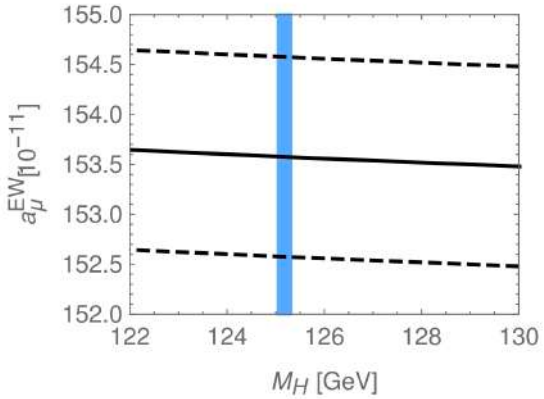
EW unchanged

Slide content by Thomas Teubner.

Electro-Weak 1-loop diagrams:





known to 2-loop (1650 diagrams, the first full EW 2-loop calculation):
Czarnecki, Krause, Marciano, Vainshtein; Knecht, Peris, Perrottet, de Rafael

agreement, a_μ^{EW} relatively small, 2-loop relevant: $a_\mu^{EW(1+2\ loop)} = (154\pm2)\times10^{-11}$

Higgs mass now known, update by Gnendiger, Stoeckinger, S-Kim,
PRD 88 (2013) 053005

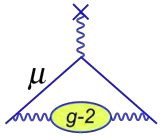
$a_\mu^{EW(1+2\ loop)} = (153.6\pm1.0)\times10^{-11}$ ✓

Uncertainty dominated by hadronic contributions.

compared with $a_\mu^{QED} = 116\ 584\ 718.951\ (80)\times10^{-11}$

Dispersive HLbL

Slide content by Aida El-Khadra.



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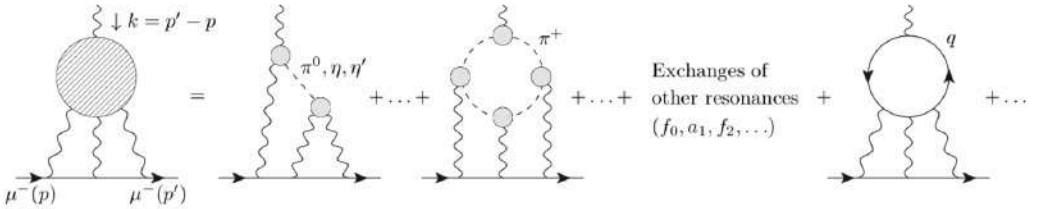
Had never been achieved before the theory initiative…

Dispersive approach:

[Colangelo at al, 2014; Pauk & Vanderhaegen 2014; …]

- model independent
- significantly more complicated than for HVP
- provides a framework for data-driven evaluations
- can also use lattice results as inputs

- Target: ≲ 10% total error



Contribution	PdRV(09) [471]	N/JN(09) [472, 573]	J(17) [27]	Our estimate
π^0, η, η'-poles	114(13)	99(16)	95.45(12.40)	93.8(4.0)
π, K-loops/boxes	−19(19)	−19(13)	−20(5)	−16.4(2)
S-wave $\pi\pi$ rescattering	−7(7)	−7(2)	−5.98(1.20)	−8(1)
subtotal	88(24)	73(21)	69.5(13.4)	69.4(4.1)
scalars	–	–	–	− 1(3)
tensors	–	–	1.1(1)	
axial vectors	15(10)	22(5)	7.55(2.71)	6(6)
u, d, s-loops / short-distance	–	21(3)	20(4)	15(10)
c-loop	2.3	–	2.3(2)	3(1)
total	105(26)	116(39)	100.4(28.2)	92(19)

Lattice HLbL

Slide content by Aida El-Khadra.

Had never been achieved before the theory initiative…

- Dispersive calculation of the pion TFF — Hoferichter et al. (18)

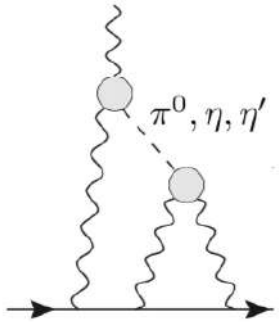
$$a_\mu^{\pi^0} = 63.0^{+2.7}_{-2.1} \times 10^{-11}$$

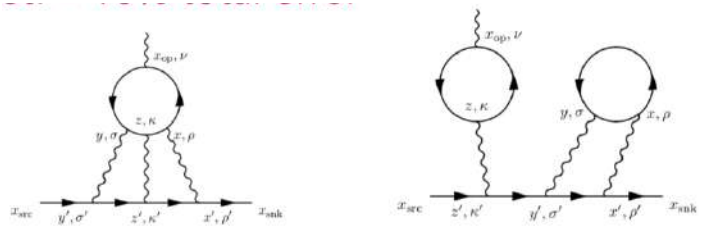
- Padé-Canterbury approximants — Masjuan & Sanchez-Puertas (17)

$$a_\mu^{\pi^0} = 63.6(2.7) \times 10^{-11}$$

- Lattice — Gérardin, Meyer, Nyffeler (19)

$$a_\mu^{\pi^0} = 62.3(2.3) \times 10^{-11}$$





RBC/UKQCD [T. Blum et al, arXiv:1911.08123, PRL 2020]:

✦ Target: ≲ 10% total error

First complete LQCD calculation of connected and leading disconnected contribution with continuum and finite volume extrapolation

$$a_\mu^{\mathrm{HLbL}} = 7.87\,(3.06)\,(1.77) \times 10^{-10}$$

➠ a_μ^{HLbL} cannot "rescue" the SM

➠ combine disp and lattice HLbL for SM prediction

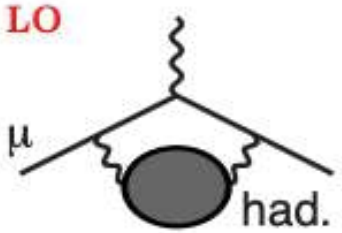
Data-driven HVP

Slide content by Aida El-Khadra.

