FERMILAB-SLIDES-18-043-PPD

Electric Field Effects on the Muon Anomalous Precession Frequency in the Fermilab Muon g-2 Experiment

Wanwei Wu

Department of Physics and Astronomy

University of Mississippi

(on behalf of the Muon g-2 Collaboration)

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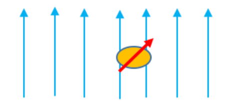
A particle with spin has a magnetic moment ($\vec{\mu}$) aligned with its spin ($\vec{S}$):

$$\vec{\mu} = g \frac{q}{2m} \vec{S}$$



Dirac, 1928

This differs from (1) by the two extra terms

$$\frac{eh}{c}(\sigma, H) + \frac{ieh}{c}\rho_1(\sigma, E)$$

in F. These two terms, when divided by the factor $2m$, can be regarded as the additional potential energy of the electron due to its new degree of freedom. The electron will therefore behave as though it has a magnetic moment $eh/2mc \cdot \sigma$ and an electric moment $ieh/2mc \cdot \rho_1 \sigma$. This magnetic moment is just that assumed in the spinning electron model. The electric moment, being a pure

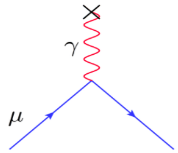
Dirac Theory predicts that $g = 2$.

However, experiments showed that $g \neq 2$.

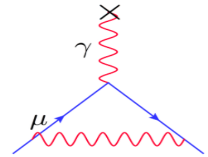
⇒ Anomalous Magnetic Dipole Moment: $a = \frac{g-2}{2}$.

g-2: Standard Model (SM)

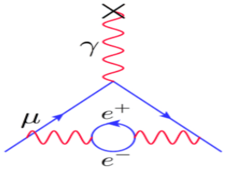
QED:



Dirac term

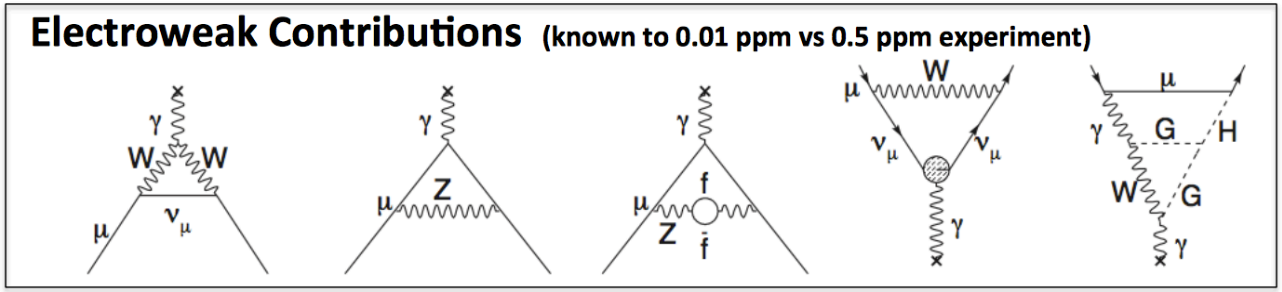


Schwinger term

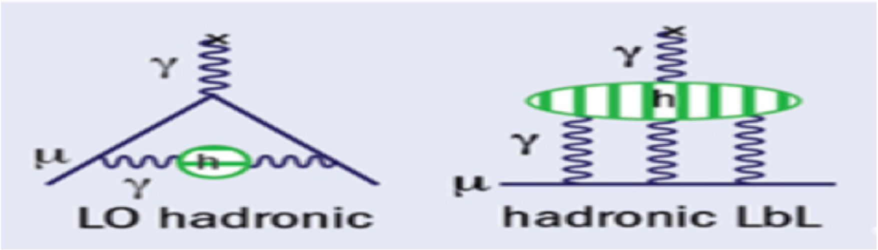


Vacuum polarization

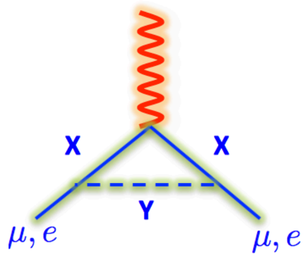
EW:



Hadron:

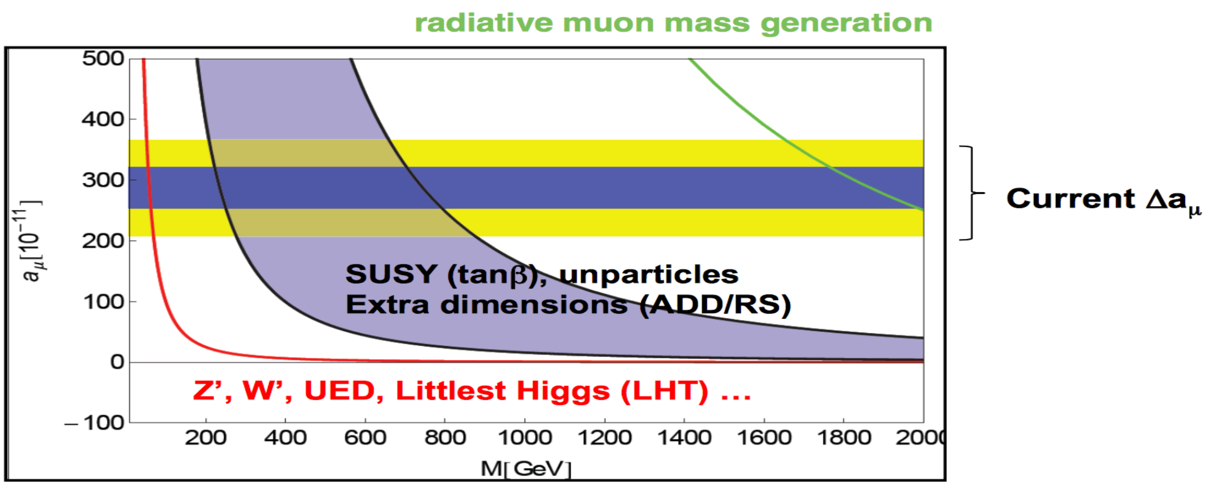


$$a_\mu^{SM} = a_\mu^{QED} + a_\mu^{EW} + a_\mu^{Had} = 116591828(50)\times 10^{-11}$$



New Physics proportional to: $\left(\frac{m_\ell}{M_{\rm NEW}}\right)^2$

a_μ represents a sum over all physics, it is sensitive to a wide range of potential new physics.



$$a_\mu^{NP} = a_\mu^{Exp} - a_\mu^{SM}$$

SM: $116591828(50)\times10^{-11}$

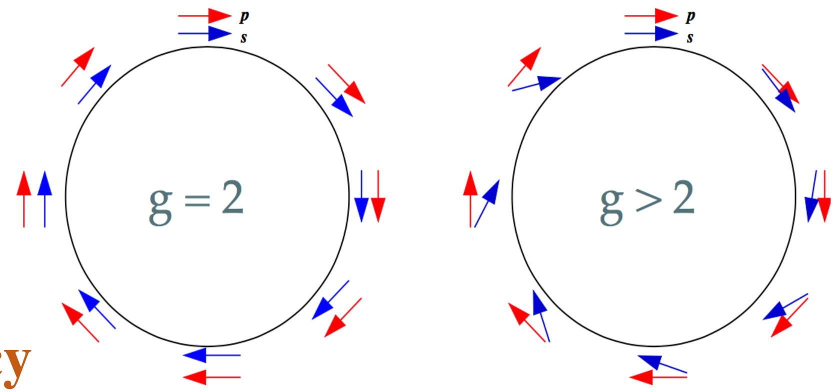
BNL g-2: $116592080(63)_{tot}\times10^{-11}$

