Measuring the Muon Anomalous Magnetic Moment to High Precision

David Flay
Electromagnetic Interactions with Nucleons and Nuclei, Paphos, Cyprus
31 October 2019
Outline

Introduction

• The Magnetic Moment and the Anomaly
• Recent Theoretical Efforts

The Muon g-2 Experiment at Fermilab

• Experimental Technique
• Overview of Operations to Date
• Analysis Status

Summary
The Magnetic Moment and the Anomaly

The Magnetic Moment

\[ \vec{\mu} = g \frac{q}{2m} \vec{s} \]

- Magnetic moment connected to spin via dimensionless g-factor
- Dirac: \( g = 2 \) for \( s = 1/2 \) particles (1928)
- Hyperfine structure experiments on hydrogen: \( g \neq 2 \) (Nafe, Nelson, Rabi 1947)
  - Anomalous contribution \( a = (g-2)/2 = \alpha/2\pi \) (Schwinger, QED, 1948)
  - Radiative corrections from virtual particles in loops
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**The Muon Anomaly \( a_\mu \)**

\[ a_\mu = \]

- Schwinger \( O(\alpha) \)
- Vacuum polarization \( O(\alpha^2) \)

\[ a_\mu = \text{QED} + \text{EW} + \text{QCD} \]
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**The Muon Anomaly $a_\mu$**

\[ a_\mu = \text{QED} + \text{EW} + \text{QCD} \]

- Schwinger $O(\alpha)$
- Vacuum polarization $O(\alpha^2)$

**Current Status**

- **Disagreement** between experiment and theory at $>3\sigma$
- **Improvements**
  - **Experiment**: more statistics, reduced systematics
  - **Theory**: focus on QCD uncertainties

**S. Corrodi**

- DHMZ10
- JS11
- HLMNT11
- FJ17
- KNT18
- DHMZ19

EXP (BNL: 540ppb)  
Fermilab g-2 goal:  
$x4$ accuracy: 140ppb

$\sim 7\sigma$  
$>3\sigma$  

$(a_\mu^M \times 10^{10}) - 11659000$
**The Magnetic Moment and the Anomaly**

Magnetic moment connected to spin via dimensionless g-factor

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- Anomalous contribution

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- Radiative corrections from virtual particles in loops

$$\mu = g q^2 m s$$

**Lattice groups making excellent progress (HVP LO, NLO, HLbL)**

**Calculation of the hadronic vacuum polarization contribution to the muon anomalous magnetic moment**


(Submitted on 22 Jan 2018)

We present a first-principles lattice QCD+QED calculation at physical pion mass of the leading-order hadronic vacuum polarization contribution to the muon anomalous magnetic moment. The total contribution of up, down, strange, and charm quarks including QED and strong isospin breaking effects is found to be $a_{\mu, QCD}^{\text{HVP,LO}} = 715.4(16.3)(9.2) \times 10^{-10}$, where the first error is statistical and the second is systematic. By supplementing lattice data for very we significantly improve the precision of our calculations. R-ratio systematic and R-ratio systematic leading-order hadronic vacuum polarization contributions to the muon $g-2$ and $\mu$ are obtained to one-loop accuracy.

Comments: 12 pages, 11 figures
Subjects: High Energy Physics – Lattice (hep-lat); Hi
Cite as: arXiv:1801.07224 [hep-lat]
(or arXiv:1801.07224v1 [hep-lat] for this version)

**Higher-order hadronic-vacuum-polarization contribution to the muon $g-2$ from lattice QCD**

B. Chakraborty, C. T. H. Davies, J. Koponen, G. P. Lepage, and R. S. Van de Water (Fermilab Lattice, HPQCD, and MILC Collaborations)

Phys. Rev. D 98, 094503 – Published 9 November 2018

**ABSTRACT**

We introduce a new method for calculating the $O(\alpha^2)$ hadronic-vacuum-polarization contribution to the muon anomalous magnetic moment from ab initio lattice QCD. We first derive expressions suitable for computing the higher-order contributions either from the renormalized vacuum polarization function $\Pi(q^2)$ or directly from the lattice vector-current correlator in Euclidean space. We then demonstrate the approach using previously published results for the Taylor coefficients of $\Pi(q^2)$ that were obtained on few-flavour QCD gauge-field configurations with physical light-quark masses. We...
The Magnetic Moment and the Anomaly

Lattice groups making excellent progress (HVP LO, NLO, HLbL)

Calculation of the hadronic vacuum polarization contribution to the muon anomalous magnetic moment

T. Blum, P.A. Boyle, V. Gülpers, T. Izubuchi, L. Jin, C. Jung, A. Jüttner, C.}

(Submitted on 22 Jan 2018)

We present a first-principles lattice QCD+QED calculation at physical pion mass of the contribution to the muon anomalous magnetic moment. The total contribution of the strong isospin breaking effects is found to be $\delta g_{\mu}^{*} = 715.4(16.3)(9.2) \times 10^{-10}$.

By supplementing lattice data for $\sin^2\theta_W$, we significantly improve the precision of our calculated systematic, R-ratio statistical, and R-ratio systematic leading-order hadronic vacuum polarization contribution to the light-quark QED correction at $\mathcal{O}(\alpha)$.

