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Plasma Wake-field Collider Issues and Advantages / Disadvantages of Having Muons and Crystals/CNTs

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Workshop on Beam Acceleration in Crystals and Nanostructures
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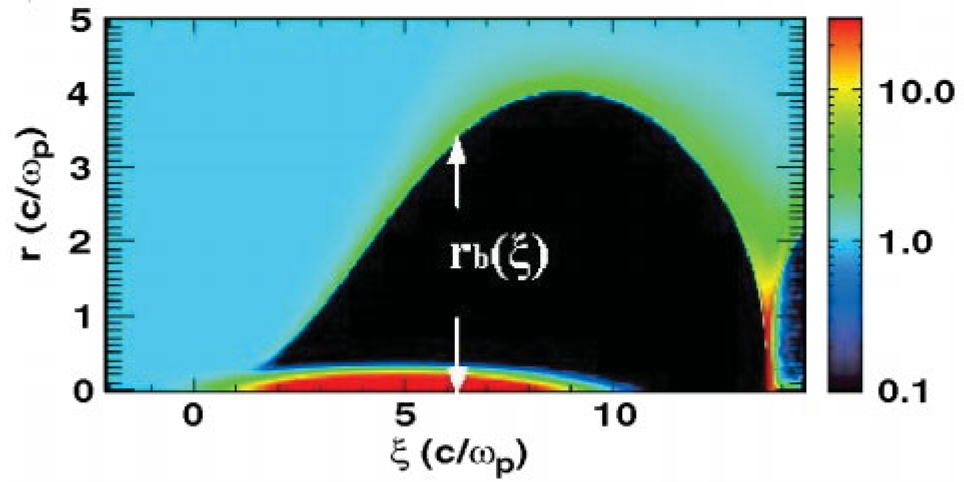
Objectives

- An introductory talk
- Overview /comparison to other accelerating methods
- Possible applications
  - Colliders
  - Compact high energy linacs
    - Acceleration of secondary particles (muons, etc.)
- Discuss major limitations
  - Accelerating gradient
  - Energy efficiency
  - Transverse and longitudinal acceptances
  - Emittance growth due to multiple scattering in plasma
  - Use of channeling
- Short overview of limitations of plasma colliders

Traditional Linacs versus Plasma-based Linacs

- Traditional linacs have high Q and intense bunch excites many HOMs
  - It limits the efficiency of energy transfer for a single bunch operation
  - Obtaining high efficiency in the energy transfer requires an operation with multiple bunches (ILC)
    - Typically, the energy transfer to a single bunch does not exceed ~1%
- Plasma based linacs have very low Q

  ⇨ Single bunch operation is the only possibility
  - In the bubble regime close to 100% efficiency of energy transfer is potentially achievable
    - Profiling of bunch density is required
    - BBU instability limits the efficiency if an acceleration to high energy is required



  - Electrons are removed from the axis in the strong bubble regime
    - This is the only regime which enables acceleration of bright collider quality bunches
    - Acceleration of bright positron bunches is presently infeasible!!!

Acceleration in Plasma and Solid Medium

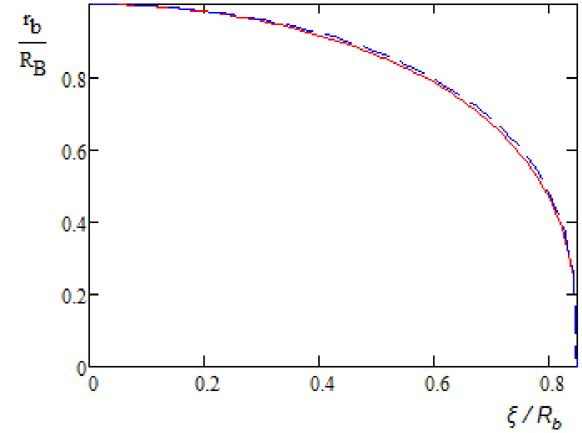
Accelerating Gradient

- For 100% e-density modulation the "maximum" electric field $\propto \sqrt{n_e}$

$$\operatorname{div}\mathbf{E} = 4\pi\rho \xrightarrow[\text{modulation}]{100\%\,\text{density}} E_0 = \frac{4\pi e n_e}{k_p} \xrightarrow{k_p=\sqrt{\frac{4\pi n_e e^2}{mc^2}}} \sqrt{\frac{4\pi n_e e^2}{r_e}}$$