$\sim 3.3\sigma$

Spin rotation of a muon in a magnetic field

- Spin precession frequency

$$\vec{\omega}_S = -\frac{qg\vec{B}}{2m} - \frac{q\vec{B}}{\gamma m}(1-\gamma)$$



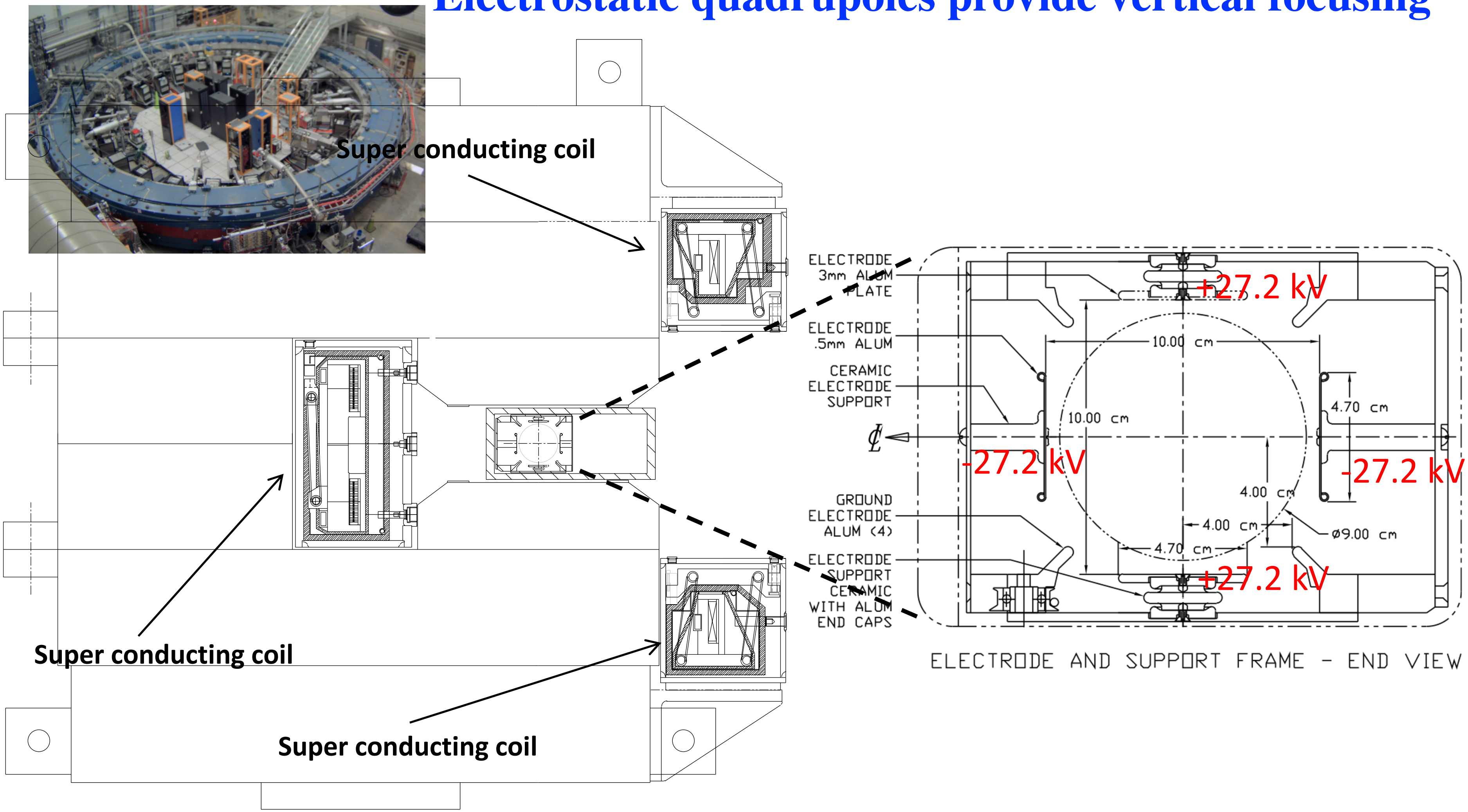
- Cyclotron rotation frequency

$$\vec{\omega}_C = -\frac{q\vec{B}}{m\gamma}.$$

Muon anomalous precession frequency:

$$\vec{\omega}_a = \vec{\omega}_S - \vec{\omega}_C = -\left(\frac{g-2}{2}\right)\frac{q\vec{B}}{m} = -a_\mu \frac{q\vec{B}}{m}.$$

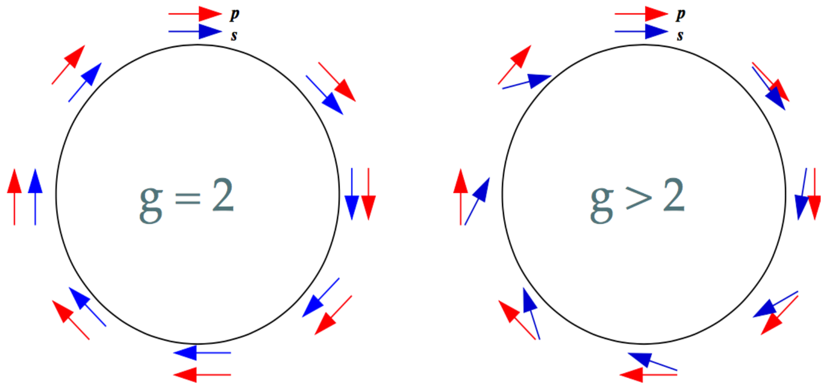
Weak focusing muon storage ring—
Electrostatic quadrupoles provide vertical focusing



Electrostatic focusing

- **Cyclotron rotation frequency**

$$\vec{\omega}_C = -\frac{q}{m}[\frac{\vec{B}}{\gamma} - \frac{\gamma}{\gamma^2-1}(\frac{\vec{\beta}\times\vec{E}}{c})]$$



- **Spin precession frequency**

$$\vec{\omega}_S = -\frac{q}{m}[(\frac{g}{2} - 1 + \frac{1}{\gamma})\vec{B} - (\frac{g}{2}-1)\frac{\gamma}{\gamma+1}(\vec{\beta}\cdot\vec{B})\vec{\beta} - (\frac{g}{2} - \frac{\gamma}{\gamma+1})(\frac{\vec{\beta}\times\vec{E}}{c})]$$

Muon anomalous precession frequency:

$$\vec{\omega}_a = -\frac{q}{m}[a_\mu\vec{B} - a_\mu(\frac{\gamma}{\gamma+1})(\vec{\beta}\cdot\vec{B})\vec{\beta} - (a_\mu - \frac{1}{\gamma^2-1})\frac{\vec{\beta}\times\vec{E}}{c}]$$

$$\vec{\omega}_a = -\frac{q}{m}[a_\mu \vec{B} - a_\mu(\frac{\gamma}{\gamma+1})(\vec{\beta}\cdot\vec{B})\vec{\beta} - (a_\mu - \frac{1}{\gamma^2-1})\frac{\vec{\beta}\times\vec{E}}{c}]$$

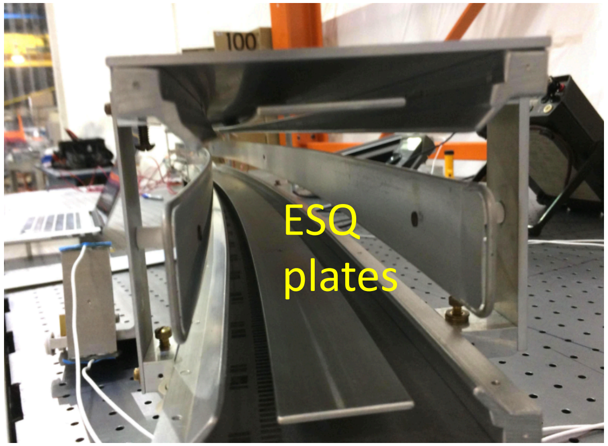
- Assuming $\vec{\beta}\cdot\vec{B} = 0$, the second term vanishes.
- By choosing $\gamma = 29.3$, the third term vanishes. The corresponding momentum (3.904 GeV/c)/radius (711.2 cm) is then called "magic" momentum/radius.

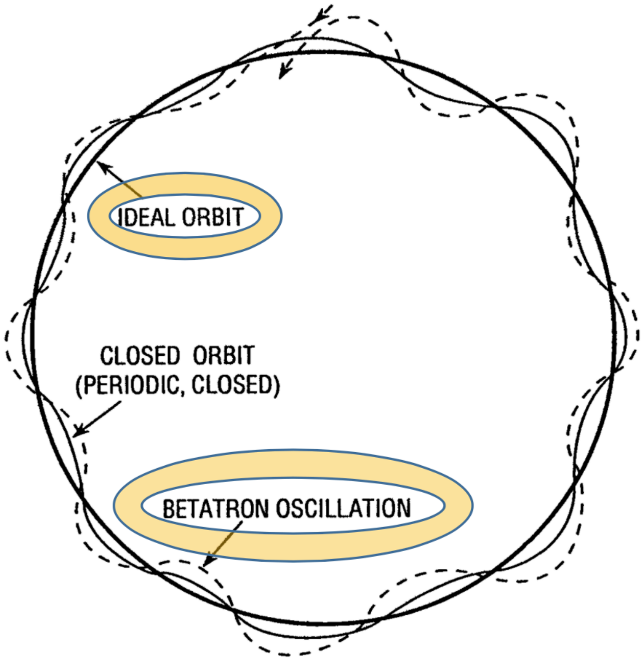
$$\omega_a = \omega_S - \omega_C = a_\mu \frac{q}{m} B$$

- We measure ω_a by using the decay positron signals and measure B by observing the Larmor frequency of stationary protons ($\omega_p = \frac{2\mu_p B}{\hbar}$) with NMR probes.
- For our final analysis, we can solve for a_μ and rewrite it as (using $\mu_e = g_e e\hbar/4m_e$):

$$a_\mu = \frac{\omega_a}{\omega_p}\frac{\mu_p}{\mu_e}\frac{m_\mu}{m_e}\frac{g_e}{2}$$

