Heavy-quark symmetry implies stable heavy tetraquark mesons $Q_i Q_j \bar{q}_k \bar{q}_l$

In the limit of very heavy quarks $Q$, novel narrow doubly heavy tetraquark states must exist.

The lightest double-beauty states composed of $bbu\bar{d}$, $bbu\bar{s}$, and $bbd\bar{s}$ will be stable against strong decays.

Heavier $bb\bar{q}_k \bar{q}_l$ states, double-charm states $cc\bar{q}_k \bar{q}_l$, mixed $bc\bar{q}_k \bar{q}_l$ states, will dissociate into pairs of heavy-light mesons.

Observing a weakly decaying double-beauty state would establish the existence of tetraquarks and illuminate the role of heavy color-antitriplet diquarks as hadron constituents.
BELLE observes $\eta_c'(3594)$ in $B \to K\bar{K}_s K^- \pi^+$ decays.

ELQ advocate $B$-meson gateways to missing charmonium levels $h(1^{1}P_1)$, $\eta_2(1^{1}D_2)$, and $\psi_2(1^{3}D_2)$.

BELLE observes $X(3872)$ in $B^+ \to K^+ \pi^+ \pi^- J/\psi$ decays ($D^0\bar{D}^{*0}$ mass!).

Figure 1(b) shows the same distribution for a large sample of generic $B$-$\bar{B}$ Monte Carlo (MC) events. Except for the prominent $J/\psi$ peak, the distribution is smooth and featureless. In the rest of this Letter we use $M(\pi^+\pi^-J/\psi)$ determined from $\Delta M + M_{J/\psi}$, where $M_{J/\psi}$ is the PDG [9] value for the $J/\psi$ mass. The spike at $\Delta M = 0.175$ GeV corresponds to a mass near 3872 MeV.

We make separate fits to the data in the $\psi$ (3580 MeV $< M_{\pi^+\pi^-J/\psi} < 3780$ MeV) and the $M = 3872$ MeV regions. The errors are statistical only.

### Table: Summary of Results

<table>
<thead>
<tr>
<th>Mass (GeV)</th>
<th>$\Delta M$ Error</th>
<th>$M_{J/\psi}$ Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X(3872)$</td>
<td>0.175 MeV</td>
<td>0.1 MeV</td>
</tr>
<tr>
<td>$h(1^{1}P_1)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\eta_2(1^{1}D_2)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\psi_2(1^{3}D_2)$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Since $X(3872)$ is observed as a vector meson, the measured mass is 589 MeV lower than the average of the $h(1^{1}P_1)$, $\eta_2(1^{1}D_2)$, and $\eta_2(1^{3}D_2)$ masses, corresponding to $M_X < M_{J/\psi}$. This mass is less than the PDG [9] systematic error.

The mass of $X(3872)$ is $3872 \pm 1.0$ MeV with a resolution of $0.5$ MeV.

Chris Quigg (Fermilab & TUM)
\( X(3872) \sim \text{Renaissance in hadron spectroscopy} \ldots \)

\[ X(3872) \neq \psi_2(1^3D_2): \quad J^{PC} = 1^{++} \]

- \(c\bar{c}\) state modified by coupling with open channels?
- Threshold “cusp” phenomenon?
  - \(D - \bar{D}^*\) molecule?
  - Tetraquark meson?
- QM superposition of several Fock states
  - Isospin violation likely
- Other new states invite hybrid \((c\bar{c}g)\) interpretations, etc.


Charged states invite tetraquark interpretations

Lo-o-o-o-ng history, dating to foundational papers of the quark model


Application to (light-)meson spectroscopy: broad scalars $a_0(980), f_0(980)$


Tetraquark interpretations of XYZ complicated by many thresholds

Tetraquark advocate: L. Maiani, “Exotic Hadrons,” CERN Heavy-hadron Spectroscopy, July 2017

Can we unambiguously demonstrate the reality of tetraquarks?
When tetraquarks resemble the helium atom . . .

