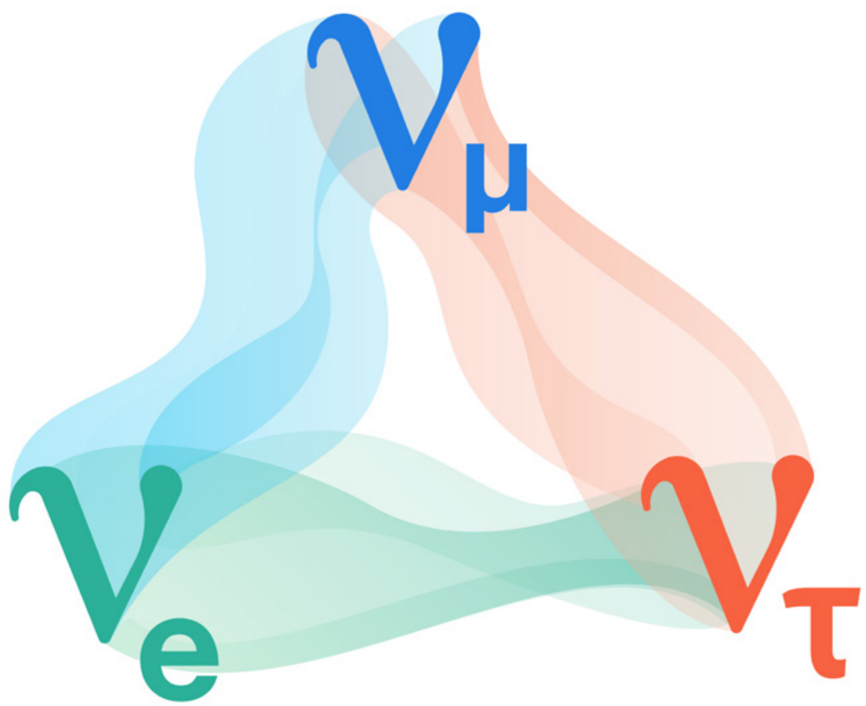
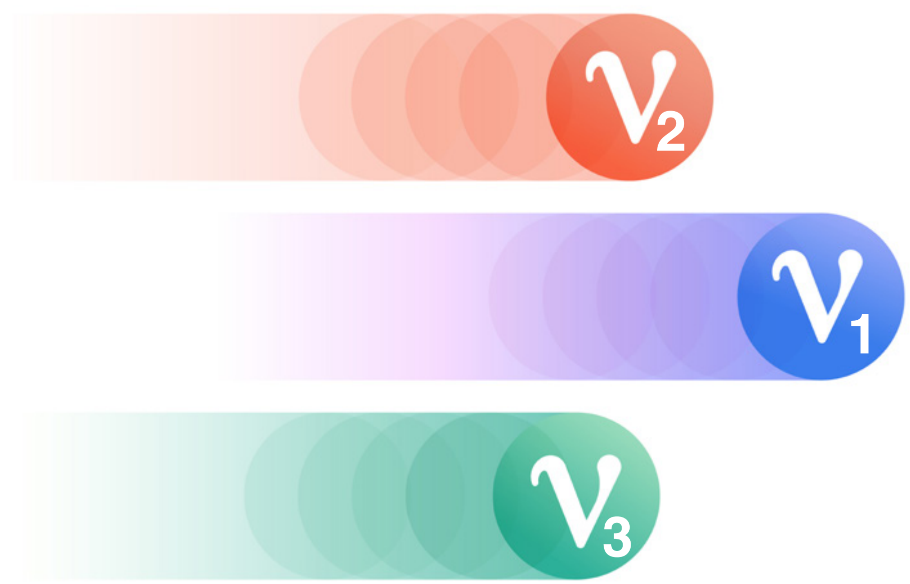
Why we need precision in Neutrino Physics

Stephen Parke
Fermilab



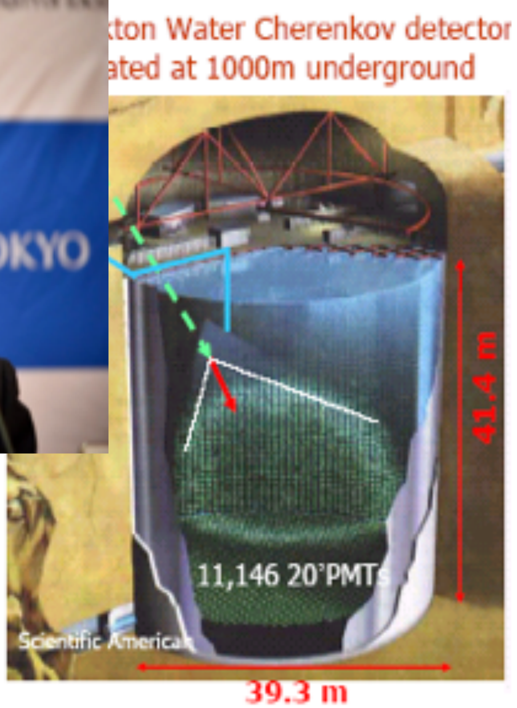


10.5281/zenodo.1173797

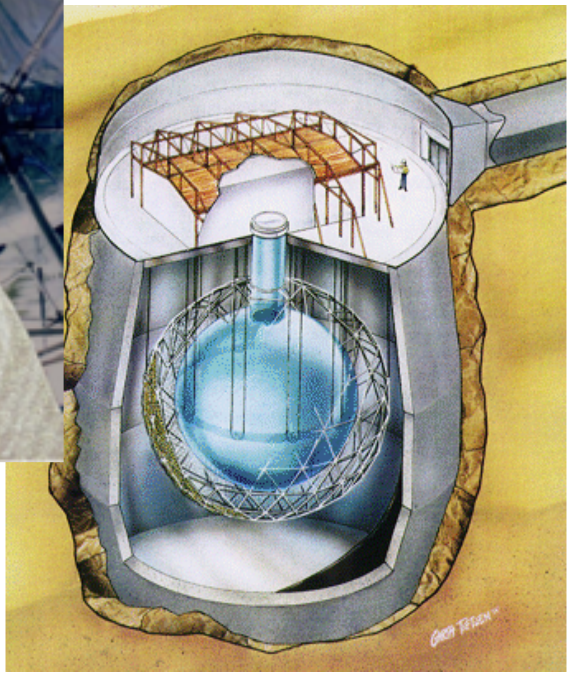
This manuscript has been authored by Fermi Research Alliance, LLC under Contract No. DE-AC02-07CH11359 with the U.S. Department of Energy, Office of Science, Office of High Energy Physics

NOBEL 2015

"for the discovery of neutrino oscillations, which shows that neutrinos have mass"



Takaaki Kajita
SuperKamiokaNDE



Art McDonald
SNO

"for the discovery of neutrino flavor transformations, which shows that neutrinos have mass"

~ vacuum oscillations

Wolfenstein matter effects dominant flavor transformations

See Smirnov arXiv:1609.02386

"All the News That's Fit to Print"

The New York

VOL. CXLVII . . . . No. 51,179 Copyright © 1998 The New York Times FRIDAY, JUNE 5, 1998

19+ years ago

Mass Found in Elusive Particle; Universe May Never Be the Same

Discovery on Neutrino Rattles Basic Theory About All Matter

By MALCOLM W. BROWNE

TAKAYAMA, Japan, June 5 — In what colleagues hailed as a historic landmark, 120 physicists from 23 research institutions in Japan and the United States announced today that they had found the existence of mass in a notoriously elusive subatomic particle called the neutrino.

The neutrino, a particle that carries no electric charge, is so light that it was assumed for many years to have no mass at all. After today's announcement, cosmologists will have to confront the possibility that much of the mass of the universe is in the form of neutrinos. The discovery will also compel scientists to revise a highly successful theory of the composition of matter known as the Standard Model.

Word of the discovery had drawn some 300 physicists here to discuss neutrino research. Among other things, they said, the finding of neutrino mass might affect theories about the formation and evolution of galaxies and the ultimate fate of the universe. If neutrinos have sufficient mass, their presence throughout the universe would increase the overall mass of the universe, possibly slowing its present expansion.

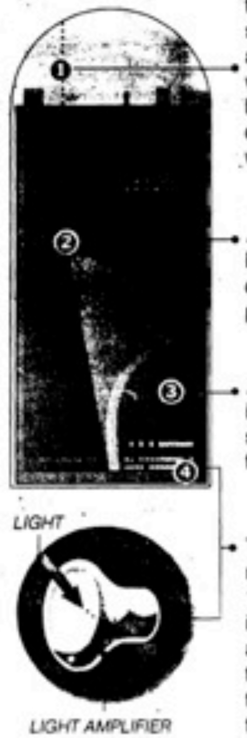
Others said the newly detected but as yet unmeasured mass of the neutrino must be too small to cause cosmological effects. But whatever the case, there was general agreement here that the discovery will have far-reaching consequences for the investigation of the nature of matter.

Speaking for the collaboration of scientists who discovered the existence of neutrino mass using a huge underground detector called Super-Kamiokande, Dr. Takaaki Kajita of the Institute for Cosmic Ray Research of Tokyo University said that all explanations for the data collected by the detector except the existence of neutrino mass had been essentially ruled out.

Dr. Yoji Totsuka, leader of the coalition and director of the Kamioka Neutrino Observatory where the underground detector is situated, 30 miles north of here in the Japan Alps, acknowledged that his group's announcement was "very strong," but said, "We have investigated all

Continued on Page A14

Detecting Neutrinos



1. Neutrinos pass through the Earth's surface to a tank filled with 12.5 million gallons of ultra-pure water . . .
2. . . . and collide with other particles . . .
3. . . . producing a cone-shaped flash of light.
4. The light is recorded by 11,200 20-inch light amplifiers that cover the inside of the tank.

And Detecting Their Mass

By analyzing the cones of light, physicists determine that some neutrinos have changed form on their journey. If they can change form, they must have mass.