Electrostatic Quadrupole System





if electrostatic quadrupole (ESQ) plates misaligned

↓

introduce E-field multipoles

↓

closed-orbit distortion

+

muon loss

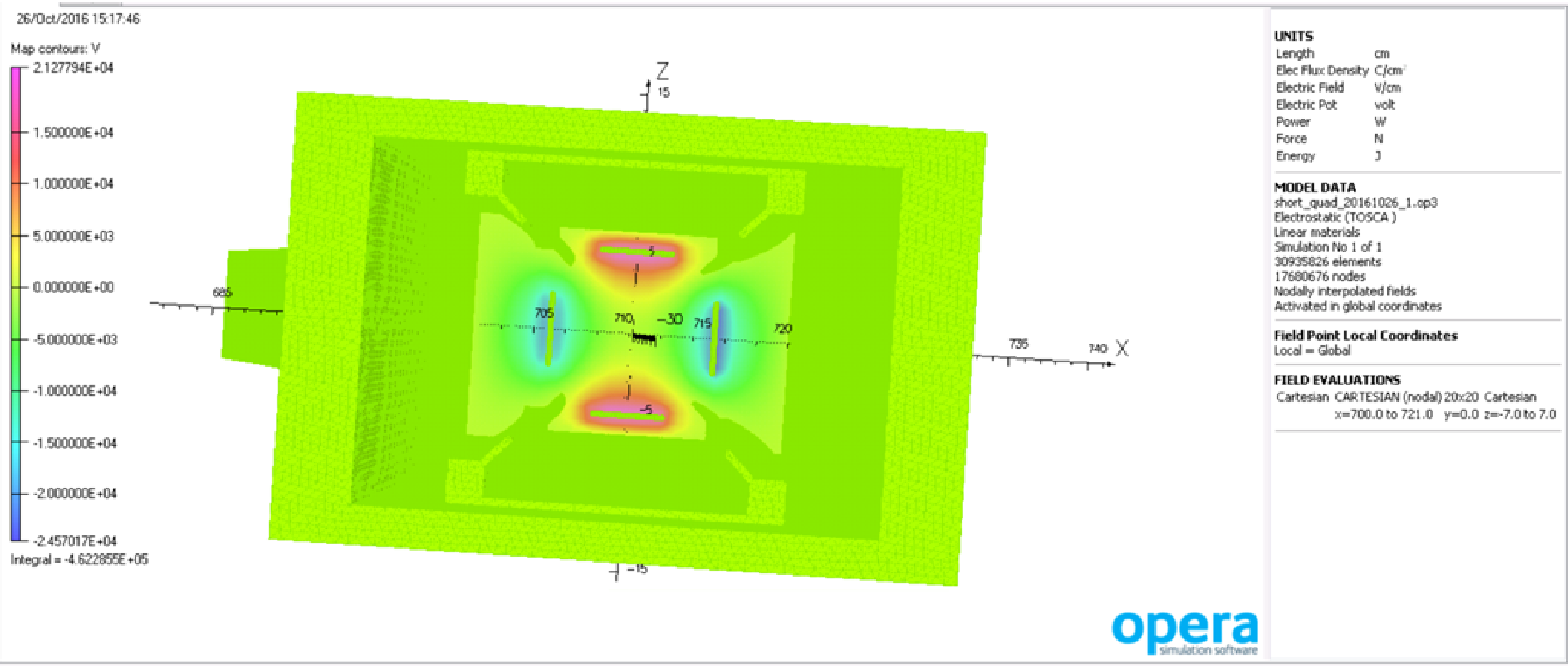
+

shifts in E-field and pitch ($\vec{\beta} \cdot \vec{B}$) corrections

+

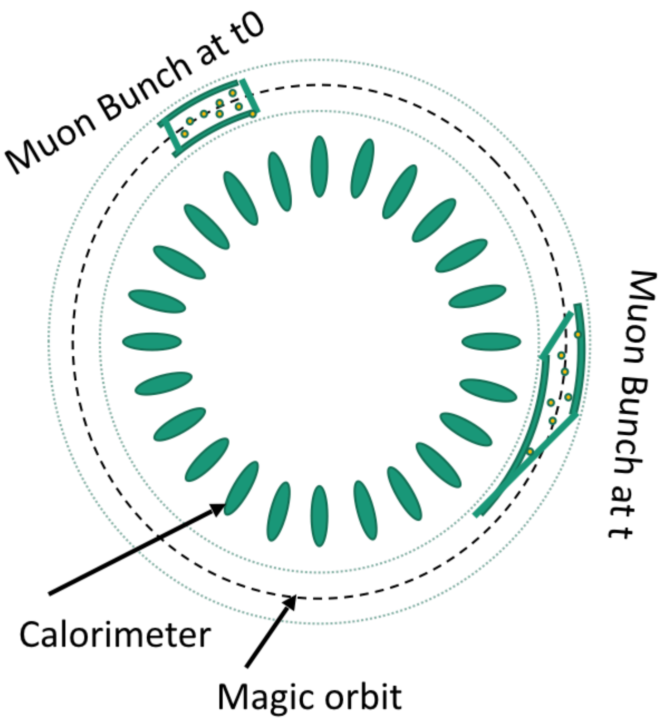
……

OPERA-3D: 3-Dimension Map



Electric Potential at the end of plates (zone map)
V=27.2kV, from Downstream theta=0°

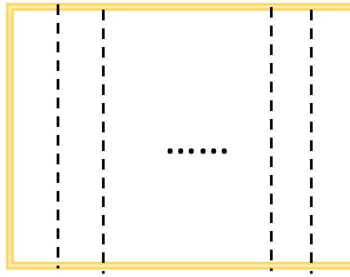
$$\vec{\omega}_a = -\frac{q}{m}[a_\mu \vec{B} - a_\mu(\frac{\gamma}{\gamma+1})(\vec{\beta}\cdot\vec{B})\vec{\beta} - \boxed{(a_\mu - \frac{1}{\gamma^2-1})\frac{\vec{\beta}\times\vec{E}}{c}}$$



- Muons are injected into the storage ring as a bunch—radial distribution
- Muons at inner equilibrium radii will go steadily ahead of those at outer equilibrium radii ->**debunching**
- Modulation of decay positron count (fast rotation signals) can be used to study the debunching
- Fast rotation analysis: use a model of the time evolution of the bunch structure to obtain the momentum (radial) distribution of decayed muons

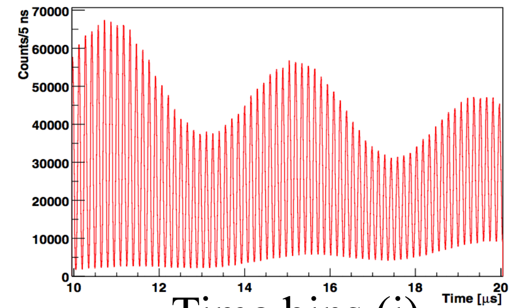
χ^2 Minimization Method

Two bin sets:



Radial bins (i) (Lx=90 mm)
(i.e., 50 bins w/width=1.8mm)

PHYSICAL REVIEW D **73,** 072003 (2006)



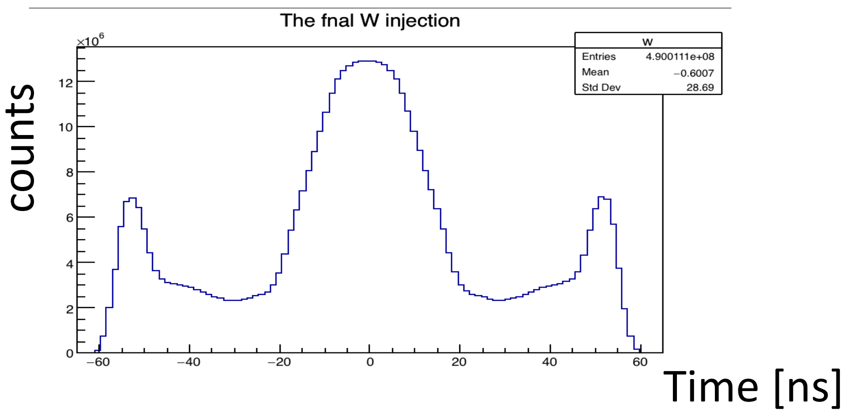
Time bins (j)
(positron count histogram)

- f_i: the content of the radial bin i, fraction of the beam oscillating around radial bin i
- N_j: $(N(j)_{obs})$ counts in time bin j
- β_{ij}: contribution from radial bin i to the counts in time bin j
- C_j: $(N(j)_{exp})$expected counts in time bin j
- Z_j: weighting factor which should equal to C_j

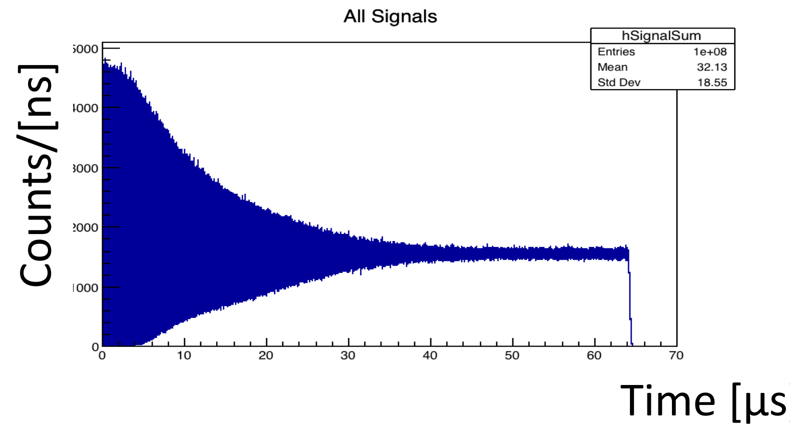
$$\chi^2 = \sum_j \frac{(N_j - C_j)^2}{Z_j} = \sum_j \frac{(N_j - \sum_i f_i \beta_{ij})^2}{Z_j}$$

Note: The geometry factors β_{ij} are known functions of ring geometry and the apparent time structure of the injected bunch (injection zero time and beam revolution time).

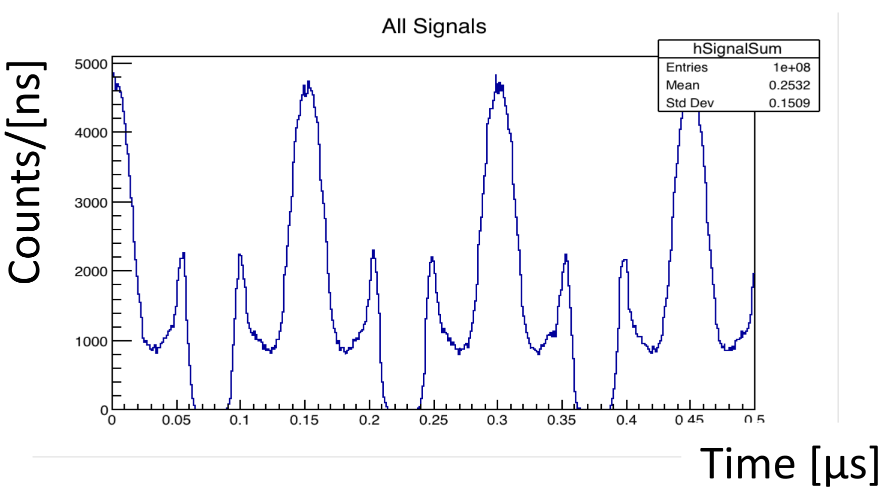
Toy MC—Injection Pulse & Signals



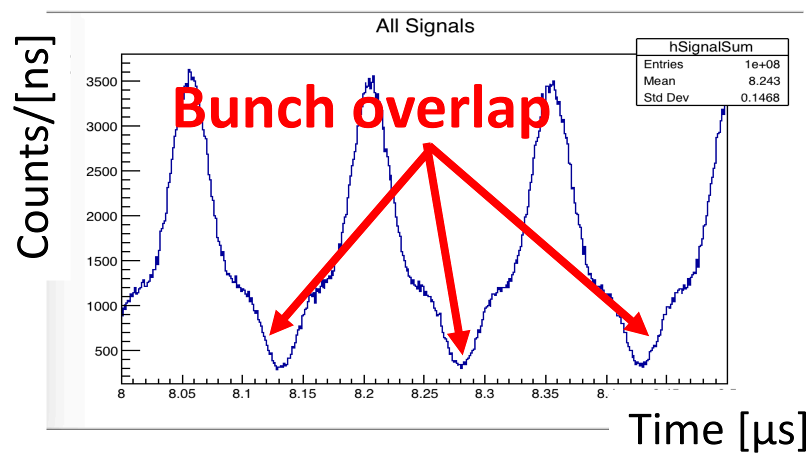
Injection Pulse



Signals seen by Detector



Signals seen by Detector (*early time*)



