Why we need precision in Neutrino Physics

Stephen Parke
Fermilab
NOBEL 2015

“for the discovery of neutrino oscillations, which shows that neutrinos have mass”

Takaaki Kajita
SuperKamiokaNDE

Art McDonald
SNO

“for the discovery of neutrino flavor transformations, which shows that neutrinos have mass”

~ vacuum oscillations

Wolfenstein matter effects dominant flavor transformations

See Smirnov arXiv:1609.02386
Mass Found in Elusive Particle; Universe May Never Be the Same

Discovery on Neutrino Rattles Basic Theory About All Matter

By MALCOLM W. BROWNE

TOKYO, Japan, June 5 — In what colleagues hailed as a historic landmark, 130 physicists from 35 research institutions in Japan and the United States announced today that they had found the existence of mass in a notoriously elusive subatomic particle called the neutrino.

The neutrino, a particle that carries no electric charge, is so light that it was assumed for many years to have no mass at all. After today's announcement, cosmologists will have to confront the possibility that much of the mass of the universe is in the form of neutrinos. The discovery will also compel scientists to revise a highly successful theory of the composition of matter known as the Standard Model.

Word of the discovery had drawn some 300 physicists here to discuss neutrino research. Among other things, they said, the findings of neutrino mass might affect theories about the formation and evolution of galaxies and the ultimate fate of the universe. If neutrinos have sufficient mass, their presence throughout the universe would increase the overall mass of the universe, possibly slowing the present expansion.

Others said the newly detected but as yet unmeasured mass of the neutrino must be too small to cause cosmological effects. But whatever the case, there was general agreement here that the discovery will have far-reaching consequences for the investigation of the nature of matter.

Speaking for the collaboration of scientists who discovered the existence of neutrino mass using a huge underground detector called Super-Kamiokande, Dr. Takaaki Kajita of the Institute for Cosmic Ray Research of Tokyo University said that all explanations for the data collect-

http://www-sk.icrr.u-tokyo.ac.jp/nu98/scan/
Zenith angle dependence (Multi-GeV)

- $X^2(\text{shape}) = 2.8/4 \text{ dof}$
- $\frac{U_{\text{Up}}}{D_{\text{Down}}} = 0.93 \pm 0.13 - 0.12$
- $X^2(\text{shape}) = 30/4 \text{ dof}$
- $\frac{U_{\text{Up}}}{D_{\text{Down}}} = 0.54 \pm 0.06 - 0.05$
- ($6.2\sigma!!$)

* Up/Down syst. error for $\mu$-like

Prediction
- Flux calculation $\leq 1\%$
- 1km rock above SK $\leq 1.5\%$
- $1.8\%$

Data
- Energy calib. for $\uparrow \downarrow 0.7\%$
- Non $\nu$ Background $\leq 2\%$
- $2.1\%$
CC: $\nu_e + D \rightarrow p + p + e^-$

NC: $\nu_x + D \rightarrow p + n + \nu_x$

ES: $\nu_e + e^- \rightarrow \nu_e + e^-$

and $\nu_\mu/\tau + e^- \rightarrow \nu_\mu/\tau + e^-$

Beacom and SP: hep-ph/0106128
Neutrinos are Everywhere!

- From Big Bang: 300 nus / cm$^3$
- 2 or more v/c << 1

**SuperNovae**
> $10^{58}$

**Sun's**
~ $10^{38}$ nu/sec

**Daya Bay**
$3 \times 10^{21}$ nu/sec

Neutrinos are Forever!!!
(except for the highest energy neutrino's)

Therefore in the Universe:
$$\frac{\partial N_\nu}{\partial t} > 0$$
Key Neutrino Questions:

• Nature of Neutrino Mass:
  • 2 comp & L violation (Majorana)
  • or 4 comp & L conserved (Dirac)

• Neutrino Standard Model:
  • Perform stringent tests 3 nu paradigm: check unitarity, ...
  • Determine size and sign of CPV
  • Determine atmospheric mass ordering
  • Does $\nu_\mu$ or $\nu_\tau$ dominate $\nu_3$ ( $\theta_{23} < \pi/4$ )

• Beyond 3 nus:
  • Steriles, Non-Standard Interactions, Lorentz violation, nuBSM, ....
Neutrino Flavor or Interaction States:

\[ W^+ \rightarrow e^+ \nu_e \quad W^+ \rightarrow \mu^+ \nu_\mu \quad W^+ \rightarrow \tau^+ \nu_\tau \]

provided \( L/E \ll 0.5 \text{ km/MeV} = 500 \text{ km/GeV} \) !!!

\( \sim 1 \) picosecond in Neutrino rest frame !!!

\( \sim \) Age of Universe / Avogadro's \#
Neutrino Mass Eigenstates or Propagation States:

\[ \nu_e = \]

Propagator: \( \nu_j \to \nu_k = \delta_{jk} e^{-i \left( \frac{m_j^2 L}{2E_{\nu}} \right)} \)

Use \( \nu_e \) content to label these states
Neutrino Mass Eigenstates or Propagation States:

$\nu_1$ most $\nu_e$

$\nu_2$

$\nu_3$ least $\nu_e$

$\nu_e =$

Solar Exp, SNO
KamiLAND
Daya Bay, RENO, ...

$\nu_\mu =$

SuperK, K2K, T2K
MINOS, NOvA
ICECUBE

$\nu_\tau =$

Unitarity
SK, Opera
ICECUBE ?
Neutrinos are fundamental particles, which cannot be broken down into smaller bits. They are produced in the nuclear reactions in the sun, particle decays in the Earth, and the explosions of stars. They are also produced by particle accelerators and in nuclear power plants.

Neutrinos are difficult but not impossible to catch. Scientists have developed many different types of particle detectors to study them.