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The Magnetic Moment

Schwinger $O(\alpha)$

Vacuum polarization $O(\alpha^2)$

$a_\mu = g_\mu - 2 = \frac{\alpha}{2\pi}\pi (\text{Schwinger, QED, 1948})$

The Muon Anomaly $a_\mu$

QCD

QED

EW

Disagreement between experiment and theory at $> 3\sigma$

Improvements

Experiment: more statistics, reduced systematics

Theory: focus on QCD uncertainties

Current Status

Lattice groups making excellent progress (HVP LO, NLO, HLbL)

2nd g-2 Theory Initiative Meeting in June 2018

Second Plenary Workshop of the Muon g-2 Theory Initiative

18 June 2018 - 22 June 2018

In the coming years, experiments at Fermilab and at J-PARC plan to reduce the uncertainties on the already very precisely measured anomalous magnetic moment of the muon by a factor of four. The goal is to resolve the current tantalizing tension between theory and experiment of three to four standard deviations. On the theory side the hadronic corrections to the anomalous magnetic moment are the dominant sources of uncertainty. They must be determined with better precision in order to unambiguously discover whether or not new physics effects contribute to this quantity.

There are a number of complementary theoretical efforts underway to better understand and quantify the hadronic corrections, including dispersive methods, lattice QCD, effective field theories, and QCD models. The Muon (g-2) Theory Initiative was formed in order to facilitate interactions between the different groups through organizing a series of workshops. The goal of this workshop is to bring together theorists from the different communities to discuss, assess, and compare the status of the various efforts, and to map out strategies for obtaining the best theoretical predictions for these hadronic corrections in advance of the experimental results.

Dates

June 18, 2018 - June 22, 2018

Timezone

GMT+2

Location

Heinrich-Hertz-Institut

Staudinger Weg 18, 14195 Berlin, Ground Floor

140ppb

~\sigma
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3rd $g$-2 Theory Initiative Meeting in Sept 2019

INT Workshop INT-19-74W

Hadronic contributions to $(g-2)_\mu$

September 9 - 13, 2019
### $a_\mu$ Theoretical Status

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### HVP (LO): Lowest-Order Hadronic Vacuum Polarization

- **Critical input** from e^+e^- colliders (data from SND, CMD3, BaBar, KLOE, Belle, BESIII), extensive physics program running to reduce δa_μ^{HVP} to ~ 0.3% in coming years
- **Progress on the lattice**: Calculations at physical π mass; approaching goal of δa_μ^{HVP} ~ 1% (cross-check with e^+e^- data)
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aμ Theoretical Status

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New *ab initio* approaches [PRD 98 094503 (2018)] finding consistent result of $(-93 \pm 13) \times 10^{-11}$ — lattice making big strides
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- Model dependent: based on χPT + short-distance constraints (operator product expansion)

- Difficult to relate to data like HVP (LO); γ* physics, π\(^0\) data (BESIII, KLOE) important for constraining models

#### Theory Progress

- New dispersive calculation approach; extend the lattice (finite volume, disconnected diagrams); Blum et al. making excellent progress

#### Summary

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### New ab initio approaches [PRD 98 094503 (2018)]

Finding consistent result of (-93 ± 13) x 10\(^{-11}\) — lattice making big strides
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- **Theory Progress**: New dispersive calculation approach; extend the lattice (finite volume, disconnected diagrams); Blum et al. making excellent progress

**Recent lattice & data-driven estimate** [PRD 100 034520 (2019)] for \(a_μ^{\pi^0-\text{pole}}\) is consistent with lowest-meson dominance, + vector phenomenological models [PRD 51 4939 (2005), PRL 83 5230 (1999), EJC 21 659 (2001), PRD 65 073034 (2002), PRD 94 053006 (2016), EJC 75 586 (2015)]
**Theoretical Status**

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**Builds confidence in HLbL term**

**aμ Theoretical Status**

New *ab initio* approaches [PRD 98 094503 (2018)] finding consistent result of $(-93 \pm 13) \times 10^{-11}$ —

- **Theory groups are making steady progress** to achieve competitive uncertainties on the same time scale as the FNAL experiment result
- **White paper** discussing all terms forthcoming

**Progress on the lattice:**
- New *ab initio* approaches [PRD 98 094503 (2018)] finding consistent result of $(-93 \pm 13) \times 10^{-11}$ —
- New vector phenomenological models [PRD 51 4939 (2005), EJC 75 586 (2015)]

**Total SM**
- $116 591 821 \pm 36$ (309 ppb)

**QED**
- $116 584 718.931 \pm 0.104$
  - *g-2* INT Workshop (2019)

**EW**
- $153.6 \pm 1.0$
  - *PRD* 88 053005 (2013)

**HVP**
- $6931 \pm 34$
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**HLbL**
- $101 \pm 26$
The Muon g-2 Experiment at Fermilab
Muon g-2: 33 Institutions, 7 countries, 203 Members
Why Fermilab?

- BNL limited by statistics (540 ppb on $9 \times 10^9$ detected $e^+$)
- E989 goal: Factor of 21 more statistics ($2 \times 10^{11}$ detected $e^+$)

**Fermilab advantages**

- ✔ Long beam line to collect $\pi^+ \rightarrow \mu^+$
- ✔ Much reduced amount of $p$, $\pi$ in ring
- ✔ 4x higher fill frequency than BNL
Measuring the Muon Anomaly

- Inject polarized muon beam into magnetic storage ring
- Measure difference between spin precession and cyclotron frequencies
- If $g = 2$, $\omega_a = 0$
- $g \neq 2$, $\omega_a = (e/m_\mu)a_\mu B$

\[
\begin{align*}
\omega_C &= -\frac{e}{\gamma m} \vec{B} \quad \text{cyclotron frequency} \\
\omega_S &= -\frac{e}{\gamma m} \vec{B} (1 + \gamma a_\mu) \quad \text{spin precession frequency (Larmor, Thomas precession)} \\
\omega_a &= \omega_S - \omega_C
\end{align*}
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- $g \neq 2$, $\omega_a = (e/m_\mu)a_\mu B$
- Using $\hbar \omega_p = 2 \mu_p |B|$:

\[
\begin{align*}
\omega_a &= \frac{\omega_p}{\mu_e} \frac{m_\mu}{m_e} \frac{g_e}{2} \\
\omega_C &= -\frac{e}{\gamma m} B \quad \text{cyclotron frequency} \\
\omega_S &= -\frac{e}{\gamma m} B (1 + \gamma a_\mu) \quad \text{spin precession frequency (Larmor, Thomas precession)} \\
\omega_a &= \omega_S - \omega_C
\end{align*}
\]

