Stable, Doubly Heavy Tetraquark Mesons
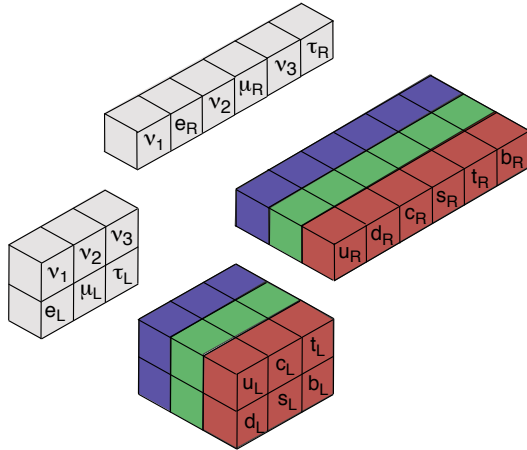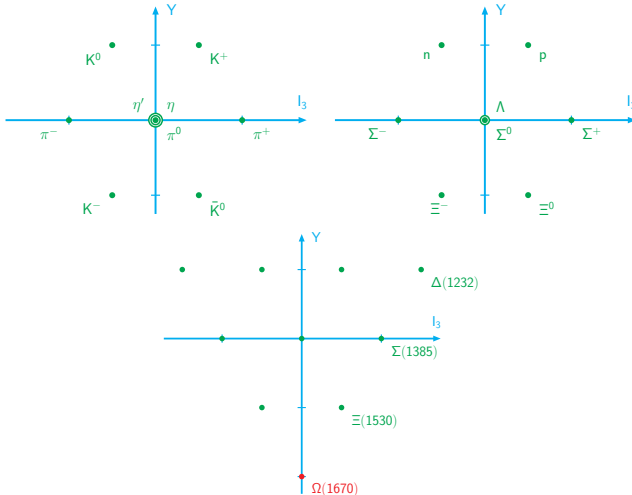
Chris Quigg

Fermilab & Nikhef
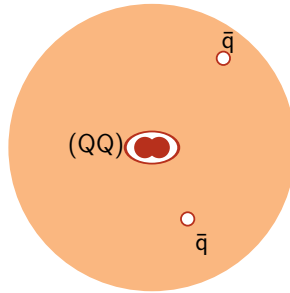
$$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \text{ gauge symmetries}$$

U(1)$_{EM}$

Ancient History: Hadron Spectroscopy & Flavor SU(3)

Gell-Mann, "The Eightfold Way" & Ne'eman, Nucl. Phys. **26**, 229 (1961): SU(3) classification symmetry

K^0 $\quad$ K^+

n $\quad$ p

η' $\;$ η $\qquad$ Λ

π^- $\quad$ π^0 $\quad$ π^+ $\qquad$ Σ^- $\quad$ Σ^0 $\quad$ Σ^+

K^- $\quad$ $\bar{K}^0$ $\qquad$ Ξ^- $\quad$ Ξ^0

I_3 $\qquad$ Y

$\Delta(1232)$

$\Sigma(1385)$

I_3

$\Xi(1530)$

$\Omega(1670)$

The Octet Model and its Clebsch-Gordan Coefficients

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1. INTRODUCTION

IN trying to understand the structure of the strong interactions, several higher symmetry schemes have been proposed.[1,2] These higher symmetries should conserve the isospin I and the hypercharge Y. Especially interesting in this respect is the octet model (unitary symmetry) proposed independently by Gell-Mann[3] and Ne'eman.[4] In this model one assumes the strongest interactions to be invariant under transformations belonging to $SU(3)$, i.e., under unimodular unitary transformations in some three-dimensional complex linear vector space ("unitary spin space"). The symmetry of these strong interactions is broken by some unknown weaker mechanism, but in such a way that the isospin and the hypercharge are still conserved. A still weaker interaction, the electromagnetic interaction, breaks this lower symmetry in such a way that only the hypercharge and the third component of isospin are conserved. In this unitary symmetry model one assigns groups of strongly interacting particles with the same quantum numbers (not the same are I, Y, I_3, and directly related ones as strangeness, charge, G parity, etc.), to irreducible representations (IR's) of the group $SU(3)$. The lowest nontrivial IR in the octet model, which is physically possible (i.e., has integer quantum numbers for the hypercharge), is the IR [8]. The eight well-known baryons N,Λ,Σ, and Ξ, as well as the eight pseudoscalar mesons, K,π,η, and $\overline{K}$, are assigned to IR's [8]. One assumes, moreover, the existence of eight vector mesons which

belong to such a representation. Perhaps the mesons ρ,ω,K^*, and $\overline{K}^*$ constitute this octet. A difficulty here is which K^* to take. There seem to be two $(K\pi)$ resonances, one[5] at 730 MeV and the other[6] at 888 MeV. One favors the 888-MeV resonance because it seems to have all the correct quantum numbers. The next higher IR can contain 10 particles. It is suggested[7] that the familiar (3,3) pion-nucleon resonance, the $N_{3/2}^*$ (1385 MeV), the recently discovered[8,9] $I = \frac{1}{2}$, $\Xi\pi$ resonance at 1532 MeV and a still unknown baryon Ω^-($Y = -2$, $I = 0$, at 1685 MeV) belong to this IR [10]. A discovery of this Ω^- would be a great triumph for this octet model. Okubo[11] has derived a mass formula for the different members belonging to the same IR. For the octets (IR [8]), this formula reduces to a mass relation between the different members. This mass relation is very well satisfied for the baryons and the 730-MeV $(K\pi)$ resonance fulfills this relation. For the IR [10] this mass formula is again very well satisfied. Coleman and Glashow[12] have given a relation connecting the electromagnetic mass differences within the baryon octet. This relation is also very well satisfied.

