

# Accelerating Feldman-Cousins for NOvA using NERSC Supercomputers

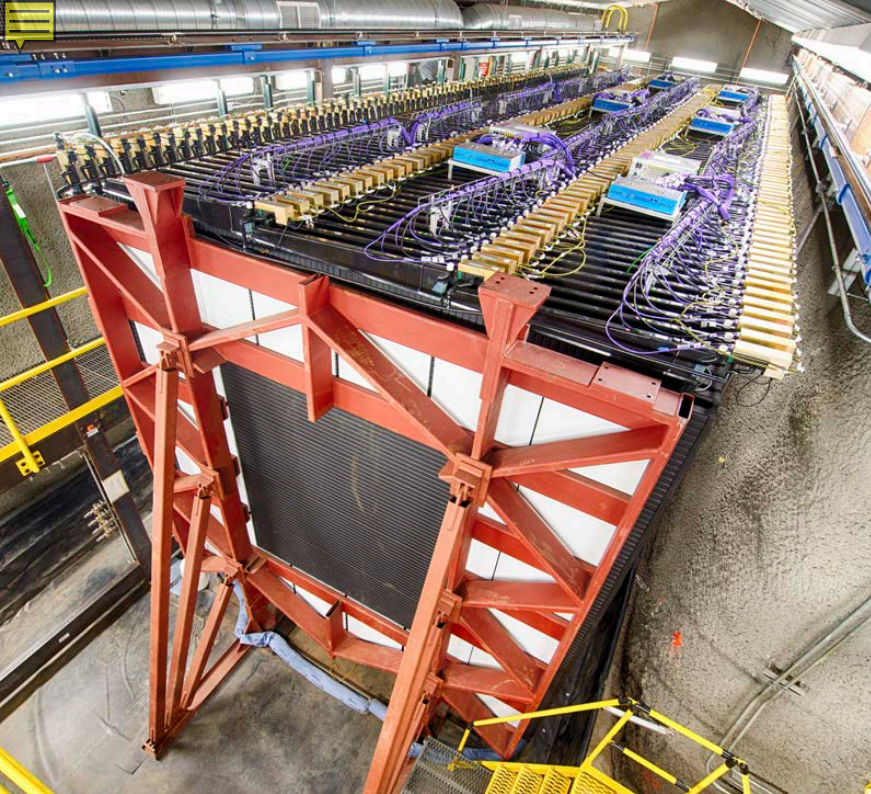
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Derek Doyle

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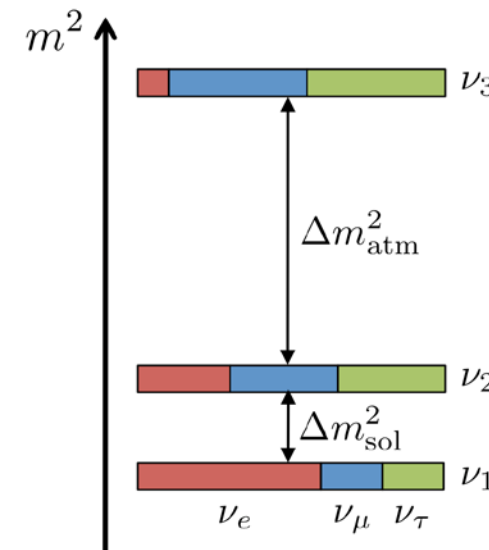
# NOvA Oscillation Analysis

- Measure parameters governing neutrino oscillations through
  - $\nu_\mu \rightarrow \nu_\mu$  and  $\nu_\mu \rightarrow \nu_e$
  - $\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu$  and  $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$
- Look for CP Violation
- Determine Neutrino Mass Ordering

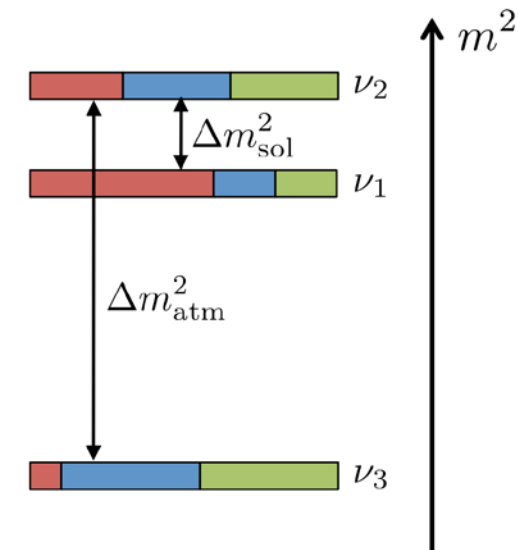


$$\begin{bmatrix} \text{Yellow} \\ \text{Orange} \\ \text{Red} \end{bmatrix} = R(\theta_{23}) \cdot R(\theta_{13}, \delta_{CP}) \cdot R(\theta_{12}) \begin{bmatrix} \text{Yellow} \\ \text{Orange} \\ \text{Red} \end{bmatrix}$$

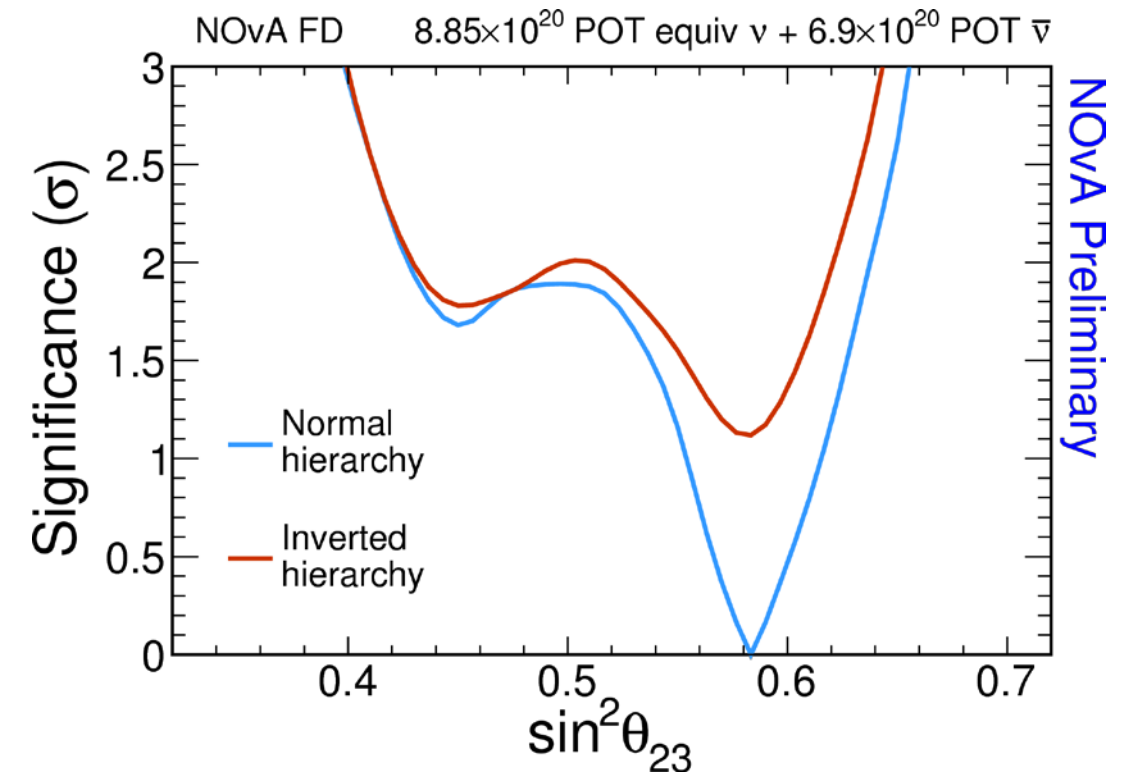
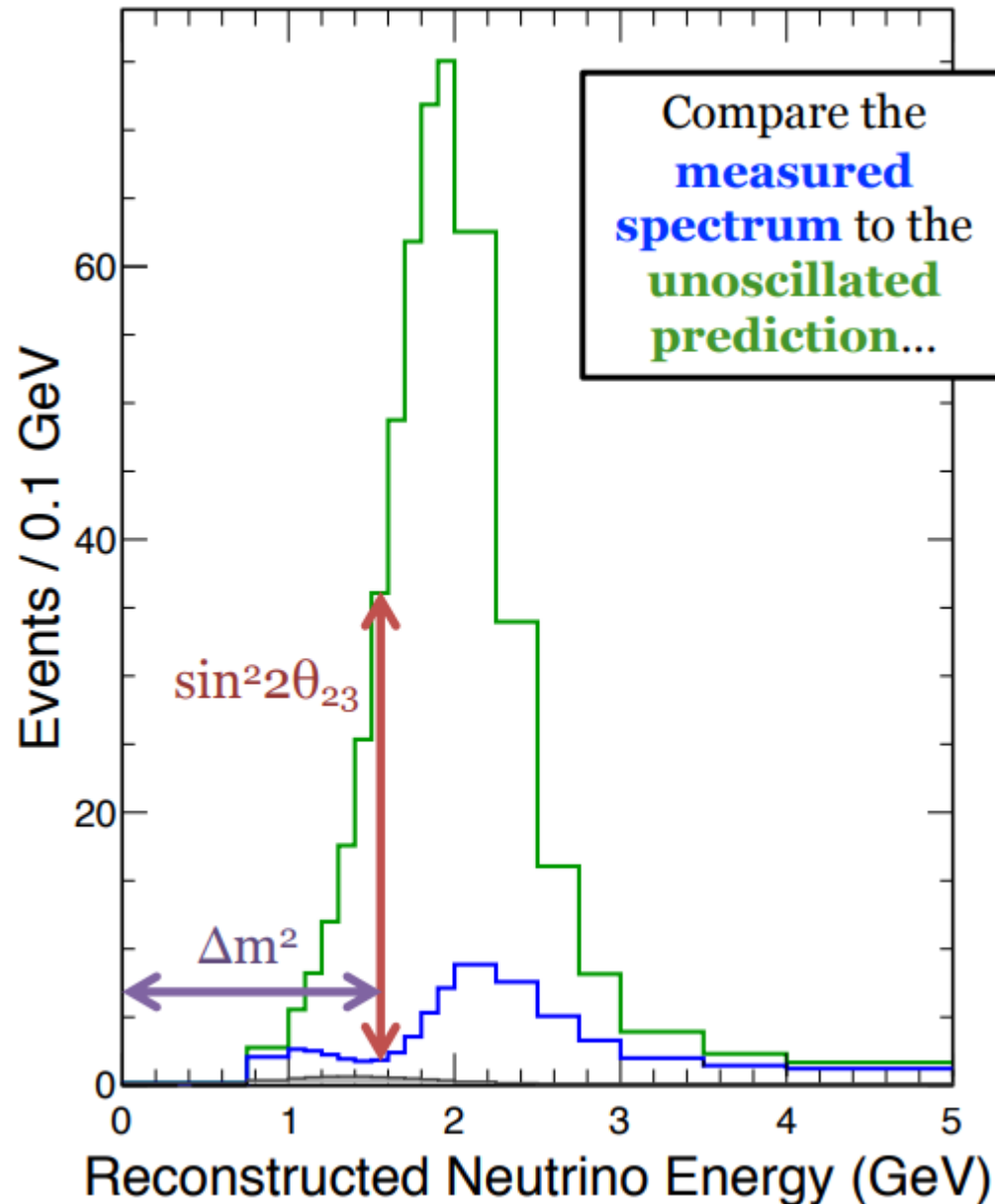
normal hierarchy (NH)