Source: University of Hawaii

The New York Times

ν98, @Takayama
June 1998

Atmospheric neutrino results from Super-Kamiokande & Kamiokande

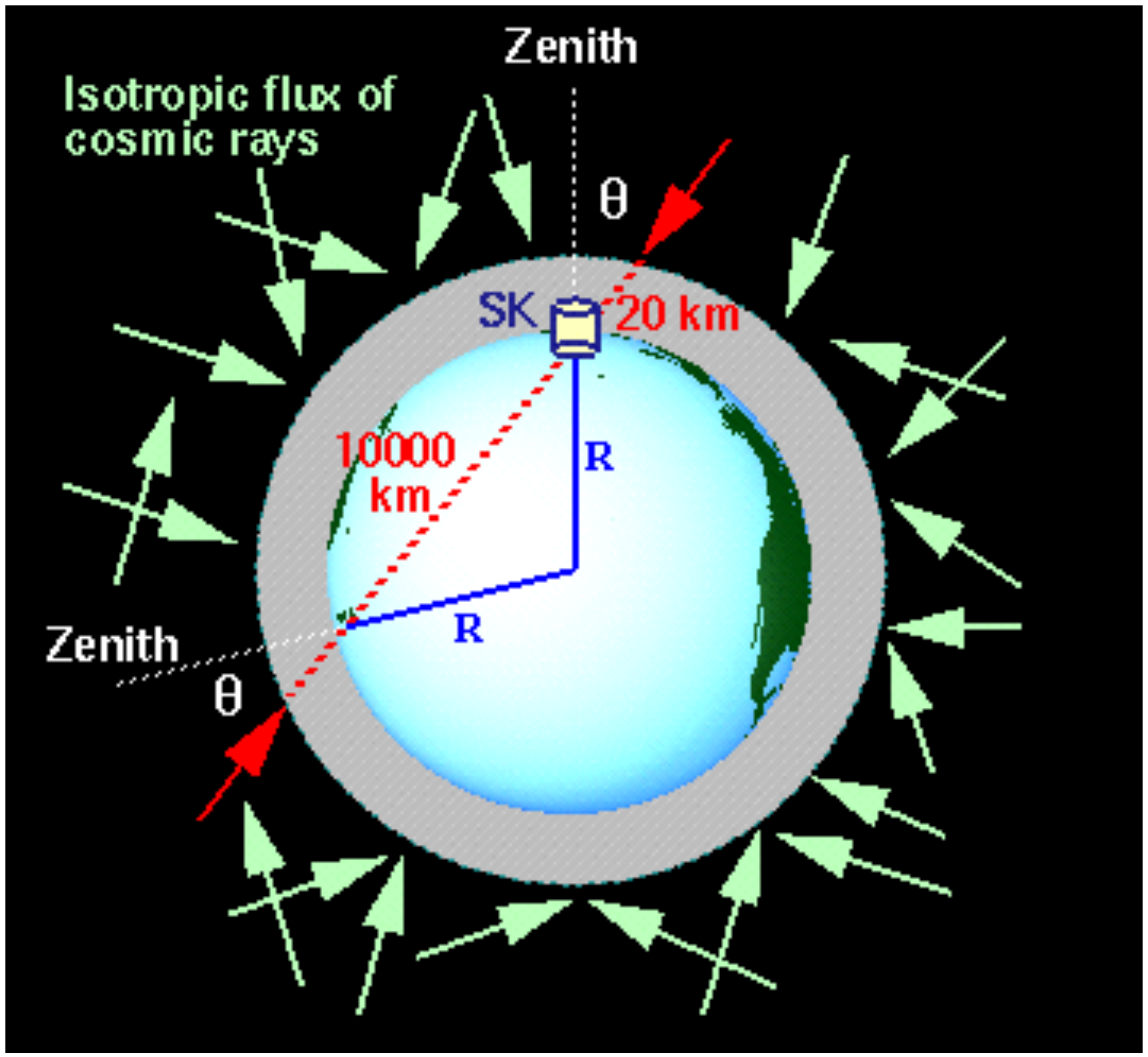
— Evidence for ν_μ oscillations —

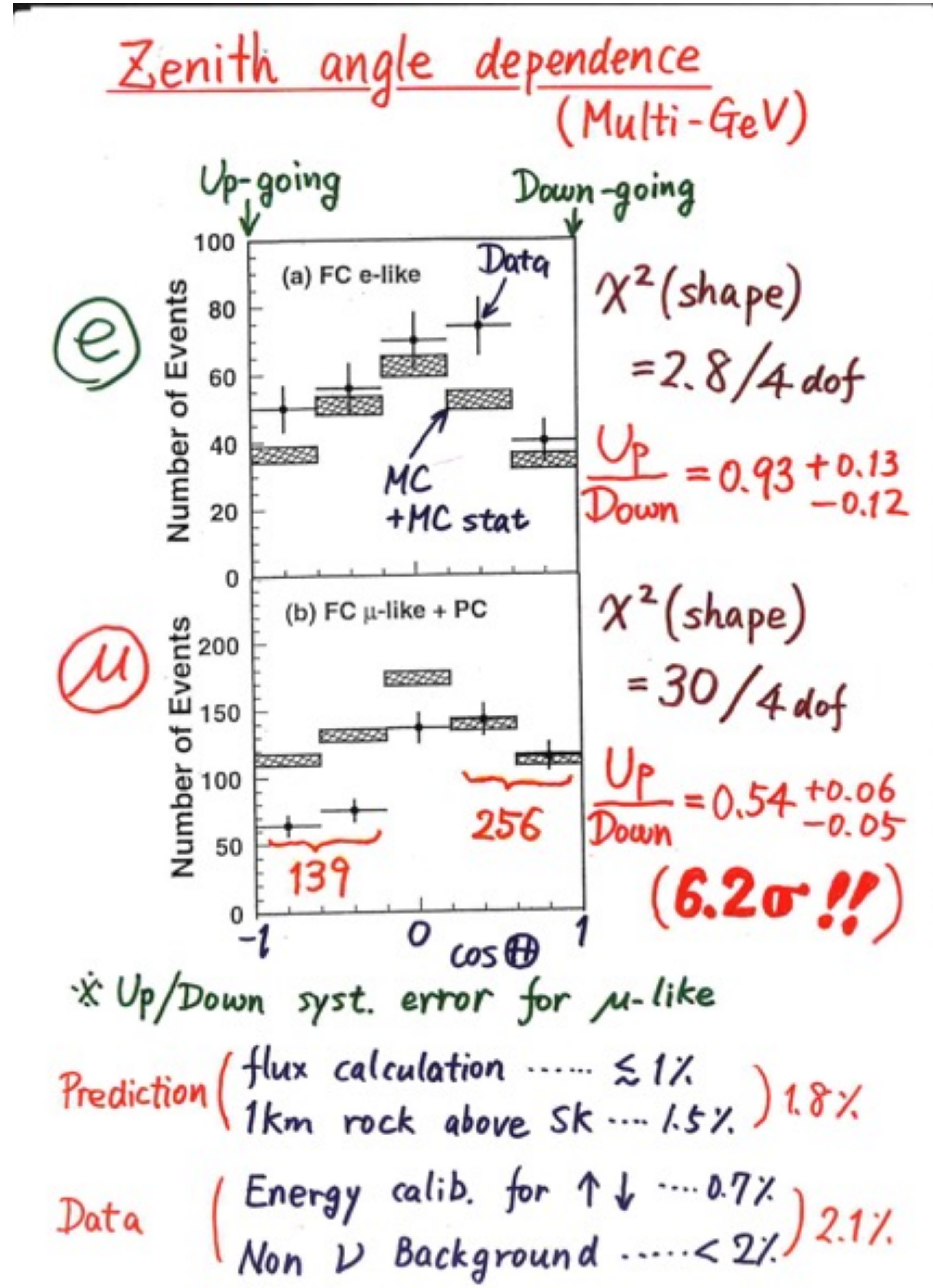
T. Kajita
Kamioka observatory, Univ. of Tokyo

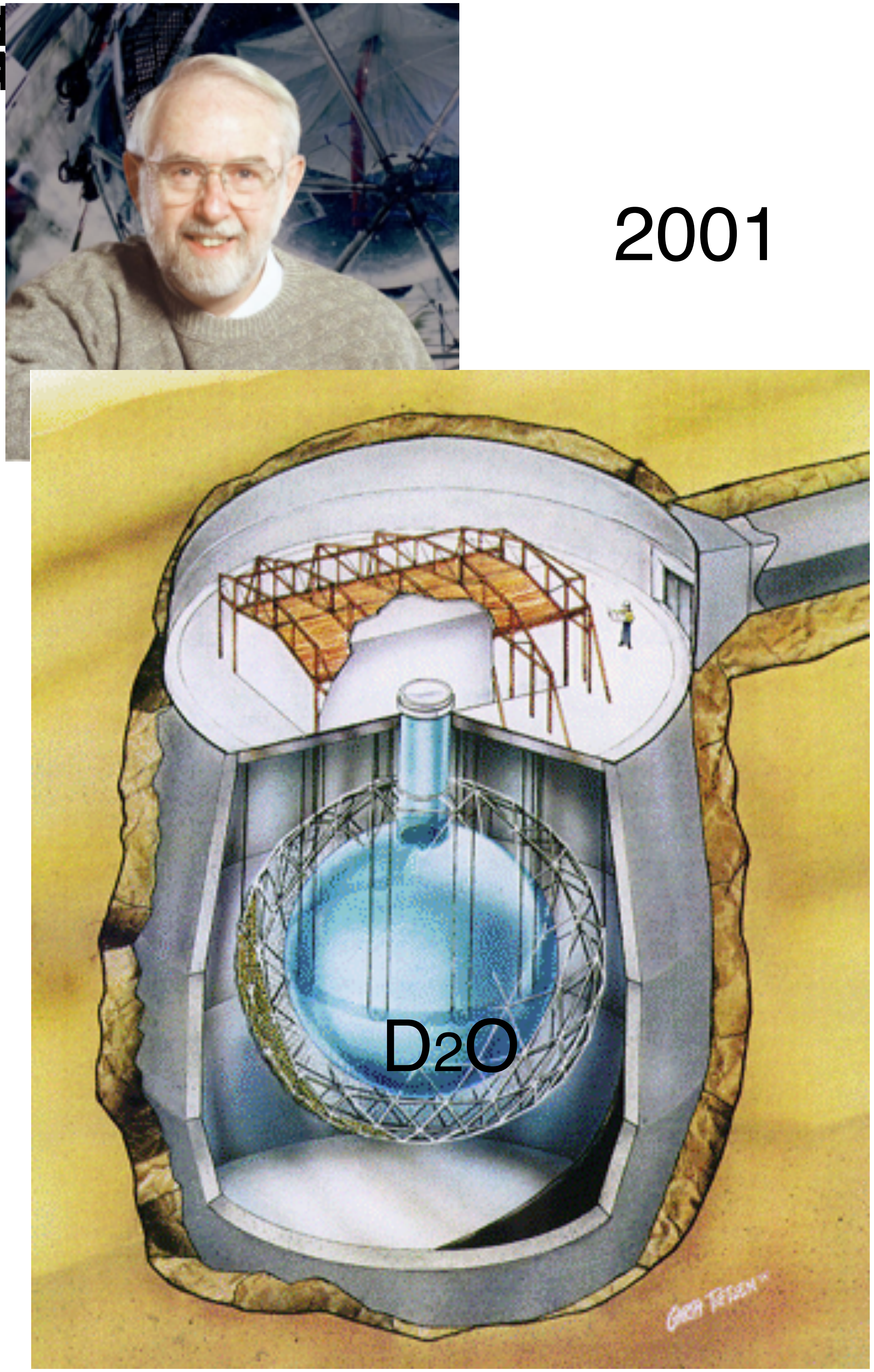
for the {Kamiokande, Super-Kamiokande} Collaborations