Signals seen by Detector (*late time*)

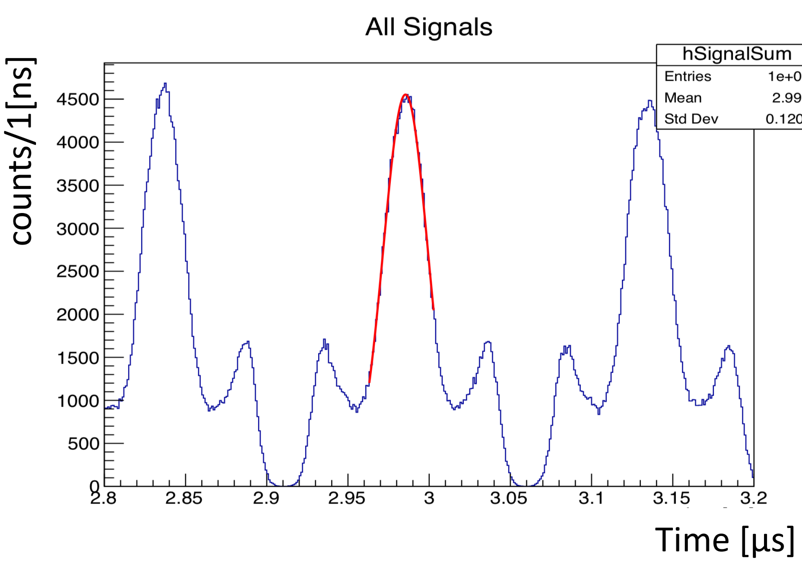
Toy MC— t_0 and T_C

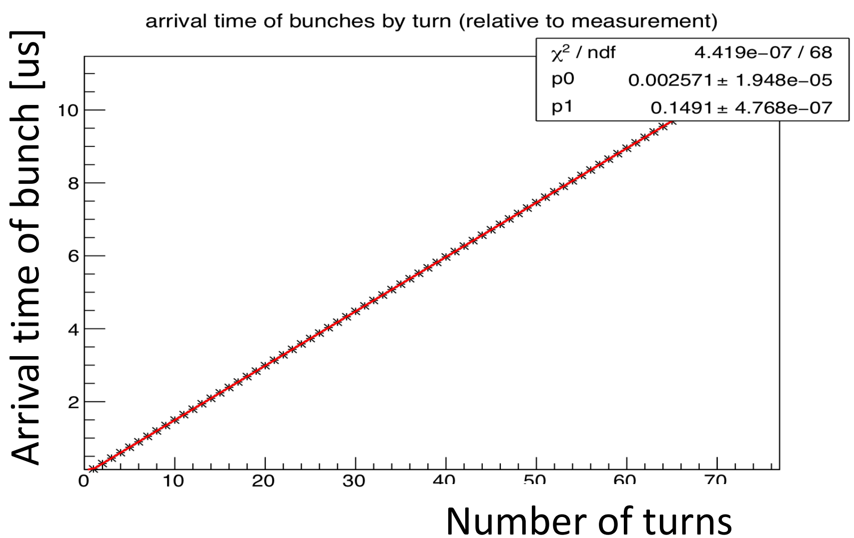
$$\chi^2 = \sum_j \frac{(N_j - C_j)^2}{Z_j} = \sum_j \frac{(N_j - \sum_i f_i \beta_{ij})^2}{Z_j}$$

Fit the arrival time of bunches by number of turns to find out the injection zero time and beam revolution time

$$t_{peak} = nT_C + t_0$$

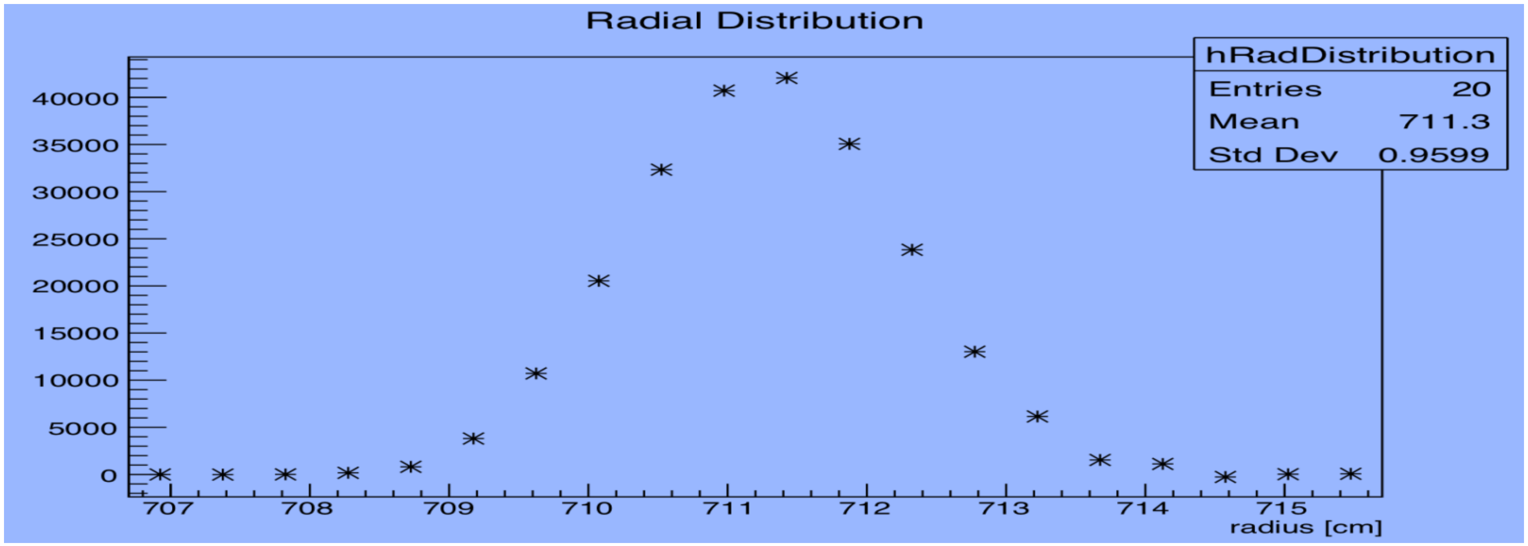
Arrival time of bunch by turns





Toy MC—Radial Distribution

Apply the χ^2 minimization analysis and solve for the radial (momentum) bin contents:

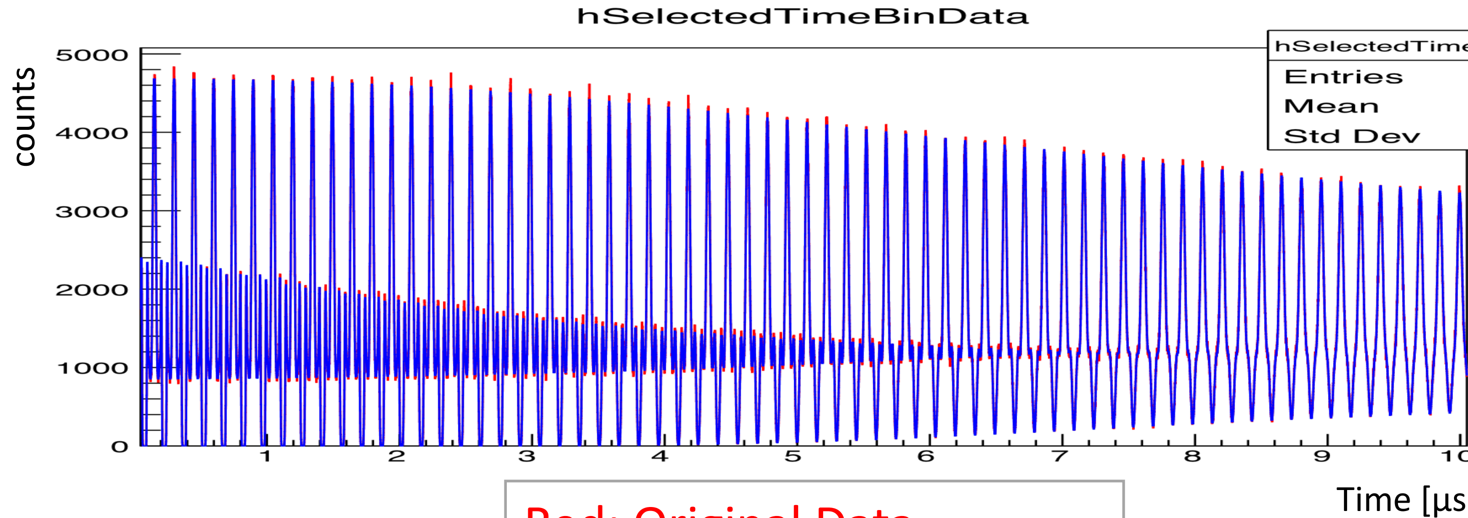


With this distribution, we are able to evaluate the electric field corrections to g-2.

