

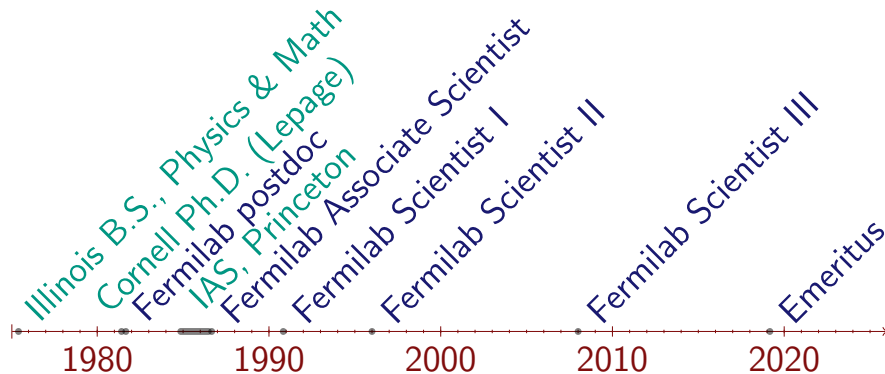
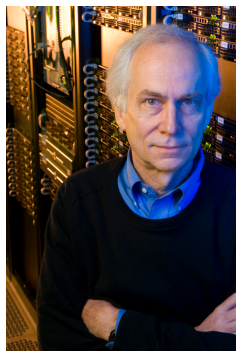
# Off Quarkonium: & the Lattice On

Chris Quigg  
*Fermilab*

*Lattice QCD at Fermilab: Celebrating the Career of Paul Mackenzie · 8 XI 2019*

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# Paul's Fermilab Timeline



APS Fellow, 1996

AAAS Fellow, 2011



# A Greeting from Tom Nash

*Ah, Paul, such memories and I am sad I can't be at your fest. Back in the day . . . there were few in HEP or computer science who saw the future as well as you did and accepted that dedicated farms of little processors could get so much done. Seems quite obvious now, what with Google, Apple, Amazon, etc., following what our field pioneered. I am really glad that ACPMAPS, which you and Estia and Andreas and colleagues suffered so much to make productive, has led to such a successful and long-lasting lattice gauge program.*



## A Greeting from Sinéad Ryan



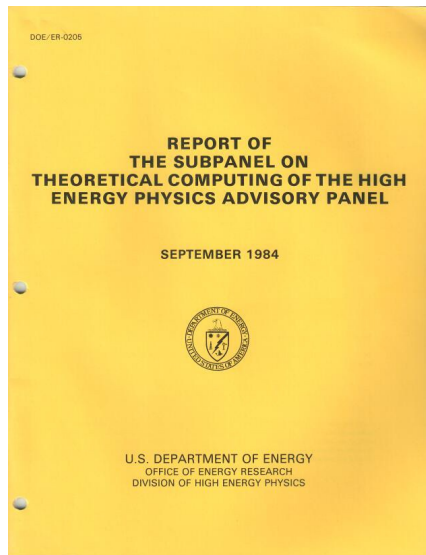
*I arrived to Fermilab in autumn of 1996, fresh from my PhD in Edinburgh. Safe to say it was something of a culture shock - I suddenly had to buy a car (and drive it to work!) not to mention coming to terms with a Chicago winter! I was intimidated too at the idea of working with people whose names I knew only from their ground-breaking papers, Paul's of course foremost amongst them, but the lattice group, and in fact the whole theory group at the Lab were so welcoming and kind that I was soon very happily settled and I can honestly say that I loved every minute of my postdoc! While at Fermilab I worked with Paul and Andreas, as well as Aida and Jim, on lattice heavy quark physics, learning a huge amount which I still rely on today. I still work on aspects of heavy quark physics - at finite temperature and in hadronic spectroscopy and those Fermilab years have had great impact and stood me in good stead. To Paul and Liz - thank you both for welcoming me to your home and for your friendship while I was at the Lab. To Paul, thank you for sharing your wisdom and intuition - it was a privilege to work with you.*

## A Greeting from Estia Eichten



*Although I am not able to attend the Paul Fest in person, I want to add my admiration for Paul's contributions to Lattice QCD. He has been a leading figure in the transformation of Lattice calculations from rough ideas into a precision tool relied on by both experimentalist and theorists. Paul and Peter Lepage showed how to define renormalized coupling constants in LQCD, thus allowing the use of Wilson loops to provide an accurate determination of  $\alpha_s$ . Paul has made many leading contributions to Lattice collaborations that have calculated masses and weak decays of B, D, and K mesons. The goal of such calculations was to provide experimentally needed results but also to determine the quark masses and CKM angles of the underlying QCD Lagrangian. In addition, Paul provided strong leadership as the chair of the Executive Committee of USQCD for many years. But all this will be discussed in much detail by other speakers. From my point of view Paul's greatest achievement was his insistence on lattice calculations having a true error budget. This has driven lattice QCD development and has been the vision that transformed the field into the precision tool that it has become.*