Compare it to the atomic field $\left(E_a = e/a_0^{\,2}\right)$:

$$\frac{E_0}{E_a} = \frac{\sqrt{4\pi n_e a_0^{\,3}}}{\alpha} \approx 486\sqrt{n_e a_0^{\,3}}$$

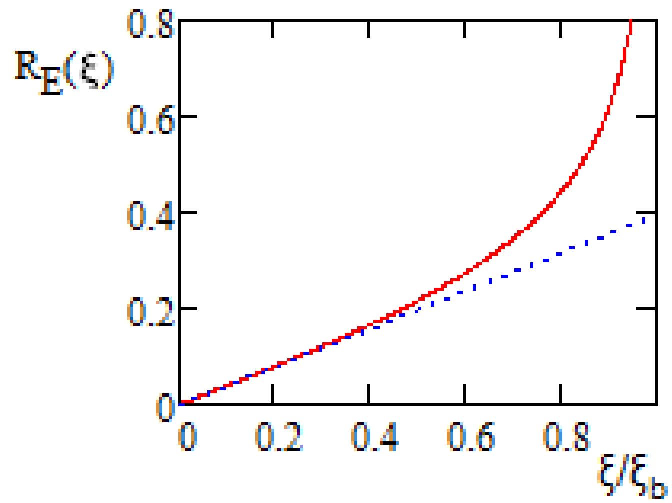


- ♦ $E_0 >> E_a$ for electron density comparable to the density in solid medium
- Strong bubble regime (Lu equation)
  - ♦ Almost spherical cavity: $\xi_b = 0.847 R_b$
  - ♦ Electric field at the axis

$$R_E(E) \equiv \frac{1}{R_b k_p}\frac{E(\xi)}{E_0} \approx \frac{0.394\xi/\xi_b}{\sqrt[3]{1-\left(\xi/\xi_b\right)^2}}$$

Linear in the bubble center

Goes to large values at its ends



Properties of Plasma Bubbles

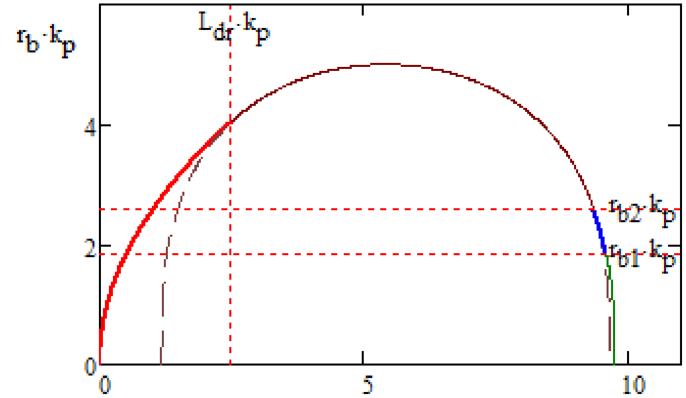
- Power transferred to the bubble is uniquely determined by ($R_b k_p$)