$$\left\langle \frac{\delta \omega_{\mathrm{a}}}{\omega_{\mathrm{a}}} \right\rangle = -2\beta^2 n(1-n)\left\langle \left(\frac{x_e}{R_o} \right)^2 \right\rangle$$

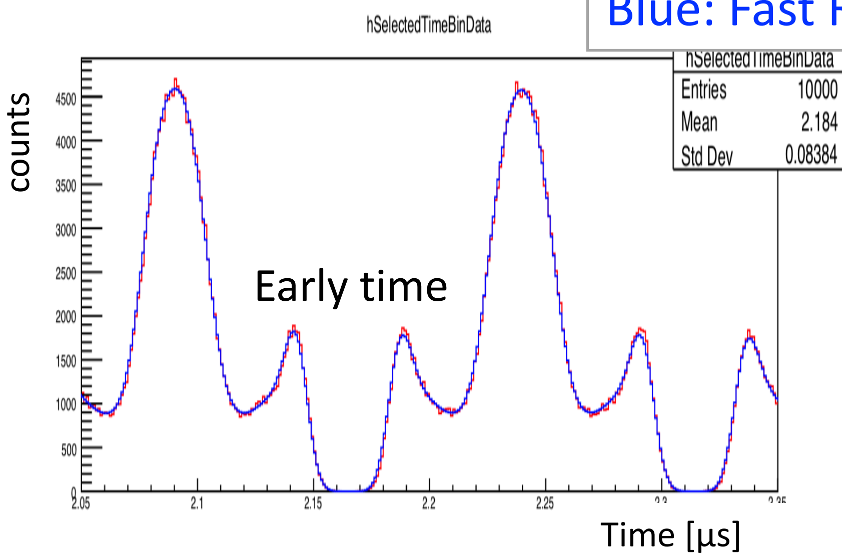
($x_e = R_{mean} - R_0$, R_0 is the “magic” radius; $n = \frac{\rho}{v_z B_0}\frac{\partial E_R}{\partial R}$)

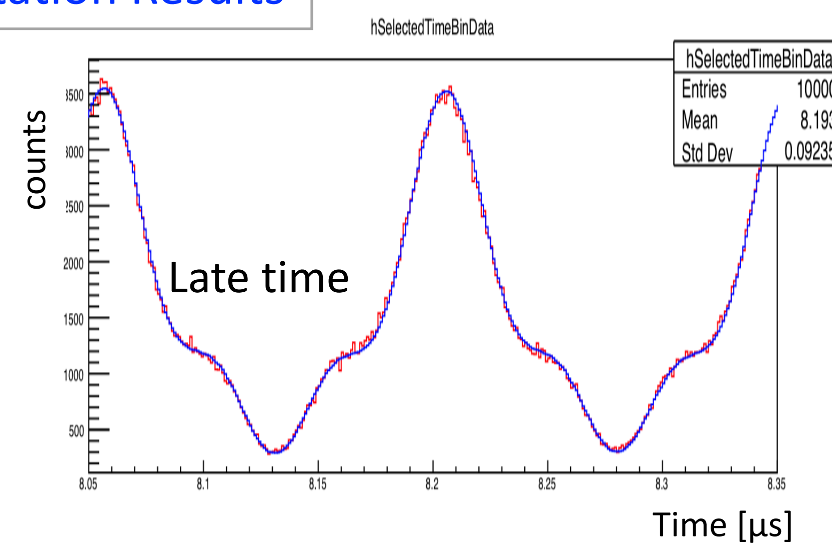
Toy MC—Check the Results



Red: Original Data

Blue: Fast Rotation Results





Conclusion

$$\left\langle \frac{\delta\omega_a}{\omega_a} \right\rangle = -2\beta^2 n(1-n)\left\langle \left(\frac{x_e}{R_o}\right)^2 \right\rangle$$

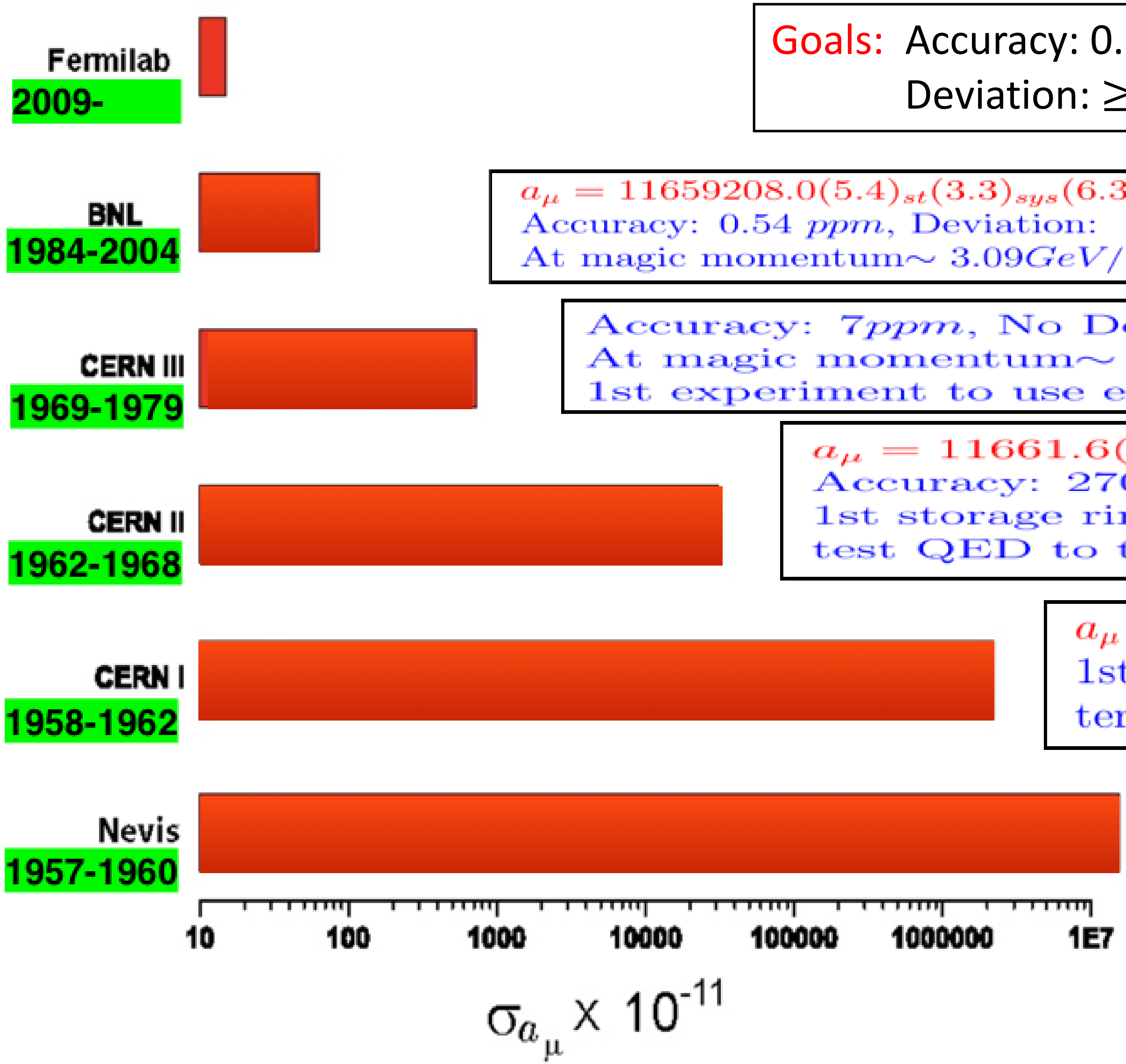
($x_e = R_{mean} - R_0$, R_0 is the "magic" radius; $n = \frac{\rho}{v_z B_0}\frac{\partial E_R}{\partial R}$)

- The electric field plays a very important role in the Fermilab Muon g-2 Experiment in many ways, i.e., muon orbits, muon losses, E-field correction.
- The electric field has a significant effect on the muon anomalous precession frequency:
  - We need to align the electrostatic quadrupole plates very carefully;
  - We need to know the 3D electric field map;
  - We need to know the muon momentum/radius distribution through the so-called fast rotation analysis.
- The Experiment is running and it will be a great time to study muon anomaly including its electric field corrections.

—Thank You!