http://www-sk.icrr.u-tokyo.ac.jp/nu98/scan/


UTokyo
UTokyo


Zenith
Isotropic flux of cosmic rays
θ
SK
20 km
10000 km
R
R
Zenith
θ


Zenith angle dependence
(Multi-GeV)
Up-going
Down-going
(a) FC e-like
Data
Number of Events
MC +MC stat
e
χ²(shape) =2.8/4 dof
Up/Down =0.93 +0.13 −0.12
(b) FC μ-like + PC
μ
χ²(shape) =30/4 dof
Up/Down =0.54 +0.06 −0.05
(6.2σ !!)
139
256
-1 0 cos Θ 1
※ Up/Down syst. error for μ-like
Prediction (flux calculation …… ≲1% ; 1km rock above SK … 1.5%) 1.8%
Data (Energy calib. for ↑↓ …0.7% ; Non ν Background ……< 2%) 2.1%

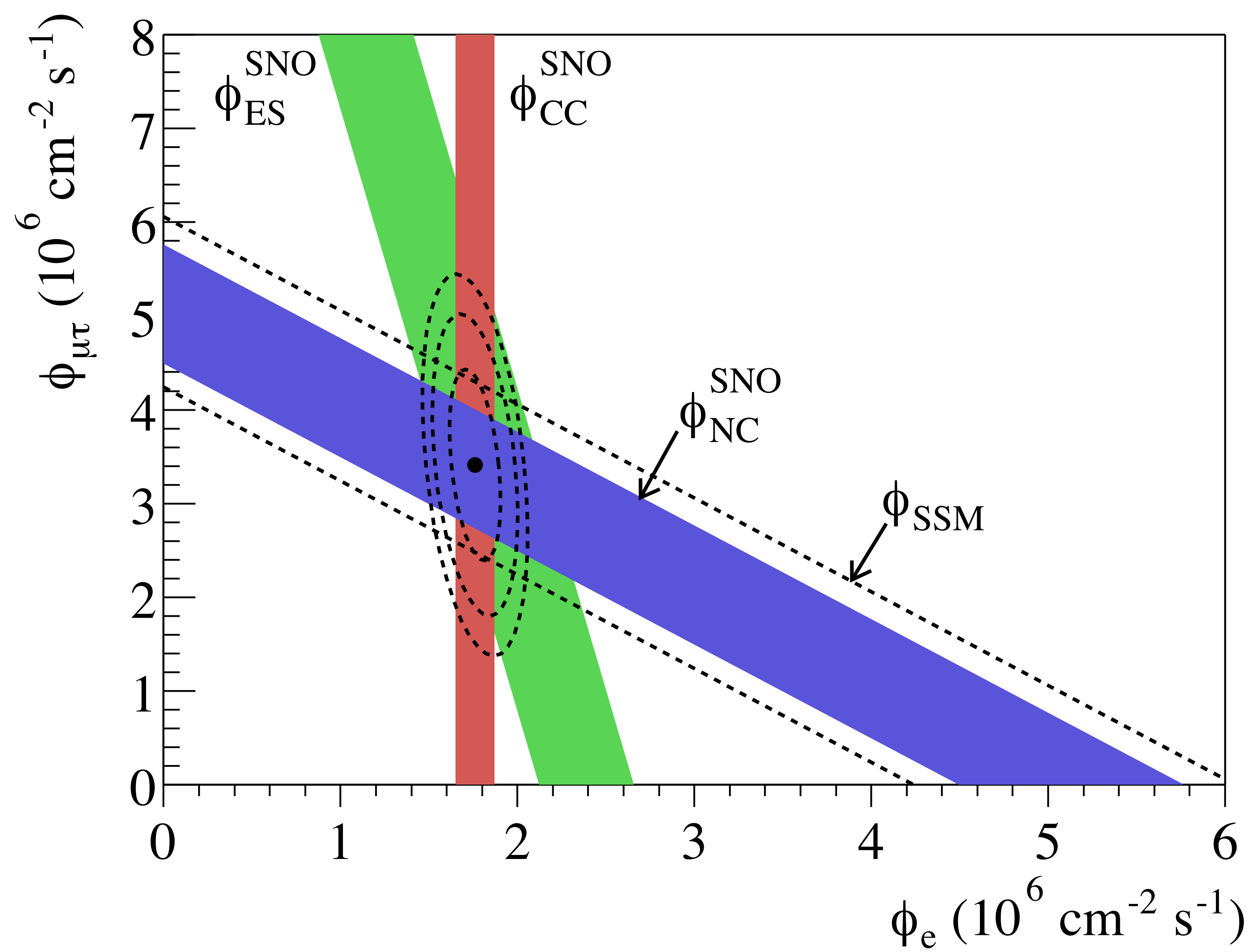


2001

CC: $\nu_e + D \rightarrow p + p + e^-$

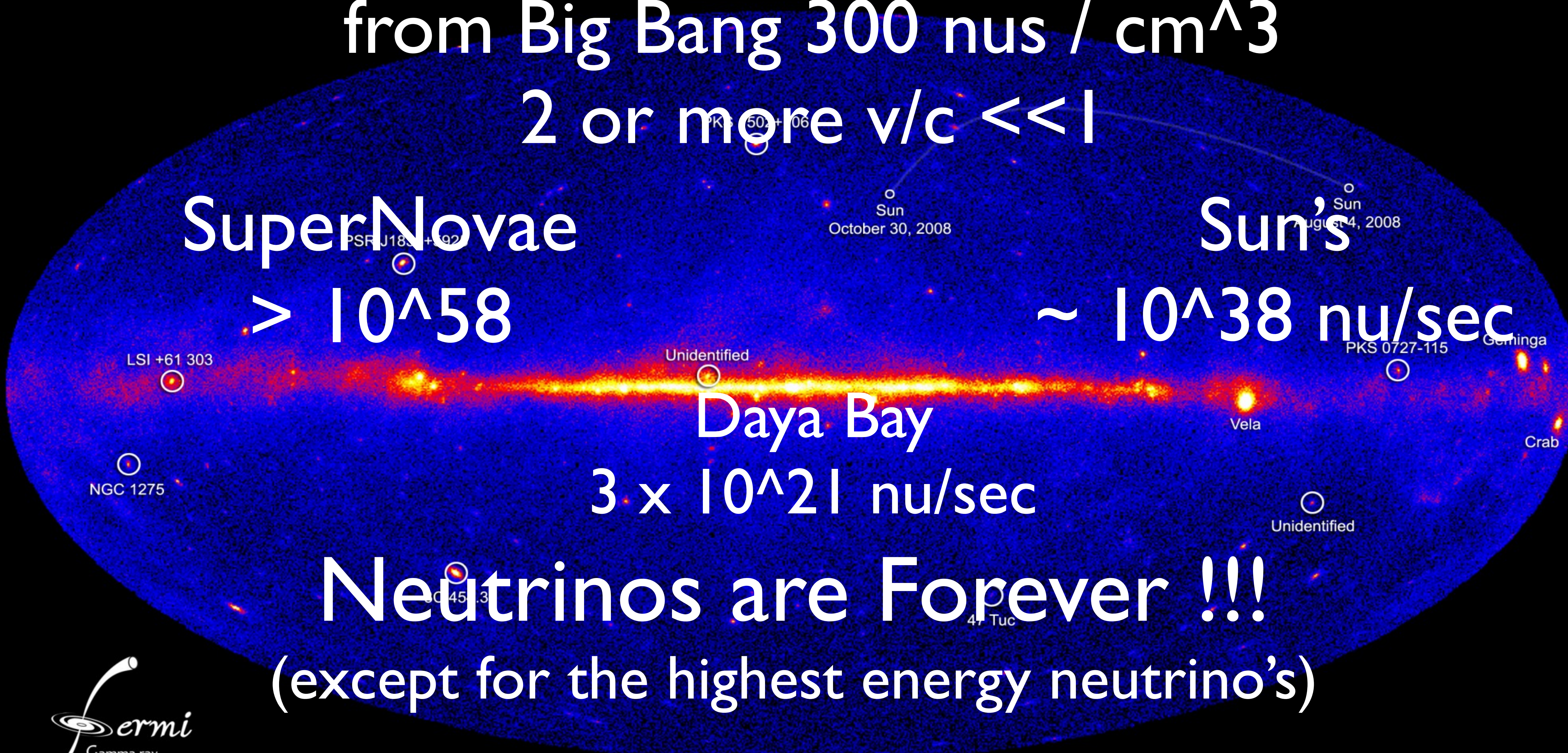
NC: $\nu_x + D \rightarrow p + n + \nu_x$

ES: $\nu_e + e^- \rightarrow \nu_e + e^-$
and $\nu_{\mu/\tau} + e^- \rightarrow \nu_{\mu/\tau} + e^-$



Beacom and SP: hep-ph/0106128

Neutrinos are Everywhere !



from Big Bang 300 nus / cm^3

2 or more v/c <<1

SuperNovae

> 10^58

Sun's

~ 10^38 nu/sec

Daya Bay

3 x 10^21 nu/sec

Neutrinos are Forever !!!