The main purpose of this paper is to derive the

Ancient History: Hadron Spectroscopy & Quarks

1964: Gell-Mann ("quarks") and Zweig ("aces") noticed that observed multiplets could be constructed from elementary spin-$\frac{1}{2}$ flavor triplets—isospin doublet (u, d) and strange isospin singlet s, according to the rules meson $= q\bar{q}$ and baryon $= qqq$.

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A LECTURE NOTE & REPRINT SERIES

The Quark Model

J.J.J. KOKKEDEE

W. A. BENJAMIN, INC. PUBLISHERS
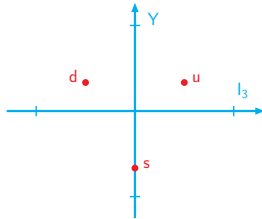
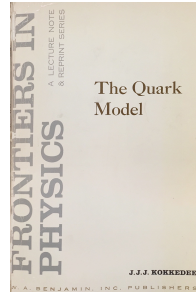"Of course, the whole quark idea is ill-founded. So far, quarks have escaped detection. This fact could simply be taken to mean that they are extremely massive and therefore difficult to produce, but it could also be an indication that quarks cannot exist as individual particles but, like phonons in a crystal, can have meaning only inside the hadrons. In either case, nevertheless, the dynamical system of such quarks binding together to give the observed hadrons that has the properties demanded by the applications, is very difficult to understand in terms of conventional concepts. The quark model should, therefore, at least for the moment, not be taken for more than what it is, namely the tentative and simplistic expression of an as yet obscure dynamics underlying the hadronic world."

—Jaap Kokkedee, 1969

SU(3) flavor algebra:
$$\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus \mathbf{8};$$
$$\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{1} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{10}$$

Establishing the quark paradigm: pointlike constituents

Bjorken scaling in deeply inelastic scattering
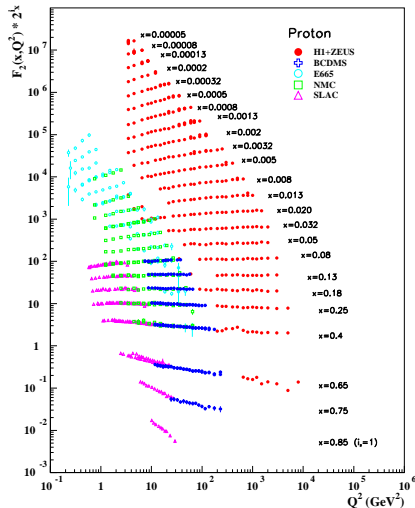
$R \equiv \sigma(e^+ e^- \to \text{hadrons})/\sigma(e^+ e^- \to \mu^+ \mu^-)$
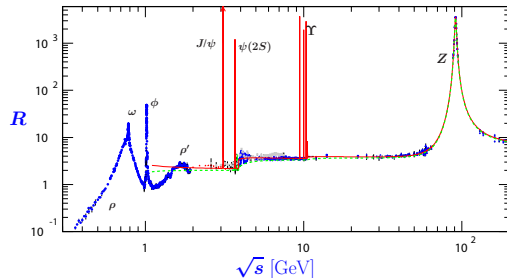
$$R = 3 \sum_{\text{flavors}} e_q^2$$

Idea of quarks as color triplets resolves spin-statistics problem of $J^P = \frac{3}{2}^+ \, \Omega^-(sss)$, suggests $SU(3)_{\text{color}}$ symmetry, color gauge theory.

Color confinement, perturbation theory, concrete quarks

Asymptotic freedom

$$1/\alpha_s(Q) \approx 1/\alpha_s(\mu) + \frac{(33 - 2n_f)}{6\pi} \ln \frac{Q}{\mu}$$
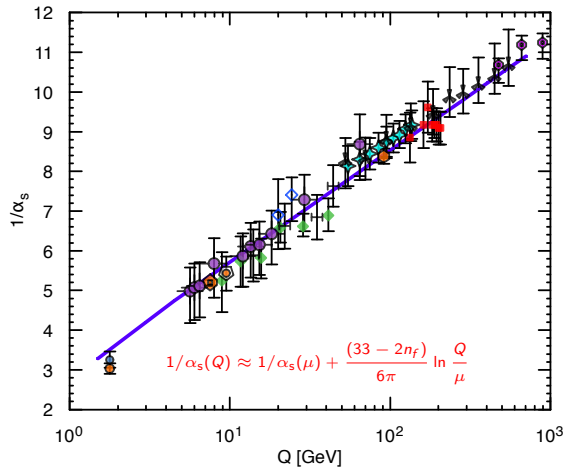
Spectrum of charmonium ($c\bar{c}$) states
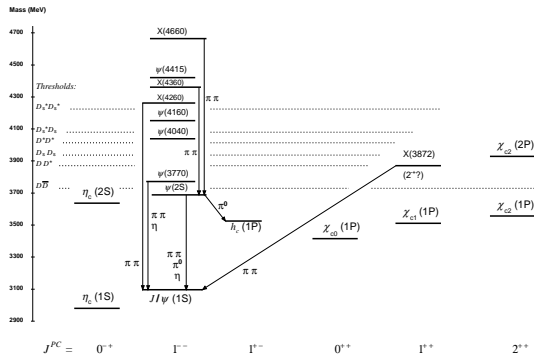
Heavy-flavor hadrons