Backup

Measurement of a_μ in History



Goals: Accuracy: 0.14 ppm
Deviation: $\geq 5\sigma$

$a_\mu = 11659208.0(5.4)_{st}(3.3)_{sys}(6.3)_{tot} \times 10^{-10}$
Accuracy: 0.54 ppm, Deviation: $\sim 3.3\ \sigma$
At magic momentum$\sim 3.09GeV/c$, use electrostatic focusing

Accuracy: $7ppm$, No Deviation was found
At magic momentum$\sim 3.09GeV/c$
1st experiment to use electrostatic focusing

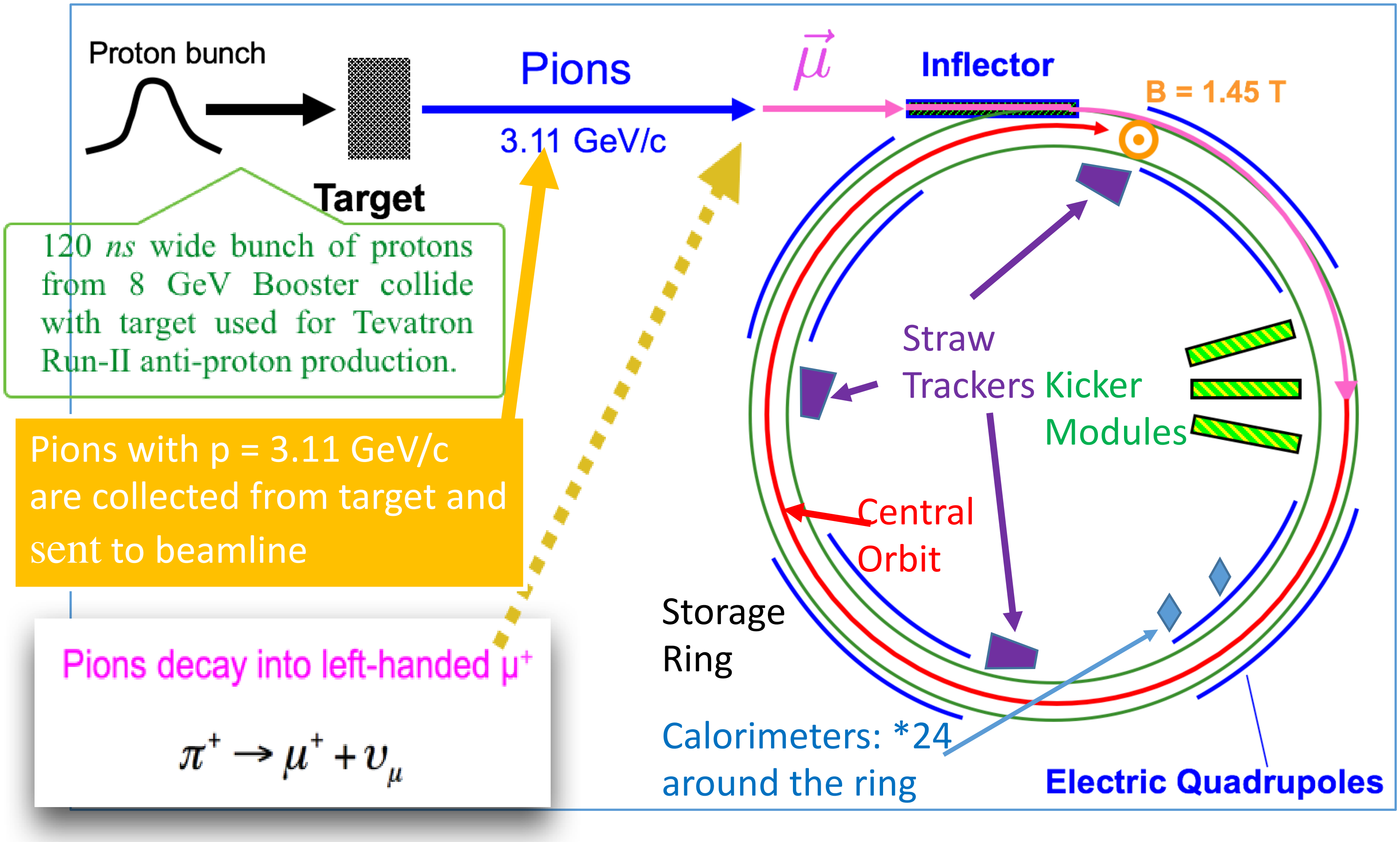
$a_\mu = 11661.6(3.1) \times 10^{-7}$, (0.27%)
Accuracy: $270ppm$, Deviation: 1.7σ
1st storage ring measurement
test QED to three-loop level

$a_\mu = 0.001145(22)$, (1.9%)
1st ovservation of $(\alpha/2\pi)$
term in the QED expansion

$a_\mu = 0.00122(8)$, (5%)
no difference with a_e

Backup

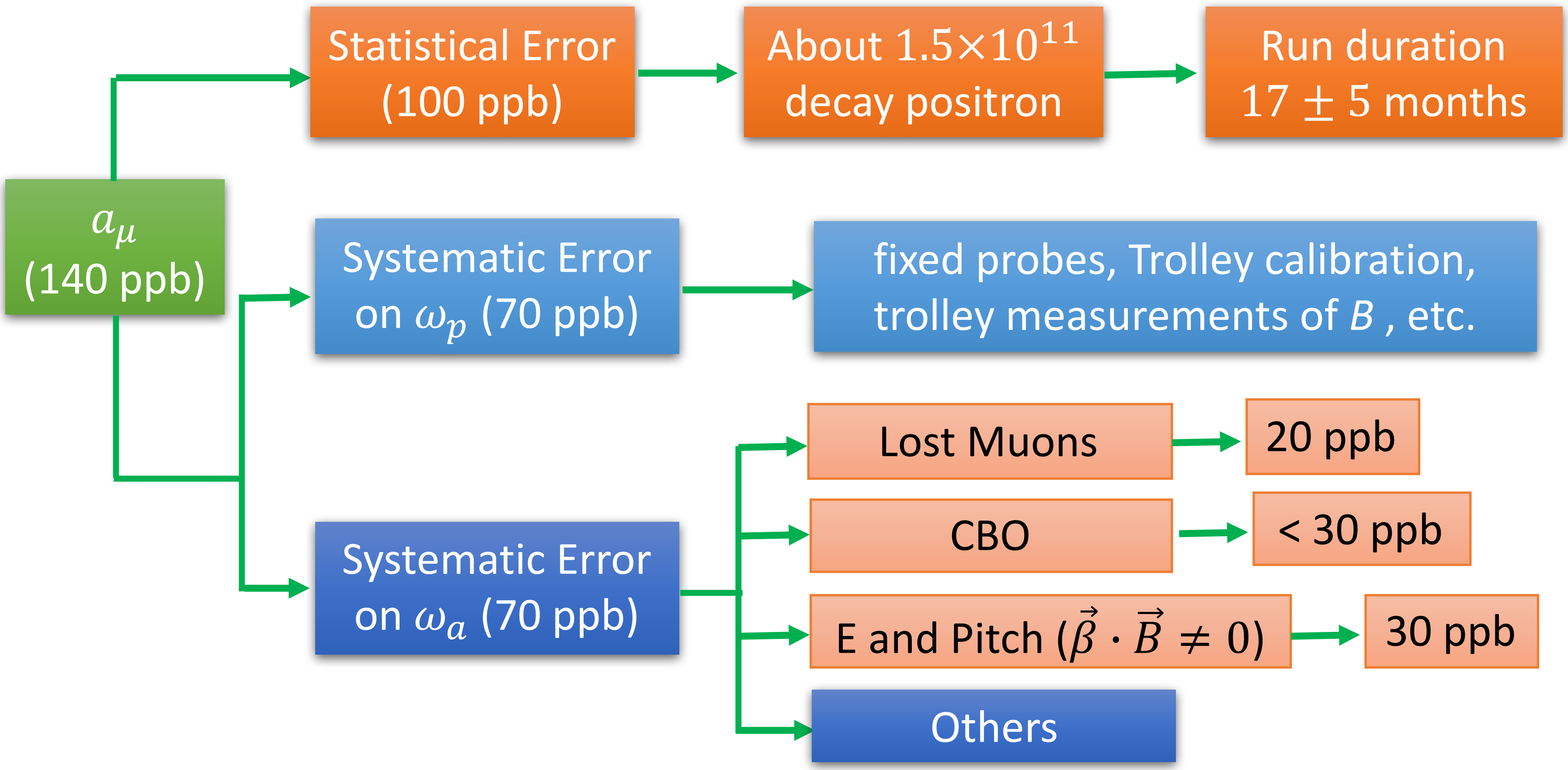
Big Picture!



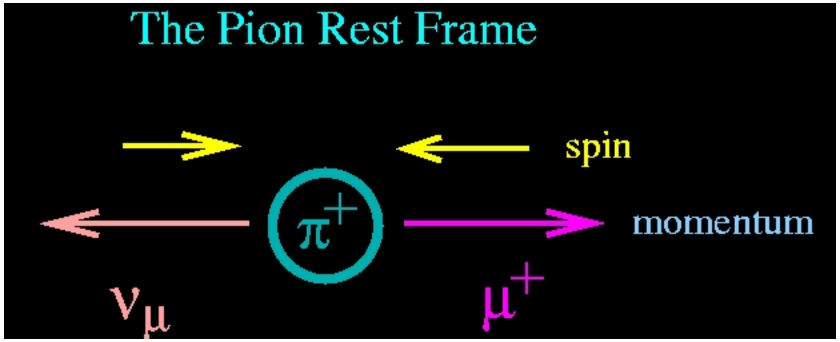
Big Picture!