First-time agreement between various groups…

Detailed comparisons by-channel and energy range between direct integration results:

	DHMZ19	KNT19	Difference
$\pi^+\pi^-$	507.85(0.83)(3.23)(0.55)	504.23(1.90)	3.62
$\pi^+\pi^-\pi^0$	46.21(0.40)(1.10)(0.86)	46.63(94)	−0.42
$\pi^+\pi^-\pi^+\pi^-$	13.68(0.03)(0.27)(0.14)	13.99(19)	−0.31
$\pi^+\pi^-\pi^0\pi^0$	18.03(0.06)(0.48)(0.26)	18.15(74)	−0.12
K^+K^-	23.08(0.20)(0.33)(0.21)	23.00(22)	0.08
$K_S K_L$	12.82(0.06)(0.18)(0.15)	13.04(19)	−0.22
$\pi^0\gamma$	4.41(0.06)(0.04)(0.07)	4.58(10)	−0.17
Sum of the above	626.08(0.95)(3.48)(1.47)	623.62(2.27)	2.46
[1.8, 3.7] GeV (without $c\bar{c}$)	33.45(71)	34.45(56)	−1.00
J/ψ, $\psi(2S)$	7.76(12)	7.84(19)	−0.08
[3.7, ∞) GeV	17.15(31)	16.95(19)	0.20
Total $a_\mu^{\mathrm{HVP,\,LO}}$	$694.0(1.0)(3.5)(1.6)(0.1)_\psi(0.7)_{\mathrm{DV+QCD}}$	692.8(2.4)	1.2



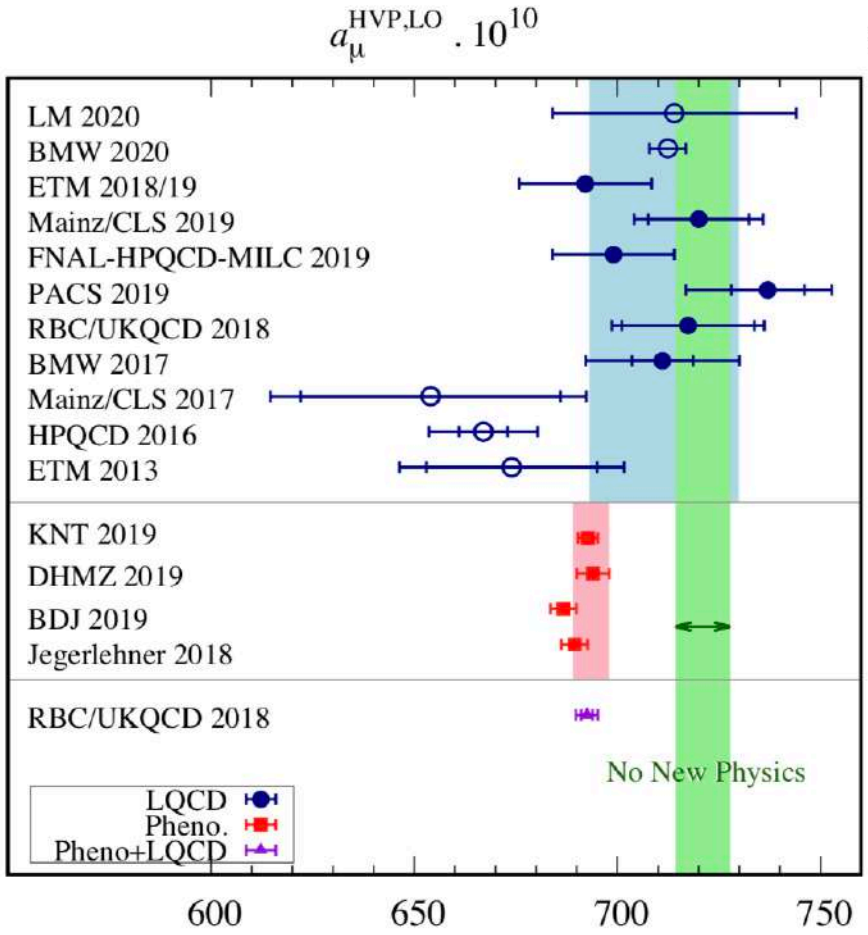
+ evaluations using unitarity & analyticity constraints for $\pi\pi$ and $\pi\pi\pi$ channels
[CHS 2018, HHKS 2019]

Conservative merging to obtain a realistic assessment of the underlying uncertainties:
- account for differences in results from the same experimental inputs
- include correlations between systematic errors

➡ $a_\mu^{\mathrm{HVP,LO}} = 693.1\,(4.0) \times 10^{-10}$

Lattice HVP

Slide content by Aida El-Khadra.



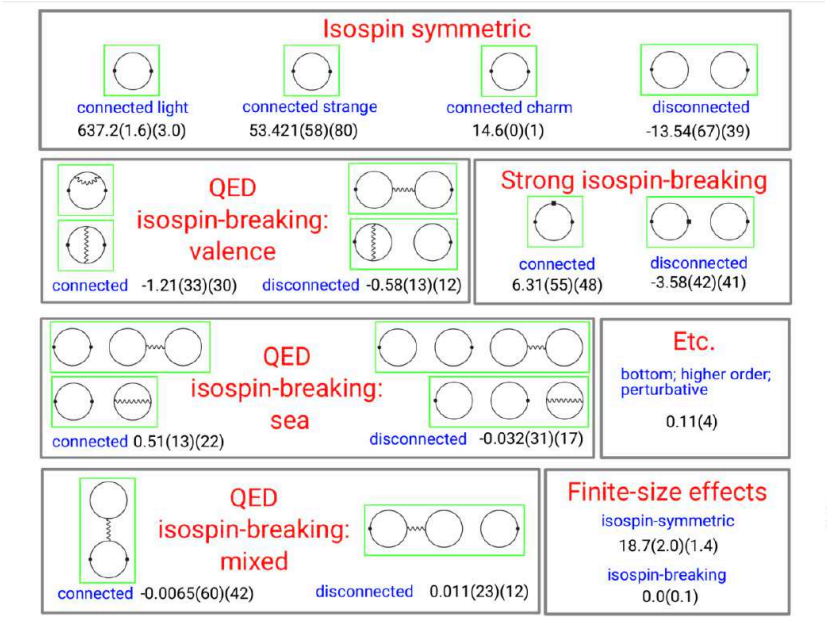
adapted from [T. Aoyama et al, arXiv:2006.04822]

