Scalable boson encoding for VQE

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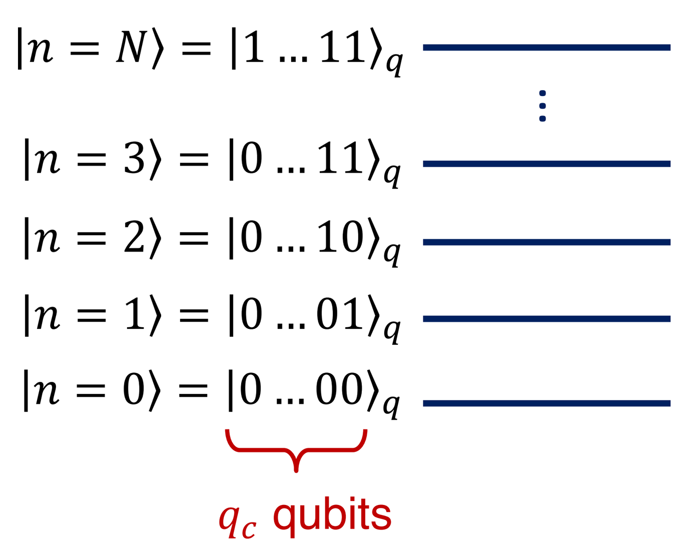
Highlights

- Number-basis binary encoding with scalable measurement scheme for boson operators
- Demonstration using a Rabi model VQE application

Number basis binary encoding

Goal: encode the states in a truncated boson Hilbert space by a finite number of qubits using the number basis



$$|\phi\rangle = \sum_{n=0}^{N} \phi_n |n\rangle$$

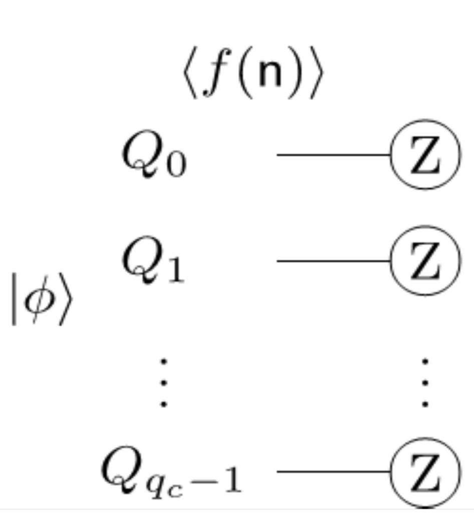
$$\mathrm{n} = \sum_{j=0}^{q_c-1} 2^{q_c-j-1}\, \mathrm{n}_j^q$$

$$\mathrm{a}^\dagger = \sum_{j=0}^{q_c-1} 2^{\frac{q_c-1}{2}} \sqrt{\sum_{k=0}^{j-1}\frac{\mathrm{n}_k^q}{2^k} + \frac{1}{2^j}}\; \sigma_j^+ \prod_{k=j+1}^{q_c-1} \sigma_k^-$$

$$\mathrm{a} = \sum_{j=0}^{q_c-1} 2^{\frac{q_c-1}{2}} \sqrt{\sum_{k=0}^{j-1}\frac{\mathrm{n}_k^q}{2^k} + \frac{1}{2^j}}\; \sigma_j^- \prod_{k=j+1}^{q_c-1} \sigma_k^+$$