# Prehistory of lattice computing



## Status of Lattice QCD

Paul B. Mackenzie

*Fermi National Accelerator Laboratory  
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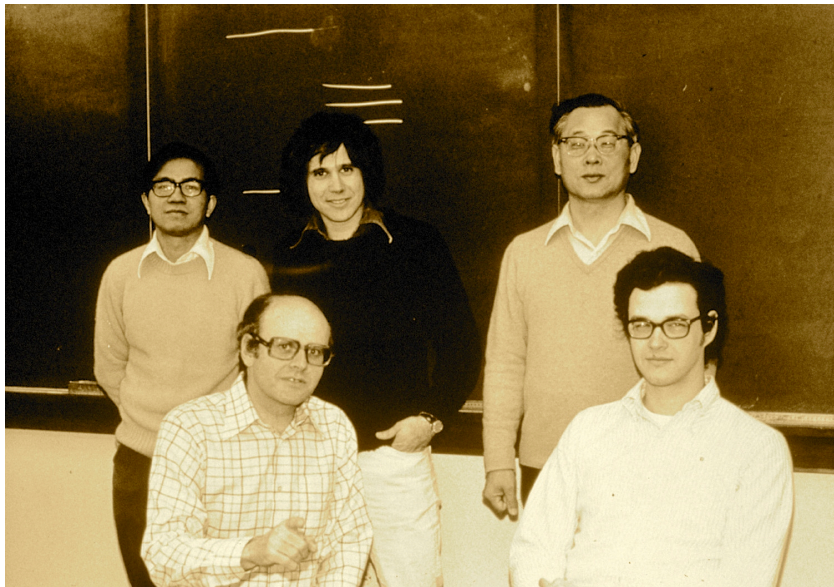
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Significant progress has recently been achieved in the lattice gauge theory calculations required for extracting the fundamental parameters of the standard model from experiment. Recent lattice determinations of such quantities as the kaon  $B$  parameter, the mass of the  $b$  quark, and the strong coupling constant have produced results and uncertainties as good or better than the best conventional determinations. Many other calculations crucial to extracting the fundamental parameters of the standard model from experimental data are undergoing very active development. I review the status of such applications of lattice QCD to standard model phenomenology, and discuss the prospects for the near future.

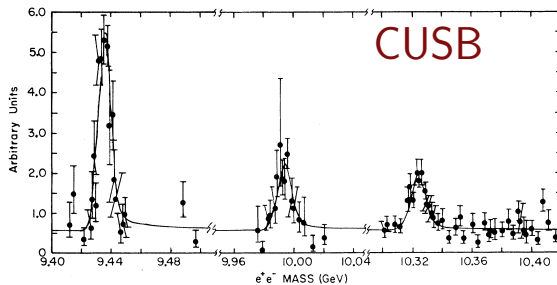
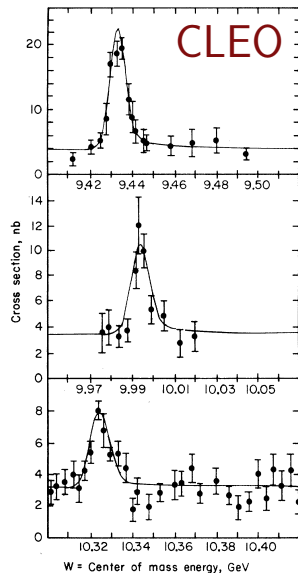
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There are now several lattice calculations ( $B_K$ ,  $m_b$ ,  $\alpha_s$ ) for which at least a first attempt has been made to examine all of the largest sources of uncertainty quantitatively. These uncertainty estimates are not yet on a par with the analysis of  $g - 2$  for the electron (although eventually they should be), but they are quite competitive with the analysis of theoretical uncertainties in short distance perturbative QCD processes.