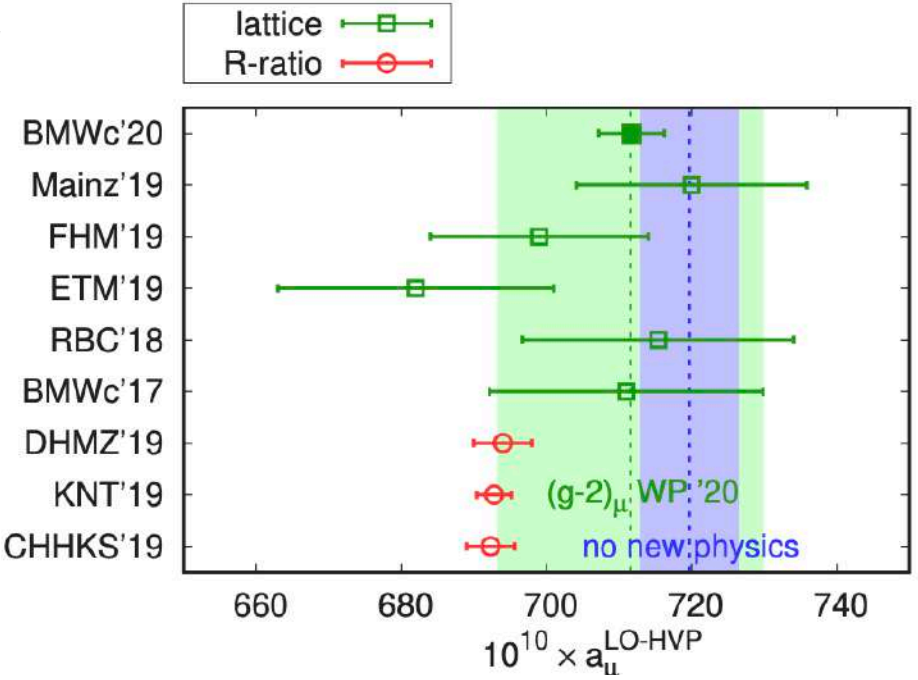
- The errors in (all but one of the) lattice QCD results are still large
- All results include contributions from connected ud, s, c, b + disconnected, QED + strong isospin breaking, and finite volume corrections.
- Lattice combination: included results shown with filled circles

$$a_\mu^{\mathrm{HVP,LO}} = a_\mu^{\mathrm{HVP,LO}}(ud) + a_\mu^{\mathrm{HVP,LO}}(s) + a_\mu^{\mathrm{HVP,LO}}(c) + a_{\mu\,\mathrm{disc}}^{\mathrm{HVP,LO}} + \delta a_\mu^{\mathrm{HVP,LO}} = 711.6\,(18.4) \times 10^{-10}$$

Lattice HVP from BMW

Slide content by Laurent Lellouch.





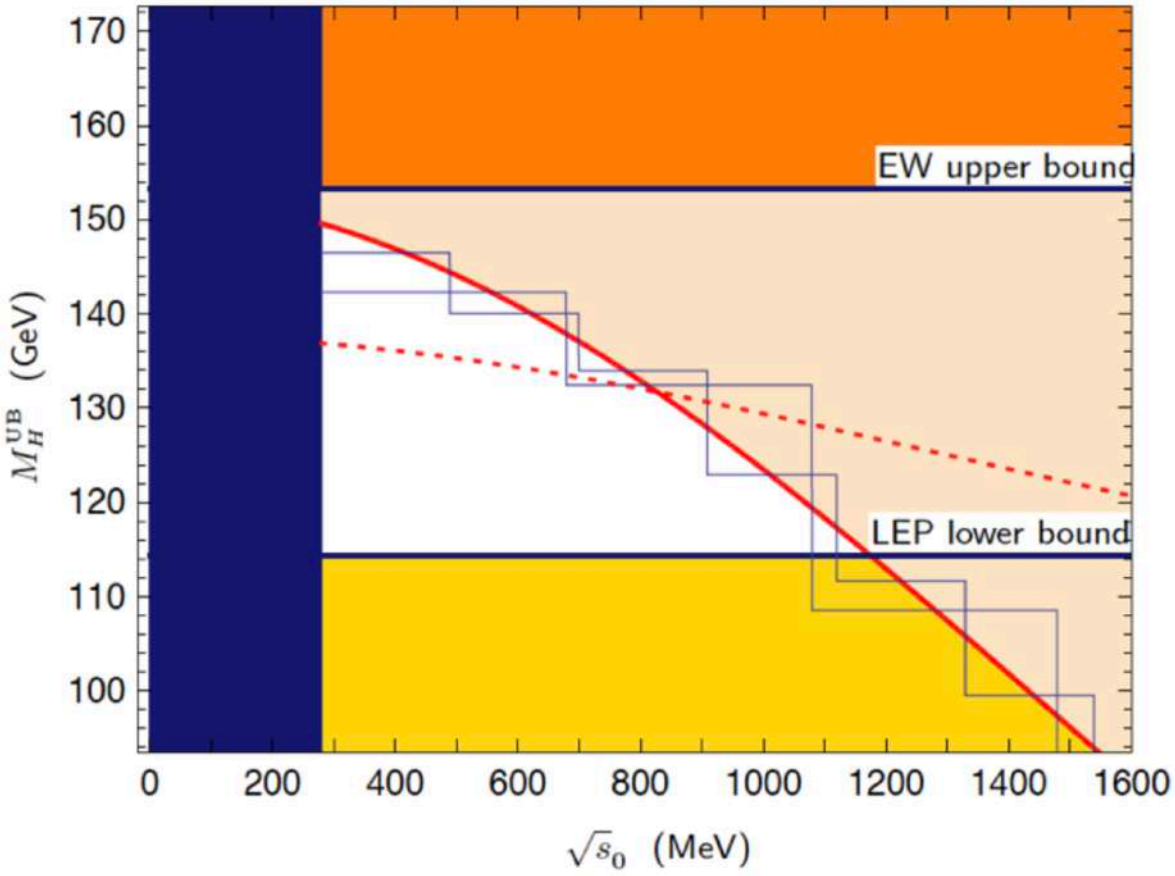
$$a_\mu^{\text{LO-HVP}} = 712.4(1.9)_{\text{stat}}(4.0)_{\text{syst}}\,[4.5]_{\text{tot}} \times 10^{-10}\ [0.6\%]$$

- Consistent with other lattice results
- Total uncertainty is $\sim \div 4$ . . .
- . . . and comparable to R-ratio
- Consistent w/ BNL experiment ("no new physics" scenario) !
- 3.1σ larger than DHMZ'19, 3.9σ than KNT'19 ?