inverted hierarchy (IH)



# Measuring Oscillation Parameters



- Comparing the data to our predictions, estimate the true parameters, exclude other regions of parameter space with some degree of certainty, conventionally in terms of  $\sigma$

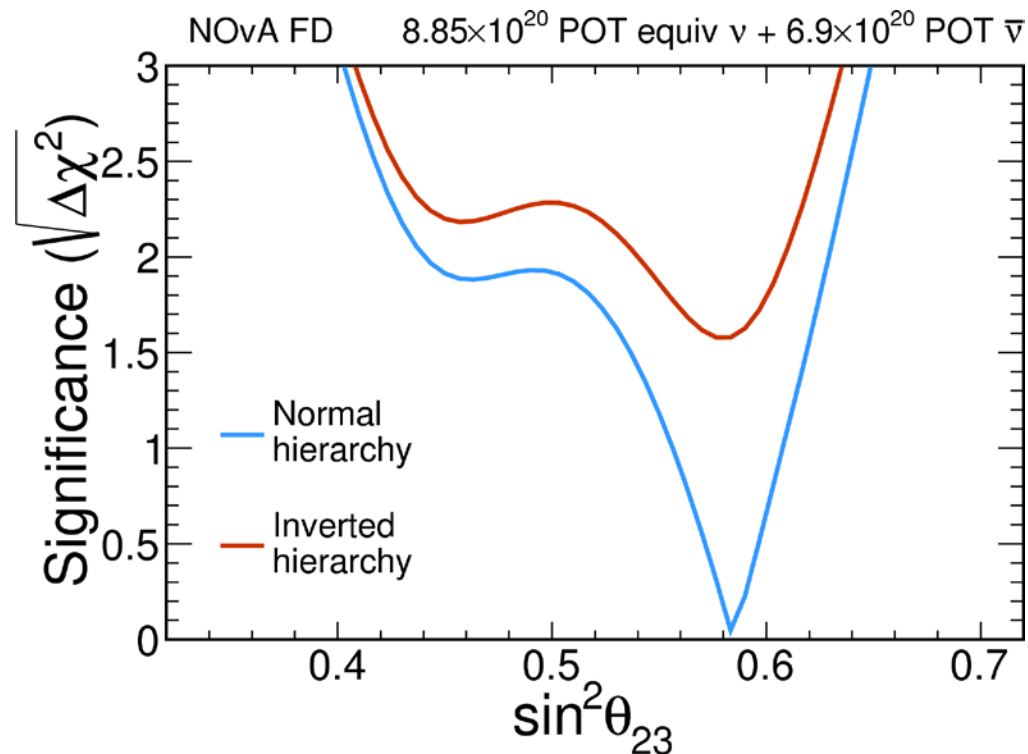


# Parameter Estimation

- Goal: Determine the likelihood that  $\vec{\theta}$  are the true parameters
  - Find  $\vec{\theta}$  that maximize the Likelihood function:  $\mathcal{L}(\vec{\theta}_{best})$

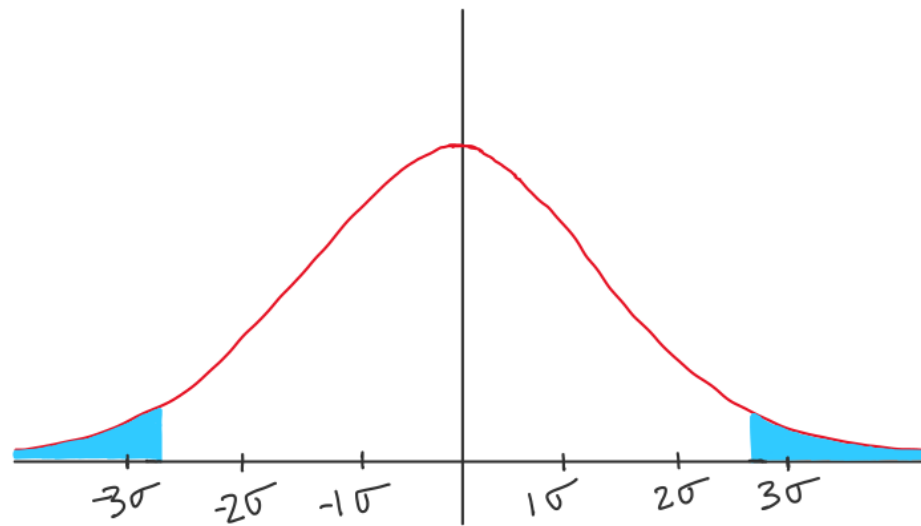
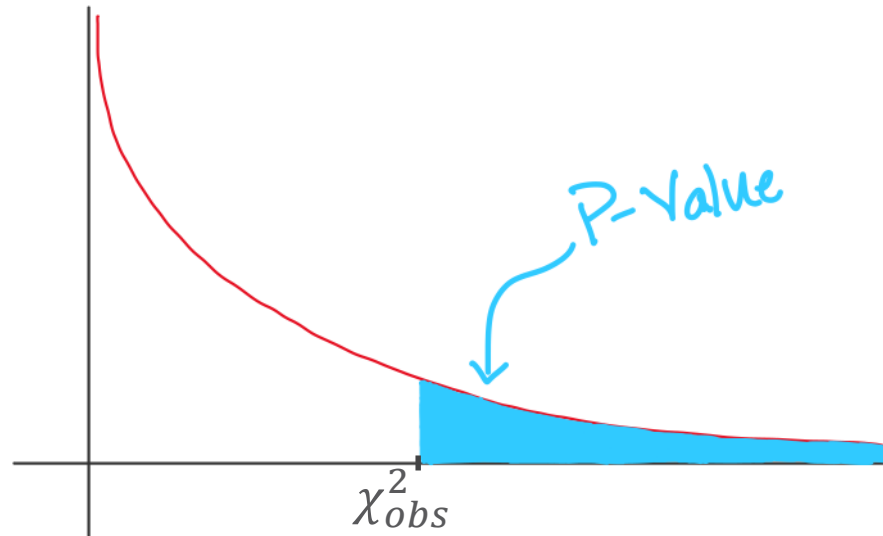
$$\chi^2(\vec{\theta}) \equiv -2 \log \mathcal{L}(\vec{\theta}) = 2 \sum_i^{bins} \left[ e_i(\vec{\theta}) - o_i + o_i \log \frac{o_i}{e_i(\vec{\theta})} \right]$$

$\vec{\theta}$ : Set of Oscillation parameters  
 $o_i$ : Observed data  
 $e_i(\vec{\theta})$ : Expected observation



- To measure the significance with which we exclude other parameters,  $\vec{\theta}$ , as the true parameters, we calculate  $\chi^2(\vec{\theta})$  and construct the test statistic,  $\Delta\chi^2 = \chi^2(\vec{\theta}) - \chi^2(\vec{\theta}_{best})$
- Conveniently, this distribution converges to a  $\chi^2$  distribution under certain assumption
- Moreover, under these assumptions,  $\sigma = \sqrt{\Delta\chi^2}$