- We measure $\omega_a$ and $\omega_p$ separately
- Aiming for 70 ppb precision on each (systematic)
- Target: $\delta a_\mu = 140$ ppb; factor of 4 improvement over BNL

Rev. Mod. Phys. 88, 035009 (2016)
Muon Beam Injection
Muon Beam Injection

- **μ⁺**

Monitoring the Incoming Muon Beam

- **Scintillating Paddles**: Monitoring temporal distribution
- **Scintillating Fibers**: Map of transverse profile, guides μ tuning into the ring
Muon Beam Injection

Inflector Magnet

- Need to cancel field in beam channel
- Prevents strong deflection of the beam
- Minimal perturbation to storage magnetic field
Muon Beam Storage and Focusing

3 Kicker Magnets
- After inflector, muons enter storage region at \( r = 77 \text{ mm} \) outside central closed orbit
- Deliver pulse in < 149 ns to muon beam
- Steer muons onto stored orbit
Muon Beam Storage and Focusing

Electrostatic Quadrupoles

- Drives the muons towards the central part of storage region vertically
- Aluminum electrodes cover ~43% of total circumference
Measuring Muon Spin Precession ($\omega_a$)

24 finely-segmented PbF$_2$ crystal calorimeters

- Self-analyzing decay: $\mu^+ \rightarrow e^+ \bar{\nu}_\mu \nu_e$
- Highest-energy $e^+$ emitted preferentially along muon spin
- Results in sinusoidally-oscillating arrival time of these $e^+$ in calorimeters
Measuring Muon Spin Precession ($\omega_a$)

Laser System

- Calibrate calorimeter gain response throughout data taking
- Demonstrated stability to $10^{-4}$/hr
Measuring Muon Spin Precession ($\omega_a$)

Muon's view of the storage region

Two straw-tracker stations
- Reconstruct the muon beam distribution from $e^+$ hits
- Tracker module: 128 straws/module
- 8 modules per station
Magnet Anatomy

B = 1.45 T (~5200 A)
• Power supply with feedback to fine-tune field in real time
12 C-shaped yokes
• 3 upper and 3 lower poles per yoke
• 72 total poles
Field Shape
• Determined by positioning of pole pieces, wedge-shaped pieces of steel, programmable surface coils

Current direction indicated by $\bigcirc$ and $x$
Monitoring and Mapping the Magnetic Field

**Pulsed NMR**

- Deliver $\pi/2$ pulse to probe, induce & record the free-induction decay (FID)
- Extracted frequency precision: 10 ppb/FID
Monitoring and Mapping the Magnetic Field

Pulsed NMR

- Deliver π/2 pulse to probe, induce & record the free-induction decay (FID)
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Fixed probes on vacuum chambers

- Measure field while muons are in ring – 378 probes outside storage region

Trolley matrix of 17 NMR probes

- Measure field in storage region during specialized runs when muons are not being stored

Electronics, Microcontroller, Communication

Positon of NMR probes
Monitoring and Mapping the Magnetic Field

Pulsed NMR

- Deliver $\pi/2$ pulse to probe, induce & record the free-induction decay (FID)
- Extracted frequency precision: $10 \text{ ppb/FID}$

Fixed probes on vacuum chambers

- Measure field while muons are in ring – 378 probes outside storage region

Trolley matrix of 17 NMR probes

- Measure field in storage region during specialized runs when muons are not being stored

- **Trolley** probes calibrated to free-proton Larmor frequency
  - Calibrate trolley probes using a special probe that uses a water sample
  - Measurements in specially-shimmed region of ring
Systematic Uncertainty Comparison: E821 and E989

$$a_\mu = \frac{\omega_a}{\tilde{\omega}_p} \frac{\mu_p}{\mu_e} \frac{m_\mu}{m_e} \frac{g_e}{2}$$

- New hardware (calorimeters, trackers, NMR)
- Improved analysis techniques
- Reduce uncertainties by at least a factor of 2.5

<table>
<thead>
<tr>
<th>Category</th>
<th>E821 (ppb)</th>
<th>E989 Goal (ppb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Field Calibration</td>
<td>50</td>
<td>35</td>
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<tr>
<td>Trolley Measurements</td>
<td>50</td>
<td>30</td>
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<tr>
<td>Fixed Probe Interpolation</td>
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<td>30</td>
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<td>Muon Convolution</td>
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<td>10</td>
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<tr>
<td>Time-Dependent Fields</td>
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<td>Others</td>
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<td>50</td>
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<tr>
<td><strong>Quadrature Sum</strong></td>
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<td><strong>70</strong></td>
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<td>120</td>
<td>20</td>
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<tr>
<td>Lost Muons</td>
<td>90</td>
<td>20</td>
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<tr>
<td>Pileup</td>
<td>80</td>
<td>40</td>
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<td>Horizontal CBO</td>
<td>70</td>
<td>&lt; 30</td>
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<tr>
<td>E-field/pitch</td>
<td>110</td>
<td>30</td>
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<tr>
<td><strong>Quadrature Sum</strong></td>
<td><strong>214</strong></td>
<td><strong>70</strong></td>
</tr>
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</table>

$$\omega_p \text{ Goal: Factor of 2.5 Improvement}$$
Run 1 Overview

- Data taking period: April—July 2018
- A number of changing conditions as we optimized hardware
- Accumulated ~ 1.1 x BNL statistics (after data quality cuts) — $\delta\omega_{a}(\text{stat}) \sim 410 \text{ ppb}$
- Field uniformity ~ 2x better than BNL
Run 2 Overview

