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Quantum Computing for Neutrino-nucleus Scattering with NISQ Devices

Andy C. Y. Li (Fermilab Quantum Institute)

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Alessandro Roggero (UW)

Andy C. Y. Li (Fermilab)

Joseph Carlson (LANL)

Rajan Gupta (LANL)

Gabriel N. Perdue (Fermilab)

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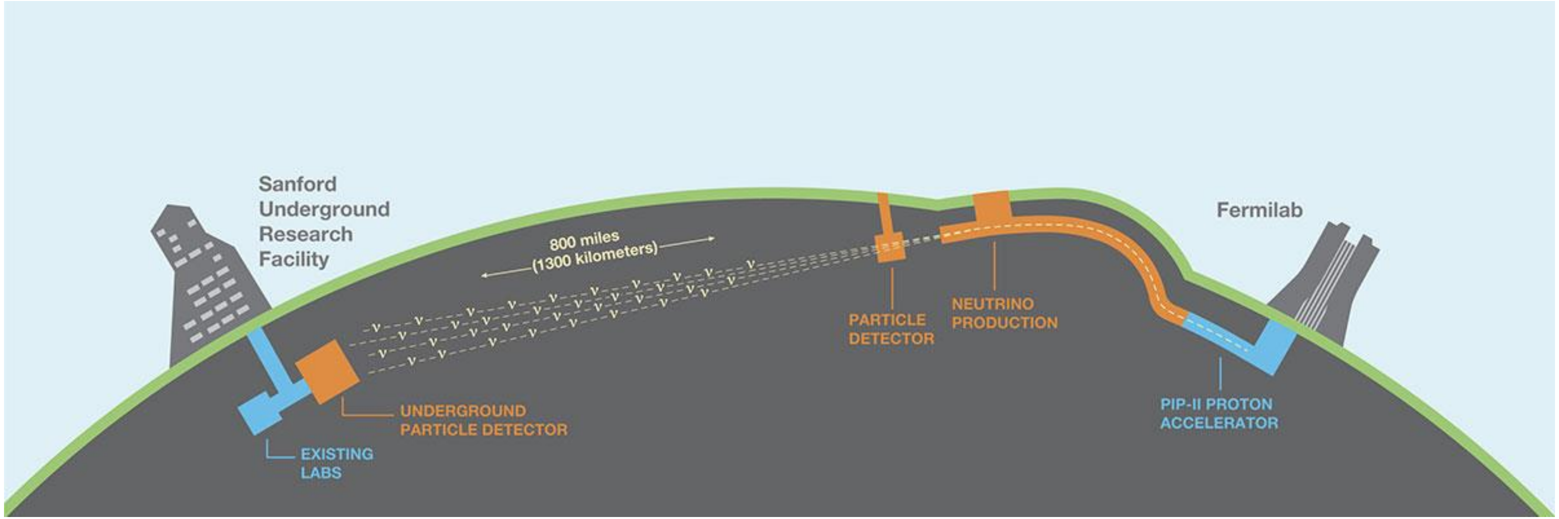
Neutrino-nucleus scattering

- Accelerator Neutrino Experiments, e.g. DUNE

- Simulate scattering cross sections to predict detector efficiency and backgrounds



Simulate response function and cross sections

- Dynamical linear response function

$$S(\omega, \hat{O}) = \sum_{\nu} \left|\langle \phi_\nu | \hat{O} | \phi_0 \rangle\right|^2 \delta(E_\nu - E_0 - \omega) = \int dt \, \langle \phi_0 | \hat{O}^\dagger e^{-i\,(\hat{H} - E_0 - \omega)t} \hat{O} | \phi_0 \rangle$$

Nuclei: $\hat{H}|\phi_\nu\rangle = E_\nu|\phi_\nu\rangle$ ground state: $|\phi_0\rangle$

- $S(\omega, \hat{O}) \rightarrow$ inclusive cross sections

- Sample the final nuclei state $|\phi_\nu\rangle \rightarrow$ semi-exclusive cross sections

- Quantum advantage: bigger nuclei, wide range of kinematics

Starting point: pionless effective field theory

$$H = 2DtA - t\sum_{f=1}^{N_f}\sum_{\langle i,j\rangle}^{M}\left[c^\dagger_{i,f}c_{j,f} + c^\dagger_{i,f}c_{j,f}\right]$$