Factorized system: separate dynamics for compact “nucleus,” light quarks

\[ {}^4\text{He} \mid r_1 - r_2 \mid e_1 \]

\[ r_1, r_2 \]

\[ (QQ) \]

\[ \bar{q} q \]

\[ \bar{q} \]

\[ o \]

Attractive one-gluon exchange for \((QQ)\) in color-\(\bar{3}\)

half strength of \(Q \bar{Q}\) attraction in color-\(1\)

also for string tension [Nakamura & Saito]

In heavy limit, idealize a stationary, structureless (color) charge
Stability in the heavy-quark limit

Dissociation into two heavy-light mesons is kinematically forbidden

\[ Q = m(Q_i Q_j \bar{q}_k \bar{q}_l) - [m(Q_i \bar{q}_k) + m(Q_j \bar{q}_l)] = \]
\[ \Delta(q_k, q_l) - \frac{1}{2} \left( \frac{2}{3} \alpha_s \right)^2 [1 + O(v^2)] \overline{M} + O(1/\overline{M}) , \]

light d.o.f.

\[ \overline{M} \equiv \left( \frac{1}{m_{Q_i}} + \frac{1}{m_{Q_j}} \right)^{-1} : \text{reduced mass of } Q_i \text{ and } Q_j \]

\[ \Delta(q_k, q_l) \xrightarrow{\overline{M} \to \infty} \text{independent of heavy-quark masses} \]

For large enough \( \overline{M} \), \( QQ \) Coulomb binding dominates, \( Q < 0 \)
Stability in the heavy-quark limit

Decay to doubly heavy baryon and light antibaryon?

\[
(Q_i Q_j q_k q_l) \rightarrow (Q_i Q_j q_m) + (\bar{q}_k \bar{q}_l \bar{q}_m)
\]

For very heavy quarks, negligible contributions from \(Q\) motion and spin interactions, so (spin configurations matter)

\[
m(Q_i Q_j \bar{q}_k \bar{q}_l) - m(Q_i Q_j q_m) = m(Q_x q_k q_l) - m(Q_x \bar{q}_m)
\]

RHS has generic form \(\Delta_0 + \Delta_1/M_{Q_x}\)

With \(m(\Lambda_c) - m(D) = 416.87\) MeV and \(m(\Lambda_b) - m(B) = 340.26\) MeV, we estimate \(\Delta_0 \approx 330\) MeV (asymptotic mass difference).

\[
\text{All} < m(\bar{p}) = 938\text{ MeV}
\]
No open strong decay channels in the heavy-quark limit!

As $\bar{M} \to \infty$, stable $Q_iQ_j\bar{q}_k\bar{q}_l$ mesons must exist

Implications for the real world?
Does a tiny quasistatic diquark core make sense in this world?

At large $Q_i - Q_j$ separations, $\bar{q}_k \bar{q}_l$ cloud screens $Q_i Q_j$ interaction

\[ \sim \text{rearrangement into heavy–light mesons} \]

In a half-strength Cornell potential, rms core radii are small on tetraquark scale: $\langle r^2 \rangle^{1/2} = 0.28 \text{ fm (cc)}; 0.24 \text{ fm (bc)}; 0.19 \text{ fm (bb)}. \]  

\[ \therefore \text{core-plus-light (anti)quarks idealization should be reliable.} \]
Beyond the heavy-quark limit . . .

Use heavy-quark-symmetry relations,

\[
\begin{align*}
    m(\{Q_iQ_j\}\{\bar{q}_k\bar{q}_l\}) - m(\{Q_iQ_j\}q_y) &= m(Q_x\{q_kq_l\}) - m(Q_x\bar{q}_y) \\
    m(\{Q_iQ_j\}[\bar{q}_k\bar{q}_l]) - m(\{Q_iQ_j\}q_y) &= m(Q_x[Q_kq_l]) - m(Q_x\bar{q}_y) \\
    m([Q_iQ_j]\{\bar{q}_k\bar{q}_l\}) - m([Q_iQ_j]q_y) &= m(Q_x\{q_kq_l\}) - m(Q_x\bar{q}_y) \\
    m([Q_iQ_j][\bar{q}_k\bar{q}_l]) - m([Q_iQ_j]q_y) &= m(Q_x[Q_kq_l]) - m(Q_x\bar{q}_y)
\end{align*}
\]

+ finite-mass corrections, \( \delta m = S \frac{\vec{S} \cdot \vec{j}_\ell}{2\mathcal{M}} + \frac{\mathcal{K}}{2\mathcal{M}} \)

(hyperfine + light d.o.f.) to estimate tetraquark masses
Masses, etc., for ground-state hadrons containing heavy quarks