# $\chi^2$ Distribution

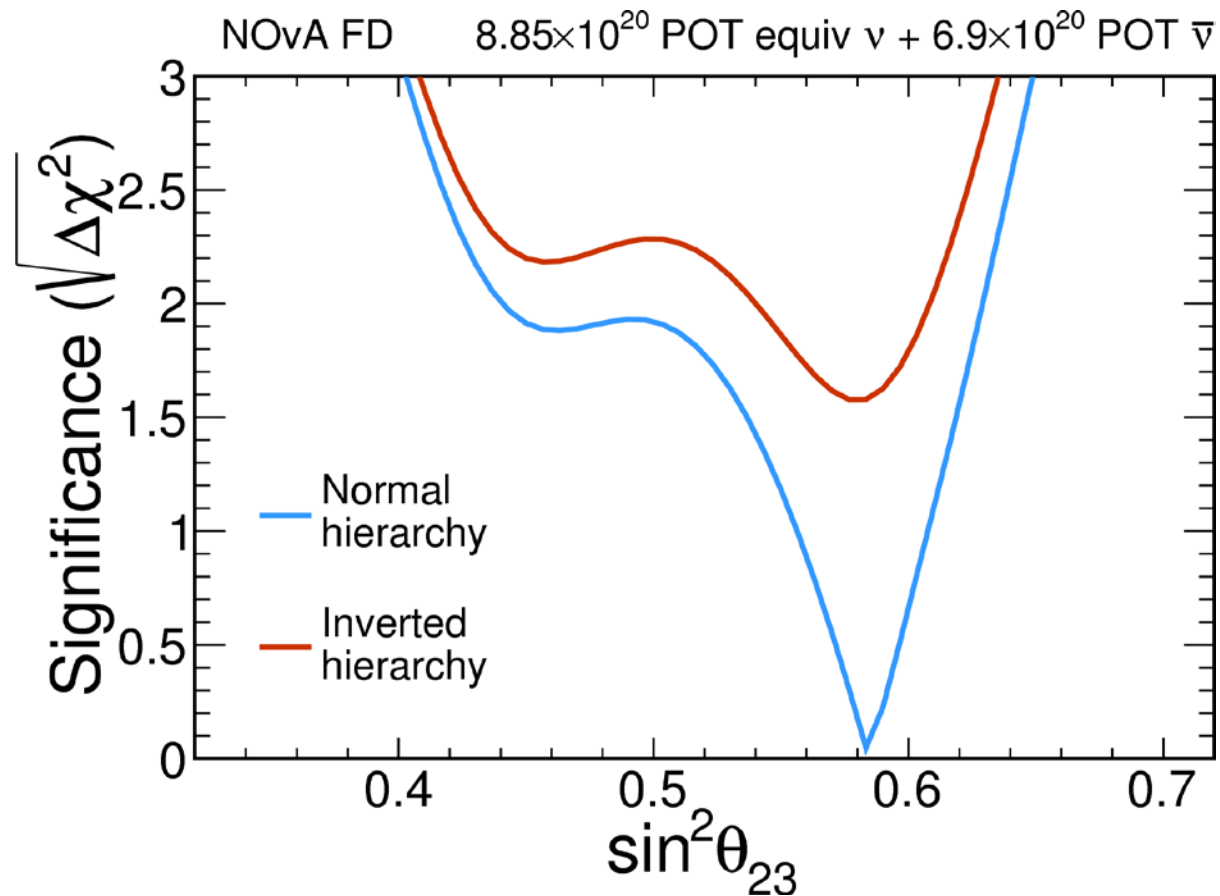


- Often used for determining goodness of fit
- Repeating experiment may result in different observations
- Want to know the probability of finding a  $\chi^2$
- **P-Value:** probability of finding a  $\chi^2$  at least as large as than the one you measure
- Tells you how probable it was to measure a  $\chi^2$  by accident

$$p\text{-value} = \int_{\chi_{obs}^2}^{\infty} f(\chi) d\chi$$

- **Critical Value:** value of test statistic corresponding to a given p-value
- Report a significance in terms of the critical value of a standard normal distribution corresponding to that p-value

# Case Study: 1D Profile



- At each point in parameter space, calculate a  $\Delta\chi^2(\vec{\theta})$

$$\Delta\chi^2 = \chi^2(\vec{\theta}) - \chi^2(\vec{\theta}_{best})$$

- We end up with the trajectory that minimizes  $\Delta\chi^2$  along one axis
  - Profile over other parameters
- Can we use a standard  $\chi^2$ ?

# Wilks' theorem

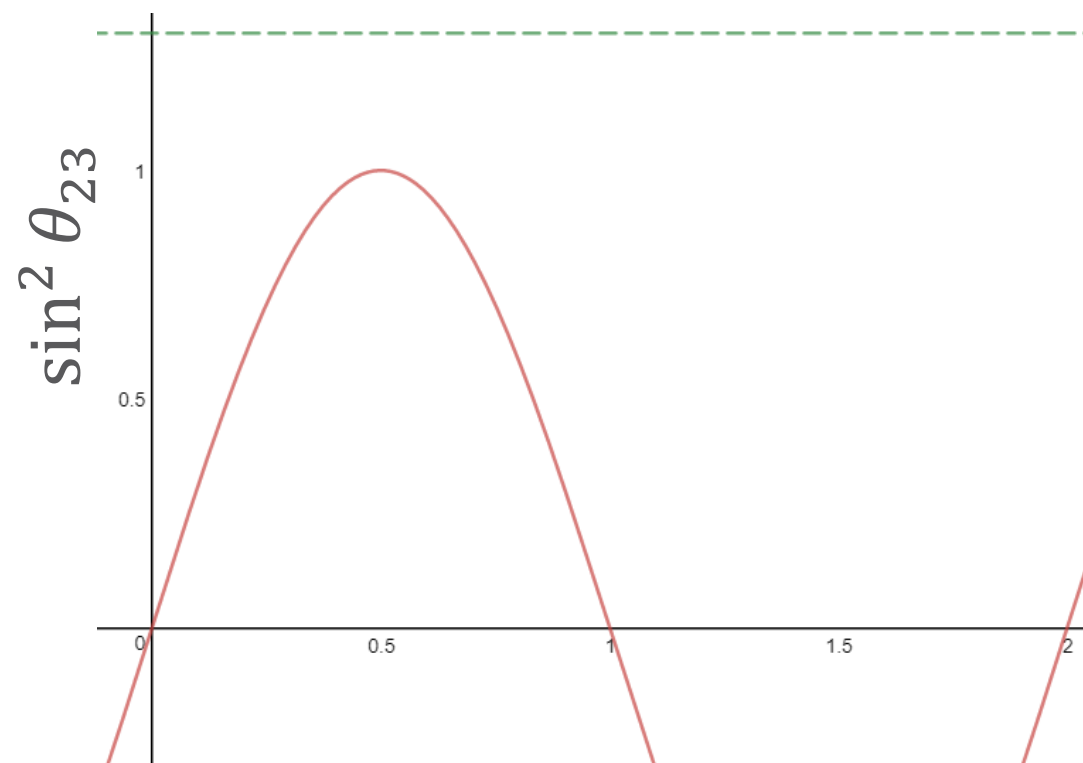
- A  $\Delta\chi^2$  distribution converges to a standard  $\chi^2$  if two conditions are met:
  1. Large Statistics
  2. Parameters are sufficiently far from physical boundaries

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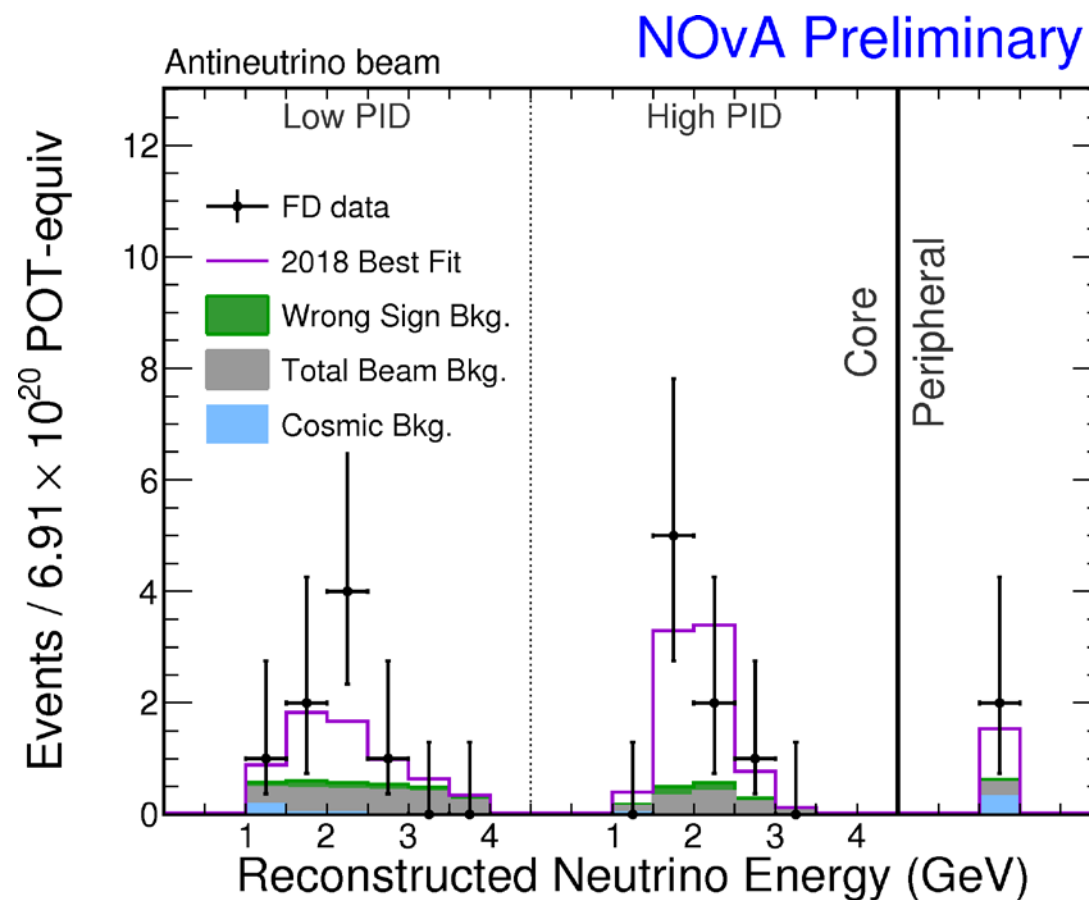
# Wilks' theorem