# Charmonium Modelers at Newman Lab

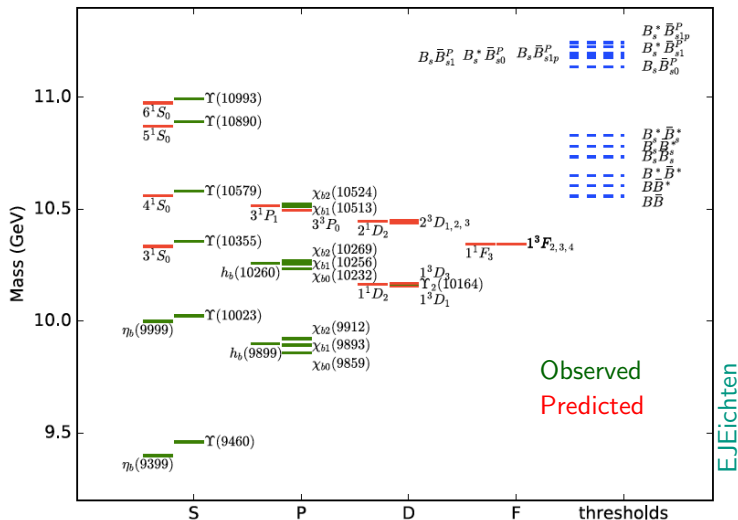


# CESR resolves three narrow $\Upsilon$ states (1979–80)



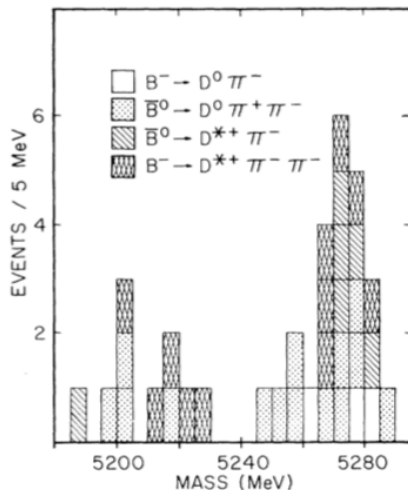
$\Upsilon(4S)$  launches  $B$  physics (1980)

# Rich spectrum of $(b\bar{b})$ levels





# Reconstruction of $B$ Mesons (CLEO, 1983)



PDG:  $I, J, P$  still need confirmation!

# Paul and quarkonium off the lattice . . .

## Quantum Chromodynamic Corrections to the Gluonic Width of the $\Upsilon$ Meson

Paul B. Mackenzie<sup>(a)</sup> and G. Peter Lepage

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(Received 3 August 1981)

The corrections to the gluonic width of  $\Upsilon$ -like mesons in quantum chromodynamics (QCD) through first order in  $\alpha_s$  are calculated, with the result  $\Gamma_g = \Gamma_g^0 [1 + (3.8 \pm 0.5) \times \alpha_s(M_\Upsilon)/\pi]$ , using the  $\overline{\text{MS}}$  scheme definition of  $\alpha_s$ , renormalized at  $M_\Upsilon$ . With this correction, the measured value of the leptonic branching ratio of the  $\Upsilon$  can be used to determine  $\alpha_s$ , or, equivalently, the QCD scale parameter  $\Lambda_{\overline{\text{MS}}}$ . The result is  $\Lambda_{\overline{\text{MS}}} = 100$  MeV, with experimental errors of  $^{+34}_{-20}$  MeV and comparable theoretical uncertainties. Analysis of the  $\psi$  and  $\psi'$  data is consistent with this value.

PACS numbers: 13.25.+m, 12.40.Cc, 13.65.+i, 14.40.Pe

## Quarkonium annihilation rates

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Paul B. Mackenzie

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Rogério Rosenfeld and Jonathan L. Rosner

*Enrico Fermi Institute and Department of Physics, University of Chicago, Chicago, Illinois 60637*

(Received 8 September 1987; revised manuscript received 11 January 1988)

Recent measurements of ratios of quarkonium annihilation rates are used to evaluate the strong fine-structure constant  $\alpha_s$ . Expressions are presented for QCD radiative corrections with  $\alpha_s$  referred to the quark-mass scale. We find  $\alpha_s(m_b) = 0.179^{+0.009}_{-0.008}$  from the ratio  $\Gamma(\Upsilon \rightarrow \gamma gg) / \Gamma(\Upsilon \rightarrow ggg)$ . The corresponding range of  $\Lambda_{\overline{\text{MS}}}^{(4)}$  (the QCD scale factor for four light-quark flavors) is 146–210 MeV, where  $\overline{\text{MS}}$  denotes the modified-minimal-subtraction scheme. The experimentally more precise but theoretically more questionable ratio of the gluonic and muonic widths of  $J/\psi$  and  $\Upsilon$  yields  $\alpha_s(m_c) = 0.29 \pm 0.02$ ,  $\alpha_s(m_b) = 0.189 \pm 0.008$  when  $v^2/c^2$  corrections to these ratios for  $J/\psi$  and  $\Upsilon$  are parametrized linearly. Further predictions are made for ratios of rates.

# Paul and quarkonium on the lattice ...

## Determination of the Strong Coupling Constant from the Charmonium Spectrum

Aida X. El-Khadra, George Hockney, Andreas S. Kronfeld, and Paul B. Mackenzie

*Theoretical Physics Group, Fermi National Accelerator Laboratory, P.O. Box 500, Batavia, Illinois 60510*

(Received 27 January 1992)

Lattice gauge theory techniques have recently achieved sufficient accuracy to permit a determination of the strong coupling constant from the  $1P$ - $1S$  splitting in the charmonium system, with all systematic errors estimated quantitatively. The present result is  $\alpha_{\overline{MS}}(5 \text{ GeV}) = 0.174 \pm 0.012$ , or, equivalently,  $\Lambda_{\overline{MS}}^{(4)} = 160_{-37}^{+47} \text{ MeV}$  ( $\overline{MS}$  denotes the modified minimal subtraction scheme).