- For our final analysis, we can solve for a_μ and rewrite it as (using $\mu_e = g_e e\hbar/4m_e$):

$$a_\mu = \boxed{\frac{\omega_a}{\omega_p}} \frac{\mu_p}{\mu_e} \frac{m_\mu}{m_e} \frac{g_e}{2}$$



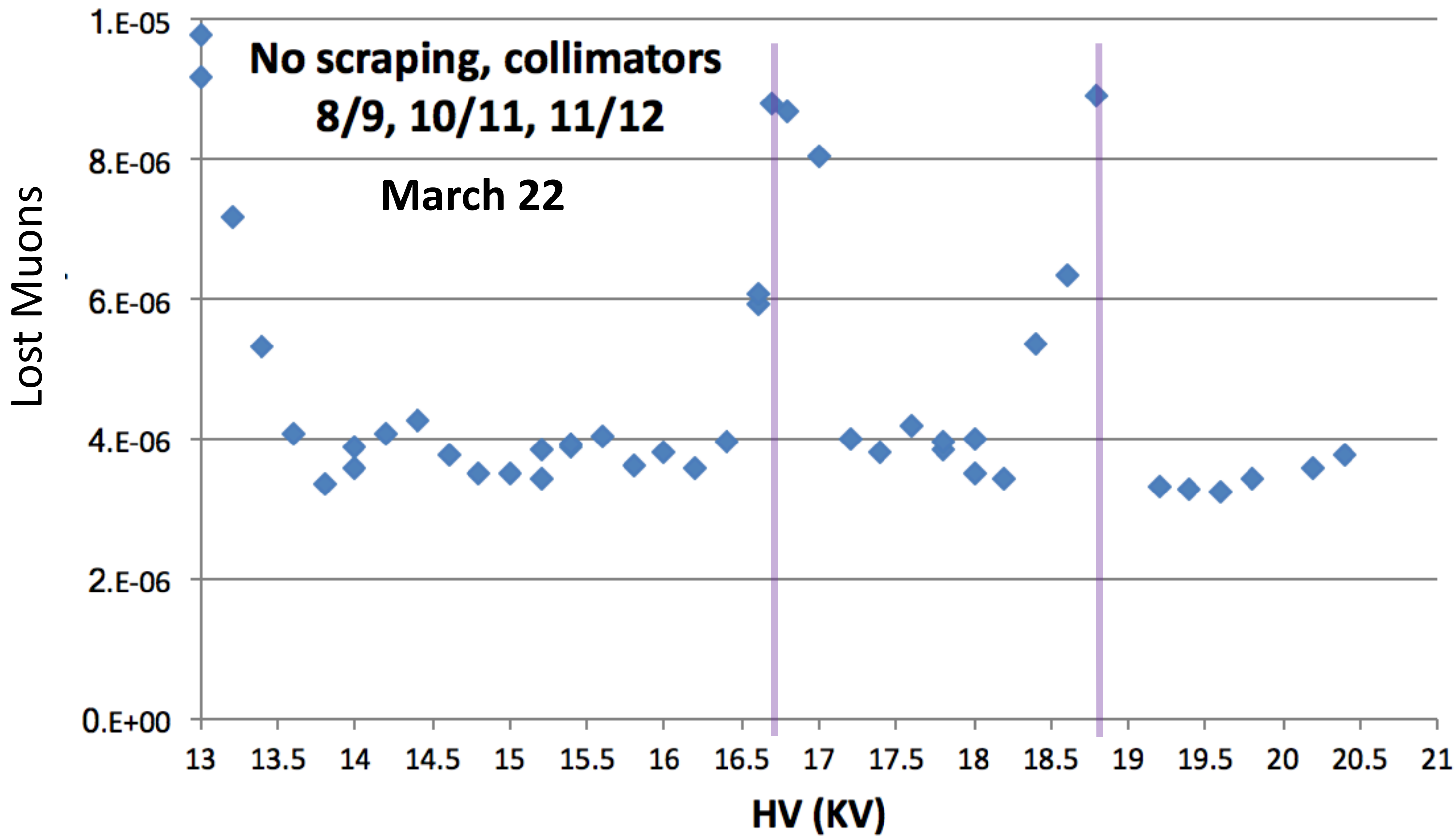
- **Muon is produced polarized**: in-flight decay, both "forward" and "backward" muons are highly polarized



Pion decay modes:

	Mode		Fraction (Γ_i/Γ)	Confidence level
Γ_1	$\mu^+\nu_\mu$	[a]	(99.98770 ± 0.00004) %	
Γ_2	$\mu^+\nu_\mu\gamma$	[b]	$(2.00\pm0.25)\times10^{-4}$	
Γ_3	$e^+\nu_e$	[a]	$(1.230\pm0.004)\times10^{-4}$	
Γ_4	$e^+\nu_e\gamma$	[b]	$(7.39\pm0.05)\times10^{-7}$	
Γ_5	$e^+\nu_e\pi^0$		$(1.036\pm0.006)\times10^{-8}$	
Γ_6	$e^+\nu_e e^+e^-$		$(3.2\pm0.5)\times10^{-9}$	
Γ_7	$e^+\nu_e\nu\overline{\nu}$		$<5\times10^{-6}$	90%

-- http://pdg.lbl.gov/2014/listings/rpp2014-list-pi-plus-minus.pdf

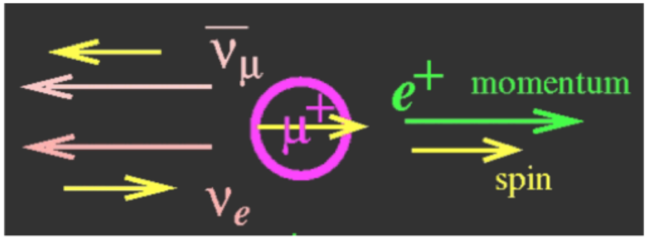


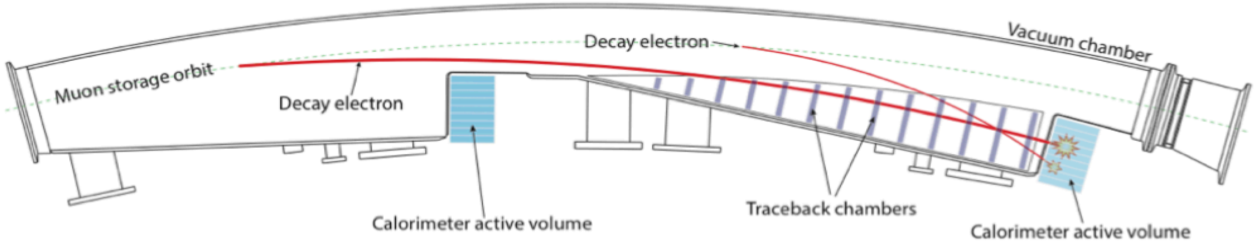
Quad scan for beam resonance study—Lost Muons

Decay Positron Signal

Muon decay:

$$\mu^+ \rightarrow e^+ + \bar{\nu}_\mu + \nu_e$$





$$N_e(t) = N_0 e^{-t/\gamma\tau}\left[1 - A\cos(\omega_a t + \phi_a)\right]$$

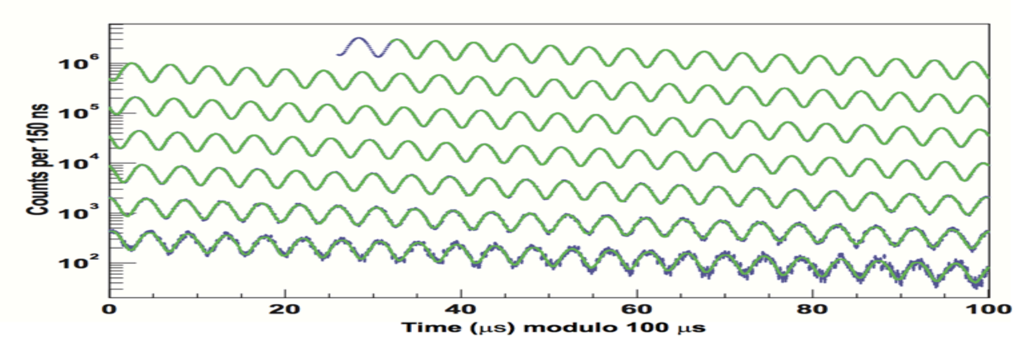
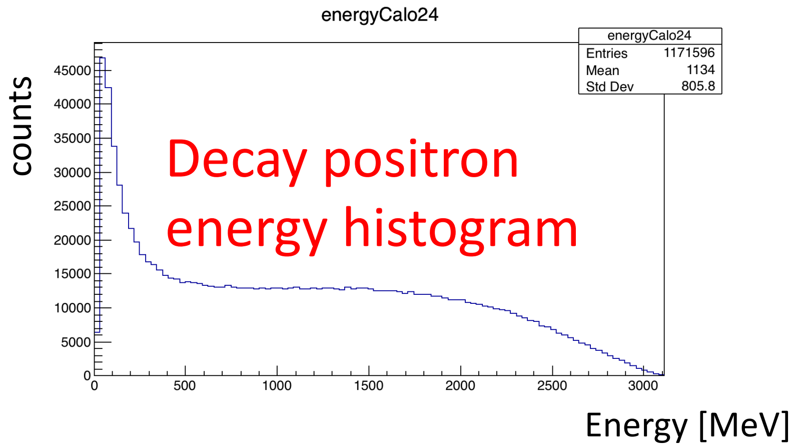
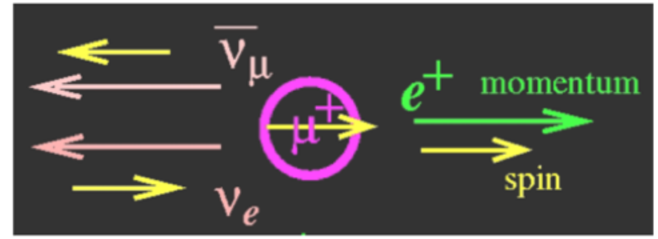


Figure 3.8: Histogram, modulo 100 μ s, of the number of detected electrons above 1.8 GeV for the 2001 data set as a function of time, summed over detectors, with a least-squares fit to the spectrum superimposed. Total number of electrons is 3.6×10^9. The data are in blue, the fit in green.