Kinetic energy

$$+\frac{1}{2}C_0\sum_{f\neq f'}^{N_f}\sum_{i=1}^{M} n_{i,f}n_{i,f'}$$

Attractive 2-body contact interaction ($C_0 < 0$)

$$+\frac{D_0}{6}\sum_{f\neq f'\neq f''}^{N_f}\sum_{i=1}^{M} n_{i,f}n_{i,f'}n_{i,f''}\,,$$

Repulsive 3-body interaction ($D_0 > 0$) to avoid collapse into deeply bound state

- Approximately reproduce binding of 3 and 4 nucleons

  [Phys. Lett. B 772 839-848 (2017), PRL 124 143402 (2020)]

- Simple model for initial study and quantum resource estimation
  - Future: need interactions involving virtual pions for accurate prediction

Dynamic linear response quantum algorithm

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Dynamic linear response quantum algorithm

Alessandro Roggero* and Joseph Carlson†
Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA

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$$S(\omega, \hat{O}) = \sum_\nu \left|\langle \phi_\nu | \hat{O} | \phi_0 \rangle\right|^2 \delta(E_\nu - E_0 - \omega)$$
$$= \sum_\nu \underline{|\langle \phi_\nu | \psi_{\hat{O}} \rangle|^2} \, \delta(E_\nu - E_0 - \omega) \, \underline{\langle \phi_0 | \hat{O}^\dagger \hat{O} | \phi_0 \rangle}$$

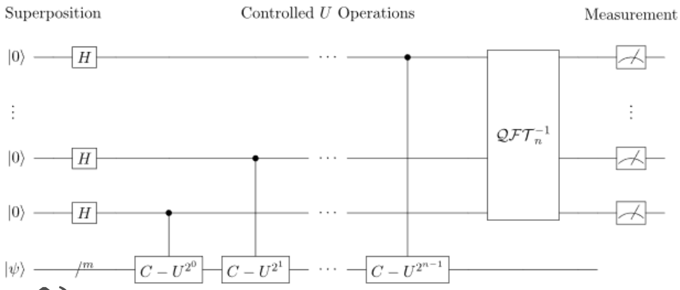
Prob. of $|\psi_{\hat{O}}\rangle$ in eigenbasis $|\phi_\nu\rangle \rightarrow$ QPE

Ground state meas.

$$|\psi_{\hat{O}}\rangle = \frac{\hat{O}|\phi_0\rangle}{\sqrt{\langle \phi_0 | \hat{O}^\dagger \hat{O} | \phi_0 \rangle}}$$

1. Qubit encoding: represent the system by qubits
2. State preparation: $|\psi_{\hat{O}}\rangle$
3. Quantum phase estimation of $|\psi_{\hat{O}}\rangle$ with $\hat{U} = e^{i(\hat{H} - E_0)}$
4. Measure ancilla qubits: probability distribution $\rightarrow S(\omega, \hat{O})$ (nuclei state by measuring the encoding qubits)

Complexity



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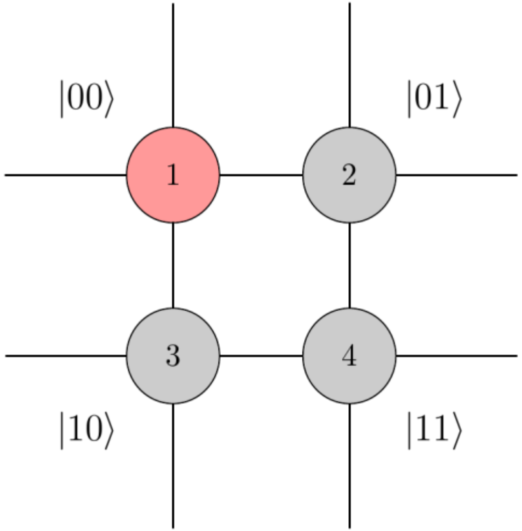
Qubit encoding efficiency

$$H = 2DtA - t\sum_{f=1}^{N_f}\sum_{\langle i,j\rangle}^{M}\left[c^\dagger_{i,f}c_{j,f} + c^\dagger_{i,f}c_{j,f}\right]$$
$$+\frac{1}{2}C_0\sum_{f\neq f'}^{N_f}\sum_{i=1}^{M} n_{i,f}n_{i,f'}$$
$$+\frac{D_0}{6}\sum_{f\neq f'\neq f''}^{N_f}\sum_{i=1}^{M} n_{i,f}n_{i,f'}n_{i,f''}\ ,$$