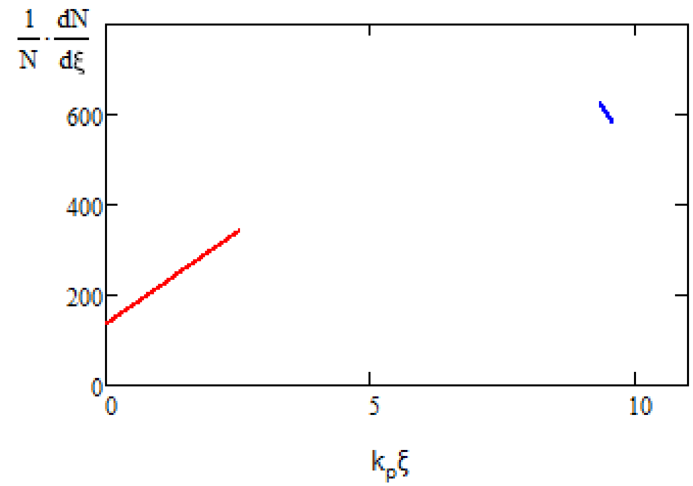
$$P = \frac{\pi^2}{4} e^2 n_e{}^2 c R_b{}^4 = \frac{mc^3}{64 r_e} \left(k_p R_b\right)^4$$

It is also the maximum power which can be transferred to the accelerated beam

- Transformer ratio grows with displacement of accelerated bunch to the cavity end
- The focusing in the plasma wave is determined by charge density at the axis.
  - In the bubble regime the equilibrium beta-function is: $\beta_f = \frac{1}{k_p}\sqrt{2\gamma\frac{M}{m}}$
  - Beam focusing from external magnets is small and can be neglected.
- Normalized acceptance of the accelerating channel

$$\varepsilon_n = \gamma \frac{R_b{}^2}{\beta_f} = \sqrt{\frac{m\gamma}{2M}} \frac{\left(k_p R_b\right)^2}{k_p}$$





Emittance growth due to scattering at ions/nucleus

- Scattering at the plasma ions results in an emittance growth

$$\delta\varepsilon_n \approx \frac{Z^2 r_e L_c}{Z_v E_{acc}/E_0}\sqrt{\frac{M}{2m}}\left(\sqrt{\gamma_{fin}}-\sqrt{\gamma_{in}}\right)$$

where Z_v is the number of valence electrons

  - This Eq. implies very strong plasma focusing. In its absence the emittance growth will be much larger
- Comparing with the bucket acceptance we obtain

$$\frac{\delta\varepsilon_n}{\varepsilon_n} \approx \frac{Z^2 r_e L_c}{Z_v\left(E_{acc}/E_0\right)\left(k_p R_b\right)}\frac{M}{m}k_p\ , \quad \gamma_{fin} \gg \gamma_{in}$$

  - Both the normalized bucket acceptance and the emittance growth increase with energy as $\sqrt{\gamma}$

    => ratio of emittance increase to the bucket size does not depend on energy
  - Electron density increase increases relative emittance growth as $\sqrt{n_e}$
  - Large Z material also increases the emittance growth

Comparison of Beam Acceleration in Plasma and Silicon

$(k_p R_b = 2,\ E_{acc}/E_0 = 1)$

	Proton plasma	Solid silicon
Nuclear density	10^{17}	$5 \cdot 10^{22}$
Z/A	1/1	14/28
Number of valence electrons	0	4
Electron density, n_e	10^{17}	$2 \cdot 10^{23}$
Basic wavelength, $2\pi k_p^{-1}$	100 μm	750 Å
Acceptance, mm mrad	$47\sqrt{\gamma m / M}$	$0.034\sqrt{\gamma m / M}$
"Maximum" field, E_0	300 MeV/cm	430 GeV/cm
Relative emittance growth, $(d\varepsilon_n/\varepsilon_n)$	$\sim 10^{-7}$	~ 0.003
Energy stored in plasma, mJ/GeV	60	0.04
Maximum number of accelerated particles*	$1.8 \cdot 10^{8}$	$1.3 \cdot 10^{5}$