Spectrum of bottomonium ($b\bar{b}$) states
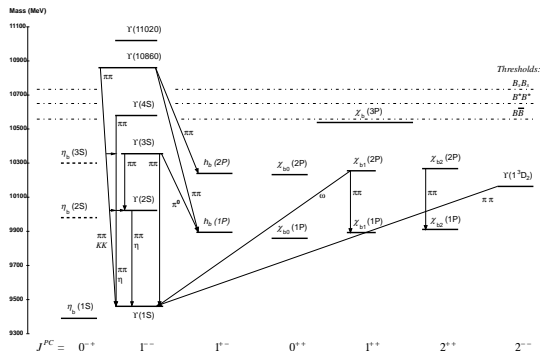
$t\bar{t}$ in the DØ experiment at Fermilab
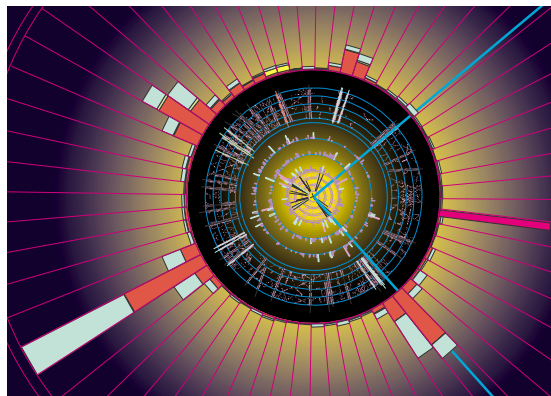
Hadron masses from Lattice QCD
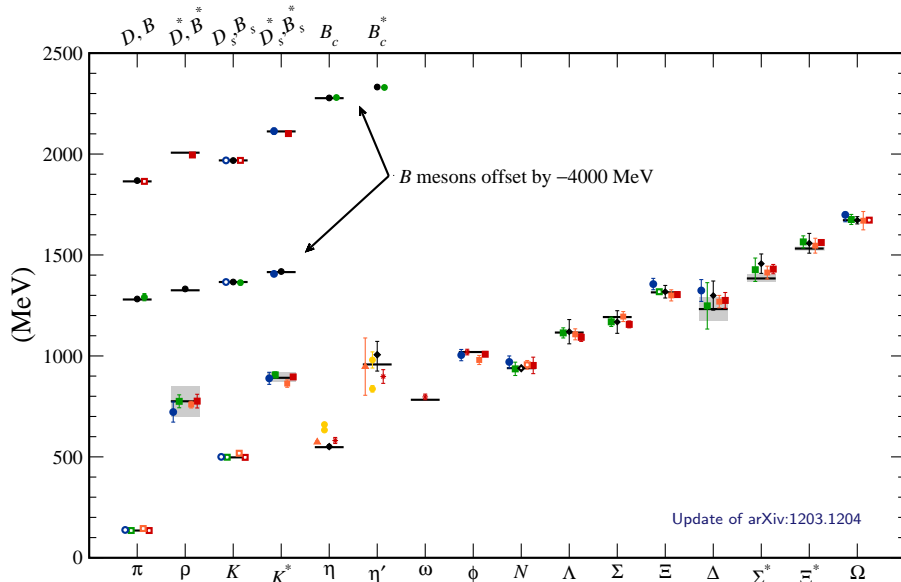
Recent Prehistory (2002–2003) . . .

BELLE observes $\eta'_c(3654)$ in $B \to KK_sK^-\pi^+$ decays (now $\eta'_c(3636)$).

ELQ advocate B-meson gateways to missing charmonium levels
$$h_c(1\,^1P_1),\ \eta_{c2}(1\,^1D_2),\ \text{and}\ \psi_2(1\,^3D_2)$$

BELLE observes $X(3872)$ in $B^\pm \to K^\pm\pi^+\pi^- J/\psi$ decays ($D^0\bar{D}^{*0}$ mass!)
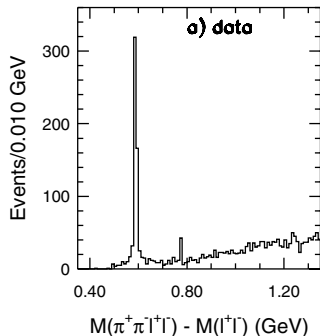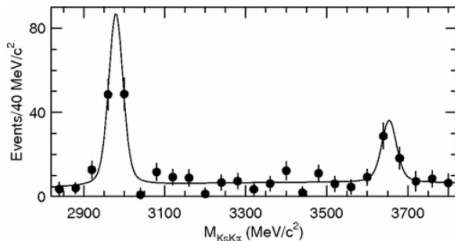
$X(3872) \leadsto$ Renaissance in hadron spectroscopy . . .