- Nucleons (fermions) $\rightarrow$ qubits
- General mapping: Jordan-Wigner, Bravyi-Kitaev [1], etc.
- Special case of fixed nucleons: lattice-location encoding
  nucleon 1: $|1\rangle_{N1} = |0\rangle_{q0}|0\rangle_{q1}, |2\rangle_{N1} = |0\rangle_{q0}|1\rangle_{q1}, \ldots$
  nucleon 2: $|1\rangle_{N2} = |0\rangle_{q2}|0\rangle_{q3}, |2\rangle_{N2} = |0\rangle_{q2}|1\rangle_{q3}, \ldots$
  ...
- Efficiency: A nucleons on a lattice with M sites and N_f fermion mode per site

  JW, BK : $N_f \times M$ qubits
  Lattice-location : $A \log_2 M$ qubits



[1]: Ann. Phys. 298, 210 (2002)

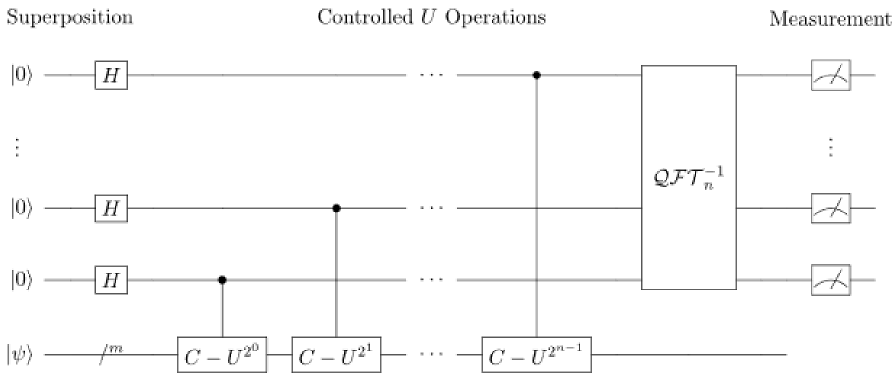
Quantum phase estimation

- QFT and Control-U circuits
  $\hat{U} = e^{i(\hat{H}-E_0)}$: system propagator

- QFT: gate cost = $O(N^2)$
  N: number of ancilla qubits

- U circuits: Trotter decompositions
  - $U_1(\tau) = e^{-i\tau K} e^{-i\tau V}$
  - $U_2^{K+V}(\tau) = e^{-i\tau K/2} e^{-i\tau V} e^{-i\tau K/2}$
  - $U_2^{V+K}(\tau) = e^{-i\tau V/2} e^{-i\tau K} e^{-i\tau V/2}$

- Control-U circuits: replace gates by their controlled version



$$H = 2DtA\left[-t\sum_{f=1}^{N_f}\sum_{\langle i,j\rangle}^{M}\left[c_{i,f}^{\dagger}c_{j,f} + c_{i,f}^{\dagger}c_{j,f}\right]\right.$$

K: kinetic energy

$$\left. + \frac{1}{2}C_0 \sum_{f\neq f'}^{N_f}\sum_{i=1}^{M} n_{i,f}n_{i,f'} + \frac{D_0}{6}\sum_{f\neq f'\neq f''}^{N_f}\sum_{i=1}^{M} n_{i,f}n_{i,f'}n_{i,f''}\right],$$