Currently being scrutinised by theory initiative for white paper round 2…

The Muon g-2 and the bounds on the Higgs boson mass

Marciano, Passera and Sirlin (2008)

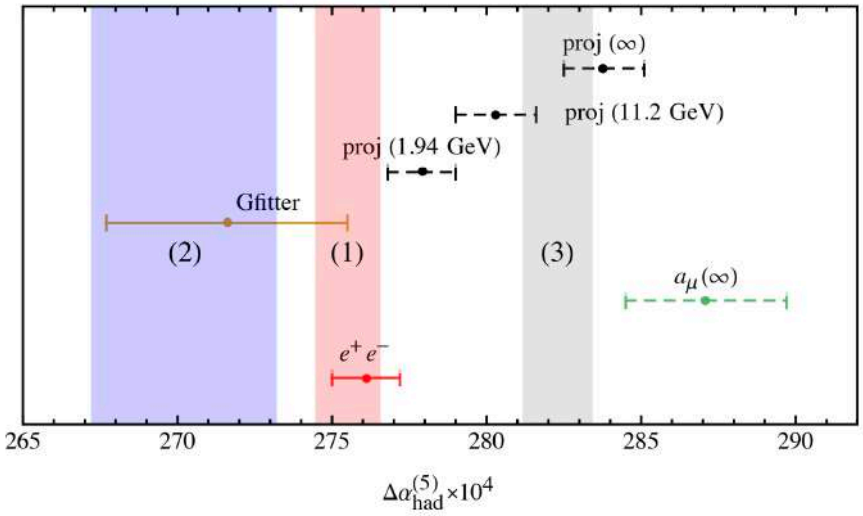


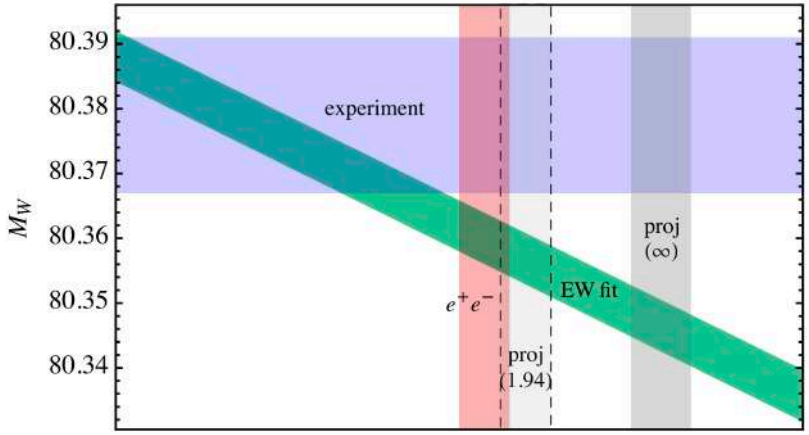
"… if the hadronic cross section is shifted up in energy regions centred above ~ 1.2 GeV to bridge the muon g–2 discrepancy, the Higgs mass upper bound becomes inconsistent with the LEP lower limit."

Hadronic vacuum polarization: $(g-2)_\mu$ versus global electroweak fits

Crivellin, Hoferichter, Manzari, Montull, arXiv:2003.04886 (2020).

- Using global EW fit (HEPFitter) results in a tension with the BMW result.
- A significant shift in HVP exacerbates tensions within the EW fit → inconsistent W mass prediction.
- Does not weaken the case for BSM physics , but to some extent shifts it from (g – 2)μ to the EW fit.





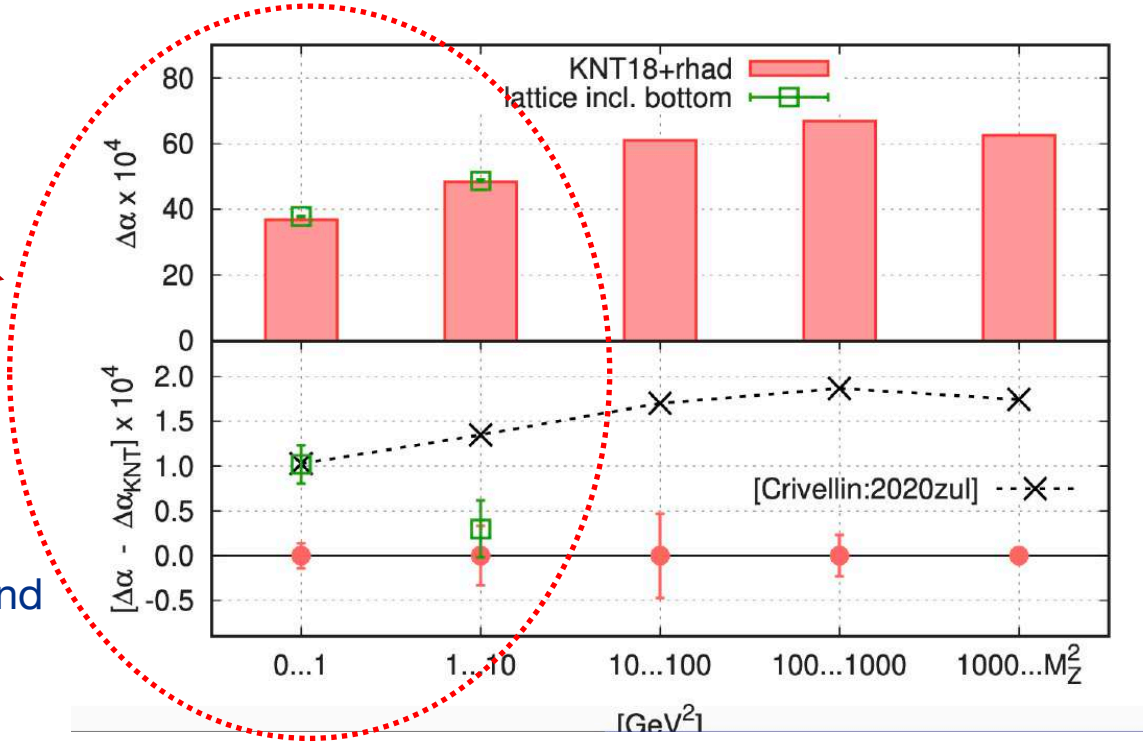
- Results cannot rule out BMW.
- Analysis uses strong assumptions → energy independent shifts of the cross section.
- Incorporating energy-dependence at low energies is crucial.