Of all particles with mass, neutrinos are the most abundant in nature. They're also some of the least interactive. Roughly a thousand trillion of them pass harmlessly through your body every second.

Neutrinos are mysterious. Experiments seem to hint at the possible existence of a fourth type of neutrino: a sterile neutrino, which would interact even more rarely than the others.

Scientists also wonder if neutrinos are their own antiparticles. If they are, they could have played a role in the early universe, right after the big bang, when matter came to outnumber antimatter just enough to allow us to exist.

Neutrinos weigh almost nothing, and they travel close to the speed of light. Neutrino masses are so small that so far no experiment has succeeded in measuring them. The masses of other fundamental particles come from the Higgs field, but neutrinos might get their masses another way.

One of the strangest aspects of neutrinos is that they don't pick just one flavor and stick to it. They oscillate as the neutrino types change. Neutrinos come in three types, called flavors. There are electron neutrinos, muon neutrinos, and tau neutrinos. One of the strangest things about neutrinos is that they don't obey the rules of quantum mechanics, which is something you might have noticed if you've ever quantum entangled yourself, which is illegal. A unitary matrix links the flavors to their masses.
\[ U_{\mu_1} \quad W^+ \quad V_{td} \quad W^+ \]

\[ \nu_1 \quad \mu^- \quad t \quad d \]

Rates: \[ |U_{\mu_1}|^2 \quad \& \quad |V_{td}|^2 \]
\[
\begin{pmatrix}
U_{e1} & U_{e2} & U_{e3} \\
U_{\mu1} & U_{\mu2} & U_{\mu3} \\
U_{\tau1} & U_{\tau2} & U_{\tau3}
\end{pmatrix}
= U_{23}(\theta_{23}, 0) U_{13}(\theta_{13}, \delta) U_{12}(\theta_{12}, 0)
\]

Why this order ???

\[
= \begin{pmatrix}
1 & 0 & 0 \\
0 & c_{23} & s_{23} \\
0 & -s_{23} & c_{23}
\end{pmatrix}
\begin{pmatrix}
c_{13} & 0 & s_{13} e^{i\delta_{cp}} \\
0 & 1 & 0 \\
-s_{13} e^{-i\delta_{cp}} & 0 & c_{13}
\end{pmatrix}
\begin{pmatrix}
c_{12} & s_{12} & 0 \\
-s_{12} & c_{12} & 0 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
e^{i\eta_1} & 0 & 0 \\
0 & e^{i\eta_2} & 0 \\
0 & 0 & 1
\end{pmatrix}
\]

Disappearance:
\[
\nu_{\mu} \rightarrow \nu_{\mu} \quad \nu_{e} \rightarrow \nu_{e} \quad \nu_{e} \rightarrow \nu_{e}
\]
500 km/GeV \quad 500 km/GeV \quad 15 km/MeV

Appearance:
\[
\nu_{\mu} \rightarrow \nu_{e}
\]
500 km/GeV
unitary matrix

\[
\begin{pmatrix}
  c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\
  -c_{23}s_{12} - s_{13}s_{23}c_{12}e^{i\delta} & c_{23}c_{12} - s_{13}s_{23}s_{12}e^{i\delta} & c_{13}s_{23} \\
  s_{23}s_{12} - s_{13}c_{23}c_{12}e^{i\delta} & -s_{23}c_{12} - s_{13}c_{23}s_{12}e^{i\delta} & c_{13}c_{23}
\end{pmatrix}
\]
$\nu_1$, $\nu_2$ Mass Ordering:

– solar mass ordering

\[ |\Delta m_{21}^2| = |m_2^2 - m_1^2| = 7.5 \times 10^{-5} \text{ eV}^2 \]

\[ L/E = 15 \text{ km/MeV} = 15,000 \text{ km/GeV} \]

SNO  \[ m_2 > m_1 \]
\( \nu_3, \, \nu_1/\nu_2 \) Mass Ordering:

- atmospheric mass ordering

\[
|\Delta m^2_{31}| = |m^2_3 - m^2_1| = 2.5 \times 10^{-3} \text{ eV}^2
\]

\( L/E = 0.5 \text{ km/MeV} = 500 \text{ km/GeV} \)

Unknown: NO\( \nu \)A, JUNO, ICECUBE, DUNE, T2HKK....
Summary:

<table>
<thead>
<tr>
<th>(\sin^2 \theta_{23})</th>
<th>0.40</th>
<th>0.50</th>
<th>0.60</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\nu_3)</td>
<td>![Pie Chart for (\nu_3) at 0.40]</td>
<td>![Pie Chart for (\nu_3) at 0.50]</td>
<td>![Pie Chart for (\nu_3) at 0.60]</td>
</tr>
</tbody>
</table>

\[
\delta = \pm \pi/2
\]

<table>
<thead>
<tr>
<th>(\nu_2)</th>
<th>(\nu_1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>![Pie Chart for (\nu_2)] at (\delta = \pm \pi/2)</td>
<td>![Pie Chart for (\nu_1)] at (\delta = \pm \pi/2)</td>
</tr>
</tbody>
</table>

\[
\nu_2 \text{ variation}
\]

<table>
<thead>
<tr>
<th>(\nu_2)</th>
<th>(\nu_1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>![Pie Chart for (\nu_2)] at (\delta = 0)</td>
<td>![Pie Chart for (\nu_1)] at (\delta = 0)</td>
</tr>
</tbody>
</table>

\[
\nu_1 \text{ variation}
\]

\[
\nu_e = \text{blue}, \quad \nu_\mu = \text{cyan}, \quad \nu_\tau = \text{red}, \quad \pi = \text{black}
\]
Leptons:

\[ U_{\mu 1}, \quad W^+ \]

\[ \nu_1 \rightarrow \mu^- \]

\[ 0.08 < |U_{\mu 1}|^2 < 0.24 \]

variation in \( \delta \) only!