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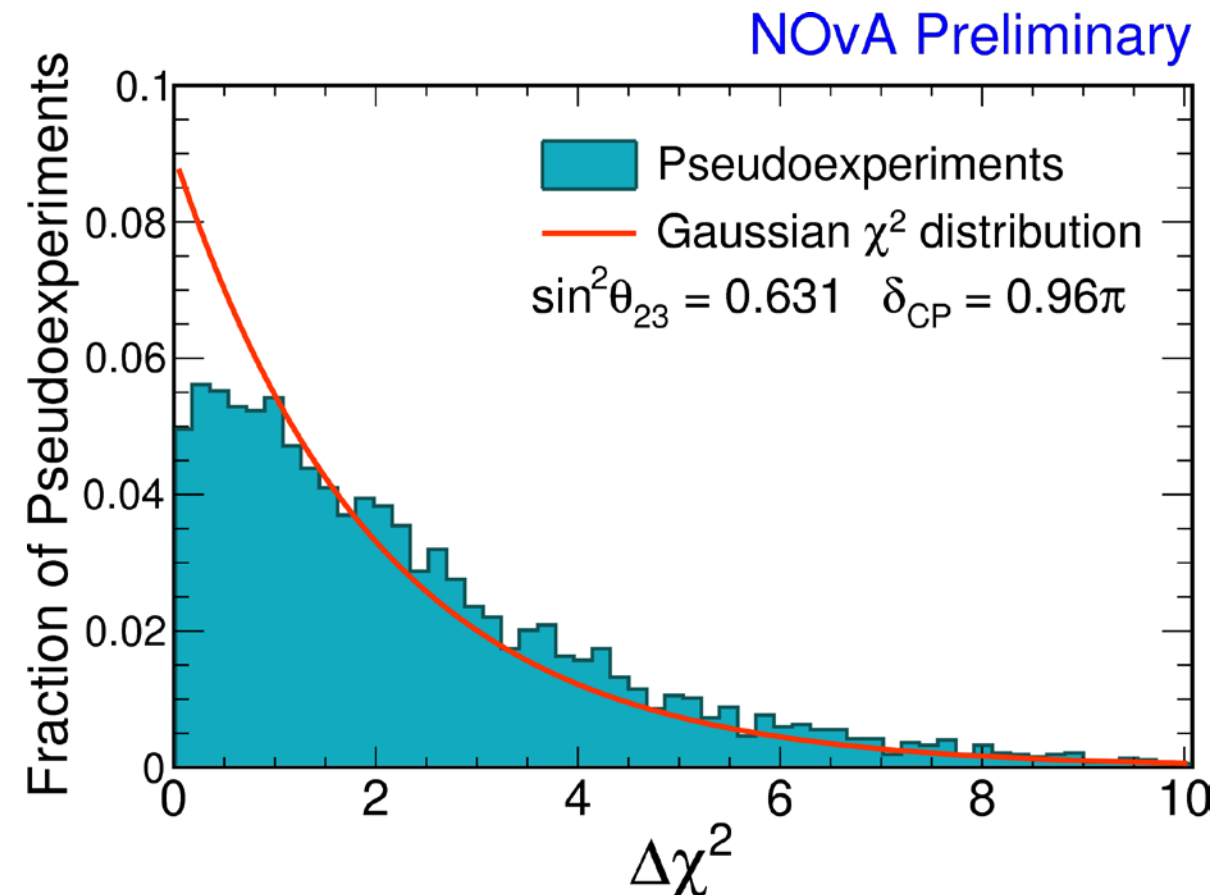
Total $\bar{\nu}_e$ Observed	18
Total Prediction	15.9
Wrong-sign	1.1
Beam Bkgd.	3.5
Cosmic Bkgd.	0.7
Total Bkgd.	5.3





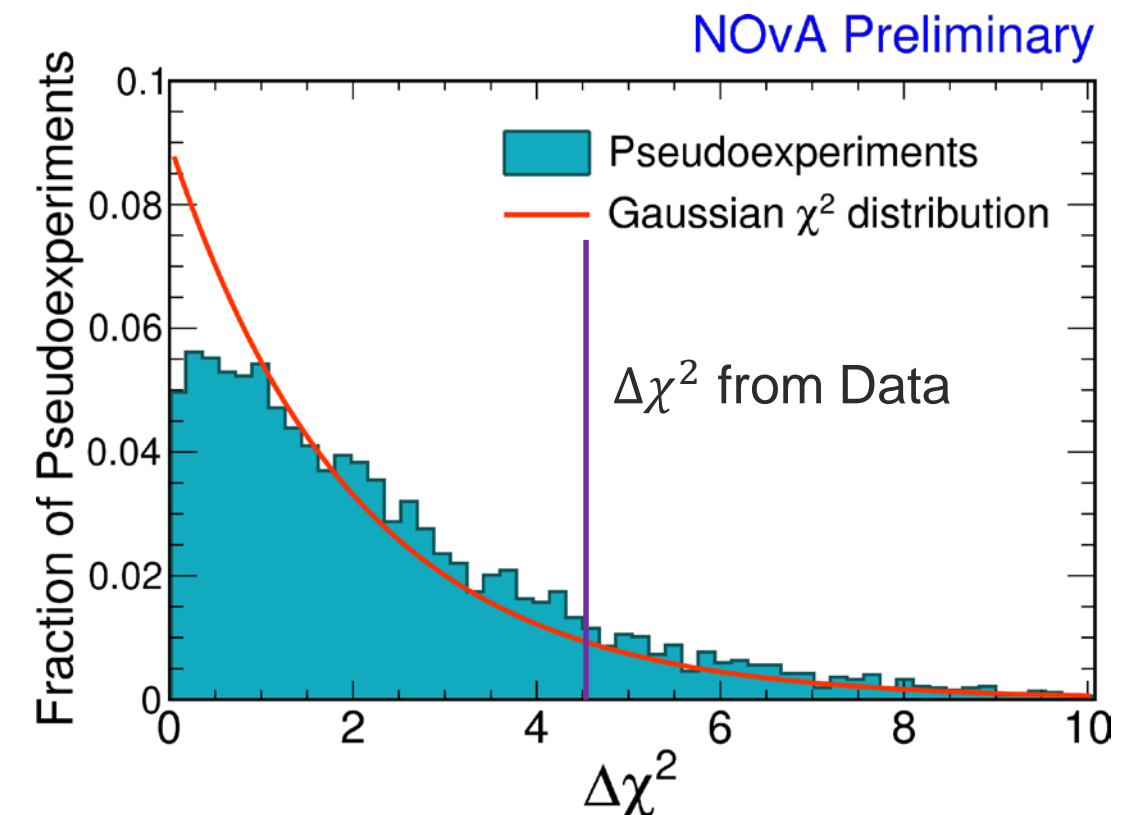
# Feldman-Cousins Procedure

- Wilks' result is a convenience if his conditions are met
- If we could find a true  $\Delta\chi^2$  distribution, we could calculate a p-value from it just like any other distribution function and quote a true significance
- So let's do that... empirically
- Recall a p-value is like proportion of your test statistic that is distributed beyond a certain value
- So if we can find a similar proportion of an empirical distribution, we can quote a significance