- Data taking period: March—July 2019
- Contiguous data set
- Accumulated ~ 1.9 x BNL statistics
  (before data quality cuts)
- Field uniformity in very good condition

Run 3

- To start in mid-November
- Aim to triple statistics accumulated to date
- Direct continuation of Run 2

Typical Field Map

Dipole Moment

RMS = 17.5 ppm

Azimuthal average
250-ppb contours

Accumulated statistics
Run 1 Analysis Status — $\omega_a$

$$a_\mu = \frac{\omega_a \mu_p m_\mu g_e}{\tilde{\omega}_p \mu_e m_e 2}$$
Run 1 Analysis Status: $\omega_a$

- Account for a number of effects that can affect the extraction of $\omega_a$

$$N(t) = N_0 e^{-t/\tau} \left[ 1 - A \cos \left( \omega_a t + \phi \right) \right]$$
Run 1 Analysis Status: $\omega_a$

- Account for a number of effects that can affect the extraction of $\omega_a$

$$N(t) = N_0 e^{-t/\tau} \left[ 1 - A \cos (\omega_a t + \phi) \right]$$

**Detector effects**

- Gain changes over time in calorimeters affects phase of signal: $N \rightarrow N(t)$, $A \rightarrow A(t)$, $\phi \rightarrow \phi(t)$
- Laser system provides corrections
Run 1 Analysis Status: $\omega_a$

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**Detector effects**

- Gain changes over time in calorimeters affects phase of signal: $N \rightarrow N(t)$, $A \rightarrow A(t)$, $\phi \rightarrow \phi(t)$
- Laser system provides corrections

**Event pileup**

- Low-energy events can mimic high-energy events in calorimeter
- Spin precession phase varies with energy — apparent high-energy decay carries phase of low-energy decays
Run 1 Analysis Status: $\omega_a$

- Account for a number of effects that can affect the extraction of $\omega_a$

$$N(t) = N_0 e^{-t/\tau} \left[ 1 - A \cos (\omega_a t + \phi) \right]$$

**Beam dynamics**

**Muon losses**

- Muons can leave storage ring by decaying or escaping
- Exhibit specific signature in multiple calorimeters
- Amplitude $N_0$ scaled by:

$$\Lambda(t) = 1 - K_{\text{loss}} \int_0^t e^{t'/\tau} L(t') dt'$$
Run 1 Analysis Status: $\omega_a$

- Account for a number of effects that can affect the extraction of $\omega_a$

$$N(t) = N_0 e^{-t/\tau} \left[ 1 - A \cos \left( \omega_a t + \phi \right) \right]$$

Beam dynamics

Muon losses

- Muons can leave storage ring by decaying or escaping
- Exhibit specific signature in multiple calorimeters
- Amplitude $N_0$ scaled by:

$$\Lambda(t) = 1 - K_{\text{loss}} \int_0^t e^{t'/\tau} L(t') dt'$$

Coherent betatron oscillations (CBO)

- Acceptance of calorimeters affected by coherent radial beam motion
- Amplitude $N_0$ scaled by:

$$C(t) = 1 - e^{-t/\tau_{\text{CBO}}} A_1 \cos \left( \omega_{\text{CBO}} t + \phi_1 \right)$$
Run 1 Analysis Status: $\omega_a$

Simple five-parameter fit

FFT of fit residuals

$\chi^2/\text{ndf}: 3983/4152$

precision: 1.35 ppm
Run 1 Analysis Status: $\omega_a$

Simple five-parameter fit

FFT of fit residuals

Big improvements when accounting for CBO, lost muons, ...
Multiple analysis techniques — more than what’s shown here!

Big improvements when accounting for CBO, lost muons,…

14-parameter fit
Run 1 Analysis Status: Relative Unblinding for $\omega_a$

- Doubly-blinded in $\omega_a$ measurement: Clock tuned to 40 MHz ± 25 ppm
- Analyzers’ results come with random frequency offset $\omega_a \rightarrow \omega_a ± 25$ ppm
- Recently compared results on subset of data at a common blinded value
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- Analyzers’ results come with random frequency offset $\omega_a \rightarrow \omega_a \pm 25$ ppm
- Recently compared results on subset of data at a common blinded value

- Consistent results at common blinded value builds confidence in our analyses
Run 1 Analysis Status — $\omega_p$

$$a_\mu = \frac{\omega_a}{\tilde{\omega}_p} \frac{\mu_p}{\mu_e} \frac{m_\mu}{m_e} \frac{g_e}{2}$$
Run 1 Analysis Status: $\omega_p$ — Field Calibration

- In the experiment, need to extract $\omega_p$; however, don’t have free protons
  - Need a calibration
- Field at the proton differs from the applied field

$$\omega_{p, \text{meas}} \approx \omega_{p, \text{free}}$$
Run 1 Analysis Status: $\omega_p$ — Field Calibration

- In the experiment, need to extract $\omega_p$; however, don’t have free protons
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\[
\omega_p^{\text{meas}} = \omega_p^{\text{free}} \left[ 1 - \sigma (\text{H}_2\text{O}, T) \right]
\]

Protons in H$_2$O molecules, diamagnetism of electrons screens protons => local B changes

- $\sigma = 25\,691(11) \times 10^{-9}$ at 25 deg C [P.J. Mohr et al, Rev. Mod. Phys. 84, 1527 (2012)]
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\omega_p^{\text{meas}} = \omega_p^{\text{free}} \left[ 1 - \sigma(H_2O, T) - \left( \frac{\varepsilon}{4\pi} - \frac{1}{3} \right) \chi(H_2O, T) \right]
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Protons in $H_2O$ molecules, diamagnetism of electrons screens protons $\Rightarrow$ local $B$ changes