PACS numbers: 12.38.Gc, 12.38.Aw, 14.40.Gx

## High-Precision Lattice QCD Confronts Experiment

C. T. H. Davies,<sup>1</sup> E. Follana,<sup>1</sup> A. Gray,<sup>1</sup> G. P. Lepage,<sup>2</sup> Q. Mason,<sup>2</sup> M. Nobes,<sup>3</sup> J. Shigemitsu,<sup>4</sup>  
H. D. Trotter,<sup>3</sup> and M. Wingate<sup>4</sup>

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<sup>3</sup>*Physics Department, Simon Fraser University, Vancouver, British Columbia, Canada*

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C. Aubin,<sup>5</sup> C. Bernard,<sup>5</sup> T. Burch,<sup>6</sup> C. DeTar,<sup>7</sup> Steven Gottlieb,<sup>8</sup> E. B. Gregory,<sup>6</sup> U. M. Heller,<sup>9</sup> J. E. Hetrick,<sup>10</sup>  
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(MILC Collaboration)

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<sup>8</sup>*Department of Physics, Indiana University, Bloomington, Indiana 47405, USA*

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<sup>10</sup>*University of the Pacific, Stockton, California 95211, USA*

<sup>11</sup>*Department of Physics, University of California, Santa Barbara, California 93106, USA*

M. Di Pierro,<sup>12</sup> A. El-Khadra,<sup>13,14,15</sup> A. S. Kronfeld,<sup>14</sup> P. B. Mackenzie,<sup>14</sup> D. Menscher,<sup>13</sup> and J. Simone<sup>14</sup>

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<sup>12</sup>*School of Computer Science, Telecommunications and Information Systems, DePaul University, Chicago, Illinois 60604, USA*

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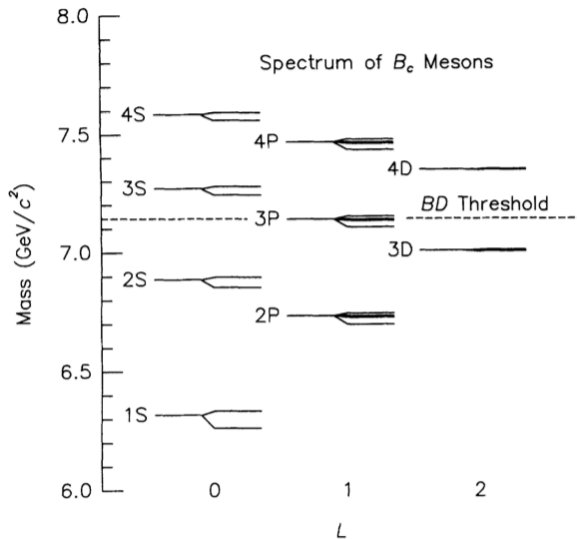
(Received 7 April 2003; published 15 January 2004)

The recently developed Symanzik-improved staggered-quark discretization allows unquenched lattice-QCD simulations with much smaller (and more realistic) quark masses than previously possible. To test this formalism, we compare experiment with a variety of nonperturbative calculations in QCD drawn from a restricted set of “gold-plated” quantities. We find agreement to within statistical and systematic errors of 3% or less. We discuss the implications for phenomenology and, in particular, for heavy-quark physics.