* 100% energy efficiency is implied

- In plasma bubble regime the linear power density in a plasma is uniquely determined by $(R_b k_p)$

$$\frac{dA}{ds} = \frac{mc^2}{64 r_e}\left(k_p R_b\right)^4$$

Challenges for acceleration in a solid medium

- To excite a plasma wave, we need to have the rms beam sizes less or about k_p^{-1}, or <120 Å
  - Longitudinal size may be larger (if envelope instability is used), but in this case
    - an efficiency of the excitation is strongly suppressed
    - and the modulation depth is reduced (AWAKE)
- Electric field is so large that it can induce impact ionization of inner shell of the atoms
  - Not a problem in a proton/hydrogen plasma
- Maximum number of accelerated particles is quite small $\le 10^5$
- Acceleration of muons is greatly complicated by small acceptances of the plasma channel
  - Justified for both transverse and longitudinal planes
  - ε_n~34 nm for 10 GeV muons ($\propto \sqrt{\gamma}$)
- Presently, multi-stage acceleration is not feasible

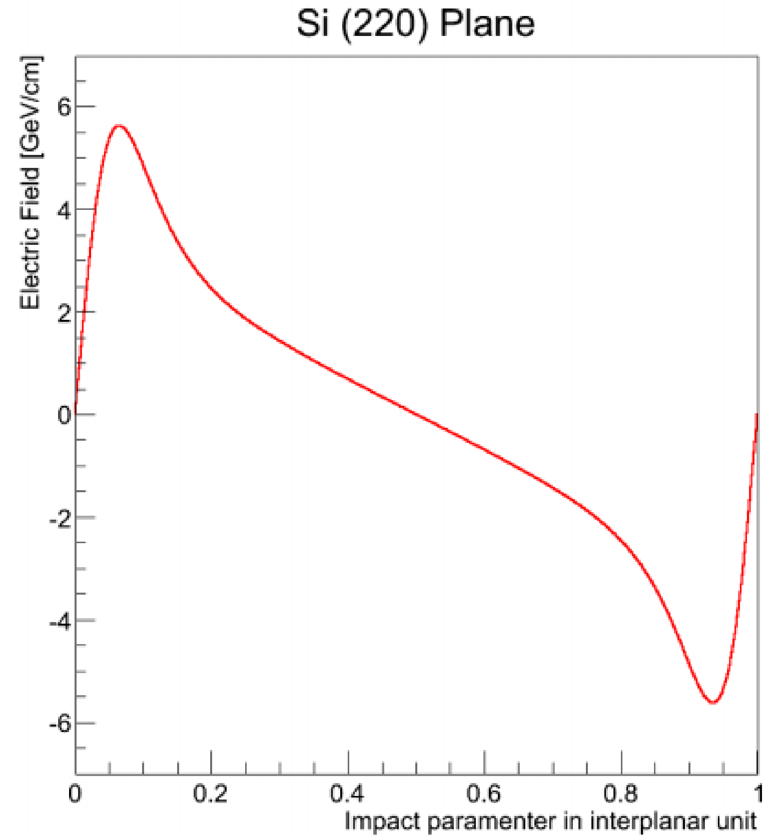
Channeling

- The channel focuces positive particles with gradient ~$4{\cdot}10^{17}$ V/cm^2
- Excitation of the plasma wave introduces non-zero charge density at the axis.
  - Its focusing is about half of the channeling and has opposite sign