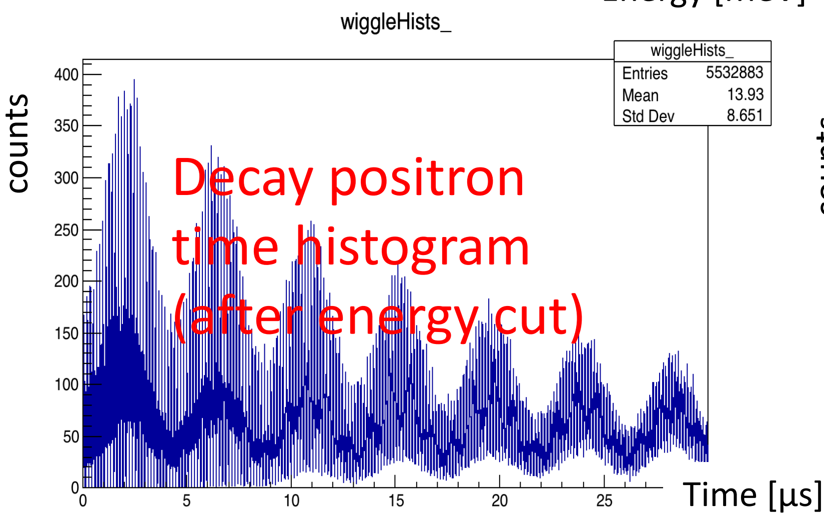
BNL g–2 PRD 2006

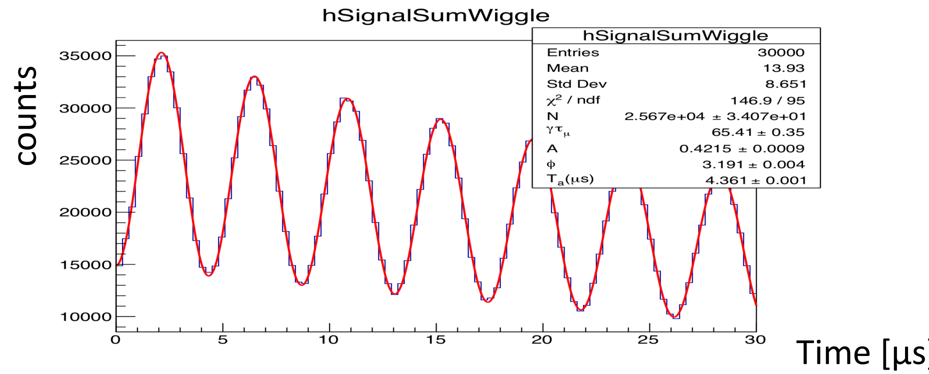
Fast Rotation Analysis





Energy Cut to select the decay positron signal



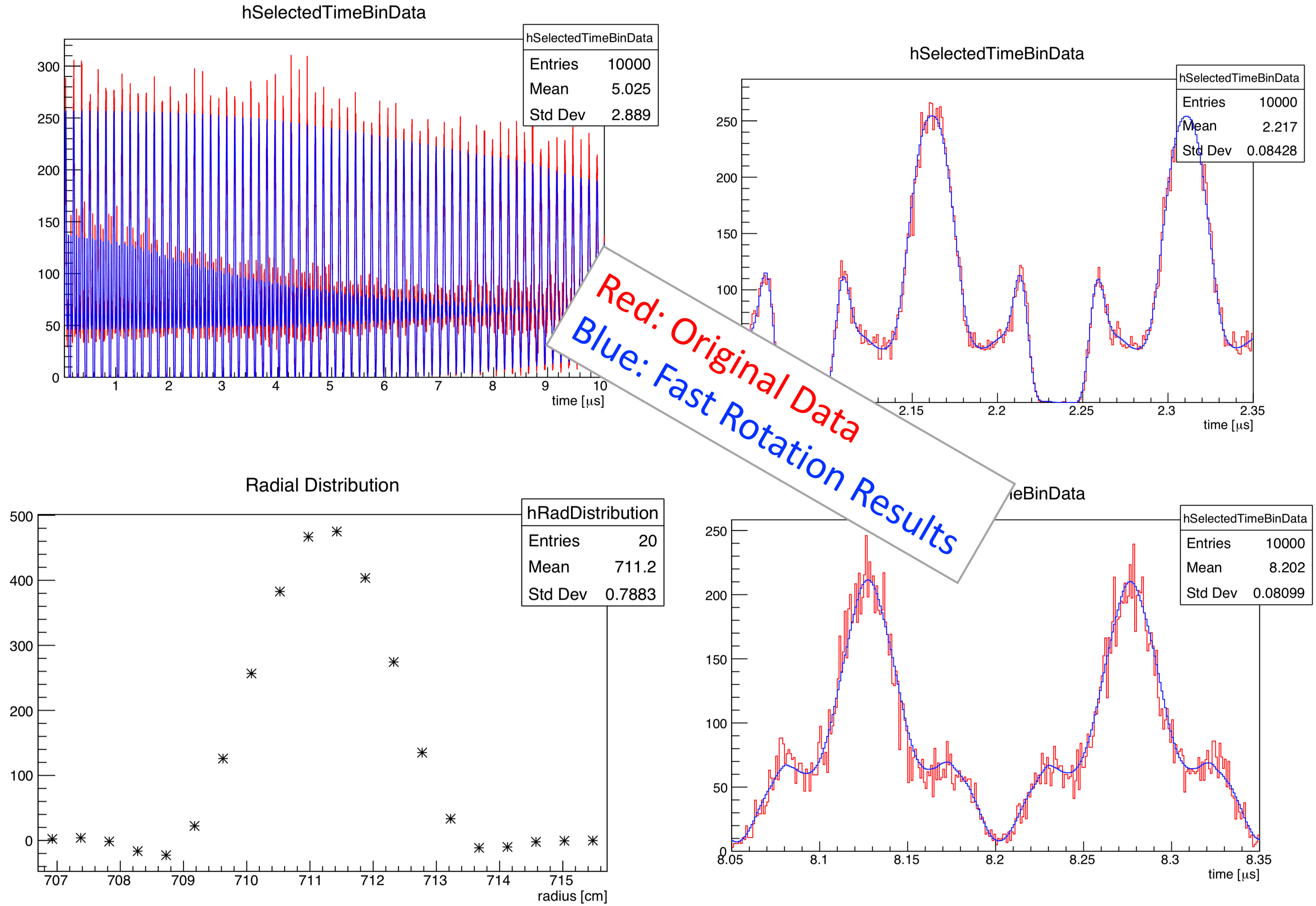


$$N_e(t) = N_0 e^{-t/\gamma\tau}\left[1 - A\cos(\omega_a t + \phi_a)\right]$$

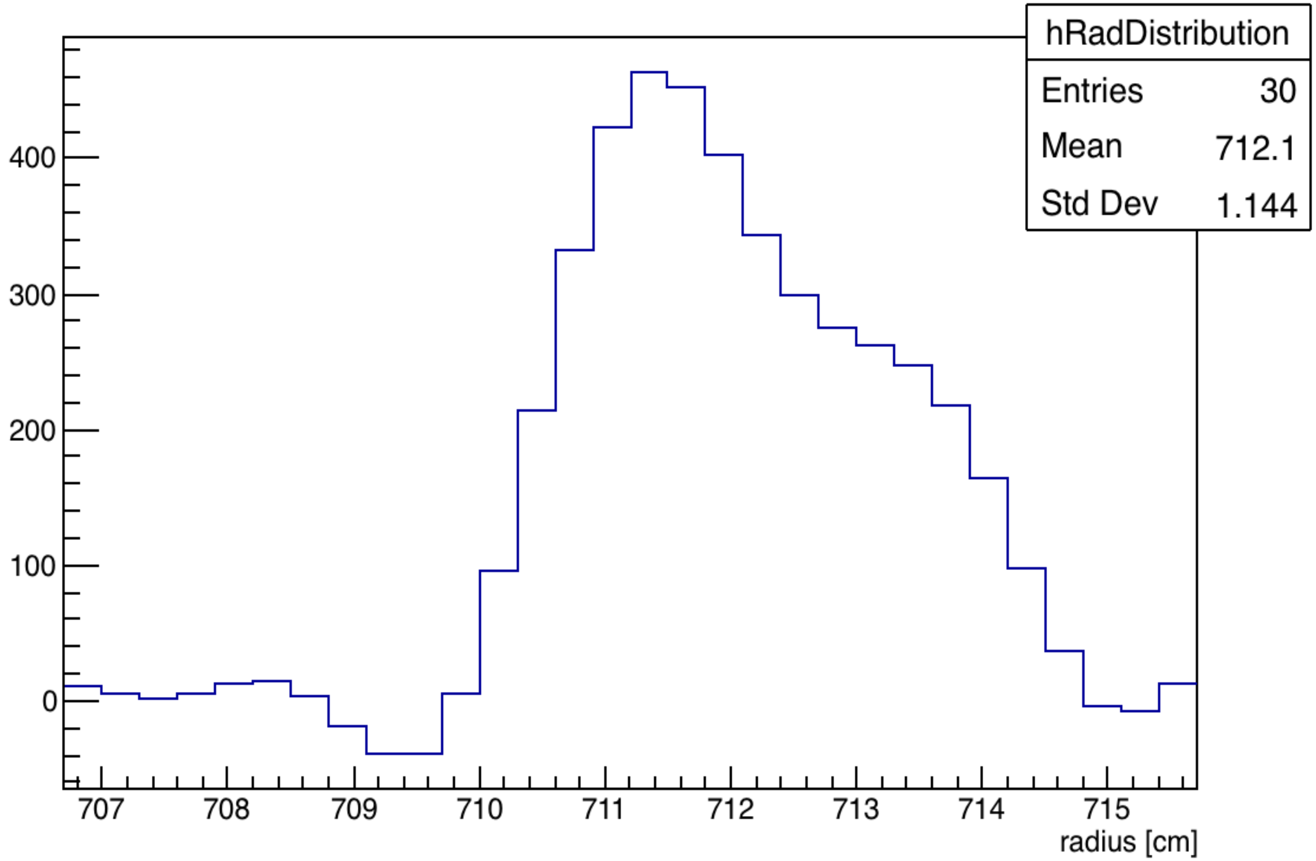
Real situations: a lot of backgrounds, i.e., muon lifetime, g-2 frequency, CBO, Muon losses ….

If we can functionalize the backgrounds, we can remove them in fast rotation analysis, i.e., τ and ω_a

Fast Rotation Analysis



Example (bunch # 0): Radius distribution



Peak at 711.5 cm