factor of 3 diff.

\[ |U_{\mu 3}|^2 = 0.4 - 0.6 \]
\[ |U_{\mu 2}|^2 = 0.26 - 0.41 \]
\[ |U_{\mu 1}|^2 = 0.08 - 0.24 \]

Quarks:

\[ |V_{ij}|^2 \text{ essentially independent of } \delta_q \]

\[ V_{td} \approx A\lambda^3(1 - 0.37e^{i\delta_q}) \]
\[ |V_{td}|^2 \approx 10^{-4} \]

\[ |V_{tb}|^2 \approx 1 \]
\[ |V_{ts}|^2 \approx \lambda^4 \approx 2 \times 10^{-3} \]
\[ |V_{td}|^2 \approx \lambda^6 \approx 8 \times 10^{-5} \]
Flavor content
NH

\[ |U_{e1}|^2 \quad |U_{e2}|^2 \quad |U_{e3}|^2 \]

\[ |U_{\mu1}|^2 \quad |U_{\mu2}|^2 \quad |U_{\mu3}|^2 \]

\[ |U_{\tau1}|^2 \quad |U_{\tau2}|^2 \quad |U_{\tau3}|^2 \]

Vary $\theta_{ij}, \delta_{CP}$
Best Fit
1$\sigma$
3$\sigma$

\[ \delta \quad \theta_{23} \text{ uncertainty} \]

Flavor content
\[ \theta_{ij} \text{ fixed at BF} \]
NH

\[ |U_{e1}|^2 \quad |U_{e2}|^2 \quad |U_{e3}|^2 \]

\[ |U_{\mu1}|^2 \quad |U_{\mu2}|^2 \quad |U_{\mu3}|^2 \]

\[ |U_{\tau1}|^2 \quad |U_{\tau2}|^2 \quad |U_{\tau3}|^2 \]

Vary $\delta_{CP}$
Best Fit
1$\sigma$
3$\sigma$

\[ \text{no } \theta_{23} \text{ uncertainty} \]
WHY?

To discover neutrino BSM, one needs precision predictions for nuSM

Determine flavor fractions of neutrino mass states

Precision Neutrino Measurements:
Determine flavor fractions of neutrino mass states

Precision Predictions for flavor ratios at ICECUBE.

Bustamante, Beacom, Winter
PRL 2015 [arXiv:1506.02645]
Neutrinos as a portal to new Physics

- ultra-light
- warm
- WIMP
- Dark matter?
- Natural seesaw
- Flavor puzzle
- Leptogenesis
- Unification

ν masses

Coherent ν-N scat.
ννββ
Solar ν
Supernova ν
Geo ν
Reactor ν
Accelerator ν
ν@LHC
Icecube HE neutrinos
Proton decay
ν matter effect

tiny! eV keV MeV GeV TeV PeV YeV
Precision Neutrino Measurements:

WHY?

- Stress Test Neutrino paradigm search for new physics
- Determine flavor fractions of neutrino mass states
- Connection to Leptogenesis
- Understanding Universe
- Test Theoretical Neutrino Models
- Determine flavor fractions of neutrino propagation states
Quarks: ASSUMES UNITARITY

Leptons:

\[ U_{e1} U_{\mu 2}^* + U_{e2} U_{\mu 2}^* + U_{e3} U_{\mu 3}^* = 0 \]

Unitarity Is assumed.

\[
|J| = 2 \times \text{Area} \\
= \left| s_{12} c_{12} s_{23} c_{23} s_{13} c_{13}^2 \sin \delta_{CP} \right|
\]

ASSUMES UNITARITY
high enough energy experiments with both an intense, well known beam of especially as there is little means to directly measure the and complementarity is vital to definitively make a order 1%, will be able to constrain the facility [62], with the uncertainty on their fluxes of the a fully fledged Neutrino Factory [61] or the nuStorm and cross sectional uncertainties are crucial for unitarity in this regime. An understanding of the neutrino flux appears and disappearance bounds. Subsequently, constrained to be unitary at a level half an order of magnitude considers new physics that enters above .

FIG. 3: 1-D 

\( \Delta \chi^2 \) 

|\[ U_{\alpha 1} U_{\beta 1}^* + U_{\alpha 2} U_{\beta 2}^* + U_{\alpha 3} U_{\beta 3}^* \] | or |\[ U_{e1} U_{e1}^* \] |

\( \Delta \chi^2 \)

\( 1 - (|U_{\alpha 1}|^2 + |U_{\alpha 2}|^2 + |U_{\alpha 3}|^2 ) \) or \( 1 - (|U_{e1}|^2 + |U_{\mu 1}|^2 + |U_{\tau 1}|^2 ) \) 

\( \alpha, \beta = e, \mu \) 

\( \alpha, \beta = e, \tau \) 

\( \alpha, \beta = \mu, \tau \) 

Columns 

\( i, j = 1, 2 \) 

\( i, j = 1, 3 \) 

\( i, j = 2, 3 \) 

Rows 

\( \alpha = e \) 

\( \alpha = \mu \) 

\( \alpha = \tau \) 

Normalisations 

\( i = 1 \) 

\( i = 2 \) 

\( i = 3 \) 

M. Ross-Lonergan + SP 
arXiv:1508.05095
a violation of probability in the calculated amplitudes. Lying physics, as non-unitarity directly corresponds to the plethora of successful experiments that have run since (PMNS) neutrino mixing matrix elements comes from the quark sector to a high precision, with the strongest normalisation constraint being the Cabibbo-Kobayashi-Maskawa (CKM) matrix. These so-called sterile neutrinos have been a major discussion point for both the theoretical and experimental tests of unitarity are valid in the quark sector, however, experimental tests of unitarity are

\[ U_{\text{PMNS}}^{\text{Extended}} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \\ U_{s_{n1}} & U_{s_{n2}} & U_{s_{n3}} \end{pmatrix} \]

\[ \sum_{i=1}^{3} |U_{ei}|^2 \leq \left( 1 - \sum_{i=1}^{3} |U_{ei}|^2 \right) \left( 1 - \sum_{i=1}^{3} |U_{\mu i}|^2 \right) \]

- \( \nu_{\mu} \) Disappearance
- \( \nu_e \) Disappearance
- \( \nu_{\mu} \rightarrow \nu_e \) Appearance
- \( \nu_e \) Disappearance
- \( \nu_{\mu} \) Disappearance

MINOS+, NOvA, T2K, atmospheric neutrinos (SK and ICECUBE)

Daya Bay, RENO, many \( \sim 10 \text{m} \) Reactor experiments & source experiments.