Energy dependence of BMW

Slide content by Laurent Lellouch.

- Crivellin et al '20, most aggressive scenario: our results suggest a 4.2σ overshoot in $\Delta^{(5)}_{\rm had}\alpha(M_Z^2)$ compared to result of fit to EWPO
- Assume same 2.8% relative deviation from R-ratio as we find in $a_\mu^{\rm LO\text{-}HVP}$
- Hypothesis is not consistent w/ BMWc '17 nor new preliminary result

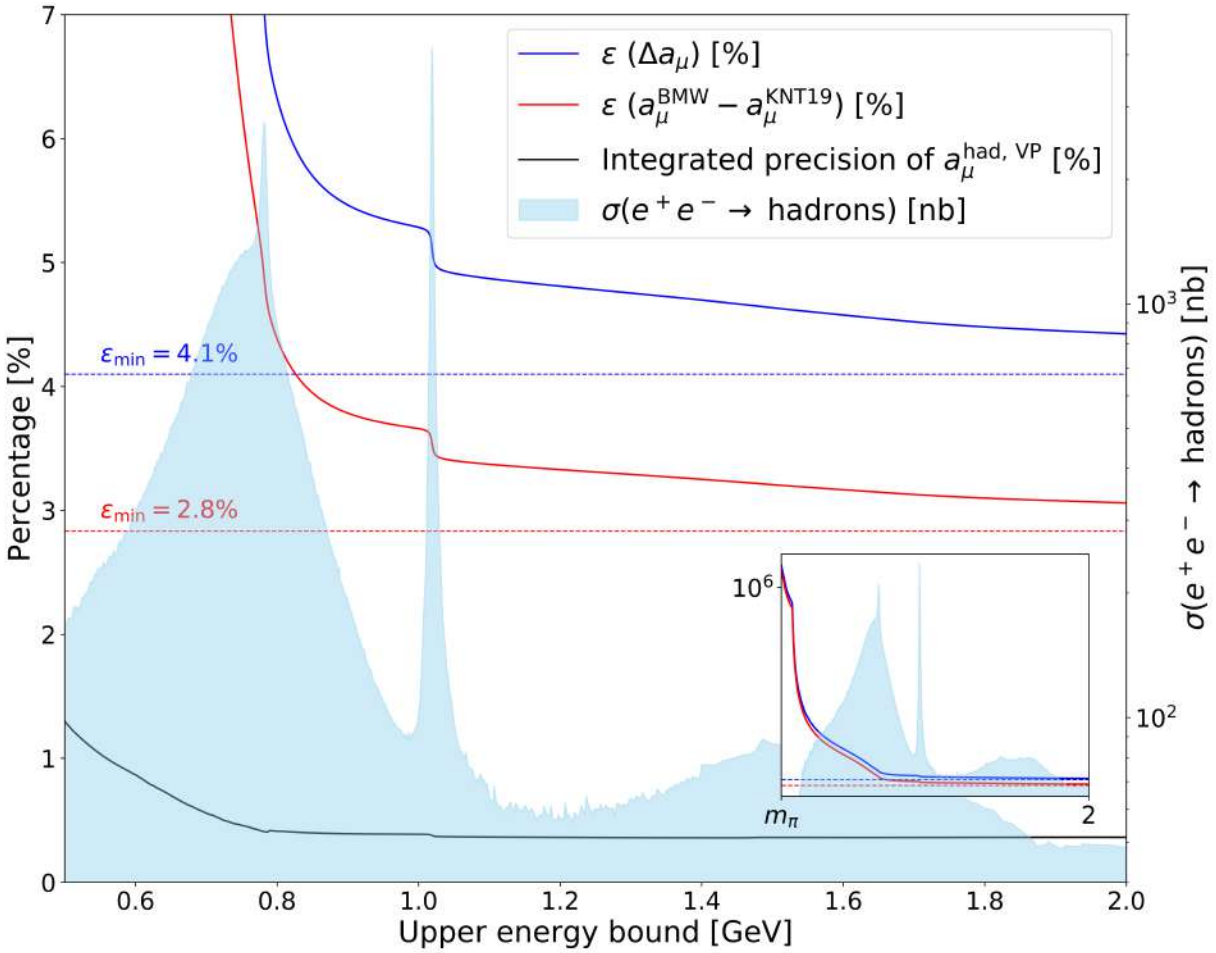
Differences between BMW and KNT are contained within low-energy, non-perturbative hadronic domain.



Note: Comparison is between time-like $\Delta\alpha$ from e^+e^- data and space-like $\Delta\alpha$ from lattice in Euclidean space-time.

How realistic are the required shifts in σ(s)?

Keshavarzi, Marciano, Passera and Sirlin, Phys. Rev. D 102 (2020) 033002

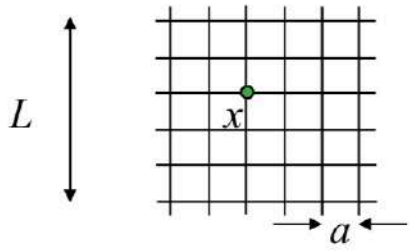


Size of missed contributions would need to be implausibly large given the robust status of the hadronic cross section measurements.