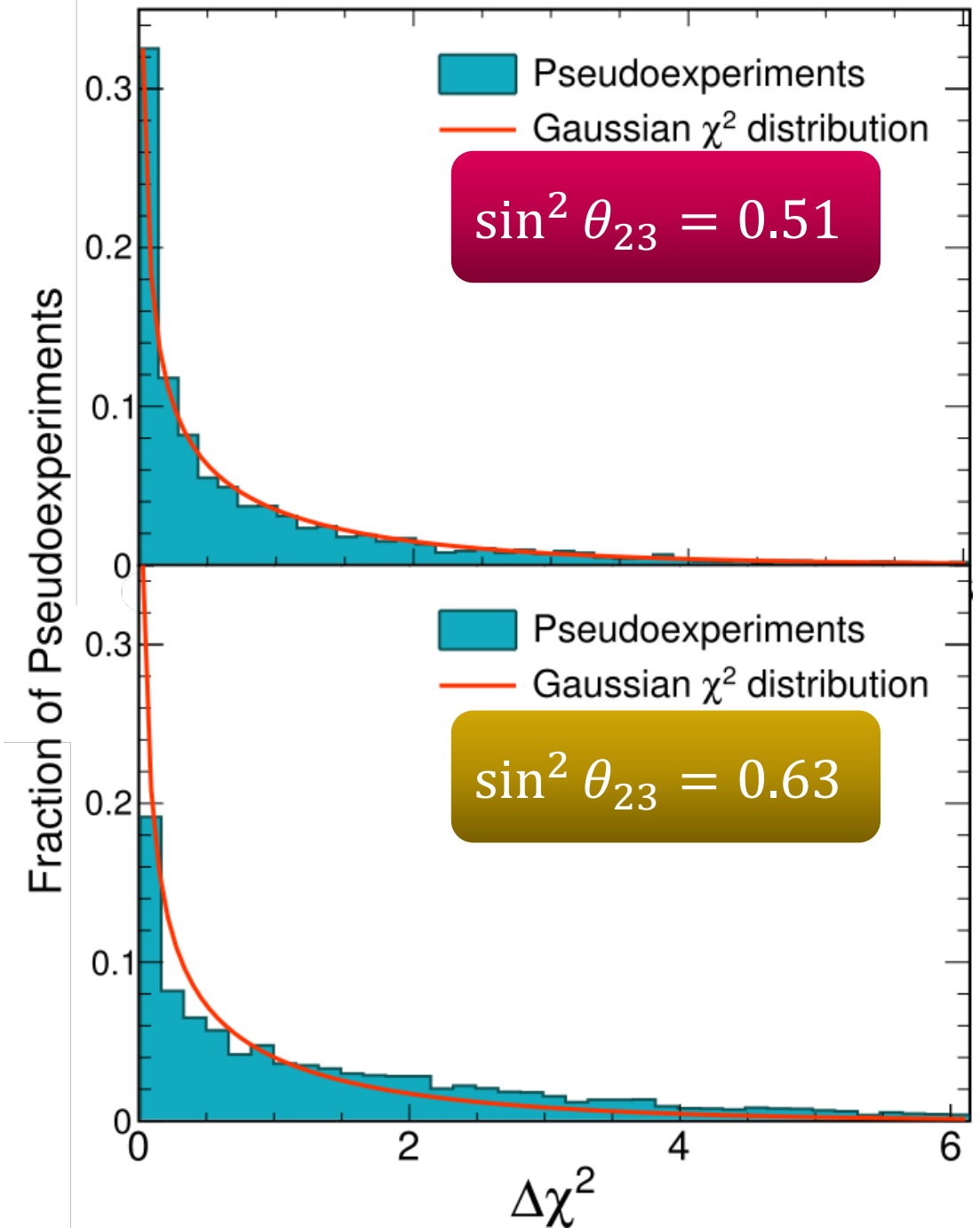
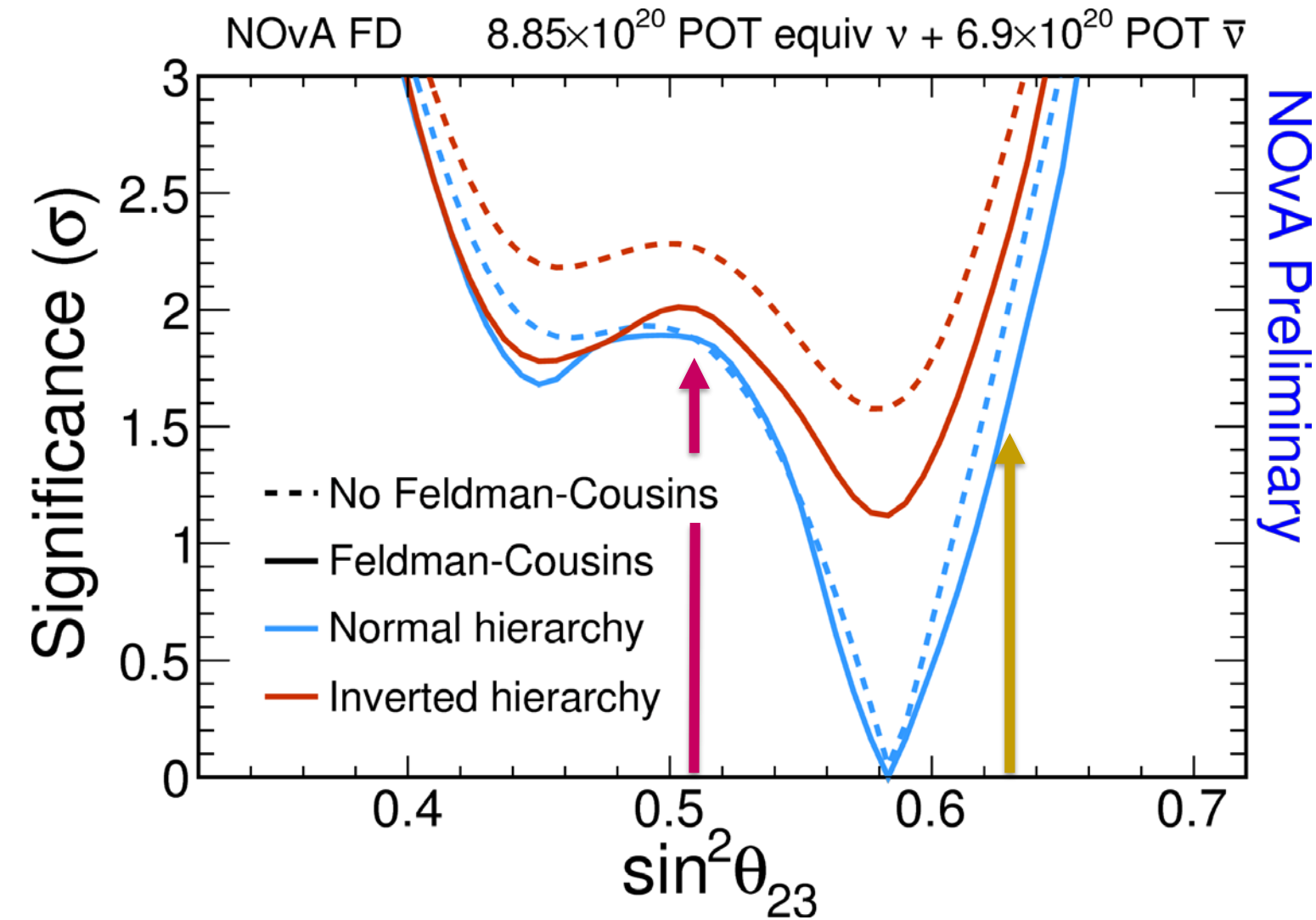
# Feldman-Cousins Procedure

- At a given point in parameter space we want to know, “does our prediction at a set of test parameters,  $\vec{\theta}_{test}$ , agree with our data?”
    - We need a  $\Delta\chi^2$  distribution where this is true in order to reject our hypothesis
    - i.e, In a world where the true parameters are  $\vec{\theta}_{test}$ , given statistical fluctuations, how would the  $\Delta\chi^2$  test statistic be distributed?
1. Generate data at  $\theta_{test}$  that is Poisson-fluctuated (pseudoexperiment) in order to get value of  $\Delta\chi^2$ 
    - a. Do this *many* times to build a distribution
  2. Find the proportion of pseudoexperiments above observed  $\Delta\chi^2$
  3. Quote a significance based off that p-value



# Correcting a profile

NOvA Preliminary



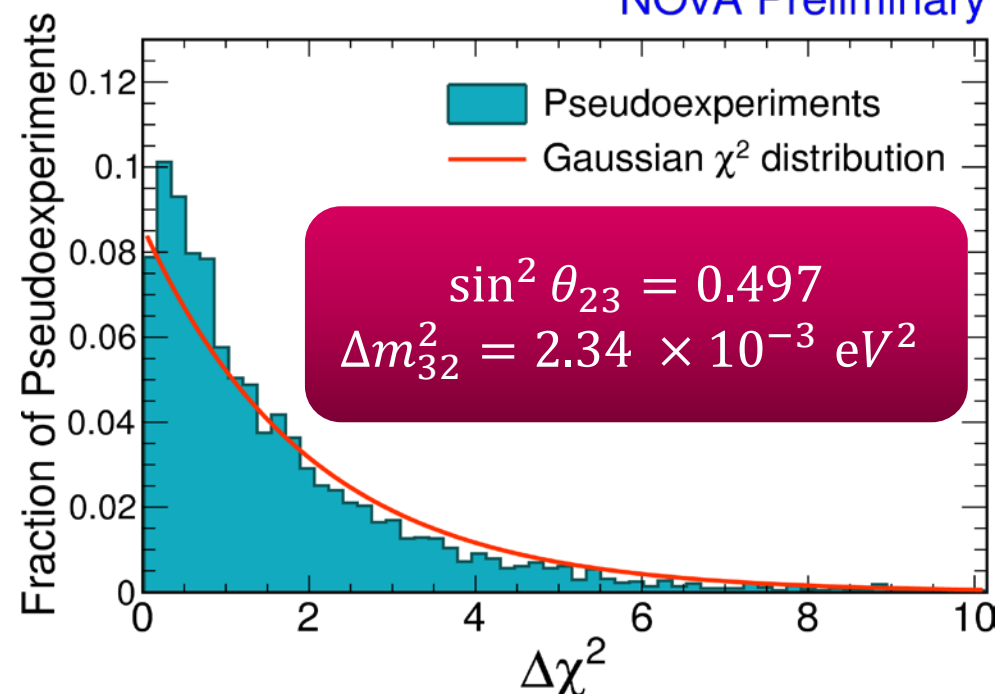
# Correcting a contour

- The same thing can be done in 2D
  - Like 1D profiles: Now we find the surface in parameter space that minimizes  $\Delta\chi^2$
  - Draw lines of constant significance (p-value)