- \( \nu_{\mu} \) Disappearance
- \( \nu_e \) Disappearance
- \( \nu_{\mu} \rightarrow \nu_e \) Appearance

Fermilab SBN Program, T2K and NOvA: DUNE & HyperK
**Precision Neutrino Measurements:**

- **CP**
- New Physics

**Stress Test**

- Neutrino paradigm
- Search for new physics

**Connection to**

- Leptogenesis
- Understanding Universe
- Test Theoretical Neutrino Models
- Determine flavor fractions of neutrino propagation states

---

**NSI**

**Figure 6.** Comparisons of the expected sensitivities to NSI parameters at DUNE and T2HK, before and after combining their respective data sets. Darker (Lighter) bands show the results when priors constraints on NSI parameters are (not) included in the fit. The vertical gray areas bounded by the dashed lines indicate the allowed regions at 90% CL (taken from the SNO-DATA lines for $f=u$ in Ref. [54]).

**Conclusions**

Neutrino physics is entering the precision Era. After the discovery of the third mixing angle in the leptonic mixing matrix, and in view of the precision measurements performed by the

---

*P. Coloma*

arXiv:1511.06357
Precision Neutrino Measurements:

### WHY?

- Stress Test Neutrino paradigm search for new physics
- Connection to Leptogenesis
- Understanding Universe
- Test Theoretical Neutrino Models
- Determine flavor fractions of neutrino mass states
Insights on the generation of the matter-antimatter asymmetry

Pedro A. N. Machado | Recent highlights from neutrino theory

which gives the cosmological matter asymmetry.

\[ \sin \theta \]

which gives \( \sin \theta \)

current experimental data. The leptonic Dirac CP violating

scalar VEVs. The predicted CP violation measures in the quar

coe

ξ

this gives a Dirac CP phase,

predicts

combined with

for the Jarlskog invariant,

testable by more precise experimental values for

[18].

mixing angle and the Cabibbo angle in the quark sector, \( \tan \theta \)

As a result of the GJ relations, our model predicts the sum rul

with the current global fit value.) Potential direct measure

Our model predicts (\( \theta_{13} \), \( \Delta m_{21}^2 \), \( \Delta m_{31}^2 \)).

Due to the presence of the doublet representation in

\[ \Delta m \]

Some predictions

1) Dependence on group and fermion representations

2) Dependence on group and fermion representations

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\[ \Delta m \]
Precision Neutrino Measurements:

- Determine flavor fractions of neutrino mass states
- Stress Test Neutrino paradigm search for new physics
- Test Theoretical Neutrino Models
- Connection to Leptogenesis Understanding Universe

WHY?
Precision Neutrino Measurements:

CP, \(\Delta m^2\), ...

Stress Test

Neutrino paradigm search for new physics

Connection to Leptogenesis

Understanding Universe

Test Theoretical Neutrino Models

Determine flavor fractions of neutrino mass states

– Typeset by Foil

WHY MEASURE THESE PARAMETERS?

➤ Lepton mixing allows for a new source of CP violation that can be studied with neutrinos

➤ CPV through \(\delta_{\text{CP}}\) may be sufficient source for leptogenesis (Nucl. Phys. B774 (2007) 1)

➤ Neutrino masses indicate new physics beyond the standard model and electroweak scale

➤ Precise values of the mixing parameters may indicate or disfavor models of flavor symmetries

Predictions from flavor symmetry forms with current measurement precision


6

Predictions from \(\cos \delta\) sum rules for discrete symmetries:

Predictions of flavor symmetry forms with projected measurement precision


Predictions of flavor symmetry forms with projected measurement precision

Stephen Parke, Fermilab

Ballet King Pascoli Prouse Wang 2016
Towards a better understanding of Osc. Prob.

Globes,
while a very useful tool,
is not enough!
Reactor $\theta_{13}$ Experiments

Daya Bay
@ Daya Bay, China

Double Chooz
@ Chooz, France

RENO
@ Yonggwang, Korea
What is $\Delta m^2_{ee}$?

H. Nunokawa, S. J. Parke and R. Zukanovich Funchal,
“Another possible way to determine the neutrino mass hierarchy,”

Daya Bay  
RENO  
D-Chooz  

Amplitude Modulation & Phase Advancement (NO) / Retardation (IO)
What is $\Delta m_{ee}^2$?

from Daya Bay: arXiv:1505.03456

from RENO arXiv:1511.05849

![Graphs and plots showing neutrino oscillation analysis from Daya Bay and RENO experiments.](Image)

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**Stephen Parke, Fermilab**

NBI, colloquium  2/5/2018  # 36
\[ \bar{\nu}_e \rightarrow \bar{\nu}_e \]

- \( P_{ee} \approx 1 - \cos^4 \theta_{13} \sin^2 2 \theta_{12} \sin^2 \Delta_{21} - \sin^2 2 \theta_{13} \sin^2 \Delta_{ee} \]

\[ \Delta \equiv \Delta m^2 L/4E \]

\[ \Delta m^2_{YY} \equiv (\frac{4E}{L}) \arcsin \left[ \sqrt{(\cos^2 \theta_{12} \sin^2 \Delta_{31} + \sin^2 \theta_{12} \sin^2 \Delta_{32})} \right] \]

**DAYA BAY**

\[ \Delta m^2_{ee} \equiv \cos^2 \theta_{12} \Delta m^2_{31} + \sin^2 \theta_{12} \Delta m^2_{32} \]

\[ + \mathcal{O}(10^{-4}) \]

\( \bar{\nu}_e \) average!