- $\sigma = 25\,691(11) \times 10^{-9}$ at 25 deg C [P.J. Mohr et al, Rev. Mod. Phys. 84, 1527 (2012)]

Magnetic susceptibility of water gives shape-dependent perturbation

- $\varepsilon = 4\pi/3$ (perfect sphere)
- $\varepsilon = 2\pi$ (infinite cylinder) when probe is perpendicular to $B$
- $\chi_{H_2O}(T = 20^\circ C) = -9049(9) \times 10^{-9}$ [world average]
Run 1 Analysis Status: $\omega_p - Field Calibration$

- In the experiment, need to extract $\omega_p$; however, don’t have free protons
  - Need a calibration
- Field at the proton differs from the applied field

$$\omega_{\text{meas}} = \omega_{\text{free}}^p \left[ 1 - \sigma \left( \text{H}_2\text{O}, T \right) - \left( \frac{\varepsilon}{4\pi} - \frac{1}{3} \right) \chi \left( \text{H}_2\text{O}, T \right) - \delta_s \right]$$

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Paramagnetic impurities in water sample

Magnetization of probe materials, geometry perturbs field experienced by protons
Run 1 Analysis Status: $\omega_p$ — Field Calibration

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- Field at the proton differs from the applied field

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Goal: Determine total correction to $\leq 35$ ppb accuracy

- Dynamic effects: radiation damping, dipolar field from protons
- Magnetization of probe materials, geometry perturbs field experienced by protons
Run 1 Analysis Status: $\omega_p$ — Field Calibration

**Plunging Probe**

- Achieved **small perturbation of plunging probe** ($\delta_s + \delta_p + \delta_{RD} + \delta_d$): (-0.2 ± 11.4) ppb
- Quantified uncertainties on plunging probe material, dynamic effects — **under budget of 35 ppb by a factor of > 2**

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### Table: Plunging Probe Perturbations

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Trolley Calibration

- **Calibration of trolley probes under control**
- Factor of $\geq 2$ improvement on uncertainties for nearly all probes compared to E821
- Uncertainty is < ~40 ppb on average per probe — **on budget**
Run 1 Analysis Status: $\omega_p$ — Field Interpolation

- Need to determine $\omega_p$ at all times while storing muons => interpolate between trolley maps using fixed probe data
Run 1 Analysis Status: $\omega_p$ — Field Interpolation

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- Example from subset of data
- Tracking algorithms showing good agreement with trolley runs
- Also tracking higher-order multipole moments — important for extracting muon-weighted field $\tilde{\omega}_p$
Summary

• **The Muon g-2 Experiment** is a highly sensitive test of the SM
  
  • Discrepancy between theory and experiment for $a_\mu > \sim 3\sigma$

  ✔ Completed Run 1 in July 2018 (1.1x BNL statistics)
  
  ✔ Analyses are mature and progressing towards a result in **early 2020**
  
  ✔ Completed Run 2 in July 2019 (1.9x BNL statistics)

  • Starting to organize analysis efforts
  
  • Run 3 starting this November: aiming to **triple** statistics to date
Thank You!
The Big Move: Transporting the Ring from BNL to FNAL

- June 2013—June 2015
- Ring deconstructed at BNL, transported by barge/flatbed trailer
- Reassembled at FNAL
- Ring successfully cooled and powered to 1.45 T in September 2015 — remarkable achievement!
Getting Muons Into the Ring: Inflector Magnet

- Outside ring: $B = 0 \text{ T}$, inside: $B = 1.45 \text{ T}$
- Need to cancel field in order to get muons in (strong deflection otherwise)
- No perturbation to field outside shield
- New inflector design with higher transmission under development
- Improve injection by 40%

Energizing coils cancels the field in the beam channel

Present inflector

New inflector coil winding mount

Super currents in passive superconducting shield prevents flux leakage
Theory Status of Hadronic Contribution to $a_\mu$

- **Critical input to HVP** from e+e- colliders (SND, CMD3, BaBar, KLOE, Belle, BESIII)

- **BESIII**: 3x more data available, luminosity measurement improvements

- **VEPP-2000**: Aiming for 0.3% (fractional) uncertainty; radiative return + energy scan

- **CMD3**: Will measure up to 2 GeV (energy scan, ISR — good cross check)

\[
a_{\mu}^{\text{had};\text{LO}} = \left( \frac{\alpha m_\mu}{3\pi} \right)^2 \int_0^\infty ds \frac{K(s)}{s^2} R(s)
\]

\[
R = \frac{\sigma_{\text{tot}}(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}
\]

- **Lattice calculations** of $a_\mu^{\text{HVP}}$ to 1% soon, 30% for HLbL in 3—5 years

---

A. Anastasi et al., arXiv:1711.03085 [hep-ex]
Physics Beyond the Standard Model?

**SUSY, TeV-Scale Models**

- Higgs measured at the LHC to be 125 GeV
- Theory: Higgs should acquire much heavier mass from loops with heavy SM particles (e.g., top quark)
  - **Supersymmetry**: new class of particles that enters such loops and cancels this contribution

- Complementary to direct searches at the LHC
  - Sensitivity to $\text{sgn}(\mu)$, $\tan(\beta)$
  - Contributions to $a_\mu$ arise from charginos, sleptons
  - LHC searches sensitive to squarks, gluinos

- Radiative muon mass generation
  - Unparticles, Extra Dimension Models, SUSY ($\tan \beta = 5$ to 50)

**Z’ Possibilities**

- Axion-like particles
- Dark photons (invisible)
- Extended Higgs/leptoquarks

Goal: 2022

D. Hertzog, Ann. Phys. (Berlin), 2015, courtesy D. Stockinger

Z’, W’, UED, Littlest Higgs
- Assumes typical weak coupling

Higgs measured at the LHC to be 125 GeV
- Theory: Higgs should acquire much heavier mass from loops with heavy SM particles (e.g., top quark)