Lattice QCD

Slide content by Aida El-Khadra.

$$\mathcal{L}_{\rm QCD} = \sum_f \bar{\psi}_f (\not{D} + m_f)\psi_f + \frac{1}{4}\mathrm{tr} F_{\mu\nu}F^{\mu\nu}$$

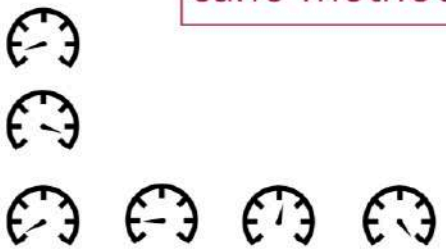


- discrete Euclidean space-time (spacing a)
  derivatives → difference operators, etc…
- finite spatial volume (L)
- finite time extent (T)

Integrals are evaluated numerically using monte carlo methods.

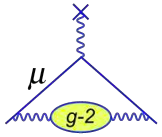
adjustable parameters

- lattice spacing: $a \to 0$
- finite volume, time: $L \to \infty, T > L$
- quark masses (m_f): $M_{H,\rm lat} = M_{H,\rm exp}$, $m_f \to m_{f,\rm phys}$
  tune using hadron masses
  extrapolations/interpolations



Dispersive HVP

Slide content by Aida El-Khadra.

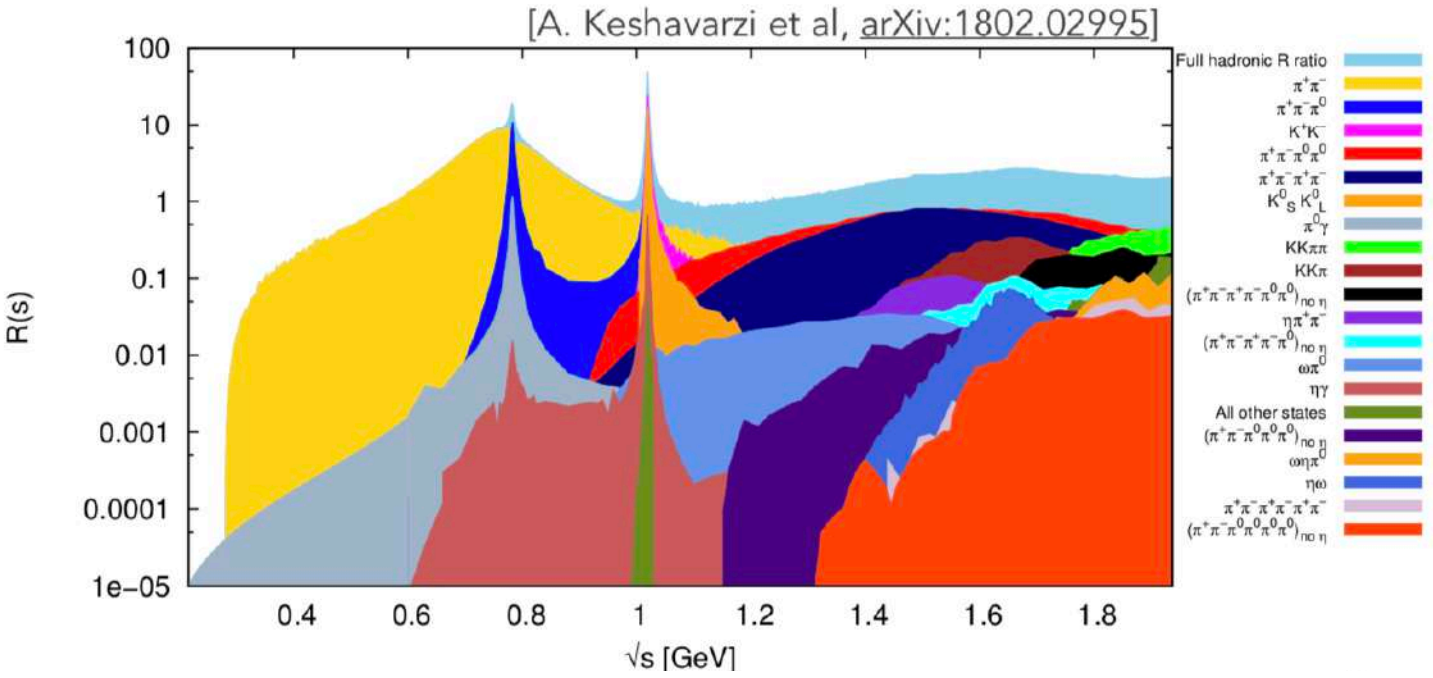


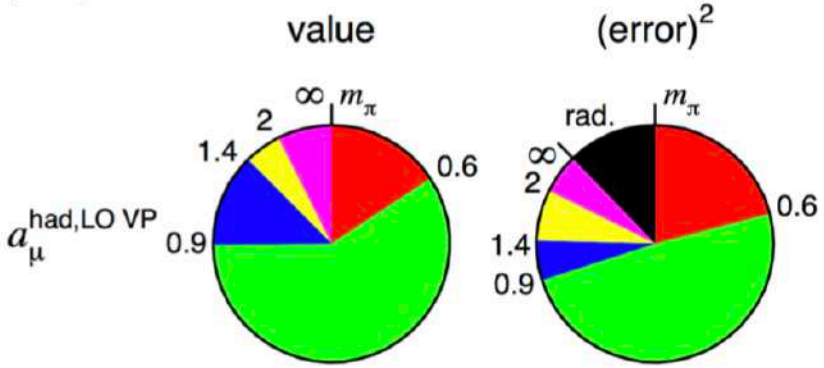
MANCHESTER 1824
The University of Manchester

- Target: ~0.2% total error
- Dispersion relation + experimental data for $e^+e^- \to \text{hadrons}$ (and τ data)
  - current uncertainty ~0.5%
  - can be improved with more precise experimental data
  - new experimental measurements expected/ongoing at BaBar, BES-III, Belle-II, CMD-3, SND, KEDR, KLOE,….
- Challenges:
  - below ~2 GeV: sum > 30 exclusive channels: 2π, 3π, 4π, 5π, 6π, $2K$, $2K\pi$, $2K2\pi$, $\eta\pi$,…. (use isospin relations for missing channels)
  - above ~1.8 GeV:
    inclusive, pQCD (away from flavor thresholds)
    + narrow resonances (J/ψ, Υ,..)
  - Combine data from different experiments/measurements:
    understanding correlations, sources of sys. error, tensions…
  - include FS radiative corrections

Dispersive HVP

Slide content by Aida El-Khadra.