3\%


SP arXiv:1601.07464
\[ \nu_\mu \rightarrow \nu_e \quad \overline{\nu}_\mu \rightarrow \overline{\nu}_e \]

\[ \nu_e \rightarrow \nu_\mu \quad \overline{\nu}_e \rightarrow \overline{\nu}_\mu \]

- Running experiments:
  
  \text{T2K (295km) and NOvA (810km)}

- Future experiments:
  
  \text{DUNE (40 ktons LAr, 1300km)}

  \text{HyperKamiokaNDE (0.5kMtons H}_2\text{O, 295km)}

  \text{0.2Mt + T2HKK}
What is DUNE/LBNF?

- DUNE/LBNF will consist of
  - An intense (1-2 MW) neutrino beam from Fermilab
  - A massive (70 kton) deep underground LAr Detector South Dakota
  - A large Near Detector at Fermilab
  - A large International Collaboration (~1000 scientist)

South Dakota

Chicago
DUNE Far Detector site

- Sanford Underground Research Facility (SURF), South Dakota
- Four caverns on 4850ft level (~1.5km underground)

Davis Campus:
- LUX
- Majorana demo.
- ...
- LZ

Yates Complex

Ross Complex
total 70-kt LAr-TPC = 4 x 17-kt modules

Fiducial = 4 x 10 kt

Ar from ~ 10 km$^3$ of air

= 300m × Area of Fermilab site (30 km$^2$)
Neutrino Oscillation Amplitudes

\[ P(\nu_\alpha \rightarrow \nu_\beta) = |A_{\alpha\beta}|^2 \]

Two Flavors:

\[ A_{\alpha\alpha} = 1 + (2i) s^2_\theta e^{+i\Delta} \sin \Delta \]

and \[ A_{\alpha\beta} = (2i) s_\theta c_\theta e^{-i\Delta} \sin \Delta \]

\[ \Delta \equiv \Delta m^2 L/4E \]
Neutrino Oscillation Amplitudes

in vacuum:

“the billion $ process”

\[ P(\nu_\mu \rightarrow \nu_e) = |A_{\mu e}|^2 \]

\[ A_{\mu e} = (2i) \left[ (s_{23}s_{13}c_{13}) \left[ c_{12}e^{-i\Delta_{32}} \sin \Delta_{31} + s_{12}e^{-i\Delta_{31}} \sin \Delta_{32} \right] \right. \]

\[ \left. + (c_{23}c_{13}s_{12}c_{12}) e^{i\delta} \sin \Delta_{21} \right] \]

maintain the symmetry: \( m_1^2 \leftrightarrow m_2^2 \) with \( \theta_{12} \rightarrow \theta_{12} \pm \pi/2 \)

Denton, Minakata, SP arXiv:1604.08167

\[ \Delta P_{CP} = 8 \left( s_{23}s_{13}c_{13} \right) \left( c_{23}c_{13}s_{12}c_{12} \right) \sin \delta \sin \Delta_{21} \sin \Delta_{31} \sin \Delta_{32} \]

\[ \Delta_{32} \approx \Delta_{31} \]

\[ A_{\mu e} \approx (2i) \left[ (s_{23}s_{13}c_{13}) \sin \Delta_{31} + (c_{23}c_{13}s_{12}c_{12}) e^{i(\delta+\Delta_{31})} \sin \Delta_{21} \right] \]
\[ \nu_\mu \rightarrow \nu_e \]

\[ A_{31} = 2s_{23}s_{13}c_{13} \sin \Delta_{31} \]
\[ A_{21} = 2c_{13}c_{23}s_{12}c_{12} \sin \Delta_{21} \]

\[ A_{\mu e} = A_{31} + e^{i(\delta + \Delta_{32})} A_{21} \]

\[ P(\nu_\mu \rightarrow \nu_e) = A_{\mu e} A_{\mu e}^* \]

\[ P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) = \bar{A}_{\mu e}^* \bar{A}_{\mu e} \]

\[ \bar{A}_{\mu e}^* = A_{31} + e^{i(\delta - \Delta_{32})} A_{21} \]

\[ \delta = 0.0 \pi \]
\[ \Delta_{32} = 0.40 \pi \]

\[ \Delta_{ij} = \Delta m_{ij}^2 L/4E \]

\[ |U_{e3}|^2 = 0.51 \quad |U_{e3}|^2 = 0.022 \]

\[ P(\nu_\mu \rightarrow \nu_e) = |\bar{A}_{\mu e}|^2 \]

Denton & Parke

Stephen Parke, Fermilab

NBI, colloquium

2/5/2018  

# 45
Matter Effects:
Neutrino Evolution in Matter:

\[ i \frac{d}{dx} \nu = H \nu \quad \text{with} \quad \nu = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} \]

\[
(2E) \quad H = U_{PMNS} \begin{bmatrix} 0 & 0 & 0 \\ 0 & \Delta m^2_{21} & 0 \\ 0 & 0 & \Delta m^2_{31} \end{bmatrix} U^\dagger_{PMNS} + \begin{bmatrix} a & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}
\]

\[ a = 2\sqrt{2}G_FN_eE \]

\textit{uniform matter}
• Solve Cubic Characteristic Eqn.

\[
\lambda^3 - (a + \Delta m_{21}^2 + \Delta m_{31}^2) \lambda^2 \\
+ [\Delta m_{21}^2 \Delta m_{31}^2 + a \left\{ (c_{12}^2 + s_{12}^2 s_{13}^2) \Delta m_{21}^2 + c_{13}^2 \Delta m_{31}^2 \right\} ] \lambda \\
- c_{12}^2 c_{13}^2 a \Delta m_{21}^2 \Delta m_{31}^2 = 0
\]

IF

• \( a = 0 \)

• or \( \Delta m_{21}^2 = 0 \)

• or \( \sin \theta_{12} = 0 \)

• or \( \sin \theta_{13} = 0 \)

THEN characteristic Eqn

FACTORIZES!