- **Supersymmetry**: new class of particles that enters such loops and cancels this contribution

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**Z’ Possibilities**

- Axion-like particles
- Dark photons (invisible)
- Extended Higgs/leptoquarks

Goal: 2022
Analysis Details: $\omega_a$

$$\frac{\delta \omega_a}{\omega_a} = \frac{\sqrt{2}}{2\pi f_a \tau_\mu N^{1/2} A}$$

Energy cut chosen to optimize figure-of-merit
Analysis Details: $\omega_a$

Energy cut chosen to optimize figure-of-merit

Project along time axis, energy cut
Analysis Details: $\omega_a$

Energy cut chosen to optimize figure-of-merit

Project along time axis, energy cut

Fit to: $N(t) = N_0 e^{-t/\tau} [1 - A \cos (\omega_a t + \phi)]$
Beam Dynamics Corrections

- Full expression for $\omega_a$:

$$\omega_a = \omega_S - \omega_C = -\frac{e}{mc} \left[ a_\mu \vec{B} - \left( a_\mu - \frac{1}{\gamma^2 - 1} \right) \vec{\beta} \times \vec{E} \right] - a_\mu \left( \frac{\gamma}{\gamma + 1} \right) \left( \vec{\beta} \cdot \vec{B} \right) \vec{\beta}$$

- Choose $\gamma = 29.3$ ($p_\mu = 3.094$ GeV/c)

Not all $\mu^+$ at this $\gamma$ → E-field correction

$$C_E = -2n(1 - n)\beta^2 \frac{\langle x_e^2 \rangle}{R_0^2}$$

Vertical beam oscillations → pitch correction

$$C_p = -\frac{n}{4} \frac{\langle y^2 \rangle}{R_0^2}$$
Quad Challenges (Run 1)

- 2 of 32 HV resistors on quad plates were flawed => did not stabilize in time for “fit start time”
- Mean of vertical muon distribution moves down by 0.6 mm
  - Investigating impact on $\omega_a$ (calculations, systematic measurements)
- Problem was fixed for Run 2

![HV's from measured plates (13.1/18.3kV)]

- Red and blue Quad plates charged up too slowly …
- Start of fit
How is a Kick Made?

- A charging power supply charges up a capacitor bank to 700 V
- Capacitors are discharged through a transformer into a **Blumlein** (a HV capacitor up to 55 kV)
- Current in **Blumlein** is discharged into a resistive load (**Z = 12.5 Ω**)
- Current delivered to plates, producing a ~200 G magnetic field, rotating muon’s momentum vector
What Affects the Beam Shape?

- **Kicker pulse** strength, shape affects structure of beam
- **Beam width** affected by dynamics

![Diagram showing Kicker Pulse](image)

- **Kicker Pulse**
  - Kick strength $\rightarrow$ beam centroid
  - Ringing kick $\rightarrow$ asymmetric dist.

![Graph showing Radial Beam Position](image)

- **Radial Beam Position [mm]**
  - CBO $\rightarrow$ radial width

- **Higher kick $\rightarrow$ lower radius**
  - Run 1
  - Run 2

![Graph comparison](image)
Kicker Challenges

- Need to deflect beam by ~11 mrad on the first turn, then turn off (< 149 ns)
- Engineering challenge never fully realized at BNL — and not yet for us

Radial beam distribution at various kick strengths; well contained, but not centered

Take away: we can live with this — we just don’t like it

Running at high voltages (~ 50 kV) burns out cables — had to back off
What Drives the $\omega_a$ Fit Start Time?

- Start fit window to extract $\omega_a$ at $\sim 30$ $\mu$s to avoid:
  - Kicker eddy currents affect the magnetic field
  - Quad scraping at early times to reduce losses

**Kicker eddy currents affect the magnetic field**

- OPERA simulation
- Magnetometer data

100 ppb level on integrated magnetic field

**Lost Muons/Decay $e^+$ with $E > 1.8$ GeV**

**PRELIMINARY**

David Flay | Measuring the Muon Anomalous Magnetic Moment to High Precision
Pulsed Nuclear Magnetic Resonance

- Apply an RF pulse for a short time to the sample at Larmor frequency — tips spins perpendicular to external B field ($\pi/2$ pulse)

- Spin precession induces an EMF in the pickup coil
  - So-called Free-Induction Decay (FID)

- Decay of signal driven by:
  - Spin-spin interactions (dephasing) (pure $T_2$)
  - Field inhomogeneities ($T_{2\ast}$)
  - Simultaneously, spins relax back to alignment with holding field (spin-lattice relaxation, $T_1$)

\[ M_z = M_0 \]
Magnetic Circuits

\[ \mathcal{E} = \oint \mathbf{f_s} \cdot d\mathbf{l} = V = IR \]
\[ \mathcal{F} = \oint \mathbf{H} \cdot d\mathbf{l} = NI \]
\[ \mathbf{B} = \mu_0 (1 + \chi_m) \mathbf{H} = \mu \mathbf{H} \]
\[ \Phi = \mathbf{B} \cdot \mathbf{A} = \mu \mathbf{H} \cdot \mathbf{A} \]
\[ \Phi \oint \frac{d\ell}{\mu A} = \mathcal{F} \Rightarrow \mathcal{R} = \oint \frac{d\ell}{\mu A} = \frac{\mathcal{F}}{\Phi} \]

Can write a similar equation for magnets

**Magnetomotive Force (mmf)**

Rewrite \( H \) in terms of \( B \)