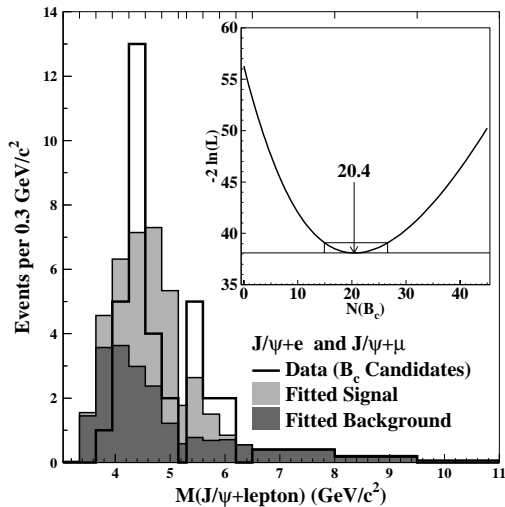
DOI: 10.1103/PhysRevLett.92.022001

PACS numbers: 12.38.Aw, 11.15.Ha, 12.38.Gc

# Anticipating $B_c$ (1994)

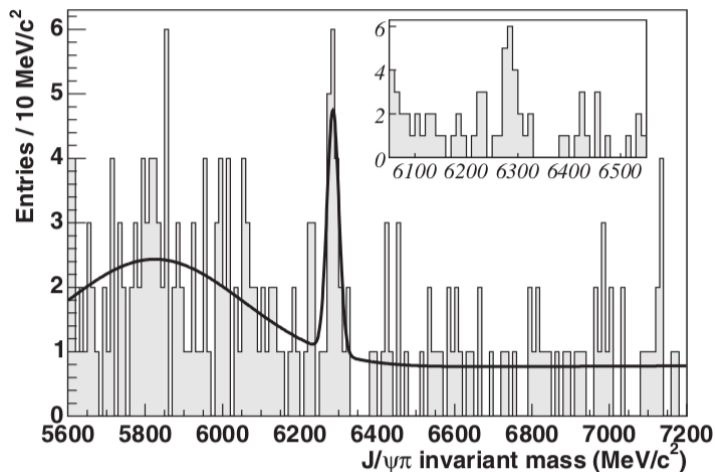


# Observation of $B_c \rightarrow J/\psi \ell \nu$ (1998)



$$M(B_c) = 6.40 \pm 0.39(\text{stat}) \pm 0.13(\text{syst}) \text{ GeV}$$

# Reconstruction of $B_c$ meson (CDF, 2006)



$M(B_c) = 6285.7 \pm 5.5 \text{ MeV}$  (Test of lattice QCD prediction (2005),  $6304 \pm 12_{-0}^{+18} \text{ MeV}$ )  
PDG (2019):  $6274.9 \pm 0.8 \text{ MeV}$

# Mesons with beauty and charm: stress test for NRQM, LQCD

$B_c$ : weak decays only

$b \rightarrow c$        $c \rightarrow s$        $b\bar{c} \rightarrow W^-$

$B_c \rightarrow J/\psi\pi$ : ( $Q\bar{Q}$ ) transmutation

Rich ( $b\bar{c}$ ) excitation spectrum;  
interpolates  $J/\psi$ ,  $\Upsilon$  ( $\neq$  masses)

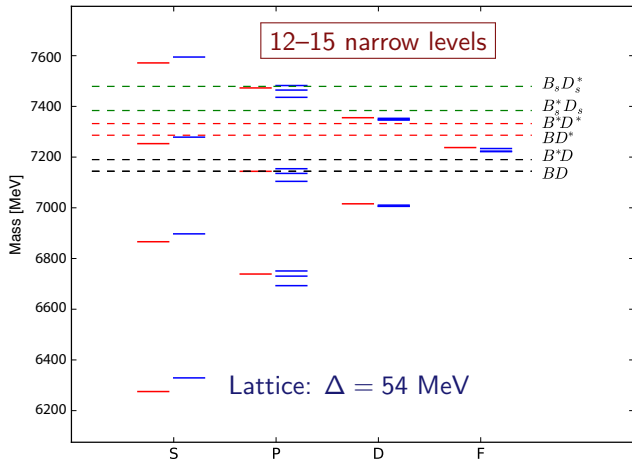
Excited states below  $BD \rightarrow B_c + \dots$

$B_c(2S) \rightarrow B_c(1S) + \pi\pi$

$P$  states:  $\gamma$  transitions

Many states observable at LHC, TeraZ

Update: Eichten & CQ (2019)  
using “frozen- $\alpha_s$ ” potential, new  
approach to spin splittings



# Observing the $B_c$ spectrum: $\pi\pi$ transitions

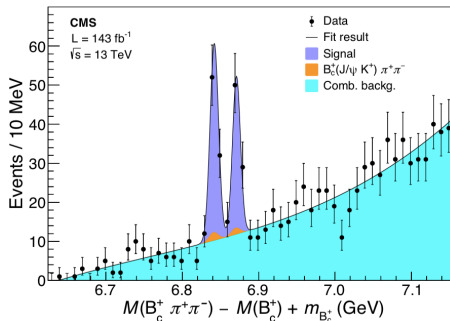
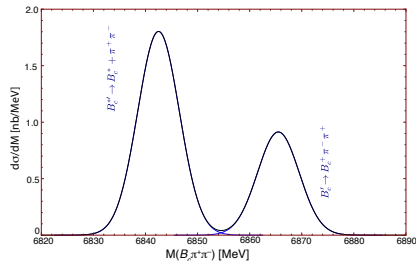
Combine predicted production rates (BCVEGPY2.2) with calculated branching fractions to obtain expectations for  $\pi\pi$  transition rates  
 $\leadsto$  peak heights:  $B_c^{*'} / B_c' \approx 2.5$

M1  $B_c^* \rightarrow \gamma B_c$  unobserved

$$[M(B_c^{*'}) - M(B_c')] - [M(B_c^*) - M(B_c)] \approx -23 \text{ MeV: } B_c^{*'} \text{ lower peak}$$