V: potential energy
Diagonal in qubit basis after JW

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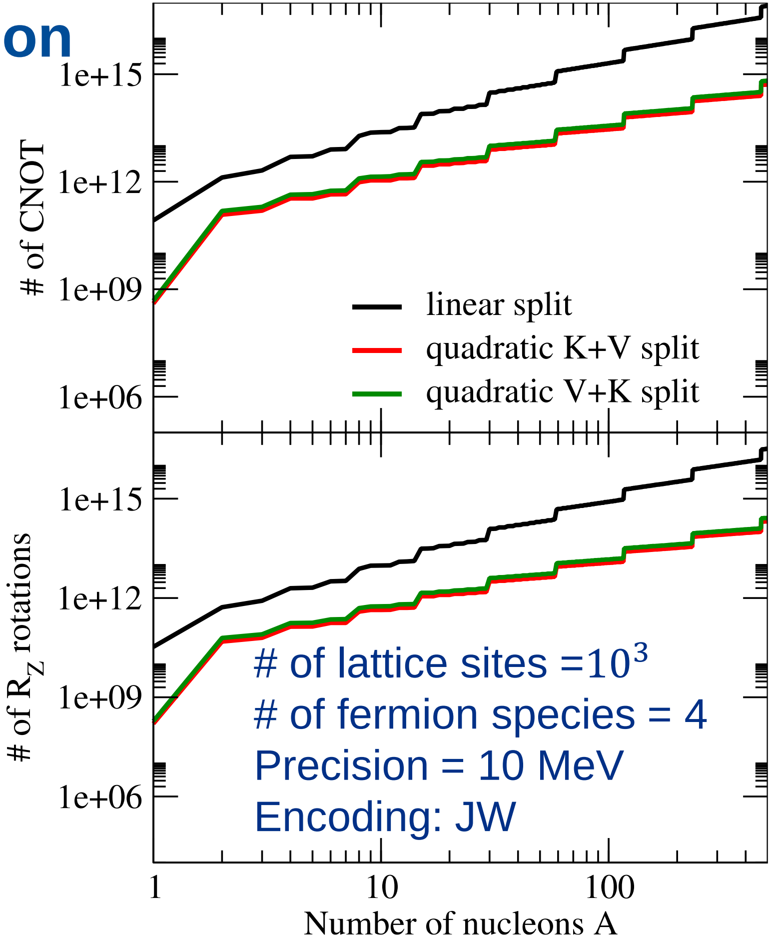
Gate counts of quantum phase estimation

- Gate counts based on 2 gates
  - CNOT: control-not, two-qubit gate
  - R_Z: rotation-Z, single-qubit gate
- Quadratic decomposition: favorable
- Gate counts → $\sim 10^{10}$
  - Final 99% fidelity: $1 - e^{\frac{\ln 0.99}{10^{10}}}$ → ~ 10^{-12} gate error rate
  - **Need error-corrected qubits for full linear response algorithm simulating realistic model**



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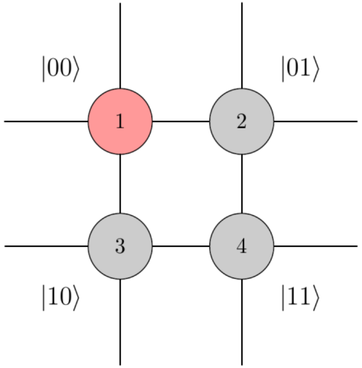
NISQ implementation of modified linear response algorithm



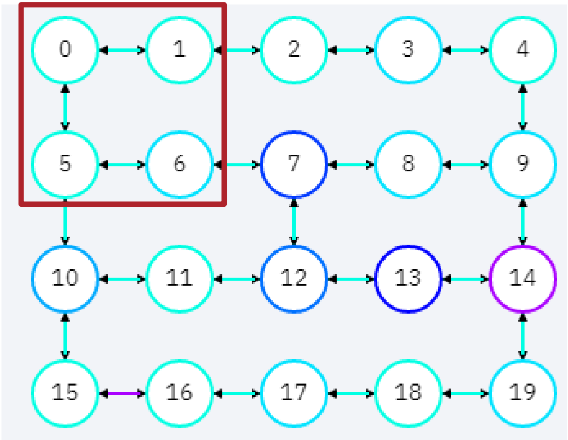
1. Qubit encoding: small # of nucleons
   - Lattice-location encoding
2. State preparation: $|\psi_{\hat{O}}\rangle = \frac{\hat{O}|\phi_0\rangle}{\sqrt{\langle\phi_0|\hat{O}^\dagger\hat{O}|\phi_0\rangle}}$
   - Approximated low-energy state $|\tilde{\phi}_0\rangle$ by a variational ansatz
3. Quantum phase estimation of $|\psi_{\hat{O}}\rangle$ with $\hat{U} = e^{i(\hat{H}-E_0)}$
   - Time evolution by $\hat{U}(t) = e^{i(\hat{H}-E_0)t}$ on a pretrained initial state
4. Measure ancilla qubits: probability distribution $\rightarrow S(\omega, \hat{O})$
   - Directly measure $S(\omega,\hat{O}) = \int dt \langle\phi_0|\hat{O}^\dagger e^{-i(\hat{H}-E_0-\omega)t}\hat{O}|\phi_0\rangle$ (no ancilla qubits)