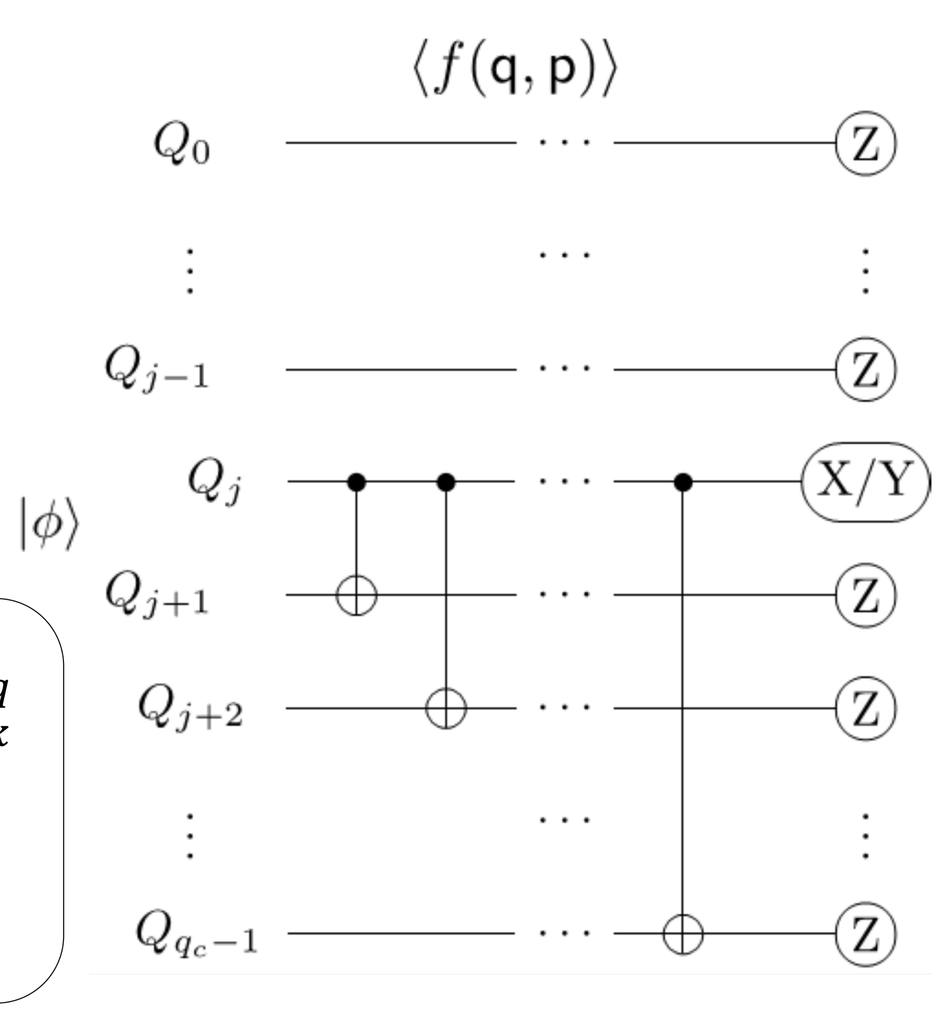
Measure $\langle f(\vec{q},\vec{p})\rangle$ with N_I-boson-mode interaction where f is a polynomial function of degree n_f	
Qubit count per mode	$q_c = \log_2(N+1)$
Circuit sampling count	$\leq O(q_c^{N_I \,\mathrm{ceil}(1+\log_2 n_f/N_I)})$
Gate count	$O(N_I \times q_c)$

- Logarithmic memory efficiency
- Shallow circuit depth for expectation value measurement
- Practical circuit sampling overhead
- Suitable for VQE applications of multi-mode systems involving a large number of bosons



Measurement circuit:
$\mathrm{CX}_{jk}\ \sigma_j^+\sigma_k^-\ \mathrm{CX}_{jk} = \sigma_j^+ n_k^q$

$U\ \sigma_j^+ \prod_{k=j+1}^{q_c-1}\sigma_k^-\ U$
$= \sigma_j^+ \prod_{k=j+1}^{q_c-1} n_k^q$

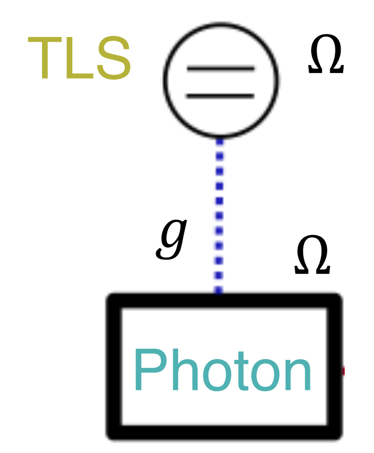


VQE of Rabi model with Rigetti's Aspen

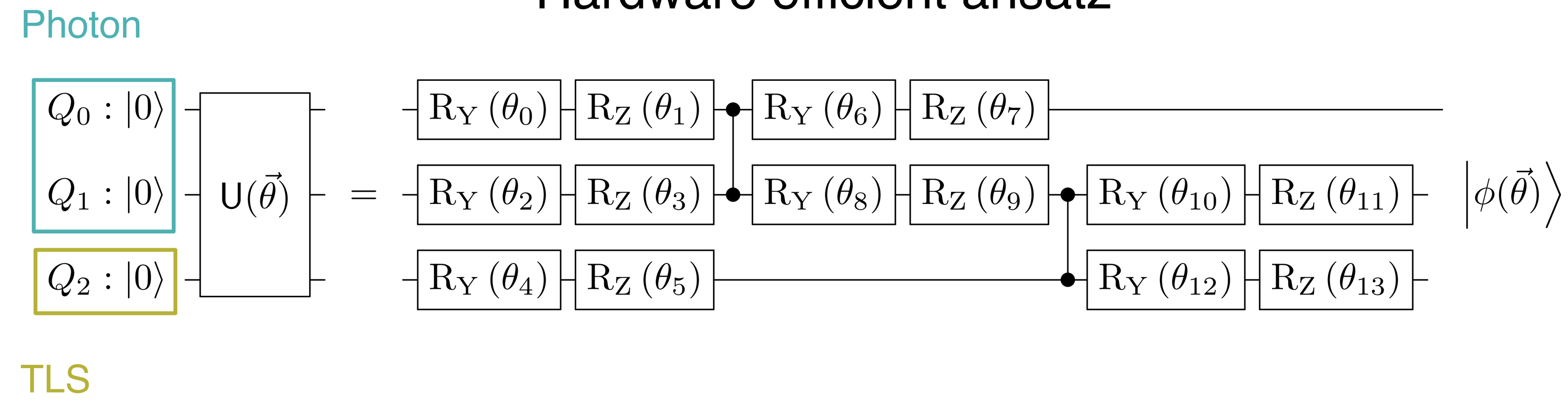
Rabi model – simple light-matter interaction