BUT

DOES NOT TRIVIALLY SIMPLIFY!

2 flavor mixing in matter

\[ ax^2 + bx + c = 0 \]

simple, intuitive, useful

3 flavor mixing in matter

\[ ax^3 + bx^2 + cx + d = 0 \]

complicated, counter intuitive, ...
Compact Perturbative Expressions For Neutrino Oscillations in Matter

Peter B. Denton\textsuperscript{a,b}, Hisakazu Minakata\textsuperscript{c,d} Stephen J. Parke\textsuperscript{a}

Addendum to “Compact Perturbative Expressions for Neutrino Oscillations in Matter”

Peter B. Denton\textsuperscript{a}, Hisakazu Minakata\textsuperscript{b}, Stephen J. Parke\textsuperscript{c}

doi: 10.5281/zenodo.1163591
Neutrino Evolution in Matter:

\[ i \frac{d}{dx} \nu = H \nu \quad \text{with} \quad \nu = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} \]

\[
(2E) H = U_{PMNS} \begin{bmatrix} 0 & 0 & 0 \\ 0 & \Delta m_{21}^2 & 0 \\ 0 & 0 & \Delta m_{31}^2 \end{bmatrix} U_{PMNS}^\dagger + \begin{bmatrix} a & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}
\]

\[ a = 2\sqrt{2}G_F N_e E \]

\[ U_{PMNS} \equiv U_{23}(\theta_{23}, 0) \ U_{13}(\theta_{13}, -\delta) \ U_{12}(\theta_{12}, 0) := U_{23}(\theta_{23}, \delta) \ U_{13}(\theta_{13}, 0) \ U_{12}(\theta_{12}, 0) \]

\[ := \] means equal after multiplying by a diagonal phase matrix on the left and/or right hand side.

\[
i \frac{d}{dx} \nu' = U_{23}^\dagger(\theta_{23}, \delta) H U_{23}(\theta_{23}, \delta) \nu' \quad \text{with} \quad \nu' = U_{23}^\dagger(\theta_{23}, \delta) \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} \]
Neutrino Evolution in Matter (conti):

\[ U_{23}^\dagger (\theta_{23}, \delta) \ H \ U_{23}(\theta_{23}, \delta) = \]

\[
\frac{1}{2E} \begin{pmatrix}
    a + s_{13}^2 \Delta m_{31}^2 + s_{12}^2 c_{13}^2 \Delta m_{21}^2 & c_{13} s_{12} c_{12} \Delta m_{21}^2 & s_{13} c_{13} \Delta m_{31}^2 - s_{12}^2 s_{13} c_{13} \Delta m_{21}^2 \\
    c_{13} s_{12} c_{12} \Delta m_{21}^2 & c_{12}^2 \Delta m_{21}^2 & -s_{13} s_{12} c_{12} \Delta m_{21}^2 \\
    s_{13} c_{13} \Delta m_{31}^2 - s_{12}^2 s_{13} c_{13} \Delta m_{21}^2 & -s_{13} s_{12} c_{12} \Delta m_{21}^2 & c_{13}^2 \Delta m_{31}^2 + s_{12}^2 s_{13}^2 \Delta m_{21}^2
\end{pmatrix}
\]

Expansions in \( s_{13} \) \( \sim 0.15 \)

\[ \begin{pmatrix} \Delta m_{21}^2 / \Delta m_{31}^2 \end{pmatrix} \sim 0.03 \]

\[ \begin{pmatrix} a / \Delta m_{31}^2 \end{pmatrix} \sim (E_\nu / 10 GeV) \]

Key observations:

- Don’t use \( \Delta m_{31}^2 \)
- Use \( \Delta m_{ee}^2 = \Delta m_{31}^2 - s_{12}^2 \Delta m_{21}^2 \)
- Subtract \( s_{12}^2 \Delta m_{21}^2 \) from all diagonal elements

Simple but major improvements in accuracy!
Neutrino Evolution in Matter (conti):

\[ U_{23}^\dagger(\theta_{23}, \delta) \, H \, U_{23}(\theta_{23}, \delta) = H_D + H_{OD} \]

\[ D=\text{diagonal} \quad \text{OD= off-diagonal} \]

\[(2E) \, H_D = \begin{bmatrix} a + s_{13}^2 \Delta m_{ee}^2 & (c_{12}^2 - s_{12}^2) \Delta m_{21}^2 \\ (c_{13}^2 \Delta m_{ee}^2) & c_{13}^2 \Delta m_{ee}^2 \end{bmatrix} \]

\[(2E) \, H_{OD}/\Delta m_{ee}^2 = s_{13} c_{13} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + c_{13} \, s_{12} \, c_{12} \left( \frac{\Delta m_{21}^2}{\Delta m_{ee}^2} \right) \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 \end{bmatrix} - s_{13} \, s_{12} \, c_{12} \left( \frac{\Delta m_{21}^2}{\Delta m_{ee}^2} \right) \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 \end{bmatrix} \]

\[\Delta m_{ee}^2 \equiv \cos^2 \theta_{12} \Delta m_{31}^2 + \sin^2 \theta_{12} \Delta m_{32}^2\]
Rotation by $U_{13}(\tilde{\theta}_{13})$