4-qubit proof-of-principle experiment

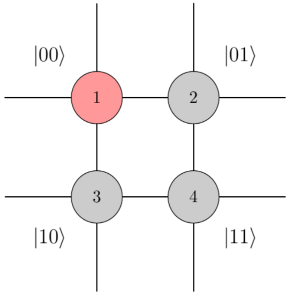
$$H = 2DtA - t\sum_{f=1}^{N_f}\sum_{\langle i,j\rangle}^{M}\left[c^\dagger_{i,f}c_{j,f} + c^\dagger_{i,f}c_{j,f}\right]$$
$$+\frac{1}{2}C_0\sum_{f\neq f'}^{N_f}\sum_{i=1}^{M} n_{i,f}n_{i,f'}$$
$$+\frac{D_0}{6}\sum_{f\neq f'\neq f''}^{N_f}\sum_{i=1}^{M} n_{i,f}n_{i,f'}n_{i,f''}\ ,$$

- Triton toy model:
  - 3 nucleons with one chosen to be static on a 2 by 2 lattice
  - 2 effective nucleons ($A = 2,\ N_f = 2,\ M = 4$)
  - Two-nucleon dynamics incorporates important information about nuclear response (arXiv:1909.06400)

IBMQ Poughkeepsie



- Lattice-location encoding: $A\log M = 4$ qubits
  - In comparison, JW needs $N_f M = 8$ qubits



$$H = 8t + \frac{U}{2} - 2t\sum_{k=1}^{4} X_k$$
$$-\frac{U}{4}(Z_1Z_4 + Z_2Z_3) - \frac{U}{4}\sum_{i<j<k} Z_iZ_jZ_k$$

$$C_0 = U, D_0 = -4U$$

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State preparation with a variational ansatz

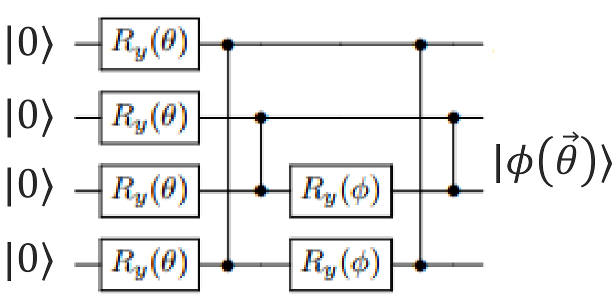
- 2-parameter variational ansatz $|\phi(\vec{\theta})\rangle$

- Trained by a noiseless simulator to minimized the energy $E(\vec{\theta}) = \langle\phi(\vec{\theta})|H|\phi(\vec{\theta})\rangle$

- Optimized state: $|\tilde{\phi}_0\rangle = \hat{O}|\phi_0\rangle$ (low-energy state)

- Run the pretrained circuit on the IBM QPU

- **QPU shows a promising result with error mitigation** (readout error mitigation and noise extrapolation)