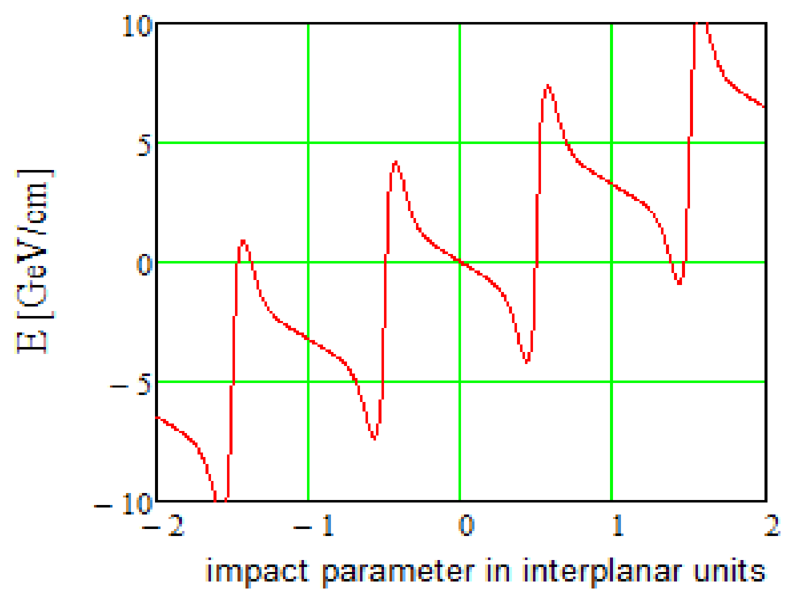
$$G = 2\pi e n_e \approx 1.8 \cdot 10^{17}\ \mathrm{V/cm}^2$$

- Only positive particles are focused
- Only axial channeling (along the crystal axis) focuses in both planes
- Combined focusing can only keep particles in the central channel
- Acceptance of the channel is much smaller than for plasma focusing in the bubble: reduction ~10^4 times

$\varepsilon_n \approx$ 5 pm for 10 GeV muons



ECHARM - E. Bagli, V. Guidi, V. A. Maisheev, PRE 81, 026708 (2010)



Dechanneling Length

- Channeling removes scattering on the nuclei however scattering on electrons is still present
  - For silicon a reduction in scattering is about 2 orders of magnitude but the channel acceptance is 4 orders of magnitude smaller than in the plasma bubble operation
- Both the normalized emittance growth due to scattering at electrons

$$\delta\varepsilon_n \approx \frac{r_e L_c}{\left(E_{acc}/E_0\right)}\sqrt{\frac{\gamma M}{2m}}$$

and the normalized acceptance of channeling $\varepsilon_n \approx k_p a_0{}^2 \sqrt{\frac{m\gamma}{2M}}$
grow as $\sqrt{\gamma}$.

  - That implies that the scattering in the channel does not limit the maximum energy if

$$\frac{r_e L_c}{\left(E_{acc}/E_0\right)}\frac{M}{m} \le k_p a_0{}^2$$

  - For acceleration of μ+, $E_{acc}/E_0>10$ is required; i.e. the acceleration at the maximum possible rate is desirable.
    It is more forgiving for positrons which have smaller mass

Plasma Based Colliders

Present Status

- Presently there is no a coherent proposal for plasma collider
  - while some preliminary ideas were published
- In the present paradigm
  - e+e- collider is excluded by inability to accelerate bright positron bunches
  - e-e- or γ-γ colliders look as a possibility but require a resolution of many technical problems before we can even discuss such a possibility
    - Presently it does not look as a competitor to the ILC or its upgrade
- Fundamentally new ideas are required to promote e+e- collider
  - This is an open quest

More about Status

- There are 2 concepts developed for plasma-based colliders
  - (1) SLAC and (2) LBL
- Both concepts claim that they can achieve a collider operation with luminosity 10^{34} cm^{-2}s^{-1} or above
- However, an examination of presented parameters shows that they are inconsistent with limitations coming from fundamental principles of physics
  - We also could not find how the present paradigm can be changed so that the required luminosity can be achieved
- Separately, each of below mentioned limitations can be mitigated, in principle, to an acceptable level
  - However, they cannot be satisfied simultaneously
- We knowingly do not consider technical limitations
  - which are also present but, potentially, can be overcame with technology development