$$a = 2\sqrt{2}G_FN_eE$$

$$\cos 2\tilde{\theta}_{13} = \frac{(\cos 2\theta_{13} - a/\Delta m_{ee}^2)}{\sqrt{(\cos 2\theta_{13} - a/\Delta m_{ee}^2)^2 + \sin^2 2\theta_{13}}}$$

$$\cos 2\tilde{\theta}_{12} = \frac{(\cos 2\theta_{12} - a'/\Delta m_{21}^2)}{\sqrt{(\cos 2\theta_{12} - a'/\Delta m_{21}^2)^2 + \sin^2 2\theta_{12}\cos^2(\tilde{\theta}_{13} - \theta_{13})}}$$

then $U_{12}(\tilde{\theta}_{12})$

$$a' \equiv a \cos^2 \tilde{\theta}_{13} + \Delta m_{ee}^2 \sin^2 (\tilde{\theta}_{13} - \theta_{13})$$

**Graphs:**

1. **NO: Matter Potentials**
   - $a, a'$ vs $E_\nu$ (GeV)
   - $\cos^2\theta_{13} \Delta m_{ee}^2$ line

2. **NO: Mixing Angles in Matter**
   - $\sin^2\tilde{\theta}_{12}$ and $\sin^2\tilde{\theta}_{13}$ curves
   - $E_\nu$ (GeV) axis

**References:**

- [Note 10](#)
- [Note 4](#)
- [Note 0](#)
Masses Squared:

\[(2E) H_D = \text{diag}(\tilde{m}_1^2, \tilde{m}_2^2, \tilde{m}_3^2)\]

\[
\begin{align*}
\tilde{m}_1^2 &= \frac{1}{2}(\Delta m_{21}^2 - \Delta \tilde{m}_{21}^2 + a') \\
\tilde{m}_2^2 &= \frac{1}{2}(\Delta m_{21}^2 + \Delta \tilde{m}_{21}^2 + a') \\
\tilde{m}_3^2 &= \Delta m_{31}^2 + (a - a')
\end{align*}
\]

\[
\Delta \tilde{m}_{21}^2 = \Delta m_{21}^2 \sqrt{(\cos 2\theta_{12} - a'/\Delta m_{21}^2)^2 + \sin^2 2\theta_{12} \cos^2 (\tilde{\theta}_{13} - \theta_{13})} \approx |\Delta m_{21}^2 \cos 2\theta_{12} - a'| \\
\text{when } |a'| \gg \Delta m_{21}^2
\]
Vacuum $\Rightarrow$ Matter

$$\Delta m_{jk}^2 \rightarrow \Delta \tilde{m}_{jk}^2$$

$$\theta_{13} \rightarrow \tilde{\theta}_{13}$$

$$\theta_{12} \rightarrow \tilde{\theta}_{12}$$

$$\theta_{23} \rightarrow \theta_{23}$$

$$\delta \rightarrow \delta$$

0th order!

$$P^\text{vac}_{\nu_\alpha \rightarrow \nu_\beta}(\Delta m_{31}^2, \Delta m_{21}^2, \theta_{13}, \theta_{12}, \theta_{23}, \delta)$$

$$\Rightarrow P^\text{mat}_{\nu_\alpha \rightarrow \nu_\beta}(\Delta \tilde{m}_{31}^2, \Delta \tilde{m}_{21}^2, \tilde{\theta}_{13}, \tilde{\theta}_{12}, \theta_{23}, \delta)$$

Intuitive and Analytically simple!
\[ U_{23}^\dagger(\theta_{23}, \delta) \ H \ U_{23}(\theta_{23}, \delta) = H_D + H_{OD} \]

**What about \( H_{OD} \)?**

\[
(2E) \frac{H_{OD}}{\Delta m_{ee}^2} = \sin(\tilde{\theta}_{13} - \theta_{13}) \ s_{12}c_{12} \ \left( \frac{\Delta m_{21}^2}{\Delta m_{ee}^2} \right) \begin{bmatrix} -\tilde{s}_{12} & \tilde{c}_{12} \\ \tilde{s}_{12} & \tilde{c}_{12} \end{bmatrix}
\]

\[
\sin(\tilde{\theta}_{13} - \theta_{13}) \approx s_{13}c_{13} \left( \frac{\alpha}{\Delta m_{ee}^2} \right) \sim 0.03 \left( \frac{E}{2 \text{ GeV}} \right) \left( \frac{\rho}{3 \text{ g.cm}^{-3}} \right) \ 0.015
\]

 Vanishes in Vacuum

\[
4 \times 10^{-4} \quad \text{for } E = 2 \text{ GeV and } \rho = 3 \text{ g.cm}^{-3}
\]

**Perturbation Theory !!!**
Figure 12. For normal ordering (NO), $\nu_\mu$ appearance: Top Left figure is for T2K, Top Right figure is NOvA, Bottom Left figure is T2HKK, and Bottom Right is DUNE. In each figure, the top panel is exact oscillation probability in matter, $P_{\text{ex mat}}$, $P_{0\text{th appx}}$, and $P_{\text{vac}}$. The Middle panel is difference between exact oscillation probabilities in matter and vacuum ($P_{\text{ex mat}}$ and $P_{\text{vac}}$), and the difference between exact and 0th DMP approximation (solid red) and exact and 1st DMP approximation (solid magenta) approximations. Bottom panel is similar to middle panel but plotting the fractional differences, $\Delta P / P$. T2K/HK, NOvA, 0th order, 1st order, 2nd order, T2HKK, DUNE.
Correlations between $\nu_\mu \rightarrow \nu_e \quad \bar{\nu}_\mu \rightarrow \bar{\nu}_e$

Normal Ordering — Inverted Ordering

$\nu_\mu \rightarrow \nu_\mu$ gives:

$$\sin^2 2\theta_{13} = 4|U_{\mu 3}|^2 (1 - |U_{\mu 3}|^2) = 0.96 - 1.00$$

$|U_{\mu 3}|^2 \leftrightarrow (1 - |U_{\mu 3}|^2)$ degeneracy!