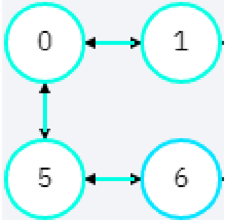


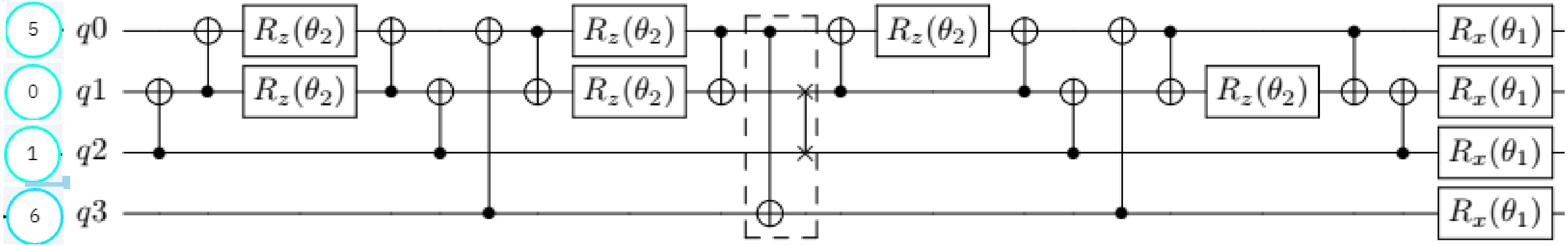
	Energy
exact g.s.	-4.843
simulator	-4.415
QPU corr	-4.4187(98)

Time evolution with 1 Trotter step

$$H = 8t + \frac{U}{2} \underbrace{- 2t \sum_{k=1}^{4} X_k}_{K} \underbrace{- \frac{U}{4}(Z_1 Z_4 + Z_2 Z_3) - \frac{U}{4} \sum_{i<j<k} Z_i Z_j Z_k}_{V}$$

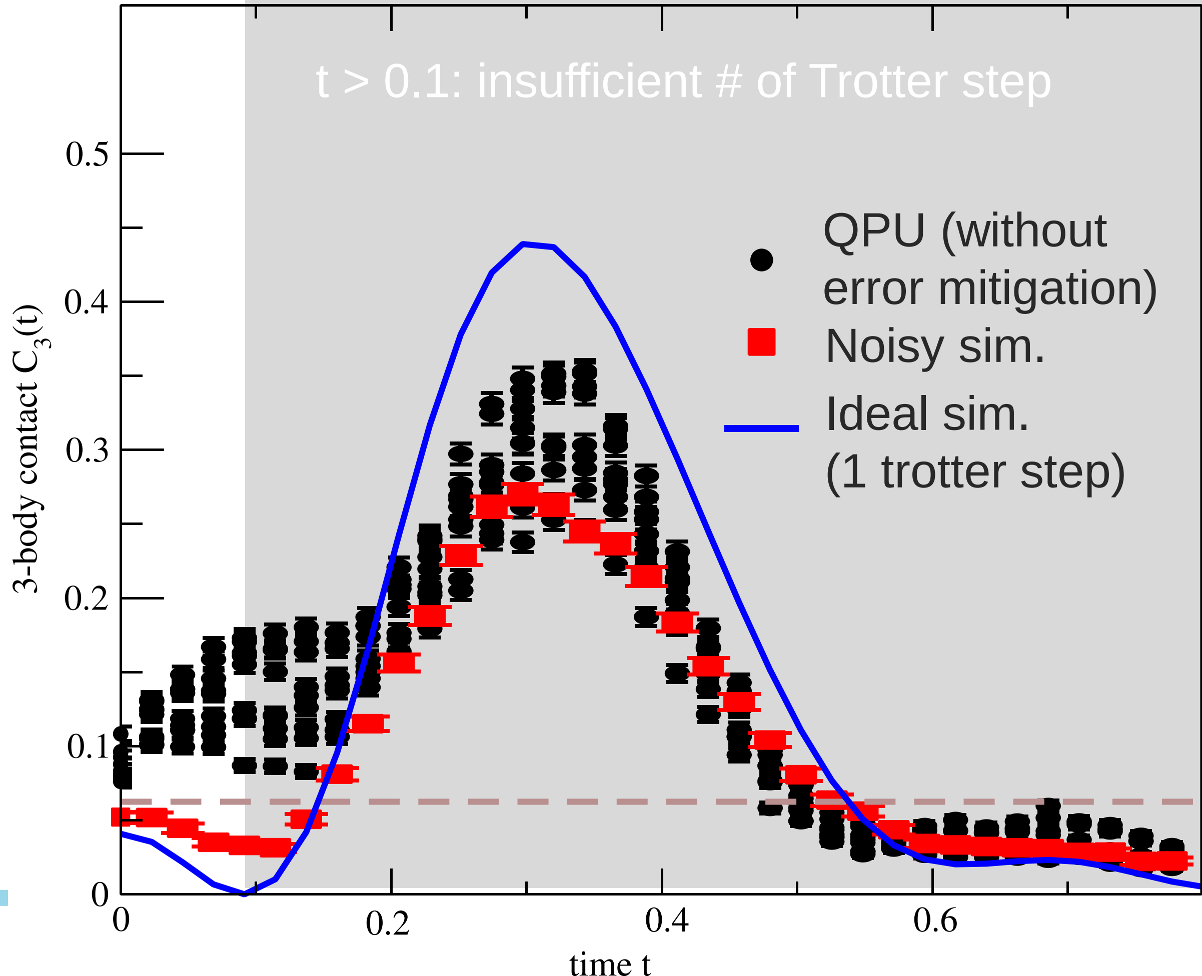
- 1^{st} order Trotter's step: $U(\tau) = e^{-i\tau K} e^{-i\tau V}$
- Initial state: pretrained state $|\tilde{\phi}_0\rangle$
- 3-body contact with : $C_3(\tau) = \left|\langle 0000|U(\tau)\tilde{\phi}_0\rangle\right|^2$
  $|0000\rangle$: all nucleons at site 1