<table>
<thead>
<tr>
<th>State</th>
<th>( j_\ell )</th>
<th>Mass ( (j_\ell + \frac{1}{2}) )</th>
<th>Mass ( (j_\ell - \frac{1}{2}) )</th>
<th>Centroid</th>
<th>Spin Splitting</th>
<th>( S ) [GeV(^2)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D(*) \ (c\bar{d}) )</td>
<td>( \frac{1}{2} )</td>
<td>2010.26</td>
<td>1869.59</td>
<td>1975.09</td>
<td>140.7</td>
<td>0.436</td>
</tr>
<tr>
<td>( D_s(*) \ (c\bar{s}) )</td>
<td>( \frac{1}{2} )</td>
<td>2112.1</td>
<td>1968.28</td>
<td>2076.15</td>
<td>143.8</td>
<td>0.446</td>
</tr>
<tr>
<td>( \Lambda_c \ (cud) )</td>
<td>0</td>
<td>2286.46</td>
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<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>( \Sigma_c \ (cud) )</td>
<td>1</td>
<td>2518.41</td>
<td>2453.97</td>
<td>2496.93</td>
<td>64.44</td>
<td>0.132</td>
</tr>
<tr>
<td>( \Xi_c \ (cus) )</td>
<td>0</td>
<td>2467.87</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>( \Xi'_c \ (cus) )</td>
<td>1</td>
<td>2645.53</td>
<td>2577.4</td>
<td>2622.82</td>
<td>68.13</td>
<td>0.141</td>
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<tr>
<td>( \Omega_c \ (css) )</td>
<td>1</td>
<td>2765.9</td>
<td>2695.2</td>
<td>2742.33</td>
<td>70.7</td>
<td>0.146</td>
</tr>
<tr>
<td>( \Xi_{cc} \ (ccu) )</td>
<td>0</td>
<td>3621.40</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>( B(*) \ (b\bar{d}) )</td>
<td>( \frac{1}{2} )</td>
<td>5324.65</td>
<td>5279.32</td>
<td>5313.32</td>
<td>45.33</td>
<td>0.427</td>
</tr>
<tr>
<td>( B_s(*) \ (b\bar{s}) )</td>
<td>( \frac{1}{2} )</td>
<td>5415.4</td>
<td>5366.89</td>
<td>5403.3</td>
<td>48.5</td>
<td>0.459</td>
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<tr>
<td>( \Lambda_b \ (bud) )</td>
<td>0</td>
<td>5619.58</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>( \Sigma_b \ (bud) )</td>
<td>1</td>
<td>5832.1</td>
<td>5811.3</td>
<td>5825.2</td>
<td>20.8</td>
<td>0.131</td>
</tr>
<tr>
<td>( \Xi_b \ (bds) )</td>
<td>0</td>
<td>5794.5</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>( \Xi'_b \ (bds) )</td>
<td>1</td>
<td>5955.33</td>
<td>5935.02</td>
<td>5948.56</td>
<td>20.31</td>
<td>0.128</td>
</tr>
<tr>
<td>( \Omega_b \ (bss) )</td>
<td>1</td>
<td>6046.1</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>( B_c \ (b\bar{c}) )</td>
<td>( \frac{1}{2} )</td>
<td>6329</td>
<td>6274.9</td>
<td>6315.4</td>
<td>54</td>
<td>0.340</td>
</tr>
</tbody>
</table>
Kinetic-energy shift differs in $Q\bar{q}$ mesons and $Qqq$ baryons . . .

Consider $\delta K \equiv K_{(ud)} - K_d$:

$$\begin{align*}
[m((cud)\bar{3}) - m(c\bar{d})] - [m((bud)\bar{3}) - m(b\bar{d})] & = \delta K \left( \frac{1}{2m_c} - \frac{1}{2m_b} \right) = 5.11 \text{ MeV} \\
\sim \delta K & = 0.0235 \text{ GeV}^2
\end{align*}$$

$m(\{cc\}(\bar{u}\bar{d})) - m(\{cc\}d) : \quad \frac{\delta K}{4m_c} = 2.80 \text{ MeV}$

$m((bc)(\bar{u}\bar{d})) - m(\{bc\}d) : \quad \frac{\delta K}{2(m_c + m_b)} = 1.87 \text{ MeV}$

$m(\{bb\}(\bar{u}\bar{d})) - m(\{bb\}d) : \quad \frac{\delta K}{4m_b} = 1.24 \text{ MeV}$

Small! (only slightly larger than isospin-breaking effects we neglect)
Estimating ground-state tetraquark masses