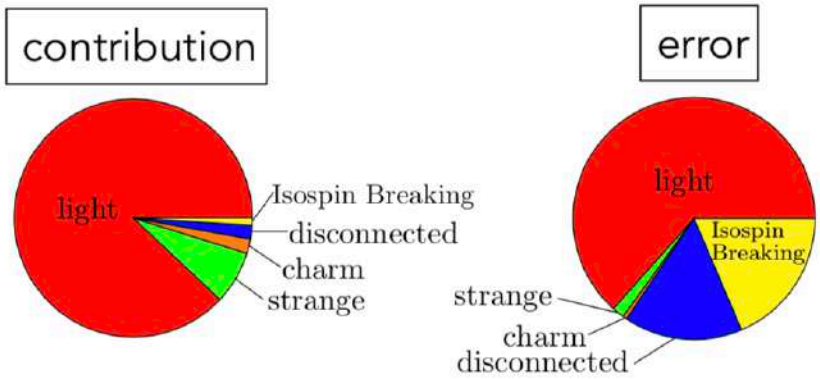


Lattice HVP

Slide content by Aida El-Khadra.

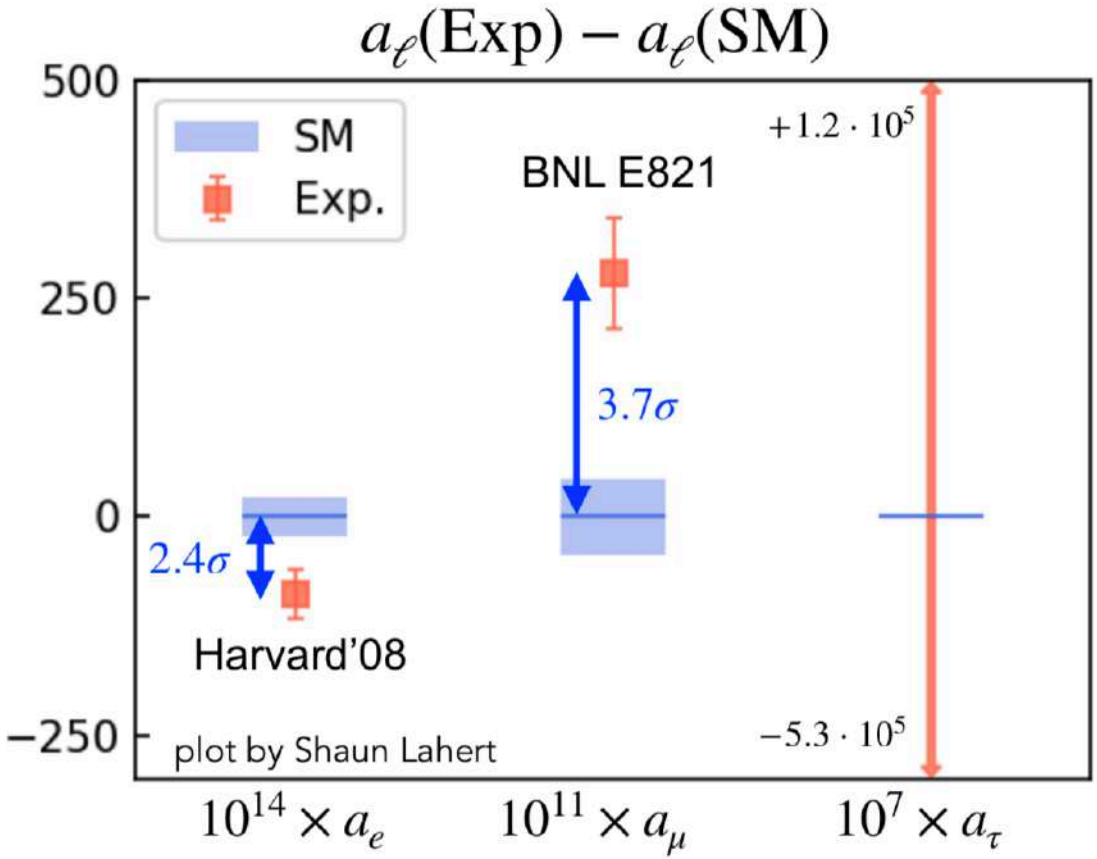
- Target: < 0.5% total error
- light-quark connected contribution, $a_{\mu,ud}^{\mathrm{HLO}}$:
  ~90% of total, with 1-3% error
- "heavy" flavor contributions, $a_{\mu,s}^{\mathrm{HLO}}$, $a_{\mu,c}^{\mathrm{HLO}}$, $a_{\mu,b}^{\mathrm{HLO}}$:
  ~8%, 2%, 0.05% of total a_{μ}^{HLO}, can be calculated with sufficient precision
- disc. contribution:
  ~2% of total a_{μ}^{HLO}, contributes ~0.3-1% error to a_{μ}^{HLO}
- Isospinbreaking (QED + $m_u \neq m_d$) corrections:
  ~1% of total a_{μ}^{HLO}, contribute ~0.3-1% error

[V. Gülpers, adapted for WP from talk @ Lattice 2019, arXiv:2001.11898]



Lepton moments summary

Slide content by Aida El-Khadra.