- Left-skewed distribution:

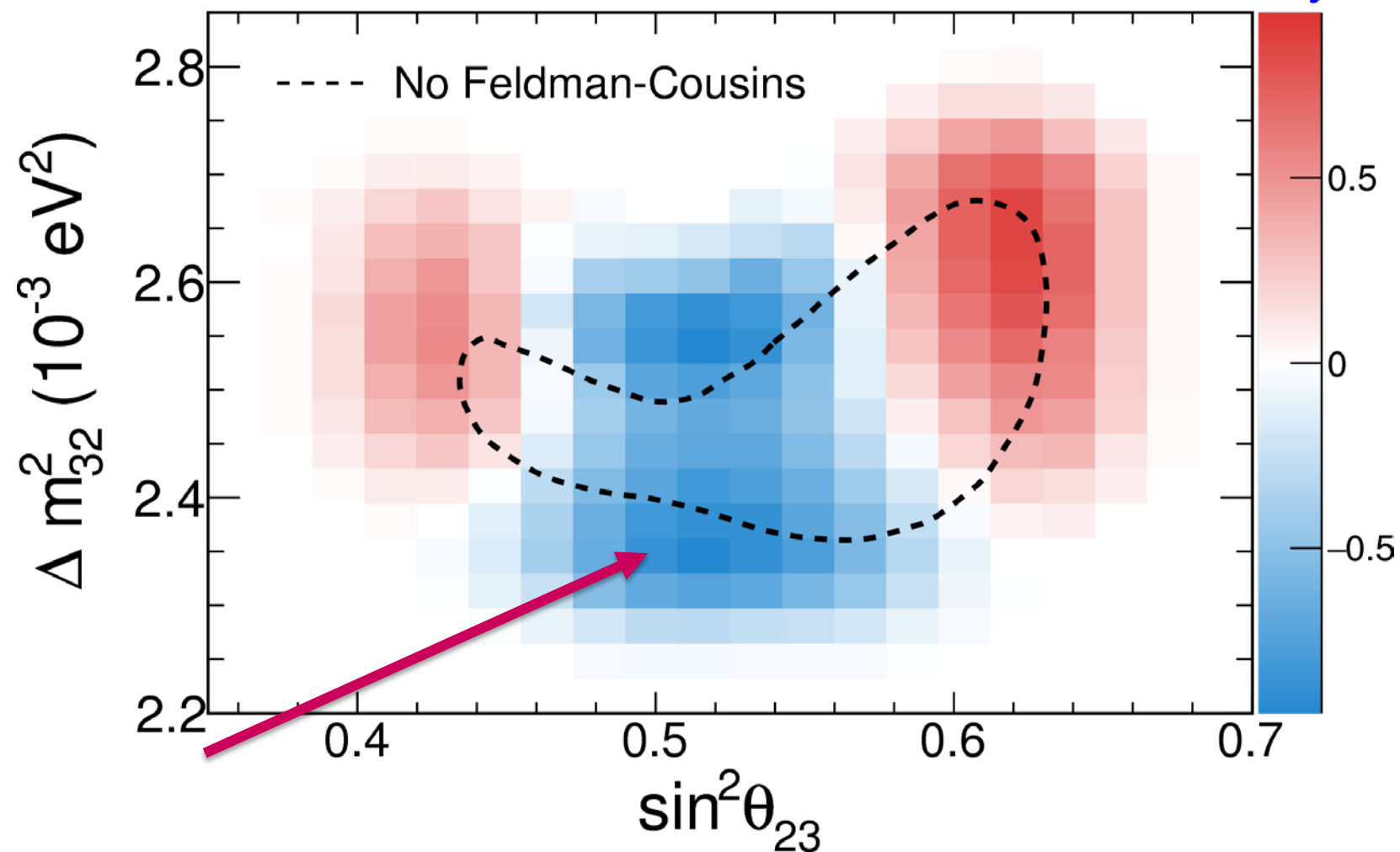
- Decreases  $\Delta\chi^2_{crit}$
- Increases ability to exclude

NOvA Preliminary



Difference between empirical and gaussian  $\Delta\chi^2_{crit}$  for a 90% Confidence Interval

NOvA Preliminary



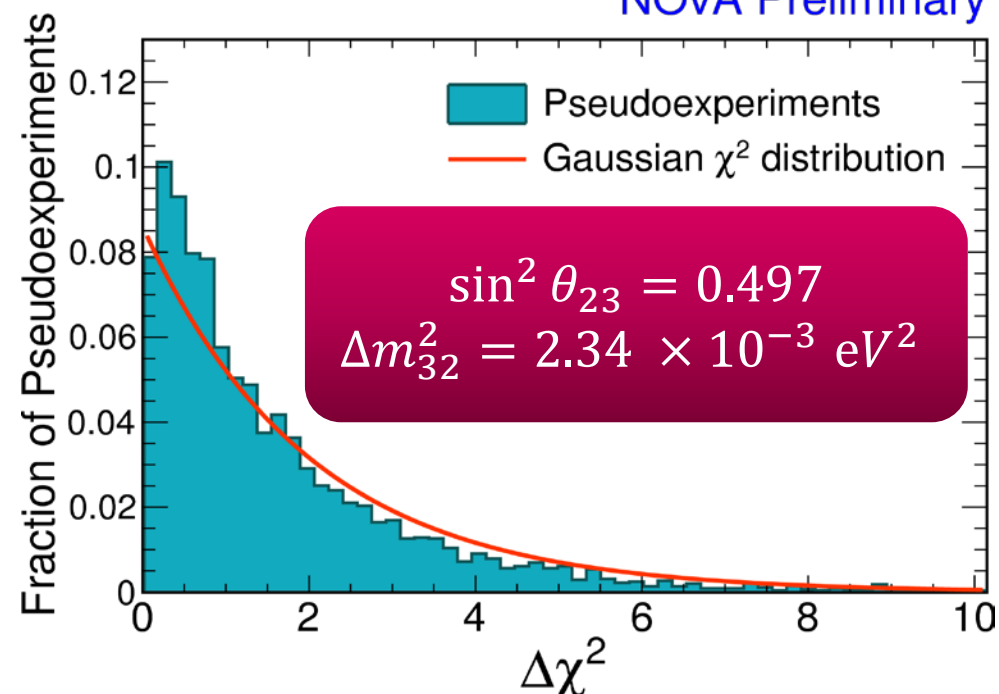
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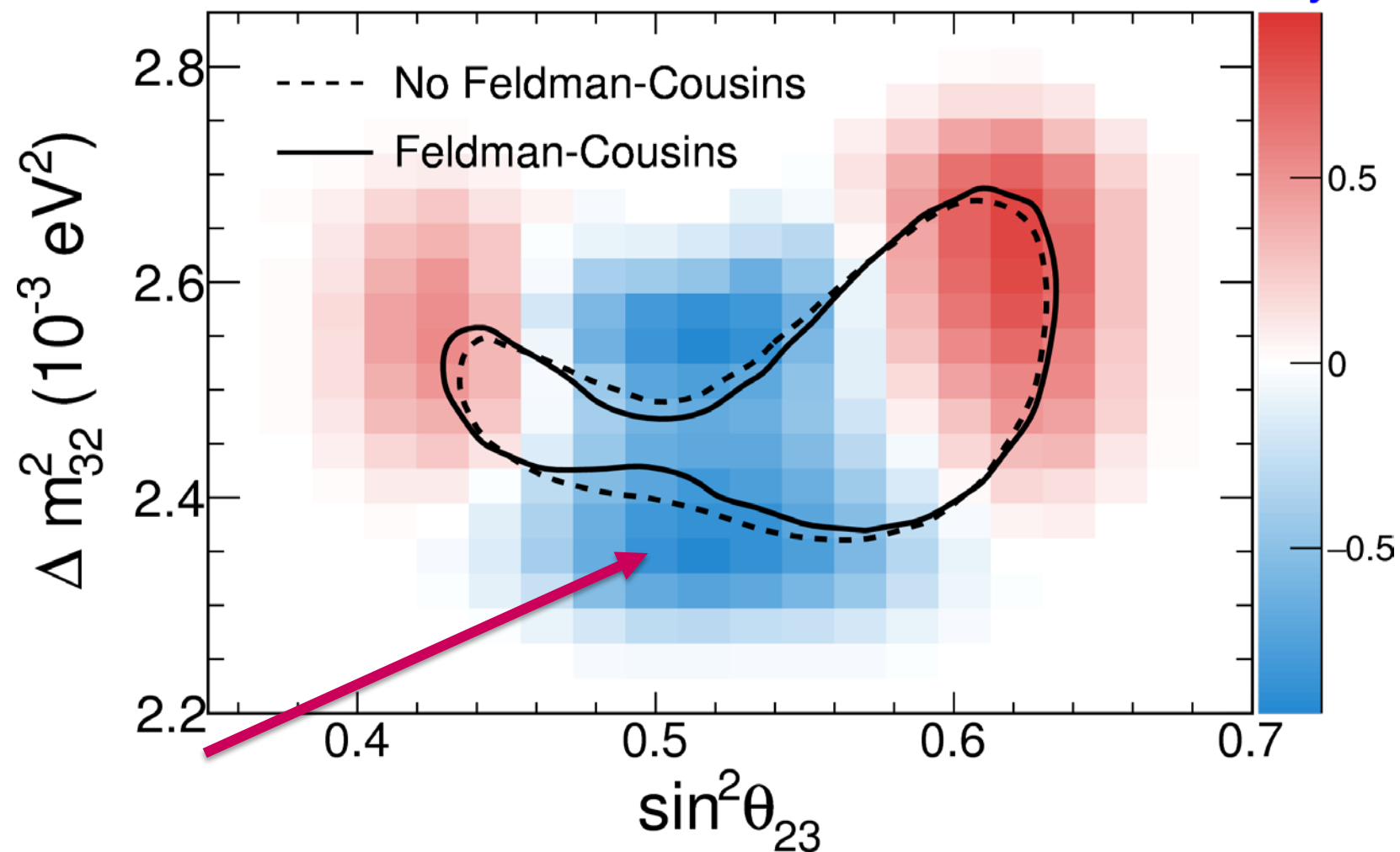
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NOvA Preliminary