RHS of

\[ m(Q_i Q_j \bar{q}_k \bar{q}_l) - m(Q_i Q_j q_m) = m(Q_x q_k q_l) - m(Q_x \bar{q}_m) \]

is determined from data

One doubly heavy baryon observed, \( \Xi_{cc} \); others from model calculations*

LHCb: \( M(\Xi_{cc}^{++}) = 3621.40 \pm 0.78 \) MeV

*We adopt Karliner & Rosner, PRD 90, 094007 (2014)

Strong decays \( (Q_i Q_j \bar{q}_k \bar{q}_l) \rightarrow (Q_i Q_j q_m) + (\bar{q}_k \bar{q}_l \bar{q}_m) \) \( \forall \) ground states

Must consider decays to a pair of heavy–light mesons case-by-case
<table>
<thead>
<tr>
<th>State</th>
<th>$J^P$</th>
<th>$j_\ell$</th>
<th>$m(Q_i Q_j q_m)$</th>
<th>HQS relation</th>
<th>$m(Q_i Q_j \bar{q}_k \bar{q}_l)$</th>
<th>Decay Channel</th>
<th>$Q$ [MeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>{cc}[$u\bar{d}$]</td>
<td>$1^+$</td>
<td>0</td>
<td>3663</td>
<td>$m({cc}u) + 315$</td>
<td>3978</td>
<td>$D^+ D^{*0}$</td>
<td>3876</td>
</tr>
<tr>
<td>{cc}[$q_k \bar{s}$]</td>
<td>$1^+$</td>
<td>0</td>
<td>3764</td>
<td>$m({cc}s) + 392$</td>
<td>4156</td>
<td>$D^+ D^{*-}$</td>
<td>3977</td>
</tr>
<tr>
<td>{cc}[$\bar{q}_k \bar{q}_l$]</td>
<td>$0^+, 1^+, 2^+$</td>
<td>1</td>
<td>3663</td>
<td>$m({cc}u) + 526$</td>
<td>4146, 4167, 4210</td>
<td>$D^+ D^0, D^+ D^{*0}$</td>
<td>3734, 3876</td>
</tr>
<tr>
<td>{bc}[$\bar{u}\bar{d}$]</td>
<td>$0^+$</td>
<td>0</td>
<td>6914</td>
<td>$m({bc}u) + 315$</td>
<td>7229</td>
<td>$B^- D^+/B^0 D^0$</td>
<td>7146</td>
</tr>
<tr>
<td>{bc}[$\bar{q}_k \bar{s}$]</td>
<td>$0^+$</td>
<td>0</td>
<td>7010</td>
<td>$m({bc}s) + 392$</td>
<td>7406</td>
<td>$B_s D$</td>
<td>7236</td>
</tr>
<tr>
<td>{bc}[$\bar{q}_k \bar{q}_l$]</td>
<td>$1^+$</td>
<td>1</td>
<td>6914</td>
<td>$m({bc}u) + 526$</td>
<td>7439</td>
<td>$B^* D/BD^*$</td>
<td>7190/7290</td>
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<tr>
<td>{bc}[$\bar{u}\bar{d}$]</td>
<td>$1^+$</td>
<td>0</td>
<td>6957</td>
<td>$m({bc}u) + 315$</td>
<td>7272</td>
<td>$B^* D/BD^*$</td>
<td>7190/7290</td>
</tr>
<tr>
<td>{bc}[$\bar{q}_k \bar{s}$]</td>
<td>$1^+$</td>
<td>0</td>
<td>7053</td>
<td>$m({bc}s) + 392$</td>
<td>7445</td>
<td>$DB_s^*$</td>
<td>7282</td>
</tr>
<tr>
<td>{bc}[$\bar{q}_k \bar{q}_l$]</td>
<td>$0^+, 1^+, 2^+$</td>
<td>1</td>
<td>6957</td>
<td>$m({bc}u) + 526$</td>
<td>7461, 7472, 7493</td>
<td>$BD/B^* D$</td>
<td>7146/7190</td>
</tr>
<tr>
<td>{bb}[$\bar{u}\bar{d}$]</td>
<td>$1^+$</td>
<td>0</td>
<td>10176</td>
<td>$m({bb}u) + 306$</td>
<td>10482</td>
<td>$B^- \bar{B}^{*0}$</td>
<td>10603</td>
</tr>
<tr>
<td>{bb}[$\bar{q}_k \bar{s}$]</td>
<td>$1^+$</td>
<td>0</td>
<td>10252</td>
<td>$m({bb}s) + 391$</td>
<td>10643</td>
<td>$\bar{B}\bar{B}_s^*$</td>
<td>$\bar{B}_s \bar{B}^*$</td>
</tr>
<tr>
<td>{bb}[$\bar{q}_k \bar{q}_l$]</td>
<td>$0^+, 1^+, 2^+$</td>
<td>1</td>
<td>10176</td>
<td>$m({bb}u) + 512$</td>
<td>10674, 10681, 10695</td>
<td>$B^- B^0, B^- B^{*0}$</td>
<td>10559, 10603</td>
</tr>
</tbody>
</table>
### Expectations for ground-state tetraquark masses, in MeV