Sensitivity to heavy new physics:

$$a_\ell^{\mathrm{NP}} \sim \frac{m_\ell^2}{\Lambda^2}$$

$$(m_\mu/m_e)^2 \sim 4 \times 10^4$$

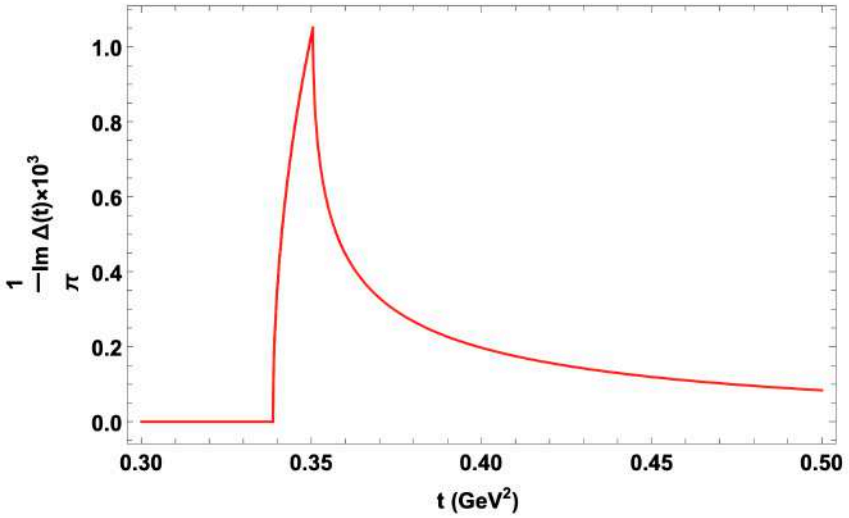
Ongoing experimental programs for improved measurements of α

[S. Guellati-Khelifa (Paris), Z. Pagel (Berkeley) @ INT workshop]

On constraints between $\Delta\alpha$ and g-2

de Rafael, arXiv:2006.13880 (2020).

- Claim that it is possible to construct spectral functions which reproduce both BMW and KNT, inducing only a very small change in $\Delta\alpha$.
- Constrain the contribution to the hadronic running that can be related to a particular dimension-6 operator (or, equivalently, to the first moment of the HVP function).
- Implication is there is a bound that restricts all effects in the hadronic running to the small piece that is related to this first moment (the first term in Eq. (6) of the attached notes). This is not the case.
- Needs to go past just first moment.
- Additionally, these shifts in the cross section at such low energies will have to be enormous.
- But, provides interesting comparison between time-like and space-like $\Delta\alpha$.

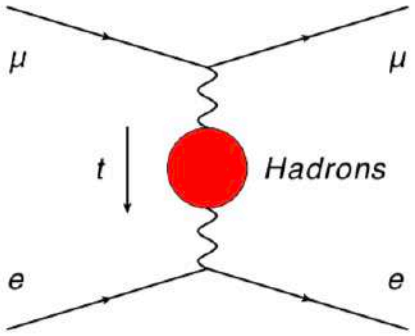


MUonE

Slide content by Massimo Passera.

μ-e elastic scattering to measure $a_\mu^{\rm HVP}$

M. Passera @ HVP KEK 2018 [A. Abbiendi et al, arXiv:1609.08987, EPJC 2017]



$$a_\mu^{\rm HLO} = \frac{\alpha}{\pi}\int_0^1 dx\,(1-x)\,\Delta\alpha_{\rm had}[t(x)]$$

$$t(x) = \frac{x^2 m_\mu^2}{x-1} < 0$$

$\Delta\alpha_{\rm had}(t)$ is the hadronic contribution to the running of α in the space-like region. It can be extracted from scattering data!

- use CERN M2 muon beam (150 GeV)
- Physics beyond colliders program @ CERN
- LOI June 2019
- Jan 2020: SPSC recommends pilot run in 2021
- goal: run with full apparatus in 2023-2024

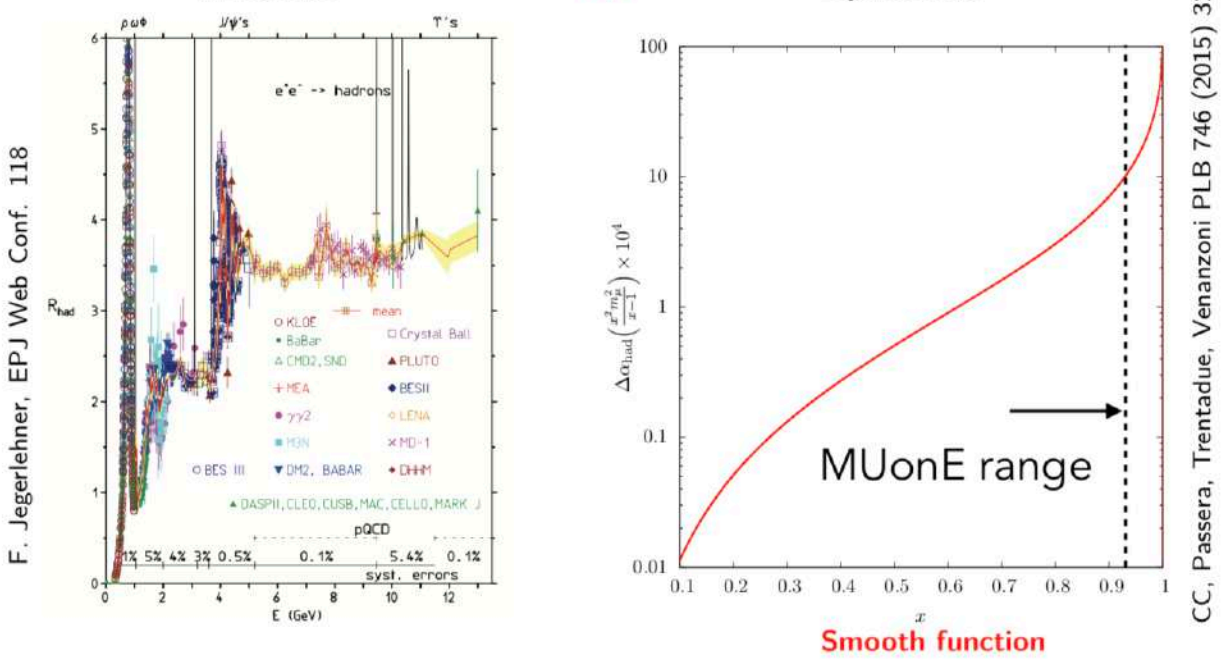
MUonE

Slide content by Massimo Passera.

μ-e elastic scattering to measure a_μ^{HVP}

C. Carloni @ g-2 INT workshop [A. Abbiendi et al, arXiv:1609.08987, EPJC 2017]

Time-like ↦ **Space-like**



- requires calculations of radiative corrections [M. Fael @ g-2 INT workshop]
- complement region not accessible to experiment with LQCD calculation [M. Marinkovic @ g-2 INT workshop]