2S  $\rightarrow \pi\pi + 1S$  transitions observed by ATLAS, CMS, LHCb

CMS separation:  $-29 \text{ MeV}$   
 LHCb:  $-31 \text{ MeV}$





# Observing the $B_c$ spectrum: $\pi\pi$ transitions

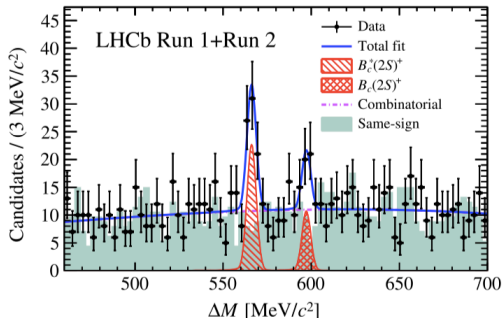
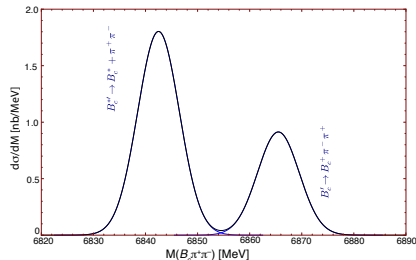
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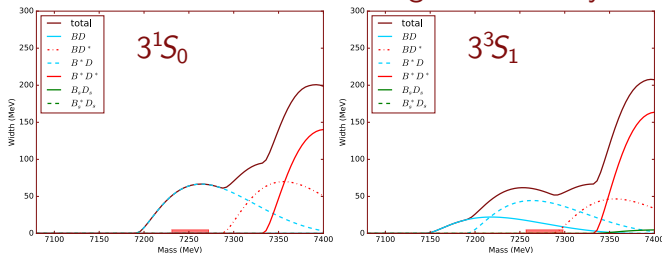
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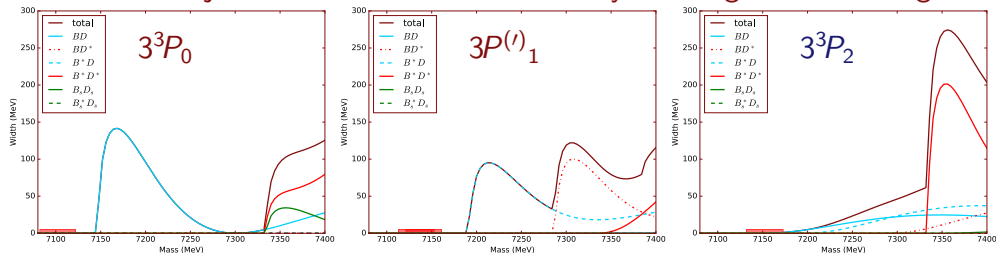


# Mesons with beauty and charm: states near flavor threshold

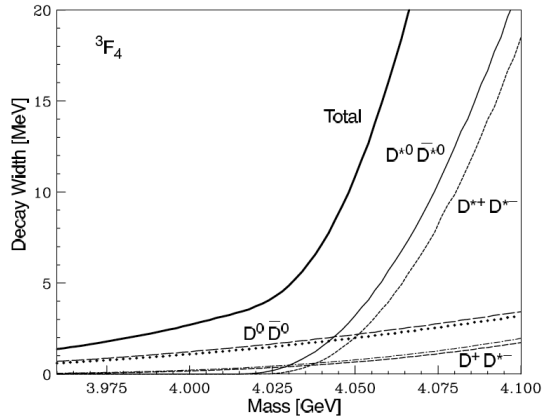
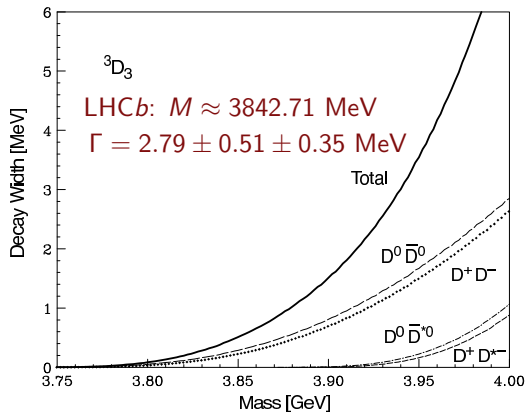
$3S$  states above threshold have significant decay widths



$3P$  states just below threshold;  $J = 1$  may have significant mixing

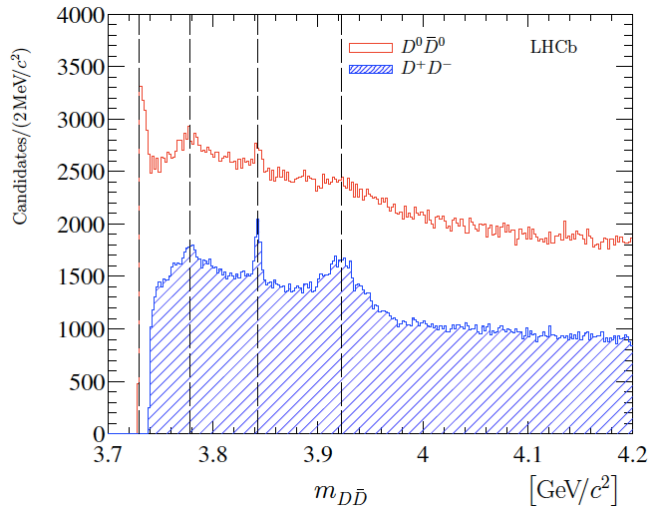


# Narrow ( $c\bar{c}$ ) states above flavor threshold



Eichten, Lane, Quigg, Phys. Rev. D **69**, 094019 (2004) / hep-ph/0401210.

# LHCb observation of $^3D_3$ candidate



Can we find  $^3F_4$ , perhaps near 4054 MeV?