Major Limitations

- The following limitations were considered
  - Limitations on the beam emittance and momentum spread coming from the SR radiated in the final focus lenses/quads
  - Emittance growth due to multiple scattering in the plasma and/or residual gas
  - Pinching/expulsion of plasma electrons by electric field of positron/electron bunch
  - Pinching/expulsion of plasma ions (in the bubble regime) by electric field of electron/ positron bunch
  - Impact ionization of ions by bunch electric field if heavy ions are used to mitigate ion pinching by electrons in the bubble regime
  - Bremsstrahlung on heavy ions in the bubble regime
  - Transverse head-tail instability supported by the transverse wake-field excited in plasma
- Below we discuss some of these limitations

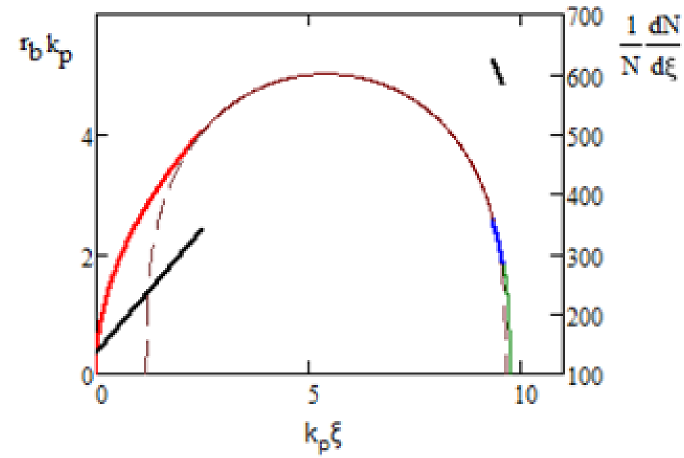
Efficiency versus Instability in Plasma Acceleration

- In optimally designed linear collider its luminosity is proportional to the beam power and weakly depends on other parameters
- Therefore, the power efficiency is the primary concern in the technology choice
- In any accelerating structure the transverse and longitudinal wakes are closely related
  - High power efficiency implies large charge in the accelerating bunch
    => That generates strong head tail effects in the course of bunch acceleration

Longitudinal and Transverse Wakes

- The value of the longitudinal wake is determined by the bubble radius at the trailing particle location (ξ)

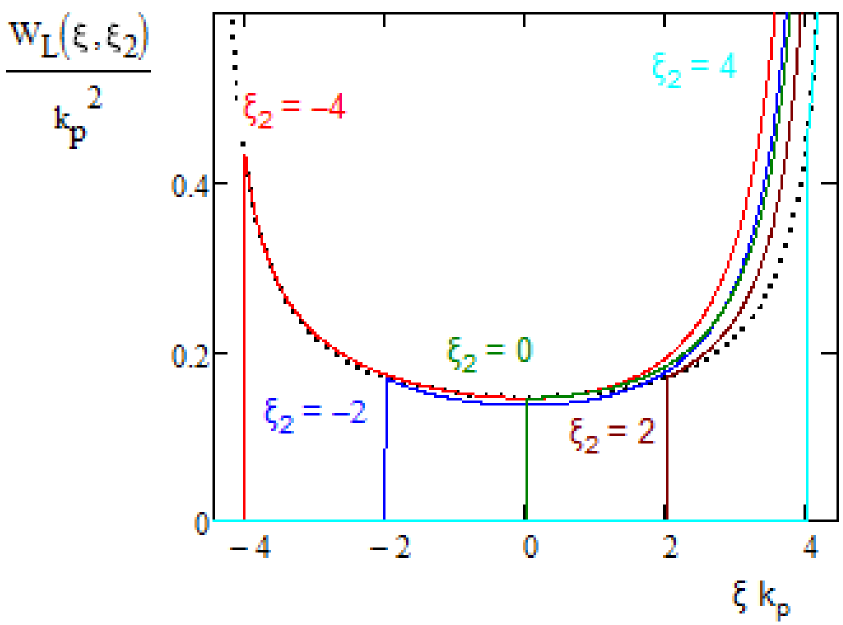
$$W_L(\xi,\xi_2) \approx \frac{4}{r_b(\xi)^2}\theta(\xi-\xi_2)$$