Consider magnetic flux

**Magnetic Reluctance**

- Analogous to resistance in an electrical circuit
  \[ V = IR \Leftrightarrow \mathcal{F} = \Phi \mathcal{R} \]
- Current flows along a path of least resistance while field lines will take a path of least reluctance
- While the emf drives electric charges (Ohm’s Law), the mmf “drives” magnetic field lines (Hopkinson’s Law)
Magnet Anatomy

- For E821, Gordon Danby had a brilliant magnet design
  \[ B = 1.45 \text{ T} \ (\sim 5200 \text{ A}) \]
- Non-persistent current: fine-tuning of field in real time

12 C-shaped yokes

- 3 upper and 3 lower poles per yoke
- 72 total poles

Shimming knobs

- Pole separation determines field: pole tilts, non-flatness affect uniformity
- Top hats (30 deg effect, dipole)
- Wedges (10 deg effect, dipole, quadrupole)
- Edge shims (10 deg effect, dipole, quadrupole, sextupole)
- Laminations (1 deg effect, dipole, quadrupole, sextupole)
- Surface coils (360 deg effect, quadrupole, sextupole, …)
Optimizing the Dipole Moment

- Want to optimize the vertical component of the field
- Step and tilt discontinuities in pole surfaces yield large variations in the field
- To reduce/remove such effects, make adjustments to pole feet, which changes the magnet gaps and tilts
  - Use 0.001—0.010” thick shims
  - Requires removal of poles from the ring
- Informed by a computer model that optimizes the pole configurations
  - Requires global continuity between pole surfaces
  - Allows only three adjacent poles to be moved at a time (preserves alignment)
Minimizing the Quad, Sext, Octu Moments

Calibrated shimming knobs
• 48 top hats
• 864 wedges
• ~8400 iron foils (on pole surfaces)

Coarse tuning: top hat & wedge adjustments (dipole, quadrupole)
• Least-squares fit to field maps predicts top hat and wedge positions

Fine tuning: iron foils (quadrupole, sextupole, ...)
• Modeled as saturated dipoles in 1.45 T field
• Computer code predicts foil width (mass) distribution to fill in the valleys of the field map
Rough Shimming Results


Goal

Vertical (cm)
R-R_0(cm)

\( \frac{(B-B_{\text{avg}})}{B_{\text{avg}}} \) (ppm)

Oct 2015

Rough Shimming Results

Aug 2016

B-field (ppm)

Vortical (cm)
R-R_0(cm)

B-field (ppm)

Vortical (cm)
R-R_0(cm)

Quad 25.13 -0.53
Sext -1.99 -0.11
Octu -1.16 -0.31
Decu 0.95 -0.07

Quad -0.62 -0.57
Sext -0.79 3.84
Octu -0.76 0.56
Decu 0.44 -1.61

50 ppm
~1400 ppm
Magnetic Field Comparison: BNL 821 and FNAL E989

- Laminations very successful in reducing field variations

- BNL E821: 39 ppm RMS (dipole), 230 ppm peak-to-peak
- FNAL rough shimming: 10 ppm RMS (dipole), 75 ppm peak-to-peak
Magnetic Field Variations

First Magnetic Field Map, Oct 14 2015

- Gradual drift from materials, pole gap changes
- 36 pairs of poles $\rightarrow$ 10-degree structure
- Pole shape:
- Pole-to-pole discontinuities
Auxiliary Field Systems

Surface Correction Coils
- Continuous PCB traces going around the ring on pole surfaces
- 100 concentric traces on upper poles, 100 on lower poles
- Programmable range: ± 20 ppm on the field
- Used to cancel higher-order multipole moments in the magnetic field (on average)

Power Supply Feedback
- Programmable current source with a range of ± 5 ppm on the field
- Uses data from fixed probe system to stabilize the field at a specified set point

Fluxgates
- Measure (x,y,z) components of transient fields in the hall
- Sensitive down to $10^{-9}$ T (DC or AC) fields
- Bandwidth up to 1 kHz
Magnet Insulation

- Temperature variations in the hall affect the quality of the magnetic field
  - Observed ~20 ppm/deg C effects on the dipole moment during the run
  - Also affects ability to track higher-order multipoles
- Two main issues
  - Large changes in average temperature over time (2–3°C)
  - Differential changes across the magnet (~3°C)
- Two-pronged solution:
  - Improved cooling system in the hall
  - Install fiberglass insulation blanket on magnet steel
Plunging Probe Design

- Used to calibrate the **trolley** NMR probes

- **Symmetry** is very important => minimizes field perturbations => reduced systematic uncertainties

- **RF coil support**: 15-mm OD high-precision glass cylinder
  - Macor supports ensure alignment of **RF coil** (zero-χ 0.97-mm OD wire)

- **Ground shield**: 1” OD, 1-mm wall 2024-T3 Al
  - Stabilizes probe tune, reduces noise pickup

- **Vacuum compatible**
Plunging Probe: Measuring Perturbations

- Take measurements of the field using the fixed probe
- Compare measurements without and with the PP mounted on stand
- Difference with and without gives the effect

Without PP

With PP — note temperature sensor connector is fixed in same orientation as at FNAL
### Plunging Probe: Material Perturbations ($\delta_s$)

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Symbol</th>
<th>Value (ppb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>General Material Perturbation</td>
<td>$\delta_{\text{mat}} + \delta_{\text{mag}}$</td>
<td>$4.2 \pm 8.0$</td>
</tr>
<tr>
<td>SMA Cable Perturbation</td>
<td>$\delta_{\text{cable}}$</td>
<td>$-1.4 + 3.0$</td>
</tr>
<tr>
<td>Probe Temperature</td>
<td>$\delta_T$</td>
<td>$0 \pm 5$</td>
</tr>
<tr>
<td>Roll Effect</td>
<td>$\delta_{\text{roll}}$</td>
<td>$0 \pm 1$</td>
</tr>
<tr>
<td>Pitch Effect</td>
<td>$\delta_{\text{pitch}}$</td>
<td>$-4.4 \pm 4.4$</td>
</tr>
<tr>
<td>Water Sample Camber</td>
<td>$\delta_c$</td>
<td>$0 \pm 1$</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td>$\delta_s$</td>
<td>$-1.6 \pm 10.9$</td>
</tr>
</tbody>
</table>