# Observing the $B_c$ spectrum: E1 transitions

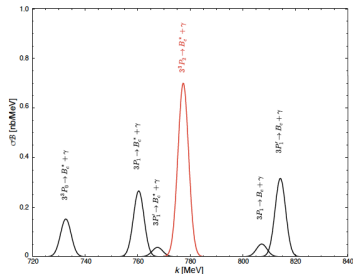
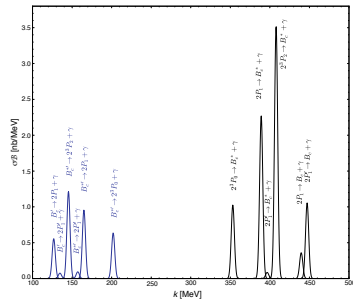
E1 spectroscopy in the  $(b\bar{b})$  family:  
LHC experiments discovered  $\chi''_{b1}, \chi''_{b2}$ .

Incentive for the search:  $2S \rightarrow 2P$  and  $2P \rightarrow 1S$  transitions, assuming missing  $B_c^* \rightarrow B_c \gamma$  in the reconstruction.

$3S, 3P$  yields  $\approx \frac{1}{4} \times 2P \rightarrow 1S$  lines, but higher  $\gamma$  energies may aid detection.

$$3^3P_2(7154) \rightarrow B_c^* \gamma(777 \text{ MeV})$$

Encourage search for  $(3,2)P(b\bar{c})$ .



# Investigating unusual body plans for hadrons

R. L. Jaffe, "Perhaps a Stable Dihyperon" (1977)  $\mathcal{C} = \langle \sum_{i < j} \lambda^{(i)} \cdot \lambda^{(j)} \sigma^{(i)} \cdot \sigma^{(j)} \rangle \quad (uds)^2 \mathbf{490}$

## Evidence against a Stable Dibaryon from Lattice QCD

Paul B. Mackenzie

*Institute for Advanced Study Princeton, New Jersey 08540*

and

H. B. Thacker

*Fermi National Accelerator Laboratory, Batavia, Illinois 60510*

(Received 9 September 1985)

We have used standard numerical techniques of lattice quantum chromodynamics to look for evidence of the proposed doubly strange spin-zero dibaryon (the  $H$  particle), and to determine the splitting between the mass of the  $H$  and the mass of two  $\Lambda$ 's, its lightest possible strong-decay channel. We find that the dibaryon is above the two- $\Lambda$  threshold, making it unstable to strong decay.



Quarkonium-associated states:  $M \gtrsim$  threshold:  $X(3872)$  etc.

Mostly narrow, seen in hadronic transitions or decays

What are they?

Quarkonium (+ coupled-channels, thresholds)

Threshold effects

New body plans:

quarkonium hybrids ( $q\bar{q}g$ )

two-quark–two-antiquark states, including

dimeson “molecules”

tetraquarks

diquarkonium · hadroquarkonium

and superpositions!

(crypto)pentaquarks



# Toward a new symmetry of QCD

## Static effective field theory: $1/m$ corrections

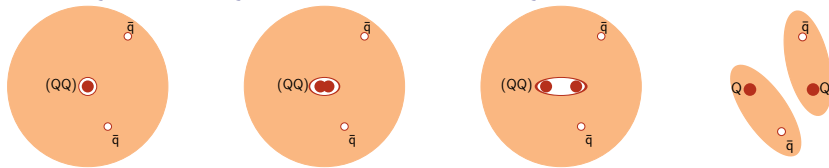
Estia Eichten <sup>1</sup> and Brian Hill <sup>2</sup>

*Fermi National Accelerator Laboratory, P.O. Box 500, Batavia, IL 60510, USA*

Received 10 April 1990

The static approximation, which is the zeroth order approximation in an expansion in the inverse of the mass of a heavy quark, has previously been formulated in terms of an effective field theory action. In this formulation, corrections to the approximation can be systematically included by the addition of higher dimensional operators to the action. We determine the coefficients to one loop of the dimension-five operators incorporating the  $1/m$  corrections to the theory.

Heavy quark symmetry  $\Rightarrow$  stable heavy tetraquarks  $Q_i Q_j \bar{q}_k \bar{q}_l$



*HQS relates DHTQ mass to masses of  $QQq$ ,  $Qqq$ ,  $Q\bar{q}$ .*

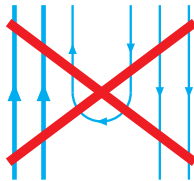
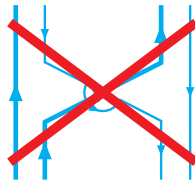
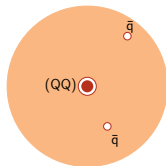
*Lightest  $bb\bar{u}\bar{d}$ ,  $bb\bar{u}\bar{s}$ ,  $bb\bar{d}\bar{s}$  states: (likely) no strong decays.*