- The short-range wake theorem binds the $\perp$ and || wakes

$$W_\perp\left(\tilde{\xi}\right) \approx \frac{2}{\tilde{r}_b^{\,2}}\int_0^{\xi} W_L(s)ds$$

That yields: $W_\perp(\xi,\xi_2) \approx \dfrac{8(\xi-\xi_2)}{r_b(\xi)r_b^{\,3}(\xi_2)}\theta(\xi-\xi_2)$



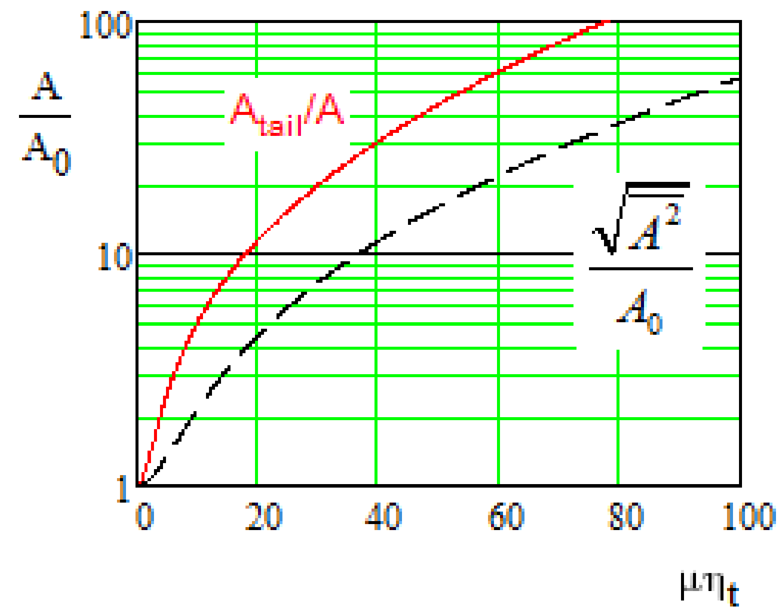
- High value of accelerating efficiency requires the trailing bunch to be located at the end of the cavity.
- Accounting that the $\perp$ wake diverges at the bucket end as $1/r_b(\xi)$ yields that the beam breakup (head-tail) instability is greatly amplified with efficiency increase

Efficiency versus Instability Relationship

- We will characterize the ⊥ wake strength by parameter η_t equal to the wake deflecting force at the bunch end for uniformly displaced bunch to the focusing force coming from plasma
- Relationship between ⊥ and || wakes binds η_t and power efficiency η_P:

$$\boxed{\eta_t \approx \frac{{\eta_P}^2}{4(1-\eta_P)}}$$

- The product of parameter η_p and the betatron phase advance in the course of acceleration μ uniquely characterize the instability growth
  - For 1 TeV collider $\mu = 10^3$ we require the rms amplitude growth to be not more than 3 for a single perturbation
    - => $\mu\eta_t < 10$ => $\eta_t < 0.01$
    - => $\eta_p \approx 18\%$.