(except for the highest energy neutrino's)

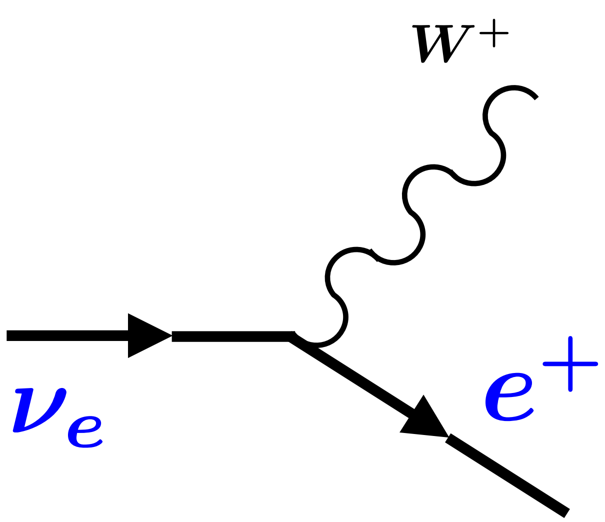
therefore in the Universe: $\frac{\partial N_\nu}{\partial t} > 0$

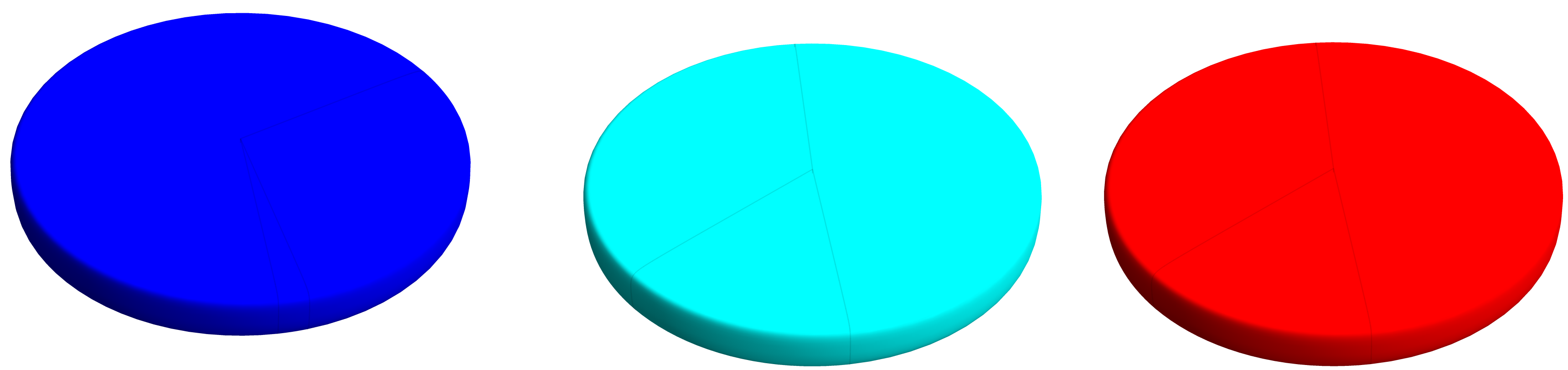
Key Neutrino Questions:

- **Nature of Neutrino Mass:**
  - 2 comp & L violation (Majorana)
  - or 4 comp & L conserved (Dirac)
- **Neutrino Standard Model:**
  - **Perform stringent tests 3 nu paradigm**: check unitarity, ...
  - **Determine size and sign of CPV**
  - **Determine atmospheric mass ordering**
  - **Does ν_μ or ν_τ dominate ν_3 ($\theta_{23} >< \pi/4$)**
- **Beyond 3 nus:**
  - **Steriles**, Non-Standard Interactions, Lorentz violation, nuBSM, ...

Neutrino Flavor or Interaction States:

$W^+ \to e^+\nu_e$ $\quad$ $W^+ \to \mu^+\nu_\mu$ $\quad$ $W^+ \to \tau^+\nu_\tau$





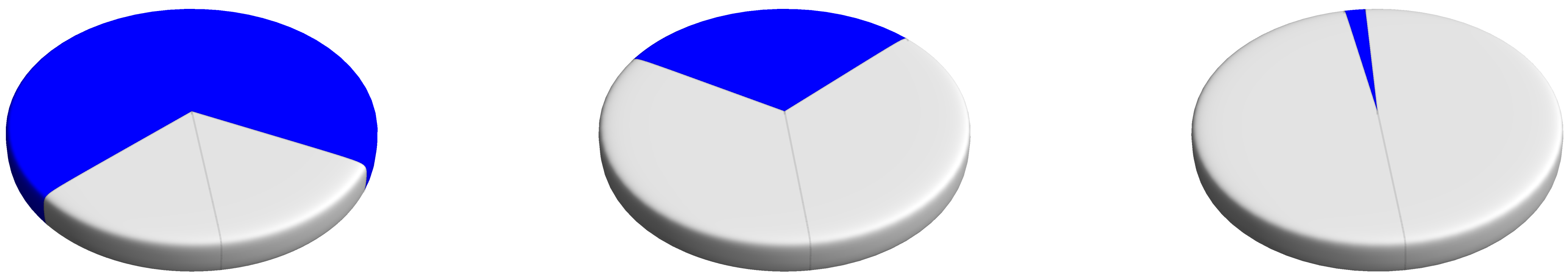
ν_e $\quad$ ν_μ $\quad$ ν_τ

provided $L/E \ll \mathbf{0.5}$ km/MeV $= \mathbf{500}$ km/GeV !!!

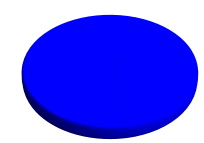
$\sim$ 1 picosecond in Neutrino rest frame !!!

$\sim$ Age of Universe / Avogadro's #

Neutrino Mass Eigenstates or Propagation States:



ν_1 ν_2 ν_3

$\nu_e =$ 

$$\text{Propagator } \nu_j \to \nu_k \;=\; \delta_{jk}\, e^{-i\left(\frac{m_j^2 L}{2E_\nu}\right)}$$

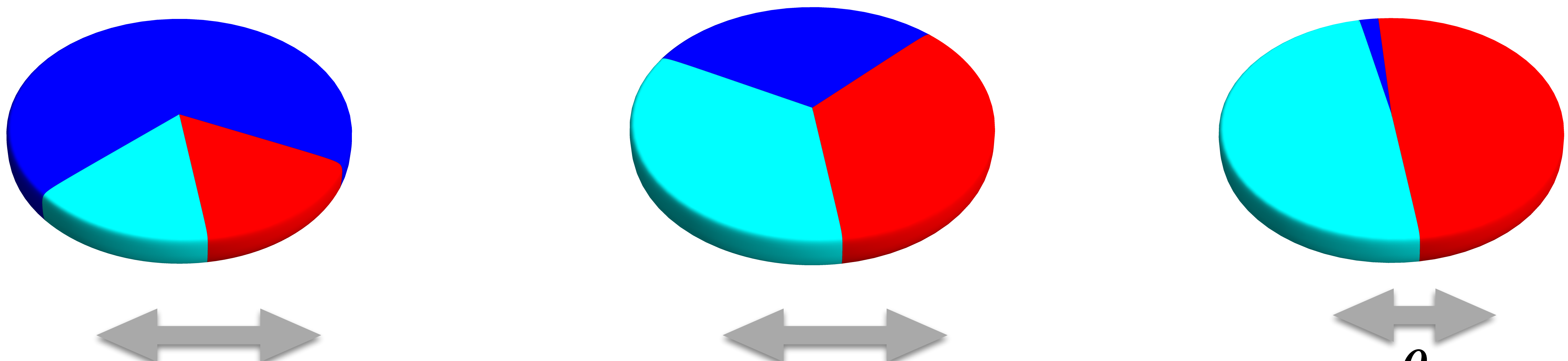
Use ν_e content to label these states

Neutrino Mass Eigenstates or Propagation States:

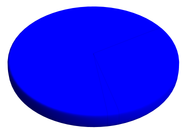
ν_1 most ν_e

ν_2

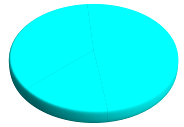
ν_3 least ν_e



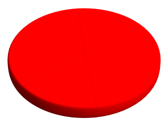
δ,θ_{23} | δ,θ_{23} | θ_{23}

$\nu_e =$ 

Solar Exp, SNO
KamiLAND
Daya Bay, RENO, …

$\nu_\mu =$ 

SuperK, K2K, T2K
MINOS, NOvA
ICECUBE

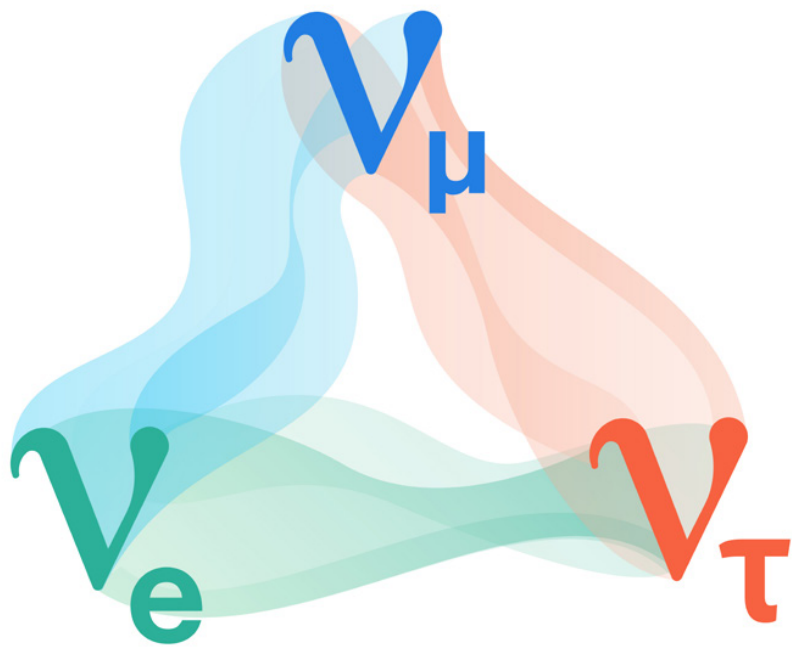
$\nu_\tau =$ 

Unitarity
SK, Opera
ICECUBE ?