$$H = \Omega a^\dagger a + \frac{\Omega}{2}\sigma_z + \frac{g}{2}(a^\dagger + a)\sigma_x$$

The photon mode (truncated to up to three photons) is encoded using two qubits.



Hardware efficient ansatz

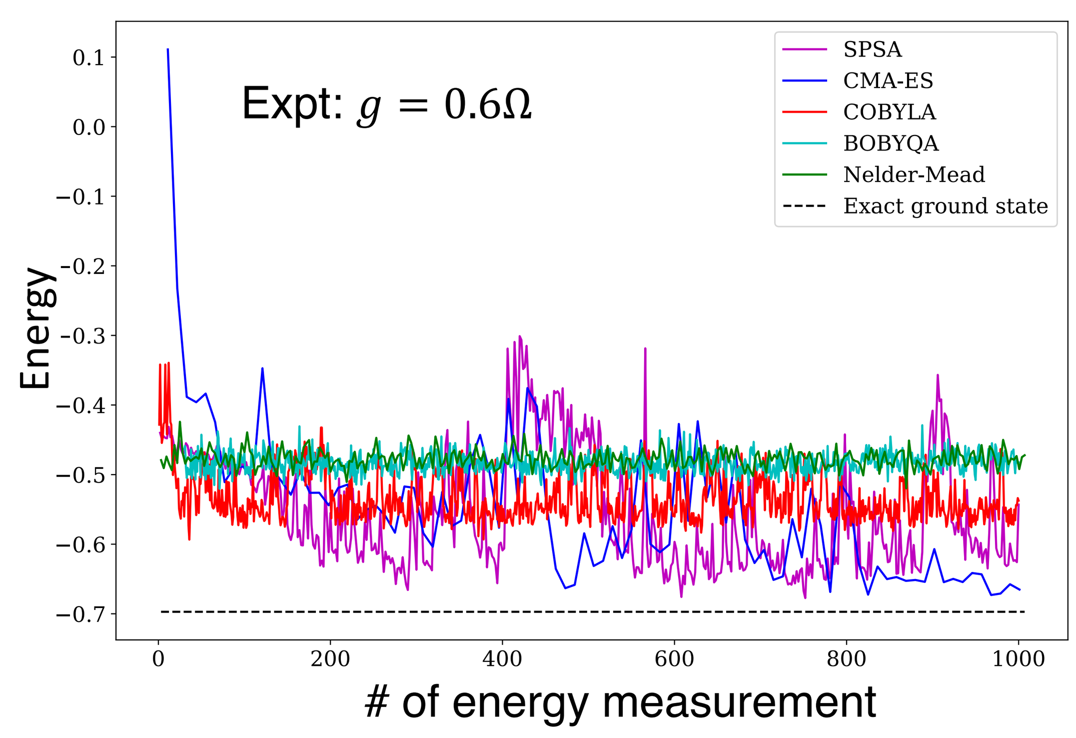


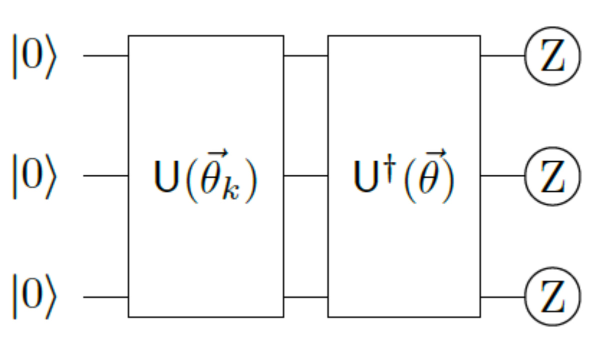
Optimization for low-energy spectrum

Cost function: $C_j = \underbrace{\langle\psi(\vec{\theta})|H|\psi(\vec{\theta})\rangle}_{\text{energy}} + \underbrace{p\sum_{k=0}^{j-1}|\langle\psi_k|\psi(\vec{\theta})\rangle|^2}_{\text{overlap penalty}}$