*Heavier  $bb\bar{q}_k\bar{q}_l$ ,  $cc\bar{q}_k\bar{q}_l$ ,  $bc\bar{q}_k\bar{q}_l \rightarrow Q\bar{q} + Q\bar{q}$  might be seen as “double-flavor” resonances near threshold.*

*Observing a weakly decaying double-beauty state would establish the existence of tetraquarks and illuminate the role of heavy color- $\bar{3}$  diquarks as hadron constituents.*

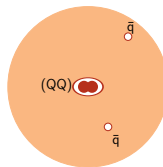
Eichten & CQ 1707.09575

# No open strong decay channels in the heavy-quark limit!



As  $\bar{M} \rightarrow \infty$ , stable  $Q_i Q_j \bar{q}_k \bar{q}_l$  mesons must exist

Implications for the real world?



$$\langle r^2 \rangle^{1/2} = 0.28 \text{ fm}(cc), 0.24 \text{ fm}(bc), 0.19 \text{ fm}(bb)$$

# HQS relations for ground-state tetraquark masses

Assumed: compact diquark, light degrees of freedom “same” for all  $(QQ)$

$$m(Q_i Q_j \bar{q}_k \bar{q}_l) - m(Q_i Q_j q_m) = m(Q_x q_k q_l) - m(Q_x \bar{q}_m)$$

+ finite-mass corrections

RHS is determined from data

One doubly heavy baryon observed,  $\Xi_{cc}$ ; others from model calculations\*

$$\text{LHCb: } M(\Xi_{cc}^{++}) = 3621.40 \pm 0.78 \text{ MeV}$$

\*We adopt Karliner & Rosner, *PRD* **90**, 094007 (2014)

Strong decays  $(Q_i Q_j \bar{q}_k \bar{q}_l) \not\rightarrow (Q_i Q_j q_m) + (\bar{q}_k \bar{q}_l \bar{q}_m) \forall$  ground states

Consider decays to pairs of heavy–light mesons case-by-case

# Expectations for ground-state tetraquark masses, in MeV

| State                           | $J^P$           | $m(Q_i Q_j \bar{q}_k \bar{q}_l)$ | Decay Channel   | $Q$ [MeV]     |
|---------------------------------|-----------------|----------------------------------|---|---------------|
| $\{cc\}[\bar{u}\bar{d}]$        | $1^+$           | 3978                             | $D^+ D^{*0}$ 3876                                       | 102           |
| $\{cc\}[\bar{q}_k \bar{s}]$     | $1^+$           | 4156                             | $D^+ D_s^{*+}$ 3977                                     | 179           |
| $\{cc\}\{\bar{q}_k \bar{q}_l\}$ | $0^+, 1^+, 2^+$ | 4146, 4167, 4210                 | $D^+ D^0, D^+ D^{*0}$ 3734, 3876                        | 412, 292, 476 |
| $[bc][\bar{u}\bar{d}]$          | $0^+$           | 7229                             | $B^- D^+ / B^0 D^0$ 7146                                | 83            |
| $[bc][\bar{q}_k \bar{s}]$       | $0^+$           | 7406                             | $B_s D$ 7236  | 170           |
| $[bc]\{\bar{q}_k \bar{q}_l\}$   | $1^+$           | 7439                             | $B^* D / B D^*$ 7190/7290                               | 249           |
| $\{bc\}[\bar{u}\bar{d}]$        | $1^+$           | 7272                             | $B^* D / B D^*$ 7190/7290                               | 82            |
| $\{bc\}[\bar{q}_k \bar{s}]$     | $1^+$           | 7445                             | $D B_s^*$ 7282  | 163           |
| $\{bc\}\{\bar{q}_k \bar{q}_l\}$ | $0^+, 1^+, 2^+$ | 7461, 7472, 7493                 | $B D / B^* D$ 7146/7190                                 | 317, 282, 349 |
| $\{bb\}[\bar{u}\bar{d}]$        | $1^+$           | 10482                            | $B^- \bar{B}^{*0}$ 10603                                | <b>-121</b>   |
| $\{bb\}[\bar{q}_k \bar{s}]$     | $1^+$           | 10643                            | $\bar{B} \bar{B}_s^* / \bar{B}_s \bar{B}^*$ 10695/10691 | <b>-48</b>    |
| $\{bb\}\{\bar{q}_k \bar{q}_l\}$ | $0^+, 1^+, 2^+$ | 10674, 10681, 10695              | $B^- B^0, B^- B^{*0}$ 10559, 10603                      | 115, 78, 136  |

Cf. M. Karliner & J. L. Rosner model, Phys. Rev. Lett. **119**, 202001 (2017) [arXiv:1707.07666].  
 Estimate deeper binding, so additional  $bc$  and  $cc$  candidates.

# The $(Q_i Q_j \bar{q}_k \bar{q}_\ell)$ system

Should we expect a  $bb\bar{b}\bar{b}$  tetraquark bound state?

**Searching for beauty-fully bound tetraquarks  
using lattice nonrelativistic QCD**

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How does the color configuration depend on the quark masses?

# How has Paul accomplished so much?



Thank you, Paul!