- How much the field changes due to probe presence
- Dependence on orientation about its own axis
- Dependence on orientation relative to field axis
Plunging Probe: Water Purity ($\delta_p$)

- Impurities in the water sample will perturb the field — paramagnetic contamination (e.g., dissolved oxygen) will increase the field

- Conduct two tests:
  1. Measure field when we degas the water — that is, heat up water just enough so that oxygen escapes. Compare to nominal water sample (at same T)
  2. Compare field measurements using water from two different vendors

- Define $\delta_p = \delta_{O2} + \delta_w$

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Symbol</th>
<th>Value (ppb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oxygen Contamination</td>
<td>$\delta_{O2}$</td>
<td>1.4 ± 0.5</td>
</tr>
<tr>
<td>Different Vendors</td>
<td>$\delta_w$</td>
<td>0 ± 1</td>
</tr>
<tr>
<td>TOTAL</td>
<td>$\delta_p$</td>
<td>1.4 ± 1.1</td>
</tr>
</tbody>
</table>
Plunging Probe: Magnetic Images

- Need to account for the effect due to magnetic images of the PP in the pole pieces

- For an infinite plane with magnetic permeability $\mu_r$, the field due to the image of a material with perturbation $\Delta B$ is:

$$\Delta B' \approx \left( \frac{\mu_r - 1}{\mu_r + 1} \right) \Delta B(x, y, z') \approx \Delta B(x, y, z')$$

For $\mu_r >> 1$ ($\sim 1450$ for the magnet*). Evaluate the perturbation at image distance $z'$ in the pole piece (upper and lower)

Plunging Probe: Measuring the Images at FNAL

- Use a stage to mount a fixed probe along the axis of the PP, which can slide over fixed probe.
- Compare field measurements with and without PP installed on the stage.
- Repeat measurements at height of center trolley probe, and highest trolley probe location.
- Also conduct measurements with PP mounting rod attached/detached.

<table>
<thead>
<tr>
<th>Type</th>
<th>Height Above Midplane (mm)</th>
<th>Image + Perturbation (ppb)</th>
<th>Calculated Prediction (ppb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PP + Holder</td>
<td>35.5</td>
<td>6.1 ± 4.4</td>
<td>11.5</td>
</tr>
<tr>
<td>PP + Holder</td>
<td>-0.2</td>
<td>7.3 ± 5.4</td>
<td>4.3</td>
</tr>
<tr>
<td>Rod</td>
<td>-0.2</td>
<td>-3.1 ± 5.6</td>
<td>-12.9</td>
</tr>
<tr>
<td>TOTAL</td>
<td>-0.2</td>
<td>4.2 ± 8.0</td>
<td>-8.6</td>
</tr>
</tbody>
</table>

Rod composition may not be pure aluminum (typically up to ~20% variation in $\chi$ => imperfect predictions).
Radiation Damping

What is it?

• Precessing spins induce emf in pickup coil; this in turn generates an alternating magnetic field that acts to rotate spins back towards the main field

• Size of effect: $\delta_{RD} \sim \frac{f_0-f_L}{f_0} \eta Q M_z(t)/\tau_{RD}$
  
  - $f_0$ = resonant frequency of circuit; $f_L$ = Larmor frequency
  - $\eta$ = filling factor; $Q$ = quality factor of circuit
  - $M_z(t)$ = magnetization of sample, $\tau_{RD}$ = time scale of effect

How to quantify?

• Use coils to produce a longitudinal field

• Precise control over main field to mimic damping effect

• Vary $\pi/2$ pulse => vary $M_z(t)$ => changes $\delta_{RD}$
Calibrating the Trolley

Procedure
• Select trolley probe to calibrate

- Impose a known gradient across the trolley; compare to bare field $B_0$.
- Define $\Delta B = B(I \neq 0) - B(I = 0)$.
- Unique $\Delta B$ for each trolley probe gives position.
- Move plunging probe into volume; measure $\Delta B$ and determine distance to move plunging probe.
- Iterate until plunging probe $\Delta B$ matches trolley probe $\Delta B$.
- Perform for radial, vertical, azimuthal coordinates.
- Shim the field to be highly uniform, and measure using the PP and the trolley (rapid swapping).
Calibrating the Trolley

**Procedure**

- Select trolley probe to calibrate

- Impose a **known gradient** across the trolley; compare to **bare field** $B_0$. Define $\Delta B = B(I \neq 0) - B(I = 0)$

- Unique $\Delta B$ for each trolley probe gives position

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• Iterate until plunging probe $\Delta B$ matches trolley probe $\Delta B$

• Perform for radial, vertical, azimuthal coordinates

Imposed gradient

Trolley

Plunging Probe

Calibration Volume

Overhead View

Muon Storage Volume

$B_0$
Calibrating the Trolley

**Procedure**

- Select *trolley* probe to calibrate

- Impose a *known gradient* across the trolley; compare to *bare field* $B_0$. Define $\Delta B = B(I \neq 0) - B(I = 0)$

- Unique $\Delta B$ for each *trolley* probe gives position

- Move *plunging probe* into volume; measure $\Delta B$ and determine distance to move *plunging probe*

- Iterate until *plunging probe* $\Delta B$ matches *trolley* probe $\Delta B$

- Perform for radial, vertical, azimuthal coordinates

- Shim the field to be highly uniform, and measure using the *PP* and the *trolley* (rapid swapping)