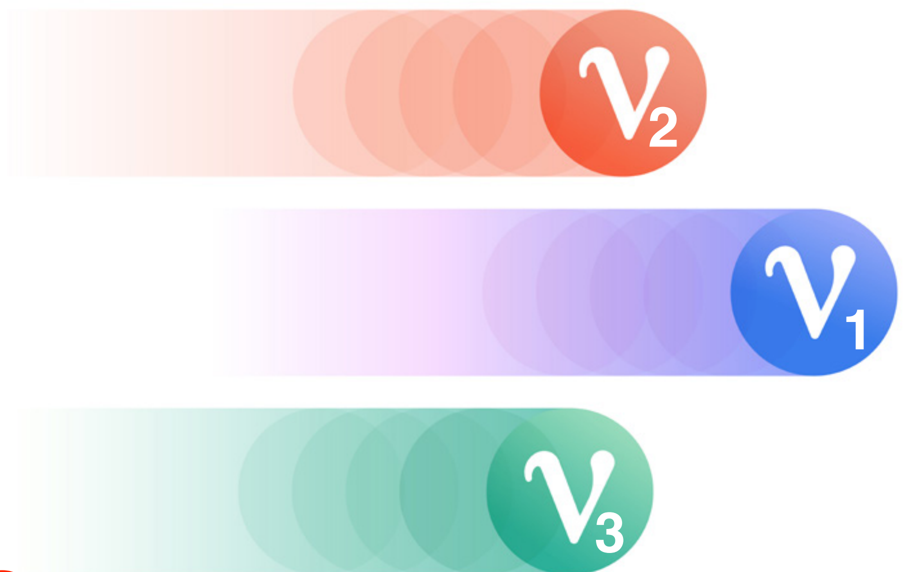
Interactions:

simple

complicated



=U



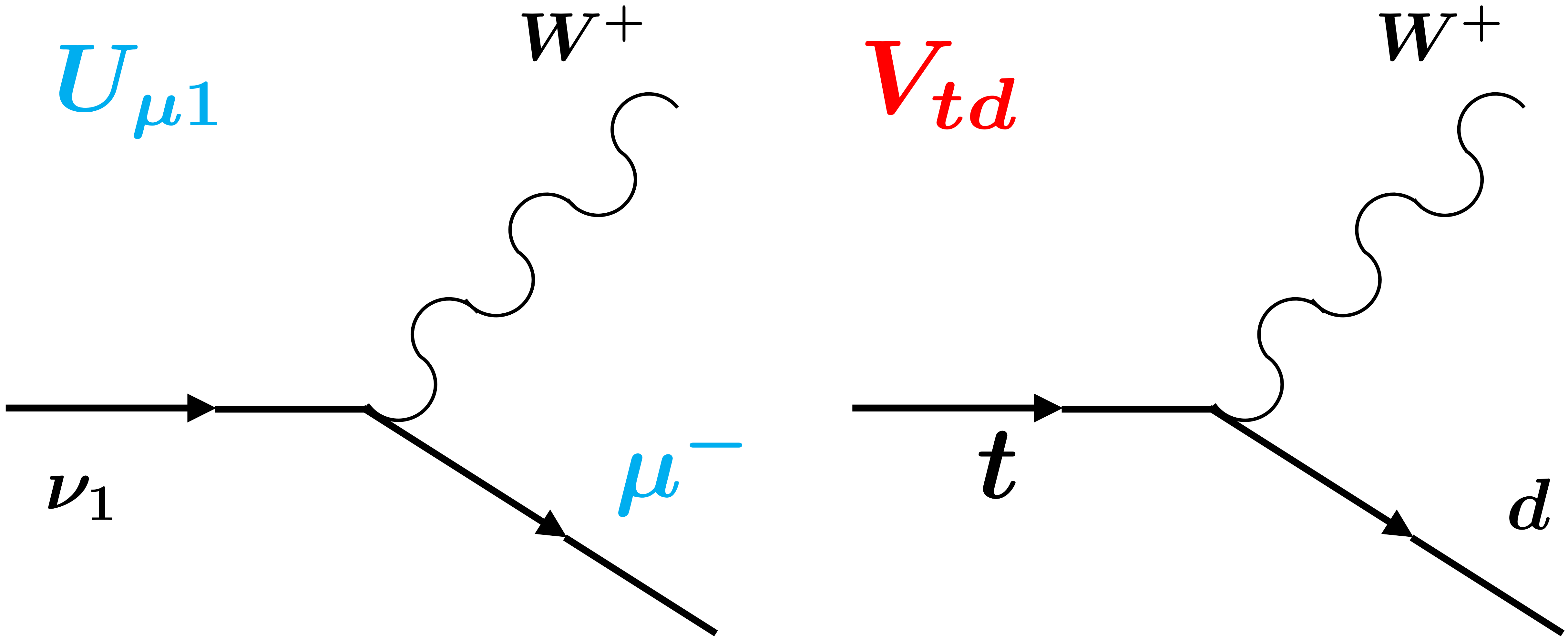
unitary matrix ?

complicated

simple

masses ?

Propagation:



Rates: $|U_{\mu 1}|^2$ & $|V_{td}|^2$

$$\begin{pmatrix} \mathbf{U}_{e1} & \mathbf{U}_{\mathbf{e2}} & \mathbf{U}_{e3} \\ U_{\mu 1} & U_{\mu 2} & \mathbf{U}_{\mu \mathbf{3}} \\ U_{\tau 1} & U_{\tau 2} & \mathbf{U}_{\tau 3} \end{pmatrix} = U_{23}(\theta_{23}, 0)\, U_{13}(\theta_{13}, \delta)\, U_{12}(\theta_{12}, 0)$$

Why this order ???

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{i\delta_{\rm cp}} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta_{\rm cp}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\eta_1} & 0 & 0 \\ 0 & e^{i\eta_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$0\nu\beta\beta$ decays

Disappearance:

$\nu_\mu \rightarrow \nu_\mu$ — 500 km/GeV

$\nu_e \rightarrow \nu_e$ — 500 km/GeV

$\nu_e \rightarrow \nu_e$ — 15 km/MeV

Appearance: $\nu_\mu \rightarrow \nu_e$

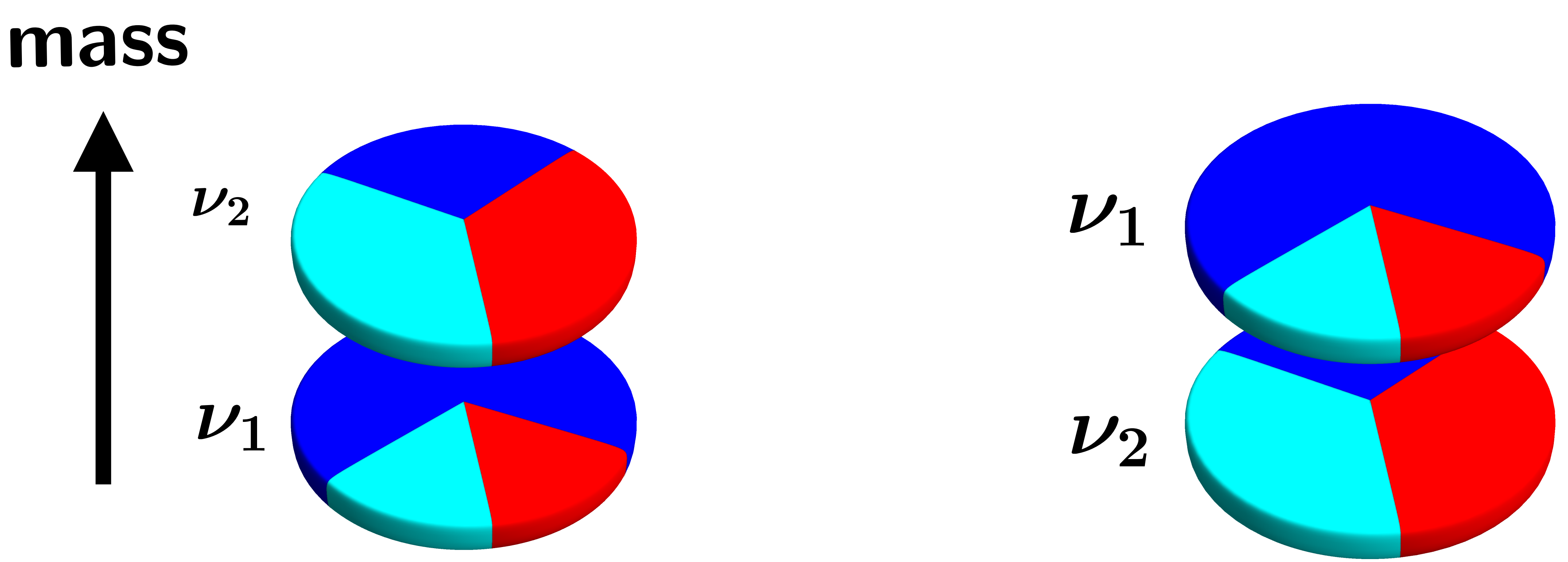
500 km/GeV

unitary matrix

$$\begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{13}s_{23}c_{12}e^{i\delta} & c_{23}c_{12} - s_{13}s_{23}s_{12}e^{i\delta} & c_{13}s_{23} \\ s_{23}s_{12} - s_{13}c_{23}c_{12}e^{i\delta} & -s_{23}c_{12} - s_{13}c_{23}s_{12}e^{i\delta} & c_{13}c_{23} \end{pmatrix}$$