$X(3872) \neq \psi_2(1\,^3D_2)$: $J^{PC} = 1^{++}$

$c\bar{c}$ state modified by coupling with open channels?

Threshold "cusp" phenomenon?

$D - \bar{D}^*$ molecule? / hadrocharmonium

Tetraquark (diquark–antidiquark) meson?

QM superposition of several Fock states

Isospin violation likely

Other new states invite hybrid ($c\bar{c}g$) interpretations, etc.

Clarity from change with heavy-quark mass ($c\bar{c}$), ($b\bar{c}$), ($b\bar{b}$)?

$X(3872) \rightsquigarrow$ Renaissance in hadron spectroscopy ...
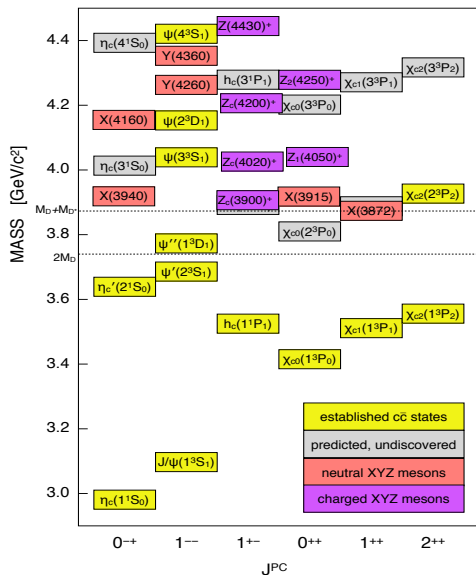
S. L. Olsen, Front. Phys. (Beijing) **10**, 121 (2015) [arXiv:1411.7738].

R. F. Lebed, R. E. Mitchell, E. S. Swanson, Prog. Part. Nucl. Phys. **93**, 143 (2017) [arXiv:1610.04528].

A. Esposito, A. Pilloni, A. D. Polosa, Phys. Rept. **668**, 1 (2016) [arXiv:1611.07920].

A. Ali, J. S. Lange, S. Stone, Prog. Part. Nucl. Phys. **97**, 123 (2017) [arXiv:1706.00610].

S. L. Olsen, T. Skwarnicki and D. Zieminska, Rev. Mod. Phys. **90**, 015003 (2018) [arXiv:1708.04012].

F.-K. Guo, C. Hanhart, Ulf-G. Meißner, Q. Wang, Q. Zhao, and B.-S. Zou. Rev. Mod. Phys. **90**, 015004 (2018).

Charged states invite tetraquark interpretations

Lo-o-o-o-ng history, dating to foundational papers of the quark model

G. Zweig, "An SU(3) model for strong interaction symmetry and its breaking," CERN-TH-401 (1964); "An SU(3) model for strong interaction symmetry and its breaking. 2," CERN-TH-412 (1964).

M. Gell-Mann, "A schematic model of baryons and mesons," Phys. Lett. **8**, 214–215 (1964).

Application to (light-)meson spectroscopy: broad scalars $a_0(980)$, $f_0(980)$

R. L. Jaffe, "Multi-Quark Hadrons. 1. The Phenomenology of $(q^2 \bar{q}^2)$ Mesons," Phys. Rev. D **15**, 267 (1977); "Multi-Quark Hadrons. 2. Methods," Phys. Rev. D **15**, 281 (1977).

Tetraquark interpretations of XYZ complicated by many thresholds

Tetraquark advocate: L. Maiani, "Exotic Hadrons," CERN *Heavy-hadron Spectroscopy,* July 2017

Can we unambiguously demonstrate the reality of tetraquarks?

Heavy-quark symmetry implies stable heavy tetraquark mesons $Q_i Q_j \bar{q}_k \bar{q}_l$

In the limit of very heavy quarks Q, novel narrow doubly heavy tetraquark states (DHTQ) must exist.

HQS relates DHTQ mass to masses of a doubly heavy baryon, heavy-light baryon, and heavy-light meson.

The lightest double-beauty states composed of $bb\bar{u}\bar{d}$, $bb\bar{u}\bar{s}$, and $bb\bar{d}\bar{s}$ will likely be stable against strong decays.

Heavier $bb\bar{q}_k\bar{q}_l$ states, $cc\bar{q}_k\bar{q}_l$ states, and mixed $bc\bar{q}_k\bar{q}_l$ states, will likely dissociate into pairs of heavy-light mesons. Some might be seen as "double-flavor" resonances near threshold.

Observing a weakly decaying double-beauty state would establish the existence of tetraquarks and illuminate the role of heavy color-$\bar{\mathbf{3}}$ diquarks as hadron constituents.

When tetraquarks resemble the helium atom . . .