Eigenstates: $|\psi_j\rangle = \underset{|\vec{\theta}\rangle}{\mathrm{argmin}}\, C_j$

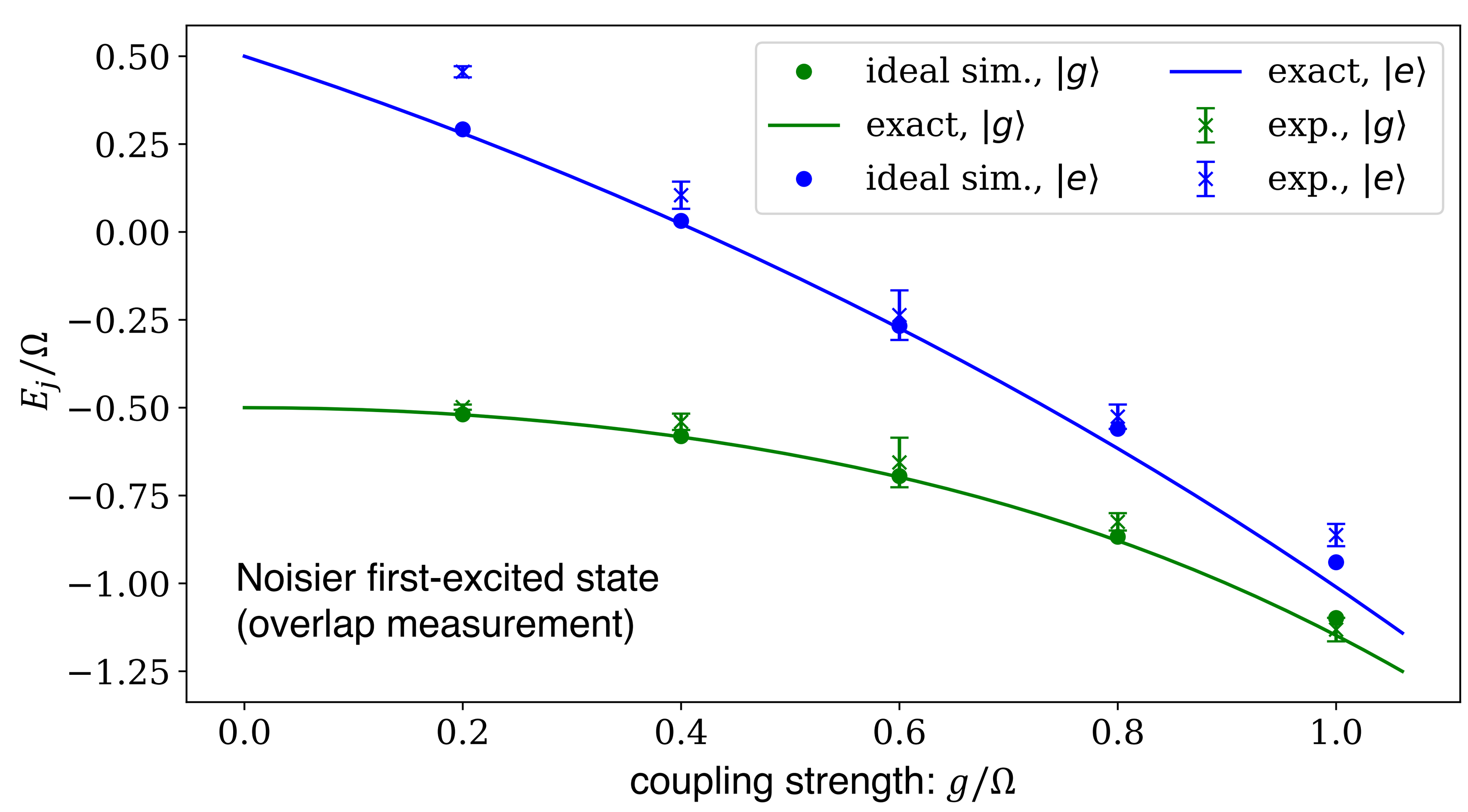
Ref: O. Higgott, D. Wang, and S. Brierley, *Quantum* 3, 156 (2019)





Optimizer	$\lvert E - E_{exact}\rvert$
CMA-ES	0.062
SPSA	0.099
COBYLA	0.165
BOBYQA	0.219
Nelder-Mead	0.223

- Low-energy spectrum: ground and first-excited states
- Narrowing of energy gap with increasing coupling g
- Discrepancy between experimental result and exact solution: hardware noise and sampling errors



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