$\nu_1, \ \nu_2$ Mass Ordering:

–solar mass ordering

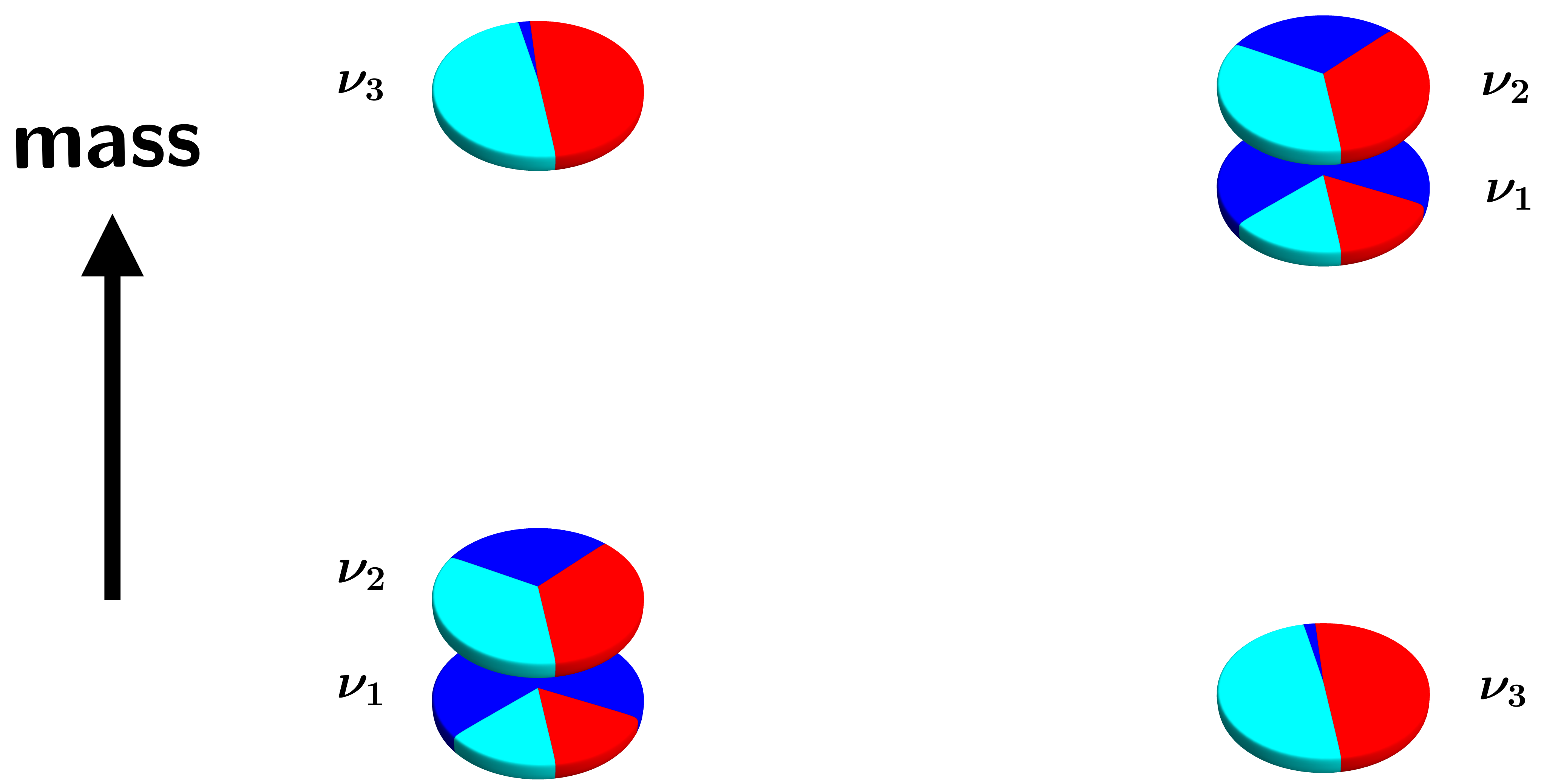


$|\Delta m^2_{21}| = |m^2_2 - m^2_1| = 7.5 \times 10^{-5} \text{ eV}^2$ $\quad L/E = 15 \text{ km/MeV} = 15,000 \text{ km/GeV}$

SNO $\quad m_2 > m_1$

$\nu_3, \ \nu_1/\nu_2$ Mass Ordering:

–atmospheric mass ordering

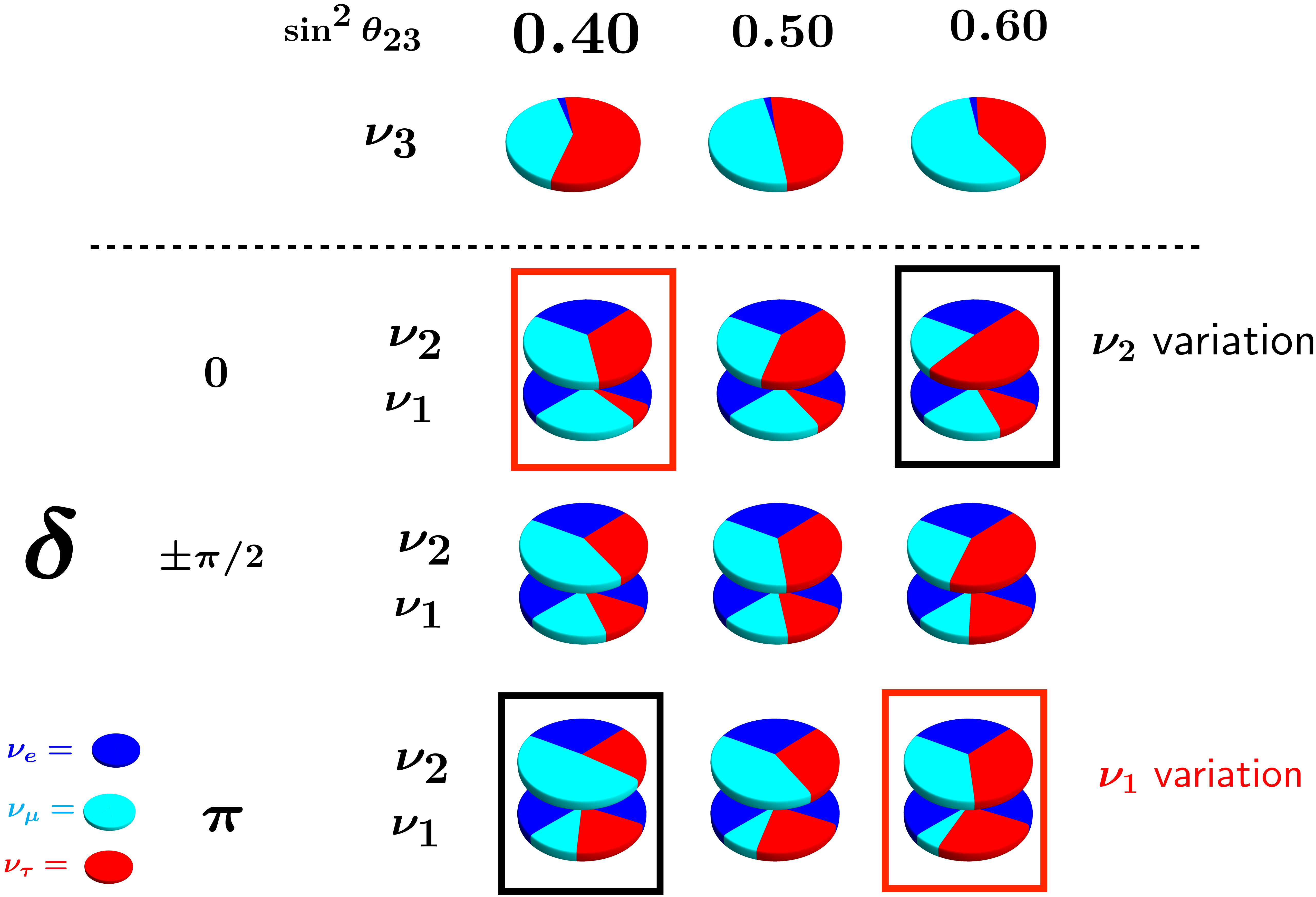


$|\Delta m^2_{31}| = |m^2_3 - m^2_1| = 2.5 \times 10^{-3}\ \mathrm{eV}^2$ $\quad$ $L/E = 0.5\ \mathrm{km/MeV} = 500\ \mathrm{km/GeV}$

Unknown: NOνA, JUNO, ICECUBE, DUNE, T2HKK....