Factorized system: separate dynamics for compact "nucleus," light quarks
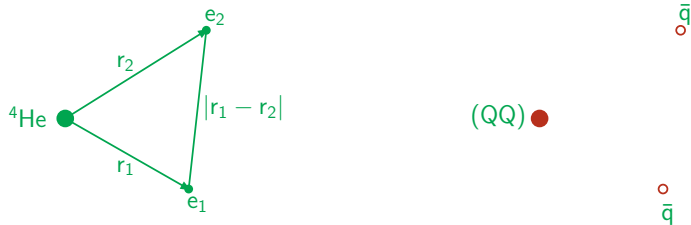
(Attractive, repulsive) one-gluon exchange for (QQ) in color-$(\bar{\mathbf{3}}, \mathbf{6})$

$\bar{\mathbf{3}}$ *half strength of* $Q\bar{Q}$ *attraction in color-*$\mathbf{1}$

also for string tension [Nakamura & Saito]

In heavy limit, idealize a stationary, structureless (color) charge

Stability in the heavy-quark limit

1) *Dissociation into two heavy-light mesons is kinematically forbidden.*

$$\mathcal{Q} \equiv m(Q_i Q_j \bar{q}_k \bar{q}_l) - [m(Q_i \bar{q}_k) + m(Q_j \bar{q}_l)] =$$

$$\underbrace{\Delta(q_k, q_l)}_{\text{light d.o.f.}} - \tfrac{1}{2}\left(\tfrac{2}{3}\alpha_{\mathsf{s}}\right)^2 [1 + O(v^2)]\overline{M} + O(1/\overline{M}) \,,$$

$\overline{M} \equiv (1/m_{Q_i} + 1/m_{Q_j})^{-1}$: reduced mass of Q_i and Q_j

$\Delta(q_k, q_l) \xrightarrow{\overline{M} \to \infty}$ independent of heavy-quark masses

For large enough $\overline{M}$, QQ Coulomb binding dominates, $\boxed{\mathcal{Q} < 0}$

Stability in the heavy-quark limit

2) *Decay to doubly heavy baryon and light antibaryon?*

$$(Q_i Q_j \bar{q}_k \bar{q}_l) \rightarrow (Q_i Q_j q_m) + (\bar{q}_k \bar{q}_l \bar{q}_m)$$

Core $Q_i Q_j$ is color-$\bar{\mathbf{3}}$, same as $\bar{Q}_x$. Up to contributions from Q motion and spin interactions,

$$m(Q_i Q_j \bar{q}_k \bar{q}_l) - m(Q_i Q_j q_m) = m(Q_x q_k q_l) - m(Q_x \bar{q}_m)$$

(spin configurations matter)   RHS has generic form $\Delta_0 + \Delta_1 / M_{Q_x}$

Using $m(\Lambda_c) - m(D) = 416.87$ MeV and $m(\Lambda_b) - m(B) = 340.26$ MeV, we estimate $\Delta_0 \approx 330$ MeV (asymptotic mass difference).

$$\boxed{\text{All} < m(\bar{p}) = 938 \text{ MeV}}$$

No open strong decay channels in the heavy-quark limit!
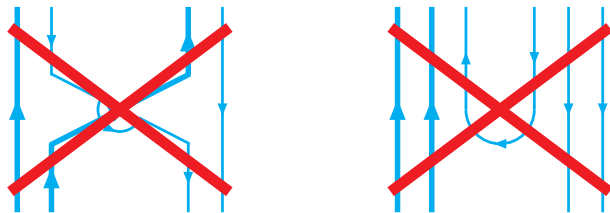
As $\overline{M} \to \infty$, stable $Q_i Q_j \bar{q}_k \bar{q}_l$ mesons must exist

Implications for the real world?

Does a tiny quasistatic diquark core make sense in our world?

At large $Q_i - Q_j$ separations, $\bar{q}_k \bar{q}_l$ cloud screens $Q_i Q_j$ interaction
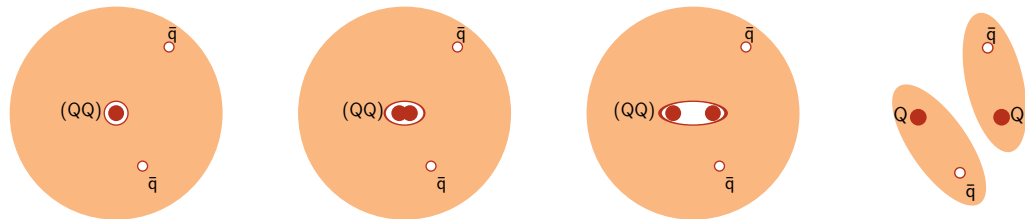
Growing separation alters $\bar{\mathbf{3}}, \mathbf{6}$ mix $\rightsquigarrow$ division into heavy–light mesons

In a half-strength Cornell potential, rms core radii are small on tetraquark scale: $\langle r^2 \rangle^{1/2} = 0.28$ fm (cc); 0.24 fm (bc); 0.19 fm (bb).   (lattice, too)

$\therefore$ core-plus-light (anti)quarks idealization should be reliable.