<table>
<thead>
<tr>
<th>State (Q_i Q_j \bar{q}_k \bar{q}_l)</th>
<th>(J^P)</th>
<th>(m(Q_i Q_j \bar{q}_k \bar{q}_l))</th>
<th>Decay Channel</th>
<th>(Q) [MeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>{cc}[\bar{u}\bar{d}]</td>
<td>1⁺</td>
<td>3978</td>
<td>(D^+ D^{*0}) 3876</td>
<td>102</td>
</tr>
<tr>
<td>{cc}[\bar{q}_k \bar{s}]</td>
<td>1⁺</td>
<td>4156</td>
<td>(D^+ D_s^{*-}) 3977</td>
<td>179</td>
</tr>
<tr>
<td>{cc}{\bar{q}_k \bar{q}_l}</td>
<td>0⁺, 1⁺, 2⁺</td>
<td>4146, 4167, 4210</td>
<td>(D^+ D^0, D^+ D^0) 3734, 3876</td>
<td>412, 292, 476</td>
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<tr>
<td>{bc}[\bar{u}\bar{d}]</td>
<td>0⁺</td>
<td>7229</td>
<td>(B^- D^+/B^0 D^0) 7146</td>
<td>83</td>
</tr>
<tr>
<td>{bc}[\bar{q}_k \bar{s}]</td>
<td>0⁺</td>
<td>7406</td>
<td>(B_s D) 7236</td>
<td>170</td>
</tr>
<tr>
<td>{bc}{\bar{q}_k \bar{q}_l}</td>
<td>1⁺</td>
<td>7439</td>
<td>(B^* D/BD^*) 7190/7290</td>
<td>249</td>
</tr>
<tr>
<td>{bc}[\bar{u}\bar{d}]</td>
<td>1⁺</td>
<td>7272</td>
<td>(B^* D/BD^*) 7190/7290</td>
<td>82</td>
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<tr>
<td>{bc}[\bar{q}_k \bar{s}]</td>
<td>1⁺</td>
<td>7445</td>
<td>(DB_s^*) 7282</td>
<td>163</td>
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<tr>
<td>{bc}{\bar{q}_k \bar{q}_l}</td>
<td>0⁺, 1⁺, 2⁺</td>
<td>7461, 7472, 7493</td>
<td>(BD/B^* D) 7146/7190</td>
<td>317, 282, 349</td>
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<tr>
<td>{bb}[\bar{u}\bar{d}]</td>
<td>1⁺</td>
<td>10482</td>
<td>(B^- \bar{B}^{*0}) 10603</td>
<td>-121</td>
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<tr>
<td>{bb}[\bar{q}_k \bar{s}]</td>
<td>1⁺</td>
<td>10643</td>
<td>(\bar{B} \bar{B}_s^<em>/\bar{B}_s \bar{B}^</em>) 10695/10691</td>
<td>-48</td>
</tr>
<tr>
<td>{bb}{\bar{q}_k \bar{q}_l}</td>
<td>0⁺, 1⁺, 2⁺</td>
<td>10674, 10681, 10695</td>
<td>(B^- B^0, B^- B^{*0}) 10559, 10603</td>
<td>115, 78, 136</td>
</tr>
</tbody>
</table>