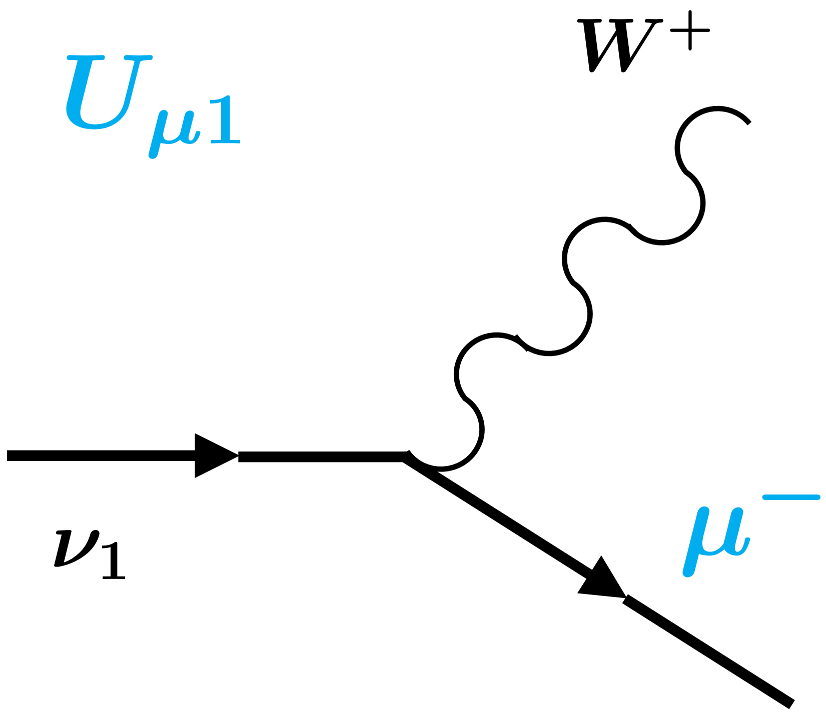
$\nu_e =$ $\quad$ $\nu_\mu =$ $\quad$ $\nu_\tau =$

Summary: Octant of θ_{23}



Leptons:

$U_{\mu 1}$



$$0.08 < |U_{\mu 1}|^2 < 0.24$$

variation in δ only !

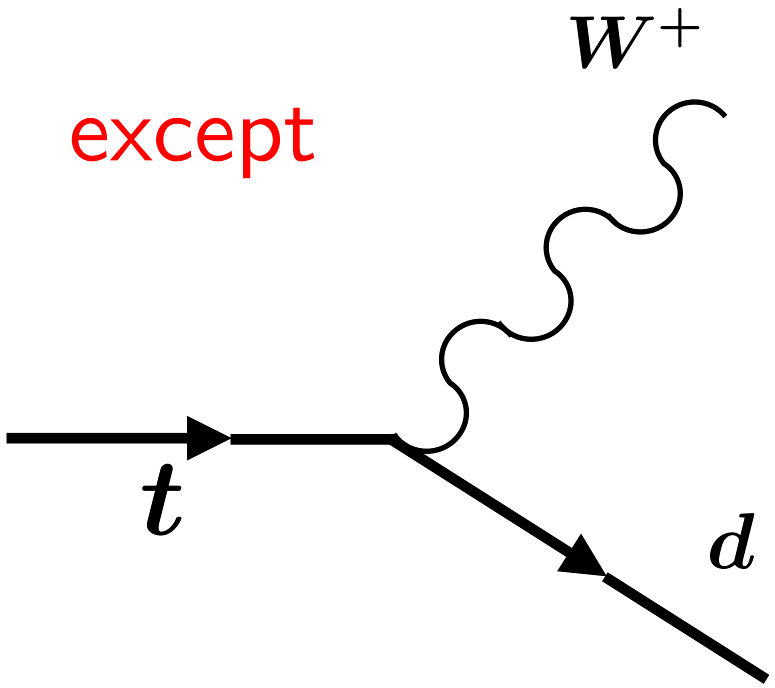
factor of 3 diff.

$$|U_{\mu 3}|^2 = 0.4 - 0.6$$
$$|U_{\mu 2}|^2 = 0.26 - 0.41$$
$$|U_{\mu 1}|^2 = 0.08 - 0.24$$

Quarks:

$|V_{ij}|^2$ essentially independent of δ_q !