Mass estimates (beyond the heavy-quark limit . . .)

Use heavy-quark-symmetry relations,

$$
\begin{aligned}
m(\{Q_iQ_j\}\{\bar{q}_k\bar{q}_l\}) - m(\{Q_iQ_j\}q_y) &= m(Q_x\{q_kq_l\}) - m(Q_x\bar{q}_y) \\
m(\{Q_iQ_j\}[\bar{q}_k\bar{q}_l]) - m(\{Q_iQ_j\}q_y) &= m(Q_x[q_kq_l]) - m(Q_x\bar{q}_y) \\
m([Q_iQ_j]\{\bar{q}_k\bar{q}_l\}) - m([Q_iQ_j]q_y) &= m(Q_x\{q_kq_l\}) - m(Q_x\bar{q}_y) \\
m([Q_iQ_j][\bar{q}_k\bar{q}_l]) - m([Q_iQ_j]q_y) &= m(Q_x[q_kq_l]) - m(Q_x\bar{q}_y) \,.
\end{aligned}
$$

$$
+ \text{ finite-mass corrections, } \delta m = \mathcal{S}\frac{\vec{S}\cdot\vec{j_\ell}}{2\mathcal{M}} + \frac{\mathcal{K}}{2\mathcal{M}}
$$

(hyperfine + light d.o.f.)   to estimate $Q_iQ_j\bar{q}_k\bar{q}_l$ masses

Masses, etc., for ground-state hadrons containing heavy quarks

State	j_ℓ	Mass ($j_\ell + \frac{1}{2}$)	Mass ($j_\ell - \frac{1}{2}$)	Centroid	Spin Splitting	$\mathcal{S}$ [GeV^2]
$D^{(*)}$ ($c\bar{d}$)	$\frac{1}{2}$	2010.26	1869.59	1975.09	140.7	0.436
$D_s^{(*)}$ ($c\bar{s}$)	$\frac{1}{2}$	2112.1	1968.28	2076.15	143.8	0.446
Λ_c (cud)$_{\bar{\mathbf{3}}}$	0	2286.46	–	–		–
Σ_c (cud)$_{\mathbf{6}}$	1	2518.41	2453.97	2496.93	64.44	0.132
Ξ_c (cus)$_{\bar{\mathbf{3}}}$	0	2467.87	–	–		–
Ξ_c' (cus)$_{\mathbf{6}}$	1	2645.53	2577.4	2622.82	68.13	0.141
Ω_c (css)$_{\mathbf{6}}$	1	2765.9	2695.2	2742.33	70.7	0.146
Ξ_{cc} (ccu)$_{\bar{\mathbf{3}}}$	0	3621.40	–	–		
$B^{(*)}$ ($b\bar{d}$)	$\frac{1}{2}$	5324.65	5279.32	5313.32	45.33	0.427
$B_s^{(*)}$ ($b\bar{s}$)	$\frac{1}{2}$	5415.4	5366.89	5403.3	48.5	0.459
Λ_b (bud)$_{\bar{\mathbf{3}}}$	0	5619.58	–	–		
Σ_b (bud)$_{\mathbf{6}}$	1	5832.1	5811.3	5825.2	20.8	0.131
Ξ_b (bds)$_{\bar{\mathbf{3}}}$	0	5794.5	–	–		
Ξ_b' (bds)$_{\mathbf{6}}$	1	5955.33	5935.02	5948.56	20.31	0.128
Ω_b (bss)$_{\mathbf{6}}$	1		6046.1			
B_c ($b\bar{c}$)	$\frac{1}{2}$	*6329*	6274.9	*6315.4*	*54*	*0.340*

Kinetic-energy shift differs in $Q\bar{q}$ mesons and Qqq baryons ...

Consider $\delta\mathcal{K} \equiv \mathcal{K}_{(ud)} - \mathcal{K}_d$:

$$[m((cud)_{\bar{3}}) - m(c\bar{d})] - [m((bud)_{\bar{3}}) - m(b\bar{d})]$$

$$= \delta\mathcal{K}\left(\frac{1}{2m_c} - \frac{1}{2m_b}\right) = 5.11 \text{ MeV}$$

$$\rightsquigarrow \delta\mathcal{K} = 0.0235 \text{ GeV}^2$$

$$m(\{cc\}(\bar{u}\bar{d})) - m(\{cc\}d): \qquad \frac{\delta\mathcal{K}}{4m_c} = 2.80 \text{ MeV}$$

$$m((bc)(\bar{u}\bar{d})) - m(\{bc\}d): \qquad \frac{\delta\mathcal{K}}{2(m_c + m_b)} = 1.87 \text{ MeV}$$

$$m(\{bb\}(\bar{u}\bar{d})) - m(\{bb\}d): \qquad \frac{\delta\mathcal{K}}{4m_b} = 1.24 \text{ MeV}$$

Small! (only slightly larger than isospin-breaking effects we neglect)

Estimating ground-state tetraquark masses

RHS of

$$m(Q_i Q_j \bar{q}_k \bar{q}_l) - m(Q_i Q_j q_m) = m(Q_x q_k q_l) - m(Q_x \bar{q}_m)$$

is determined from data

One doubly heavy baryon observed, Ξ_{cc}; others from model calculations[*]
LHC*b*: $M(\Xi_{cc}^{++}) = 3621.40 \pm 0.78$ MeV