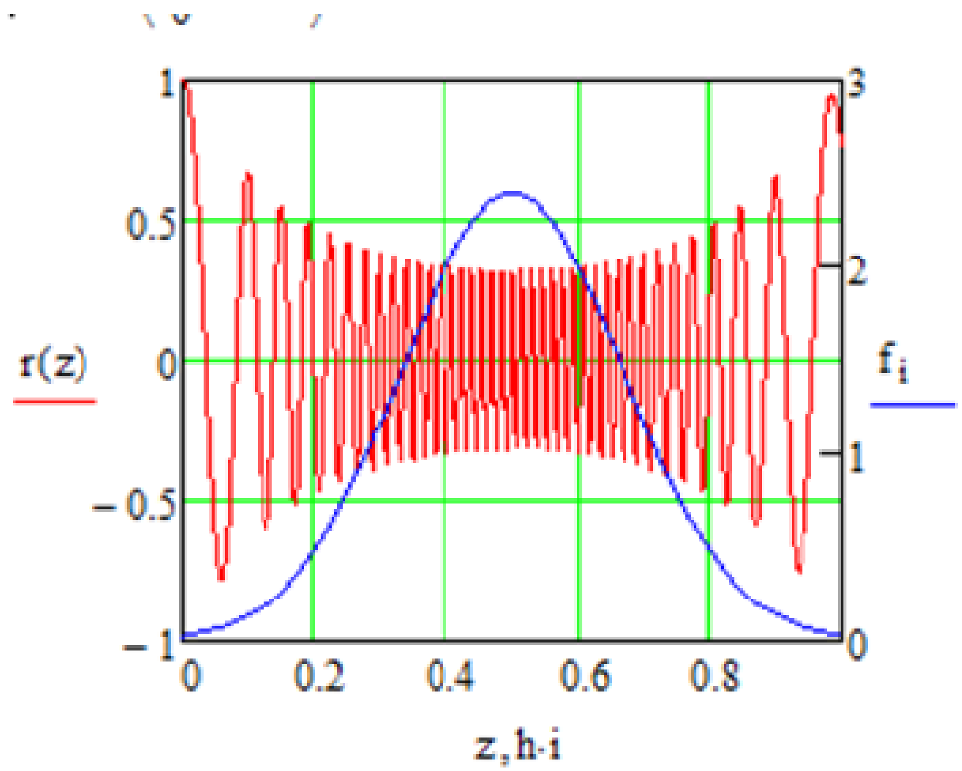
- This efficiency does not exceed efficiency of conventional accelerators

The Hollow Beam Channel

- The hollow beam channel was proposed to overcome scattering at the ions and ions pinching by the electron bunch fields. However, it has additional limitations
  - Obtaining transverse stability in the presence of very large transverse impedance of the plasma channel requires considerable momentum spread and very strong focusing which can be achieved with plasma focusing only. Such focusing cannot be supported in the hollow channel.
    - External focusing will be 3-4 orders of magnitude weaker than the required value
  - Note also that it is unclear how to create a μm radius hollow channel free of ions and gas which will be impact-ionized by the bunch field
- Acceleration in carbon nonotubes is somewhat similar to the hollow channel
  - Acceleration of small intensity bunch is feasible

What is Wrong with Positrons?

- Collapse of plasma electrons (needed for focusing) to the positron bunch center makes comparatively little correction to the beam deceleration but greatly affects the beam focusing coming from plasma
  - As result the focusing is dependent on coordinate and is strongly non-linear ⇨emittance growth
  - That prevents acceleration of positrons in the linear regime



A trajectory of a plasma electron inside of the positron bunch for $\rho_0 \ll \rho_{min}$

- Bubble regime does not have electrons on the axis => no pinching. However, it cannot be used for positrons due to their strong defocusing from ions
  - !!! Using electrons for restoring focusing of positrons has a problem with collapse of electrons to the center as it was discussed above

Conclusions

- Plasma-based acceleration is a very interesting subject. Therefore, it attracts scientists looking into how to develop and promote this technology.
- So far, we did not find how plasma-based acceleration can help to the linear colliders
  - Other applications are feasible
  - Can we generate another paradigm which will help colliders?
- Acceleration in a solid medium would be a tremendous step in development of plasma-based acceleration
  - Interesting subject
  - Potentially it can drive a small intensity beam to very high energy
  - Basic principles and limitations are clear

    More to come

New Ideas on Applications and Experimental techniques are welcomed!!!