Result of 1-Trotter-step time evolution

- Expt. result: 3-week-window collection
- Output: considerable change from run to run
- error is noticeable for a single Trotter's step
  → cannot do multiple Trotter's steps
- Error mitigation is insufficient to bring down the error



Promising result and further studies needed

1. Qubit encoding: small # of nucleons
   - Lattice-location encoding ✓
2. State preparation: $|\psi_{\hat{O}}\rangle = \frac{\hat{O}|\phi_0\rangle}{\sqrt{\langle\phi_0|\hat{O}^\dagger\hat{O}|\phi_0\rangle}}$
   - Approximated low-energy state $|\tilde{\phi}_0\rangle$ by a variational ansatz ✓
3. Quantum phase estimation of $|\psi_{\hat{O}}\rangle$ with $\hat{U} = e^{i(\hat{H}-E_0)}$
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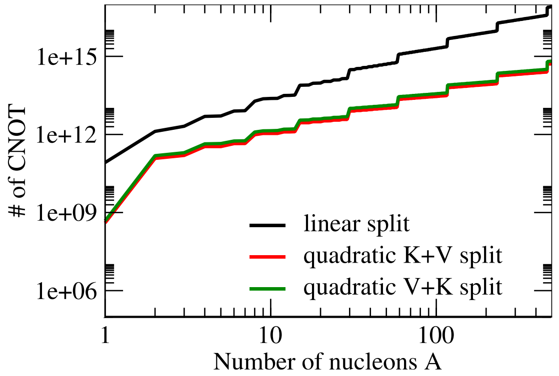
Further studies on error mitigation, hardware improvement

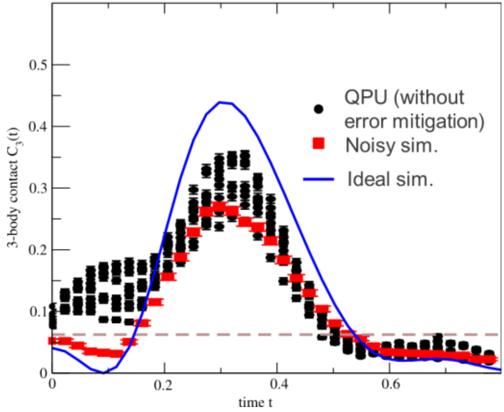
Overview

preprint: arXiv:1911.06368
(accepted by Physical Review D)

- Quantum algorithm for dynamic linear response $S(\omega, \hat{O})$
  - Inclusive/exclusive cross sections of neutrino-nucleus scattering
  - Components: state preparation and quantum phase estimation
  - Full scale studies with realistic model: potentially an important application of error-corrected quantum computer



- NISQ implementation
  - Components: ground state preparation and time evolution
  - Promising result with today hardware
  - Linear response of simple models: near-term applications with error mitigation strategies implemented and hardware improvement



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