Real-world candidates for stable tetraquarks

\[ J^P = 1^+ \{bb\}[^{\bar{u}\bar{d}}] \text{ meson, bound by } 121 \text{ MeV} \]

\[ T_{[\bar{u}\bar{d}]}^{\{bb\}} (10482)^- \rightarrow \Xi^0_{bc} \bar{p}, \ B^- D^+ \pi^-, \text{ and } B^- D^+ \ell^- \bar{\nu} \]

\[ \text{weak!} \]

\[ J^P = 1^+ \{bb\}[\bar{u}\bar{s}] \text{ and } \{bb\}[\bar{d}\bar{s}] \text{ mesons, bound by } 48 \text{ MeV} \]

\[ T_{[\bar{u}\bar{s}]}^{\{bb\}} (10643)^- \rightarrow \Xi^0_{bc} \Sigma^- \]

\[ T_{[\bar{d}\bar{s}]}^{\{bb\}} (10643)^0 \rightarrow \Xi^0_{bc} (\bar{\Lambda}, \Sigma^0) \]

SELEX \[ M(\Xi_{cc}^+) = 3519 \text{ MeV} \sim m(\{cc\}[\bar{u}\bar{d}]) = 3876 \text{ MeV}, \text{ at threshold for dissociation into a heavy-light pseudoscalar and heavy-light vector. Signatures for weak decay would include } D^+ K^- \ell^+ \nu \text{ and } \Xi_c^+ \bar{n} \quad (D^0 D^+ \gamma \text{ at } 3734 \text{ MeV}) \]
Unstable doubly heavy tetraquarks

Resonances in “wrong-sign” combinations $DD, DB, BB$?

$$\tau^{cc\{++}_{[d\bar{s}]} \rightarrow D^+ D^+_s: \textit{prima facie} \text{ evidence for a non-}q\bar{q} \text{ level}$$
Lattice studies suggest stable double-beauty tetraquarks


$J^P = 1^+ \{ bb \} [\bar{u}d]$ meson, bound by $90^{+36}_{-43}$ MeV \quad static $bb$, $m_\pi \approx 340$ MeV . . .


$J^P = 1^+ \{ bb \} [\bar{u}d]$ meson, bound by $189 \pm 10$ MeV \quad NRQCD $bb$, $m_\pi \approx 164$ MeV . . .

$J^P = 1^+$ \{$bb\} [\bar{u}s]$ and \{$bb\} [\bar{d}s]$ mesons, bound by $98 \pm 7$ MeV
Production of stable tetraquarks?

Undoubtedly rare! We offer no calculation, but note

- Large yield of $B_c$ in LHCb: $8995 \pm 103$ $B_c \rightarrow J/\psi \mu \nu \mu X$ candidates in 2 fb$^{-1}$ $pp$ collisions at 8 TeV

- CMS observation of double-$\Upsilon$ production in 8-TeV $pp$ collisions:
  \[ \sigma(pp \rightarrow \Upsilon \Upsilon + \text{anything}) = 68 \pm 15 \text{ pb} \]

Ultimate search instrument? Future $e^+e^-$ Tera-Z factory

Branching fractions $Z \rightarrow b\bar{b} = 15.12 \pm 0.05\%$, $b\bar{b}b\bar{b} = (3.6 \pm 1.3) \times 10^{-4}$

\[ \sim \text{many events containing multiple heavy quarks} \]
Other $Q_i Q_j \bar{q}_k \bar{q}_l$ configurations

All quarks heavy, one-gluon exchange prevails: No stable $QQ\bar{Q}\bar{Q}$ (equal-mass) tetraquarks in very-heavy-quark limit. Support for binding of $bb\bar{q}\bar{q}$. Study $N_c$ dependence.


Lattice–NRQCD study of $bb\bar{b}\bar{b}$: No tetraquark with mass below $\eta_b\eta_b$, $\eta_b\gamma$, $\gamma\gamma$ thresholds in $J^{PC} = 0^{++}, 1^{+-}, 2^{++}$ channels.

Heavy-quark symmetry implies stable heavy tetraquark mesons $Q_i Q_j \bar{q}_k \bar{q}_l$

In the limit of very heavy quarks $Q$, novel narrow doubly heavy tetraquark states must exist.

Mass estimates lead us to expect that the $J^P = 1^+$ set $\{bb\}[\bar{u}\bar{d}]$, $\{bb\}[\bar{u}\bar{s}]$, and $\{bb\}[\bar{d}\bar{s}]$ states should be exceedingly narrow, decaying only through the charged-current weak interaction.

Observation would herald a new form of stable matter, in which the doubly heavy color-$\mathbf{3}$ $Q_i Q_j$ diquark is a basic building block.

Unstable $Q_i Q_j \bar{q}_k \bar{q}_l$ tetraquarks with small $Q$-values may be observable as resonant pairs of heavy-light mesons.