NONLINEAR TUNE-SHIFT MEASUREMENTS IN THE INTEGRABLE OPTICS TEST ACCELERATOR

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Abstract

The first experimental run of Fermilab’s Integrable Optics Test Accelerator (IOTA) ring aimed at testing the concept of nonlinear integrable beam optics. In this report we present the preliminary results of the studies of a nonlinear focusing system with two invariants of motion realized with the special elliptic-potential magnet. The key measurement of this experiment was the horizontal and vertical betatron tune shift as a function of transverse amplitude. A vertical kicker strength was varied to change the betatron amplitude for several values of the nonlinear magnet strength. The turn-by-turn positions of the 100 MeV electron beam at twenty-one beam position monitors around the ring were captured and used for the analysis of phase-space trajectories.

INTRODUCTION

In Ref. [1], V. Danilov and S. Nagaitsev proposed a nonlinear accelerator lattice, which leads to integrable and stable nonlinear motion. This introduces large betatron tune spread where bounded motion can fill a large phase space. The nonlinear element has a special elliptic-potential, that is time-independent, which can be expressed in elliptic coordinates (ξ, η) as:

\[ U(ξ, η) = \frac{f(ξ) + g(η)}{ξ^2 - η^2}, \]

where \( f \) and \( g \) are arbitrary functions. In order for the potential to satisfy the Laplace equation these functions are expressed as:

\[ f_2(ξ) = ξ \sqrt{ξ^2 - 1}[d + t \text{acosh}(ξ)], \]
\[ g_2(η) = η \sqrt{1 - η^2}[b + t \text{acos}(η)], \]

where \( b, d, \) and \( t \), are arbitrary constants. In order to have the lowest multipole expansion term to be a quadrupole, \( d = 0 \) and \( b = \frac{5}{2}t \). The constant \( t \) is then the nonlinear potential strength. To determine the maximum attainable betatron tune spread at small amplitudes, a multipole expansion of the potential can be done. The small amplitude betatron tune can then be expressed as [2]:

\[ Q_x = Q_0 \sqrt{1 + 2t}, \]
\[ Q_y = Q_0 \sqrt{1 - 2t}, \]

where \( Q_0 \) is the unperturbed, linear-motion, working point tune. Thus for small amplitude, stable linear motion, the range of strength values need to be \(-0.5 < t < 0.5\). There are regions of the phase space that exhibit bounded nonlinear motion for larger \( t \) values as well [3].

The IOTA ring, see Fig. 1, was specifically built to experimentally demonstrate integrable nonlinear optics predicted by Danilov and Nagaitsev. The experiment will use a 150 MeV, 'pencil-like', electron beam of RMS emittance 40 nm. The beam will map out the dynamic aperture by kicking the beam at various amplitudes. The key goal of the experiment is to demonstrate large betatron tune spread of \( ΔQ_y > 0.10 \), with using a special elliptic-potential magnet, without beam loss and degradation of dynamical aperture [4].

EXPERIMENTAL SETUP

Nonlinear Magnet

The nonlinear magnet and vacuum chamber were designed and manufactured by RadiaBeam Technologies. Due to the complexity of having a varying aperture, there are eighteen individual magnets that scales as the square root of β-function along the beam path. All eighteen magnets were required to have a 50 μm alignment of the magnetic axes. Each element magnetic field is also required to be within one percent of the theoretical model [5].

The magnetic centers of each of the 18 magnets were aligned to ±50 μm using a stretch copper-beryllium wire [6]. To verify the alignment of the nonlinear elements, beam
based orbit response measurements were made. The alignment was good up to 100 µm. To verify the quadrupole component response of each magnetic element, tune response measurement was performed.

Data Collection and Condition

The FAST LINAC injected a 100 MeV electron beam with 4 mA of beam current circulating in IOTA. The RMS emittance was reported to be 25.3 nm in a coupled lattice and 96.3 nm uncoupled. This corresponds to a RMS beam size of 0.13 mm and 0.25 mm at the center of the nonlinear magnet, respectively. The beam condition was chosen to operate with nonlinear decoherence in the lattice [7]. After establishing stable beam at a working point of $Q_{x,y} = 0.30$, integer tune of 5, the nonlinear magnet is turned on. The explored nonlinear strength values were at $t = 0.22$, 0.29, 0.43. The beam is then kicked to various vertical amplitudes using a stripline kicker [8]. This would trigger an event in which the twenty-one beam position monitors (BPM) will start to record turn-by-turn data, for 2000 turns.

RESULTS

Analysis

The tune was obtained via Fast Fourier transform (FFT) from all BPM’s. However, due to fast nonlinear decoherence [9], only the first 200 turns of the data was used. To reduce uncertainty, an interpolated FFT is applied for each individual BPM signal, this allows for an uncertainty in the tune measurements down to ±0.001. The measured tune is then compared to a lattice model. Then a 6 mm aperture restriction, in a 2 m straight section of IOTA, was applied to the model. Using MAD-X, single particle tracking is done with the models at various amplitudes and their tunes were calculated.

Amplitude Dependent Tune Shift

Figure 2 is tune diagrams for each of the respective t-strength values. The information shown are the measured tunes, model tunes with and without aperture restriction, and the theoretical small amplitude tune. Table 1 is the corresponded measured tune shift for each case.