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NOvA Preliminary





# Computational considerations

- We need to generate  $\Delta\chi^2$  distributions at each point in parameter space that we're sampling
  - Profiles: 1,210 bins
  - Contours: 471 bins
- In each bin, we need at least 3,000 pseudoexperiments in  $\Delta\chi^2$  to generate an accurate empirical distribution
- In previous years, this has been done on the FermiGrid with results in ~4 weeks
- With the addition of antineutrino data and a longer list of systematics, the FermiGrid was no longer an option

Required No. Bins	Required No. Pseudoexperiments
1,681	6,724,000

# NERSC – Lawrence Berkeley National Lab

(National Energy Research Scientific Computing)

## CORI – Phase I

- Intel "Haswell" at 2.3 GHz
- Total Cores: 76,416
- Capable of 2.81 PFlops
- 32 cores share 128 GB memory
- *Each core can support 2 FC jobs*

## CORI – Phase II

- Intel "Knights Landing" (KNL) at 1.4 GHz
- Total Cores: 658,784
- Capable of 29.5 PFlops
- 68 cores share 96 GB memory
- *Each core can support 1 FC job*



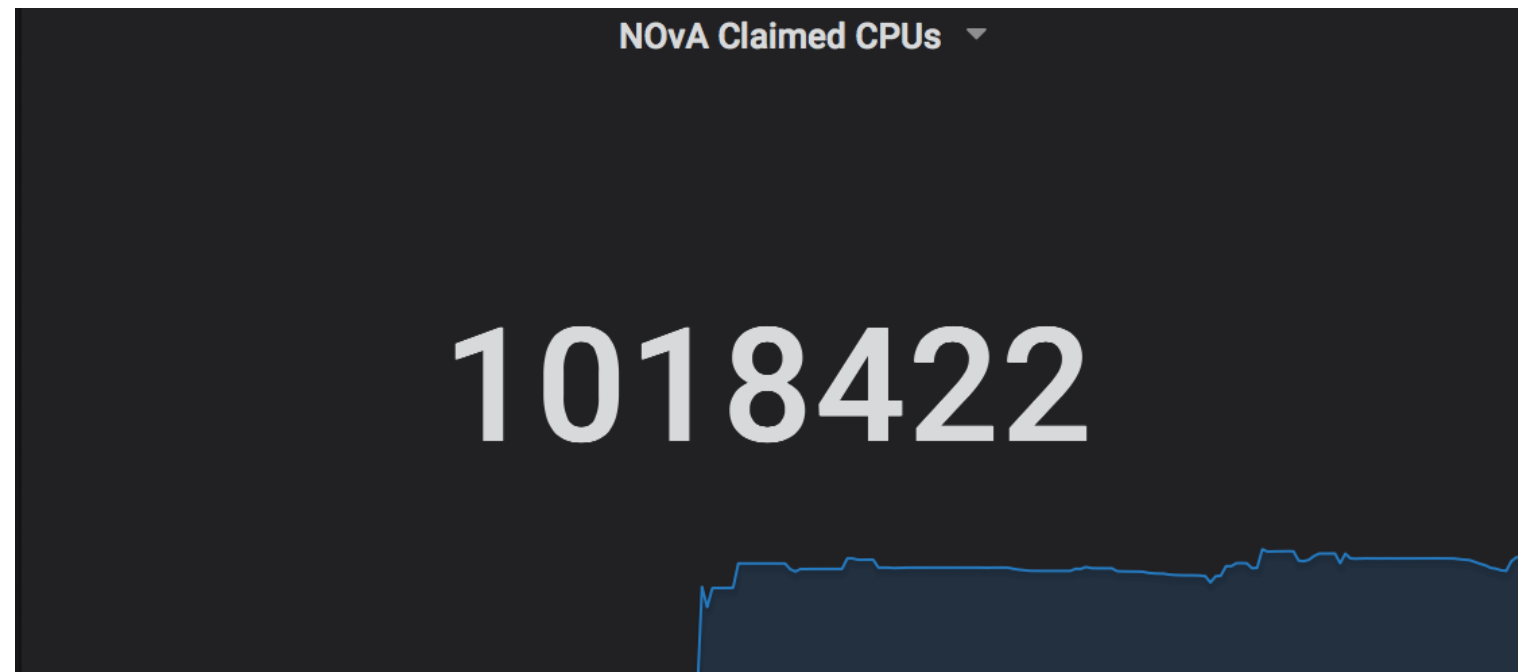
# The NERSC Advantage

## First Run

- Peaked over 1 million cores
  - Largest Condor pool ever!
- Noticed the fits were misbehaving due to added complexity
- *NERSC allowed us to correct for this unforeseen complication and quickly turn around results*

## Second Run

- Peaked ~700,000 cores
  - Second Largest Condor pool ever!
- Over 9 million total points
- 36 hours



# The NERSC Advantage

## 2017 $\nu$ Analysis\*

- Fitting 5 histograms
- 4 profiles
- 4 contours
- ~ 4 weeks to completion (FermiGrid)
- 2 minutes per  $\Delta\chi^2$  (NERSC)
- Reproduced on NERSC in a few hours

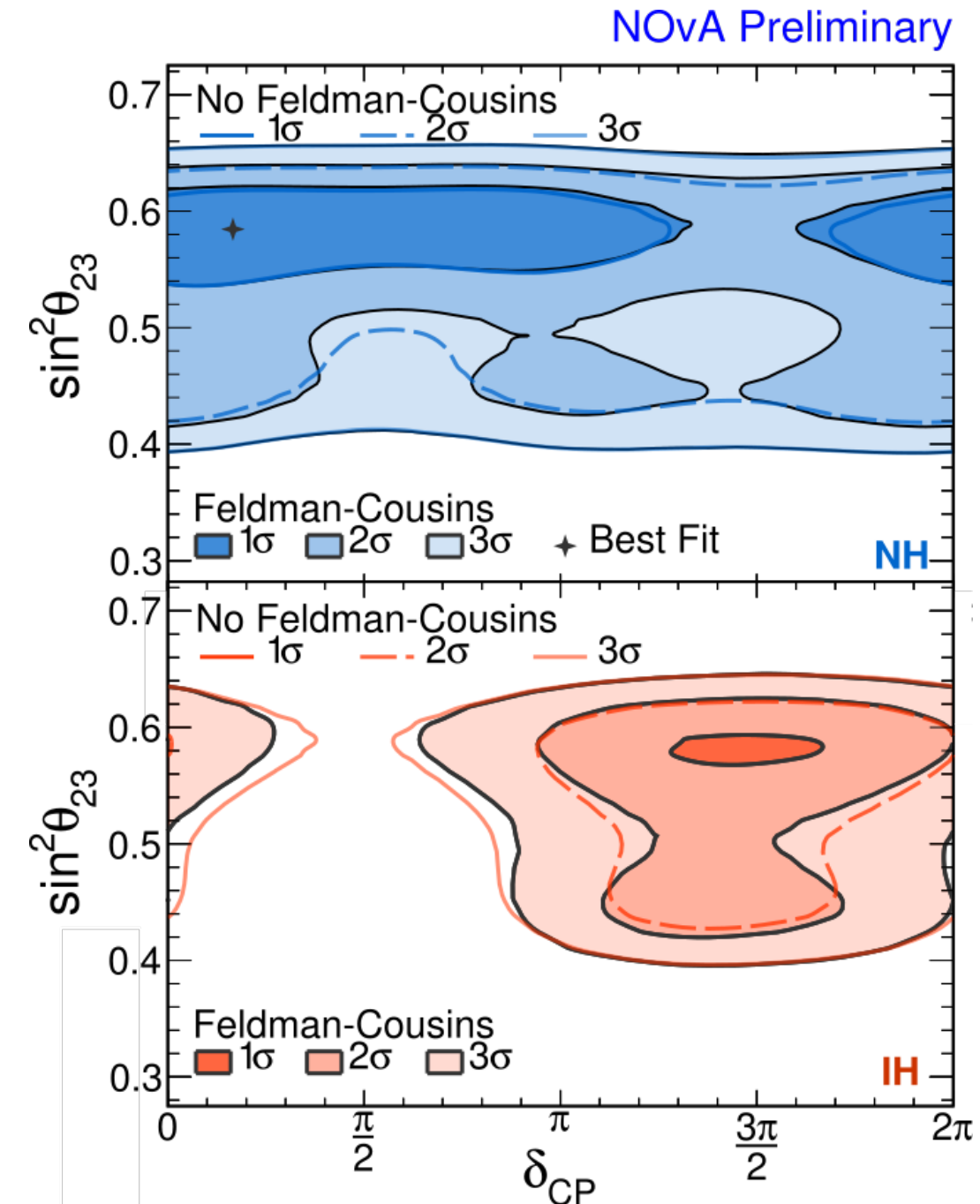
## 2018 $\bar{\nu} + \nu$ Analysis

- Fitting 10 histograms
- 10 profiles
- 4 contours
- 36 hours to completion (NERSC)
- 10 minutes per  $\Delta\chi^2$
- 36 hours to completion (NERSC)
- ~ 5 months on FermiGrid