[*]We adopt Karliner & Rosner, *PRD* **90**, 094007 (2014)

Strong decays $(Q_i Q_j \bar{q}_k \bar{q}_l) \not\rightarrow (Q_i Q_j q_m) + (\bar{q}_k \bar{q}_l \bar{q}_m) \; \forall$ ground states

Must consider decays to pairs of heavy–light mesons case-by-case

Expectations for ground-state tetraquark masses, in MeV

State	J^P	$m(Q_i Q_j \bar{q}_k \bar{q}_l)$	Decay Channel	$\mathcal{Q}$ [MeV]
$\{cc\}[\bar{u}\bar{d}]$	1^+	3978	$D^+ D^{*0}$ 3876	102
$\{cc\}[\bar{q}_k\bar{s}]$	1^+	4156	$D^+ D_s^{*+}$ 3977	179
$\{cc\}\{\bar{q}_k\bar{q}_l\}$	$0^+, 1^+, 2^+$	4146, 4167, 4210	$D^+ D^0, D^+ D^{*0}$ 3734, 3876	412, 292, 476
$[bc][\bar{u}\bar{d}]$	0^+	7229	$B^- D^+ / B^0 D^0$ 7146	83
$[bc][\bar{q}_k\bar{s}]$	0^+	7406	$B_s D$ 7236	170
$[bc]\{\bar{q}_k\bar{q}_l\}$	1^+	7439	$B^* D / B D^*$ 7190/7290	249
$\{bc\}[\bar{u}\bar{d}]$	1^+	7272	$B^* D / B D^*$ 7190/7290	82
$\{bc\}[\bar{q}_k\bar{s}]$	1^+	7445	$D B_s^*$ 7282	163
$\{bc\}\{\bar{q}_k\bar{q}_l\}$	$0^+, 1^+, 2^+$	7461, 7472, 7493	$B D / B^* D$ 7146/7190	317, 282, 349
$\{bb\}[\bar{u}\bar{d}]$	1^+	10482	$B^- \bar{B}^{*0}$ 10603	$\boxed{\mathbf{-121}}$
$\{bb\}[\bar{q}_k\bar{s}]$	1^+	10643	$\bar{B} \bar{B}_s^* / \bar{B}_s \bar{B}^*$ 10695/10691	$\boxed{\mathbf{-48}}$
$\{bb\}\{\bar{q}_k\bar{q}_l\}$	$0^+, 1^+, 2^+$	10674, 10681, 10695	$B^- B^0, B^- B^{*0}$ 10559, 10603	115, 78, 136

Cf. M. Karliner & J. L. Rosner model, Phys. Rev. Lett. **119**, 202001 (2017) [arXiv:1707.07666].
Estimate deeper binding, so additional *bc* and *cc* candidates.

Real-world candidates for stable tetraquarks

$J^P = 1^+$ $\{bb\}[\bar{u}\bar{d}]$ meson, bound by 121 MeV

(77 MeV below $B^-\bar{B}^0\gamma$)

$$\mathcal{T}^{\{bb\}}_{[\bar{u}\bar{d}]}(10482)^- \to \Xi^0_{bc}\bar{p},\ B^-D^+\pi^-,\ \text{and}\ \underbrace{B^-D^+\ell^-\bar{\nu}}_{\text{manifestly weak!}}$$

$J^P = 1^+$ $\{bb\}[\bar{u}\bar{s}]$ and $\{bb\}[\bar{d}\bar{s}]$ mesons, bound by 48 MeV

(3 MeV below $BB_s\gamma$)

$$\mathcal{T}^{\{bb\}}_{[\bar{u}\bar{s}]}(10643)^- \to \Xi^0_{bc}\overline{\Sigma}^- \qquad \mathcal{T}^{\{bb\}}_{[\bar{d}\bar{s}]}(10643)^0 \to \Xi^0_{bc}(\bar{\Lambda}, \overline{\Sigma}^0)$$

SELEX $M(\Xi^+_{cc}) = 3519$ MeV $\rightsquigarrow m(\{cc\}[\bar{u}\bar{d}]) = 3876$ MeV, at threshold for dissociation into a heavy-light pseudoscalar and heavy-light vector. Signatures for weak decay would include $D^+K^-\ell^+\nu$ and $\Xi^+_c\bar{n}$.   ($D^0D^+\gamma$ at 3734 MeV)

Lattice studies also suggest stable double-beauty tetraquarks

P. Bicudo, K. Cichy, A. Peters and M. Wagner, PRD **93**, 034501 (2016)
[arXiv:1510.03441]:
$J^P = 1^+$ $\{bb\}[\bar{u}\bar{d}]$ meson, bound by 90^{+36}_{-43} MeV $\qquad$ static bb, $m_\pi \approx 340$ MeV ...