T2K/HK: $L=295$ km, $E=0.65$ GeV

NOvA: $L=810$ km, $E=2.0$ GeV

DUNE: Same L/E as NOvA

$\propto \rho L \ \sin^2 \theta_{23}$

$\sin \delta_{NO} - \sin \delta_{IO} = \tan \theta_{23} \times \begin{cases} 0.48 & \text{T2K} \\ 1.62 & \text{NOvA} \\ 2.60 & \text{DUNE} \end{cases}$

O. Mena & SP  hep-ph/0408070
MCMC analyses including both mass orderings.

TABLE XXVII. Best-fit results and the 1\sigma contours for the parameter of interest. The contours are produced by marginalizing the likelihood with respect to all parameters other than the parameters of interest, for both four-sample (red) and five-sample (black) analyses with normal (left) and inverted (right) mass ordering hypotheses. The red line shows the critical values obtained using T2K data with the reactor constraint. The contour is produced by marginalizing its dependence on this choice of prior has been tested and found to be well-ordered in all cases. The Bayes factor for normal ordering is 6.3, but the upper octant can be computed with the method described in [90]. The Bayes factor for the upper octant is 0.305.

The two sets of intervals are in reasonable agreement.

The Bayes factor for the mass ordering and the number of νe candidates is closer to the maximum violating value of 0.025. The best-fit is closer to that obtained by the reactor experiment, but the proper coverage is taken from Ref. [86].

Inverted Ordering

Normal Ordering

Table XXVIII. Posterior probabilities for the mass ordering parameters using T2K data with the reactor constraint. The Bayes factor for normal ordering is 6.3, but the upper octant can be computed with the method described in [90]. The Bayes factor for the upper octant is 0.305.

The two sets of intervals are in reasonable agreement.
T2K & NOvA

Number of Events proportional to Oscillation Probability

\[ |U_{\mu 3}|^2 = 0.56 \]
\[ |U_{e 3}|^2 = 0.022 \]

\( \delta = 0 \)
\( \pi/2 \)
\( \pi (-\pi) \)
\( 3\pi/2 (-\pi/2) \)

1 sigma: NO IO

Needs an update!
Summary:

- from Nu1998 to now, tremendous exp. progress on Neutrino SM: more at Nu2018

- LSND Sterile Nu’s neither confirmed or ruled out at acceptable CL: - ultra short baseline reactor exp.

- Great Theoretical progress on understand many aspects of Quantum Neutrino Physics: - Oscillations, Decoherence, Osc. Probabilities in Matter, Leptogenesis, ..... 

- Still searching for convincing model of Neutrino masses and mixings: with testable and confirmed predictions!
extras
\[ P(\nu_e \rightarrow \nu_e) = 1 - \sin^2 2\tilde{\theta}_{13} \sin^2 \frac{\Delta \tilde{m}_{ee}^2 L}{4E} - \ldots \]

\[ \sin 2\tilde{\theta}_{13} = \frac{\sin^2 2\theta_{13}}{[(\cos 2\theta_{13} - a/\Delta m_{ee}^2)^2 + \sin^2 2\theta_{13}]} \]

\[ \Delta \tilde{m}_{ee}^2 = \Delta m_{ee}^2 \sqrt{(\cos 2\theta_{13} - a/\Delta m_{ee}^2)^2 + \sin^2 2\theta_{13}} \]

depth of first minimum

\[ \sin 2\theta_{13} \rightarrow \sin 2\tilde{\theta}_{13} \]

energy at first minimum

\[ \frac{\Delta m_{ee}^2 L}{2\pi} \rightarrow \frac{\Delta \tilde{m}_{ee}^2 L}{2\pi} \]
Fig. 13. For normal ordering (NO), $\bar{\nu}_e$ disappearance: Top Left figure is for T2K, Top Right figure is NOvA, Bottom Left figure is T2HKK, and Bottom Right is DUNE. In each figure, the top panel is exact oscillation probability in matter, $P_{\text{ex mat}}$, from $\text{6th order DMP}$ approximation, $P_{\text{0th appx}}$, and the vacuum oscillation probability, $P_{\text{vac}}$ (black dots). The Middle panel is difference between exact oscillation probabilities in matter and vacuum (black dots), and the difference between exact and 0th DMP approximation (solid red) and exact and 1st DMP approximation (solid magenta) approximations. Bottom panel is similar to middle panel but plotting the fractional differences, $P/P$. 

T2K/HK

NOvA

0th order

1st order

2nd order

DUNE

Top panel: $P_{\text{ex mat}}$, $P_{\text{0th appx}}$ and $P_{\text{vac}}$

Middle and bottom panels:

- black dotted lines: $\Delta P = |P_{\text{ex mat}} - P_{\text{vac}}|$
- red solid lines: $\Delta P = |P_{\text{ex mat}} - P_{\text{0th appx}}|$
- magenta solid lines: $\Delta P = |P_{\text{ex mat}} - P_{\text{1st appx}}|$

$\overline{P} = \frac{1}{2}(P_{\text{ex mat}} + P_{\text{vac}})$

$\overline{P} = \frac{1}{2}(P_{\text{ex mat}} + P_{\text{0th appx}})$

$\overline{P} = \frac{1}{2}(P_{\text{ex mat}} + P_{\text{1st appx}})$
Approximately same uncertainty on $\delta$
until **systematic uncertainties** dominate at 1st OM!

ESSnuSB, T2HKK