<table>
<thead>
<tr>
<th>t-strength</th>
<th>$\Delta Q_x$</th>
<th>$\Delta Q_y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) $t = 0.22$</td>
<td>0.0334±0.0018</td>
<td>0.0245±0.0018</td>
</tr>
<tr>
<td>(b) $t = 0.29$</td>
<td>0.0198±0.0005</td>
<td>0.0216±0.0010</td>
</tr>
<tr>
<td>(c) $t = 0.43$</td>
<td>0.0261±0.0018</td>
<td>0.0530±0.0018</td>
</tr>
</tbody>
</table>

Parameterization of Eq. (3), the tune diagram can be explored with varying nonlinear strength parameter $t$, for small amplitude kicks. This is shown in Fig. 3, where also the explored nonlinear strength values measured at small
amplitude is compared with the corresponding theoretical point.

Table 2 lists the maximum vertical kick values and the maximum amplitude in the center of the nonlinear magnet (NL). This is done for each of the respective t-strength values, comparing measurements to the model and its 6 mm aperture restriction (AR) section. Past the model maximum values, the particles are lost due to the aperture of the nonlinear magnet beam pipe. In which by design, is the smallest restriction of 5.5 mm vertically in the middle of the nonlinear magnet.

**Discussion**

At small amplitude kick, the tune at a nonlinear strength value of \( t = 0.22 \) is within 2.00%. Where as at \( t = 0.29 \) is within 2.37% and at \( t = 0.43 \) is within 8.51% of the theoretical values in Eq. (3).

For \( t = 0.22 \), the maximum measurement matches the model without aperture restriction, unlike other strength parameters. This is due to the beam fast decoherence of 200 turns. In the AR model, particle loss occurs in 300-500 turns at the restricted section. Conversely, at higher t-strength for the AR model, particle loss occurs in the order of 10s of turns and agrees with measurements. The model shows that there is much more dynamical aperture to be explored.

### Table 2: Maximum Values for Amplitude Dependent Tune Shift Measurements at Different t-strength Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Meas.</th>
<th>AR</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t = 0.22 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kick Voltage [kV]</td>
<td>4.80</td>
<td>3.39</td>
<td>4.80</td>
</tr>
<tr>
<td>Kick Angle [mrad]</td>
<td>3.43</td>
<td>2.43</td>
<td>3.43</td>
</tr>
<tr>
<td>NL Center Amplitude [mm]</td>
<td>4.48</td>
<td>3.18</td>
<td>4.48</td>
</tr>
<tr>
<td>( t = 0.29 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kick Voltage [kV]</td>
<td>3.00</td>
<td>4.68</td>
<td>5.18</td>
</tr>
<tr>
<td>Kick Angle [mrad]</td>
<td>2.14</td>
<td>2.28</td>
<td>3.71</td>
</tr>
<tr>
<td>NL Center Amplitude [mm]</td>
<td>2.98</td>
<td>3.18</td>
<td>5.19</td>
</tr>
<tr>
<td>( t = 0.43 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kick Voltage [kV]</td>
<td>2.40</td>
<td>2.40</td>
<td>4.00</td>
</tr>
<tr>
<td>Kick Angle [mrad]</td>
<td>1.71</td>
<td>1.71</td>
<td>2.86</td>
</tr>
<tr>
<td>NL Center Amplitude [mm]</td>
<td>3.65</td>
<td>3.65</td>
<td>5.19</td>
</tr>
</tbody>
</table>

For the case of \( t = 0.43 \), Figure 4 shows how the tune shifts with respect to vertical kick angle. One should note that in the model, varied horizontal kicks were also applied, where as in the measurements, a single constant horizontal kick was applied. It can be clearly seen that the experiment was limited by the aperture restriction, where the theoretical maximum tune shift at this strength would be \( \Delta Q_y = 0.085 \).

During the first run there were limitations during the experiment. For one, orbit bump measurements were made through a straight section in IOTA. This revealed an unexpected restriction of 6 mm in the section. Physical measurements of the beam pipe confirmed that this is indeed the case. This mechanical restriction did not allow for higher amplitude kicks of the beam. A replacement for the beam pipe is currently underway. The electron beam energy was also limited to 100 MeV. Consequently this lead to large energy spread due to intrabeam scattering (IBS), leading to beam loss while kicking the beam in IOTA. Minimum beam loss occurred at lower beam current (<1 mA), however, at these low currents there was excess noise in the BPM signal. During the second run, a 'pencil sized', 150 MeV electron beam will be injected into IOTA, to mitigate IBS.

Further simulation shows that at a nonlinear strength of \( t = 0.48 \), a tune shift of \( \Delta Q_y \approx 0.11 \) can be achieved.

**CONCLUSION**

During the first experimental run, the measurements are in good agreement with MAD-X simulations. The largest observed tune shift was \( \Delta Q_y = 0.053 \pm 0.0018 \), at a strength value of \( t = 0.43 \). Further simulation at larger t-strengths of 0.48 shows a tune shift of \( \Delta Q_y \approx 0.11 \). The outlook for the second run in Fall 2019 looks promising, as work on fixing the mechanical aperture restriction is underway. As well as re-commissioning the cryomodule in FAST in achieving 150 MeV, pencil-like, electron beam. In addition, IOTA lattice will be realigned and additional sextupoles will be installed, allowing for reduction in energy spread.

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REFERENCES