A. Francis, R. J. Hudspith, R. Lewis and K. Maltman, PRL **118**, 142001 (2017)
[arXiv:1607.05214]: $J^P = 1^+$ $\{bb\}[\bar{u}\bar{d}]$ meson, bound by 189 ± 10 MeV $\qquad$ NRQCD
bb, $m_\pi \approx 164$ MeV ...
$J^P = 1^+$ $\{bb\}[\bar{u}\bar{s}]$ and $\{bb\}[\bar{d}\bar{s}]$ mesons, bound by 98 ± 7 MeV

Unstable doubly heavy tetraquarks

Resonances in "wrong-sign" (double flavor) combinations DD, DB, BB?

$J^P = 1^+ \ \mathcal{T}_{[\bar{d}\bar{s}]}^{\{cc\}^{++}}(4156) \to D^+ D_s^{*+}$: *prima facie evidence* for non-$q\bar{q}$ level

Double charge / double charm

(New kind of resonance: no attractive force at the meson–meson level.)

Also, $1^+ \ \mathcal{T}_{[\bar{q}_k \bar{q}_l]}^{\{bb\}^{++}}(10681)$, $\mathcal{Q} = +78$ MeV $\quad 1^+ \ \mathcal{T}_{[\bar{q}_k \bar{s}]}^{\{bc\}^{++}}(7272)$, $\mathcal{Q} = +82$ MeV

$0^+ \ \mathcal{T}_{[\bar{u}\bar{d}]}^{\{bc\}^{++}}(7229)$, $\mathcal{Q} = +83$ MeV $\quad 1^+ \ \mathcal{T}_{[\bar{u}\bar{d}]}^{\{cc\}^{++}}(3978)$, $\mathcal{Q} = +102$ MeV

Aside: $^3D_3$ and $^3F_4$ $c\bar{c}$ mesons still to be found in $D\bar{D}$, etc.

Production of stable tetraquarks?

Undoubtedly rare! We offer no calculation, but note

- Large yield of B_c in LHCb: 8995 ± 103 $B_c \to J/\psi\mu\nu_\mu X$ candidates in 2 fb^{-1} pp collisions at 8 TeV
- CMS observation of double-Υ production in 8-TeV pp collisions: $\sigma(pp \to \Upsilon\Upsilon + \text{ anything}) = 68 \pm 15$ pb

Ultimate search instrument? Future e^+e^- Tera-Z factory
Branching fractions $Z \to b\bar{b} = 15.12 \pm 0.05\%$, $b\bar{b}b\bar{b} = (3.6 \pm 1.3) \times 10^{-4}$

$\rightsquigarrow$ many events containing multiple heavy quarks

Homework for experiment

Look for double-flavor resonances near threshold.

Discover and determine masses of doubly-heavy baryons.
  needed to implement HQS calculation of tetraquark masses
  intrinsic interest in these states: comparison with heavy–light mesons,
possible core excitations

Resolve Ξ_{cc} uncertainty (SELEX/LHCb)

Find stable tetraquarks through weak decays. Lifetime: $\sim 1/3$ ps ??

Homework for theory

Develop expectations for production.

Refine lifetime estimates for stable states.

Understand how color configurations evolve with QQ (and $\bar{q}\bar{q}$) masses.

Investigate stability of different body plans in the heavy-quark limit.
  ... up to $(Q_iQ_j)(Q_kQ_l)(Q_mQ_n)$: $B = 2$, but $Q_pQ_qQ_r$ color structure?

Other $Q_i Q_j \bar{q}_k \bar{q}_l$ configurations

All quarks heavy, one-gluon exchange prevails: No stable $QQ\bar{Q}\bar{Q}$ (equal-mass) tetraquarks in very-heavy-quark limit. Support for binding of $bb\bar{q}\bar{q}$. Study N_c dependence.

A. Czarnecki, B. Leng, M. B. Voloshin, "Stability of tetrons," arXiv:1708.04594.

Lattice–NRQCD study of $bb\bar{b}\bar{b}$: No tetraquark with mass below $\eta_b\eta_b$, $\eta_b\Upsilon$, $\Upsilon\Upsilon$ thresholds in $J^{PC} = 0^{++}, 1^{+-}, 2^{++}$ channels.

C. Hughes, E. Eichten, C. T. H. Davies, "The Search for Beauty-fully Bound Tetraquarks Using Lattice Non-Relativistic QCD," arXiv:1710.03236.

Heavy-quark symmetry implies stable heavy tetraquark mesons $Q_i Q_j \bar{q}_k \bar{q}_l$

In the limit of very heavy quarks Q, novel narrow doubly heavy tetraquark states must exist.

Mass estimates lead us to expect that the $J^P = 1^+$ $\{bb\}[\bar{u}\bar{d}]$, $\{bb\}[\bar{u}\bar{s}]$, and $\{bb\}[\bar{d}\bar{s}]$ states should be exceedingly narrow, decaying only through the charged-current weak interaction

Observation would herald a new form of stable matter, in which the doubly heavy color-$\bar{\mathbf{3}}$ $Q_i Q_j$ diquark is a basic building block.

Unstable $Q_i Q_j \bar{q}_k \bar{q}_l$ tetraquarks with small $\mathcal{Q}$-values may be observable as resonant pairs of heavy-light mesons