\* arXiv:1806.00096 [hep-ex]

# Results and Conclusions

- Feldman-Cousins Corrections have a large effect on our results
- Low statistics and physical boundaries require an empirical approach to stating a significance
- Part of a larger project: HEP Data Analytics on HPC
  - Pythia8 and detector simulation tuning for LHC experiments
  - Improve performance and precision of neutrino cross-section analyses
- We hope to develop tools made available to all members of the HEP community in achieving similar advances







# Thank you



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**ENERGY**

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Science



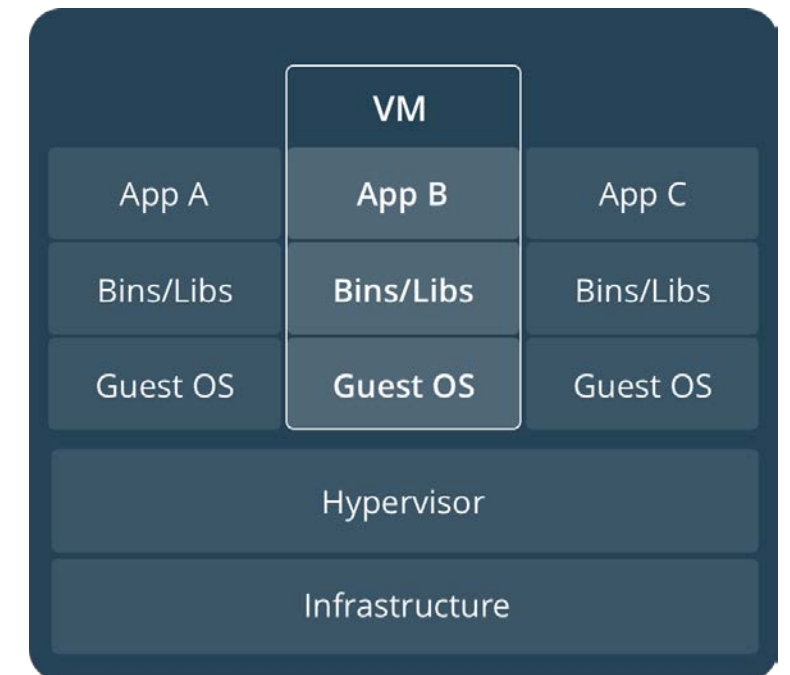
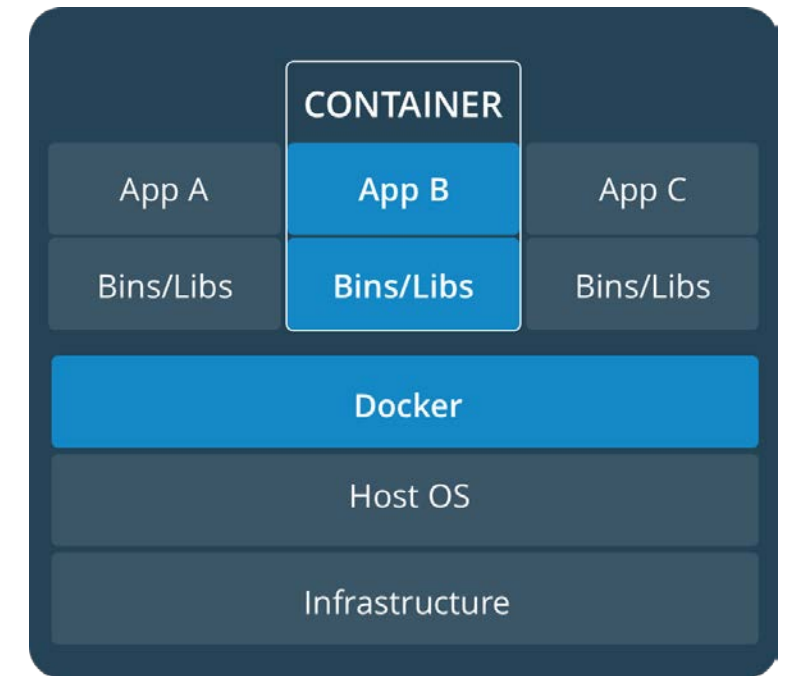
National Energy Research  
Scientific Computing Center





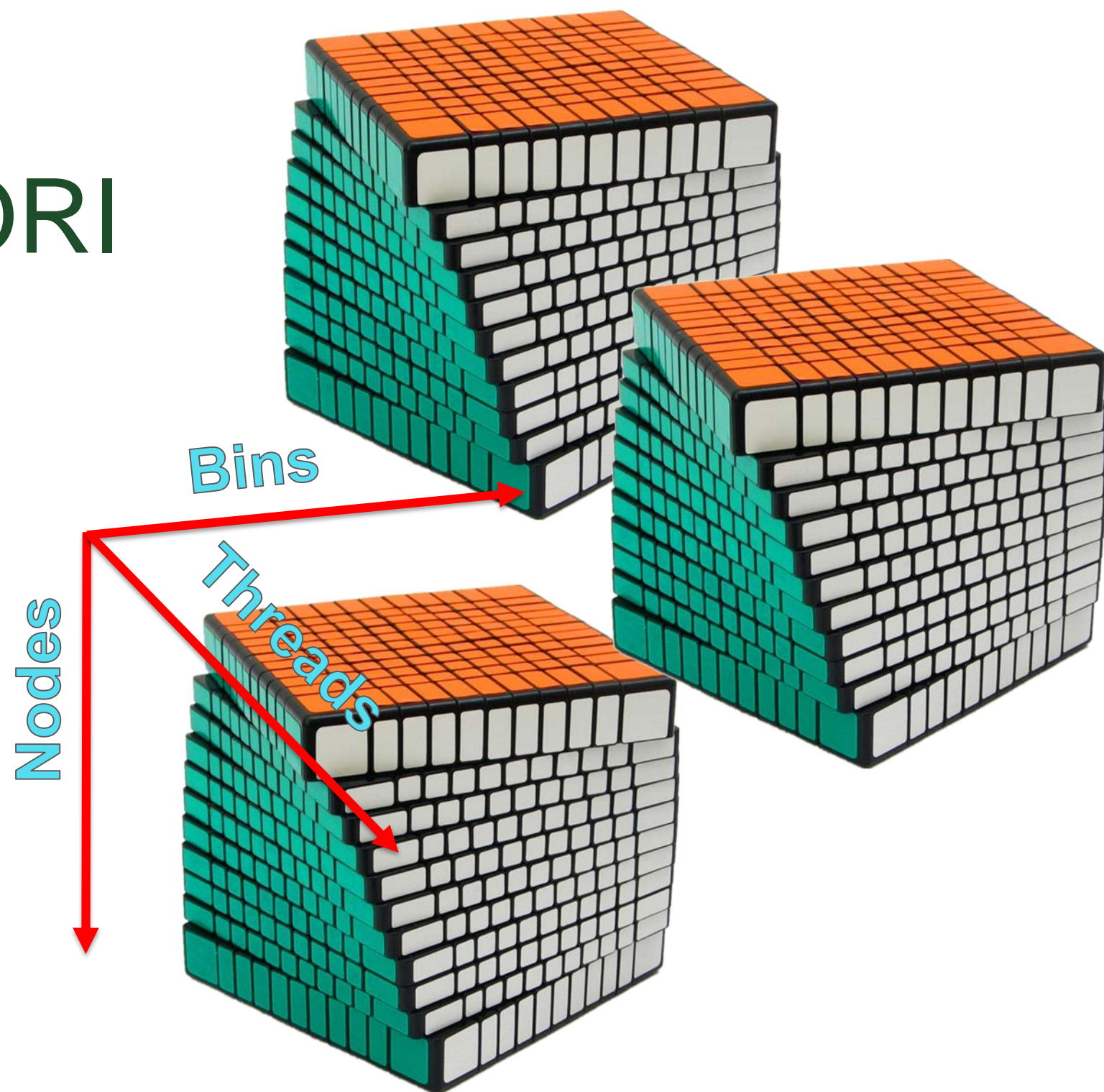
# How we run on CORI

- Docker Images: Like a Virtual Machine, but has a direct line to the machine's kernel
- What is in our image:
  - Slf6.7
  - NOvASoft
  - Spectra/inputs to the FC root macro
- Jobs submitted to CORI mount the image
  - Each job can run a separate image instance, called a *container*, on available threads



# How we run on CORI

- Submit to CORI like a giant Rubix Cube
  - Each compute node has a number of threads that each run a separate instance of the macro
  - Each macro runs over all bins few FC points in each bin
  - Scale by requesting a large number of nodes
- Wait in regular queue: ~ 3 days for big jobs
- Make a reservation: instant access to requested resources



# Inverted Hierarchy Rejection

- $H_o: \Delta m_{32}^2 < 0$
- Need a distribution where this is true
- Profile out dependencies on the value of  $\Delta m_{32}^2$
- Let  $\Delta\chi^2 = 0$  if  $\vec{\theta}$  is in Inverted Hierarchy

Observed $\Delta\chi^2$	Significance
2.47	1.77