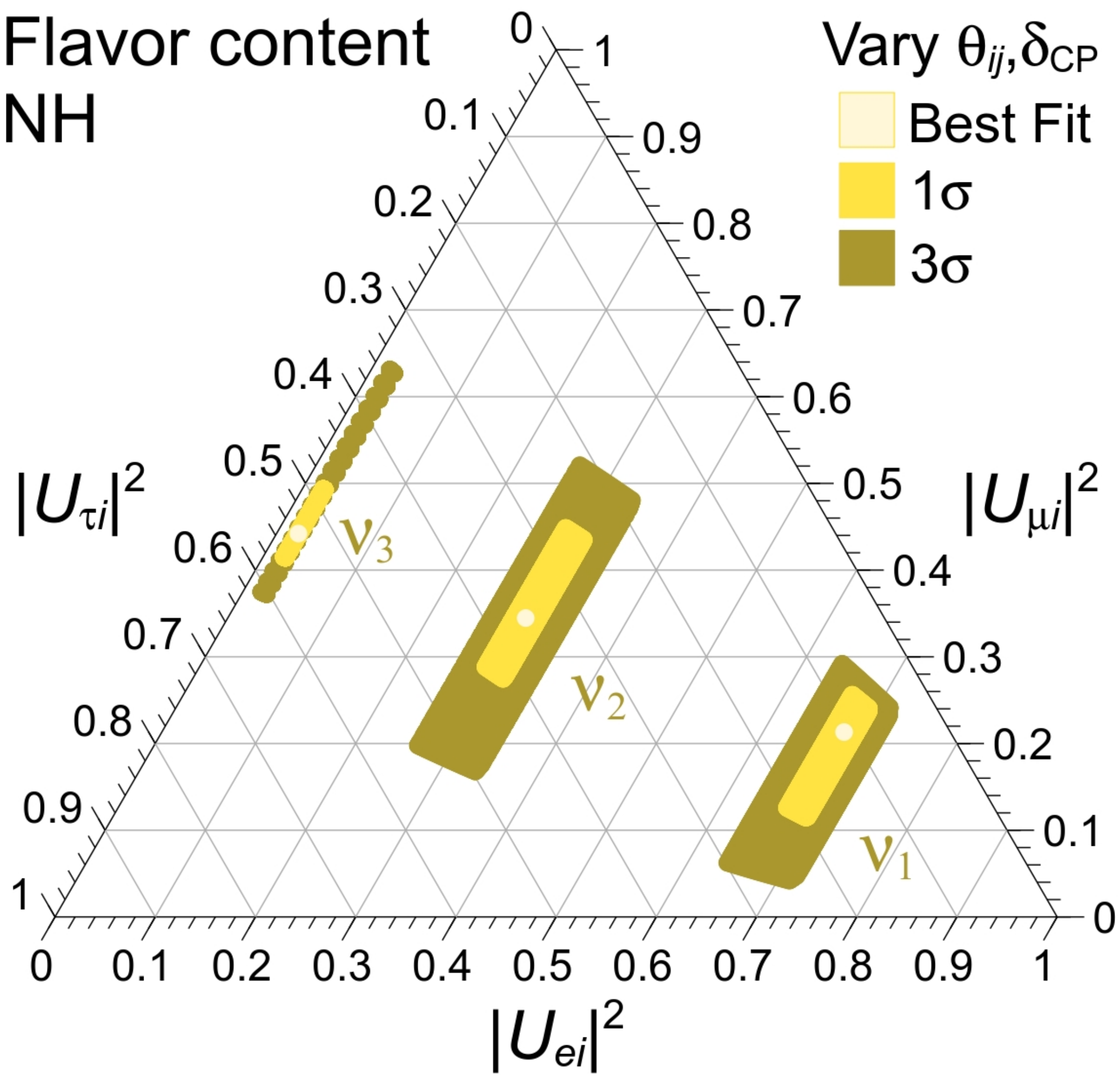
except

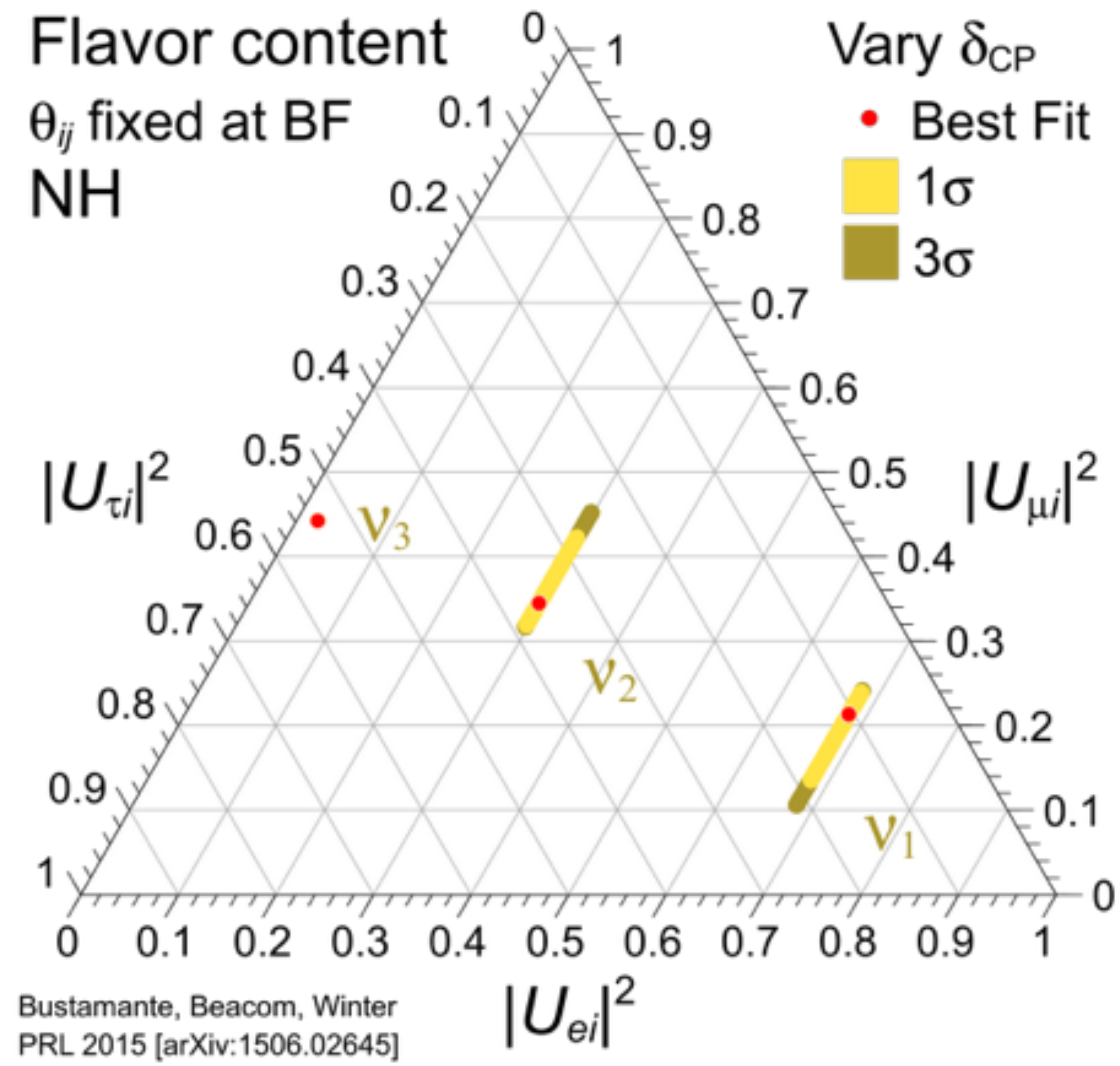


$$V_{td} \approx A\lambda^3(1 - 0.37e^{i\delta_q})$$

$$|V_{td}|^2 \approx 10^{-4}$$

$$|V_{tb}|^2 \approx 1$$
$$|V_{ts}|^2 \sim \lambda^4 \approx 2 \times 10^{-3}$$
δ_q → $|V_{td}|^2 \sim \lambda^6 \approx 8 \times 10^{-5}$





δ & θ_{23} uncertainty

no θ_{23} uncertainty

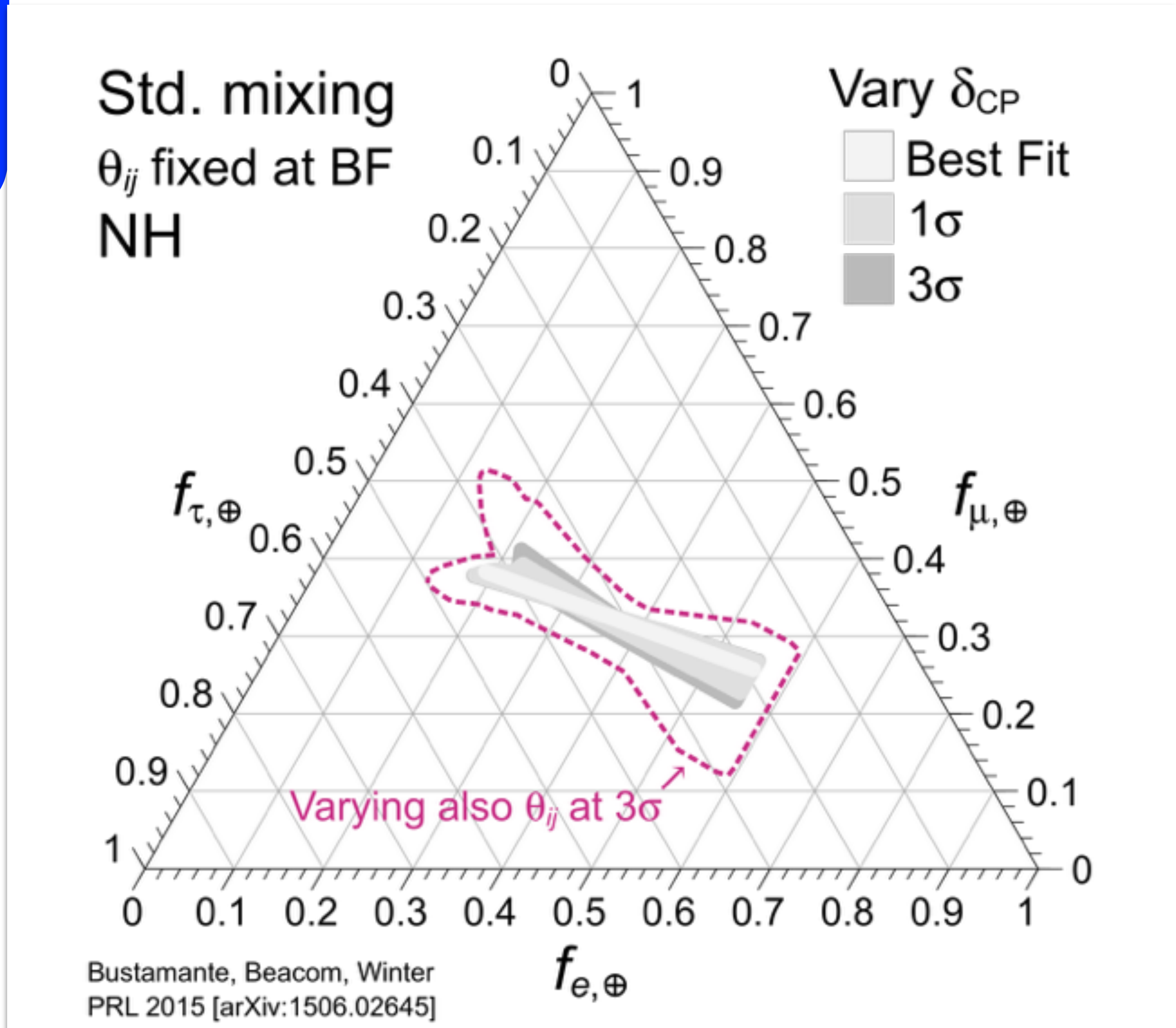
Determine flavor fractions of neutrino mass states

WHY?

Precision Neutrino Measurements:

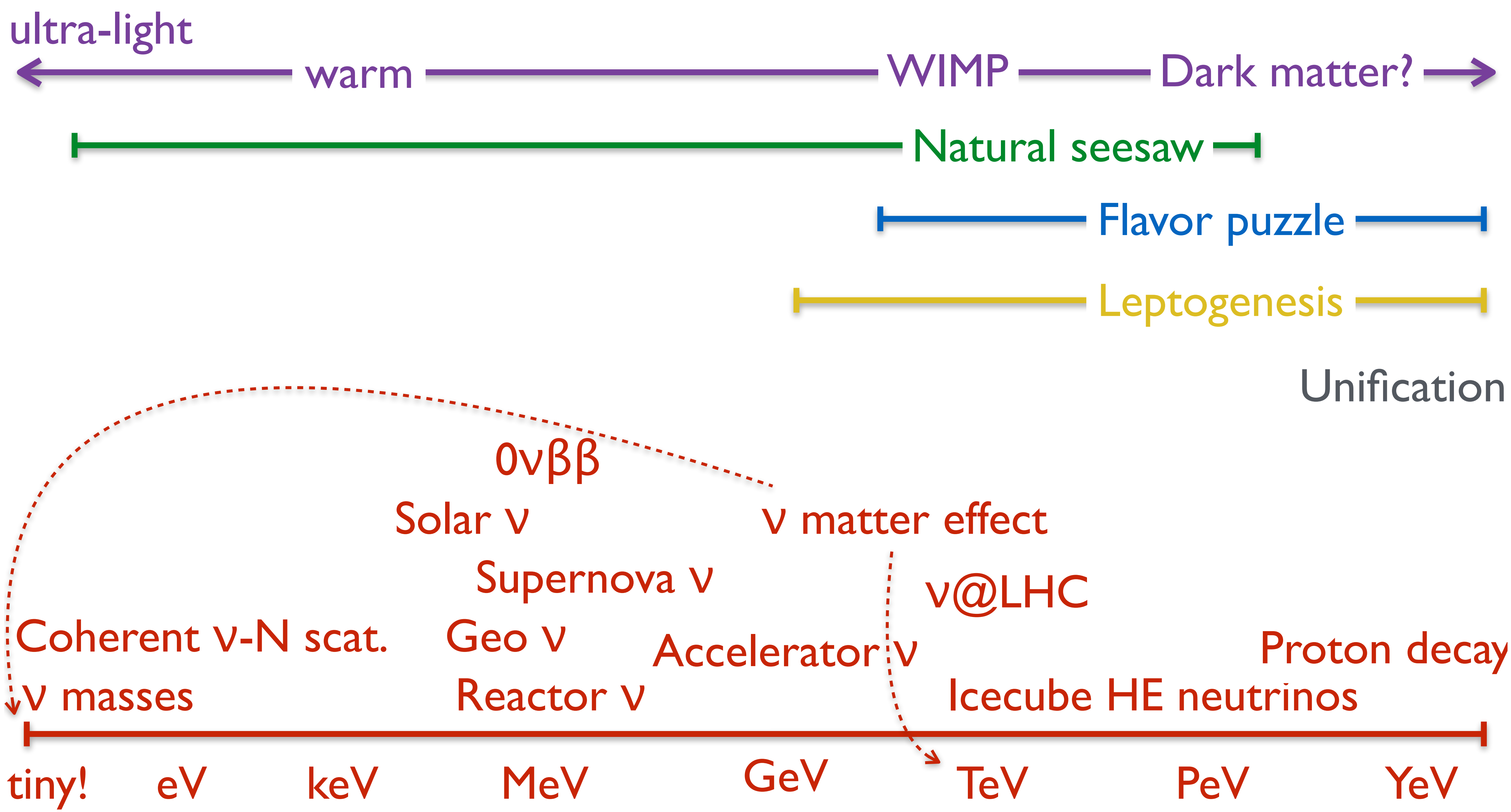
To discover neutrino BSM,
one needs precision predictions for nuSM

Determine flavor fractions of neutrino mass states



Precision Predictions for flavor ratios at ICECUBE.

Neutrinos as a portal to new Physics



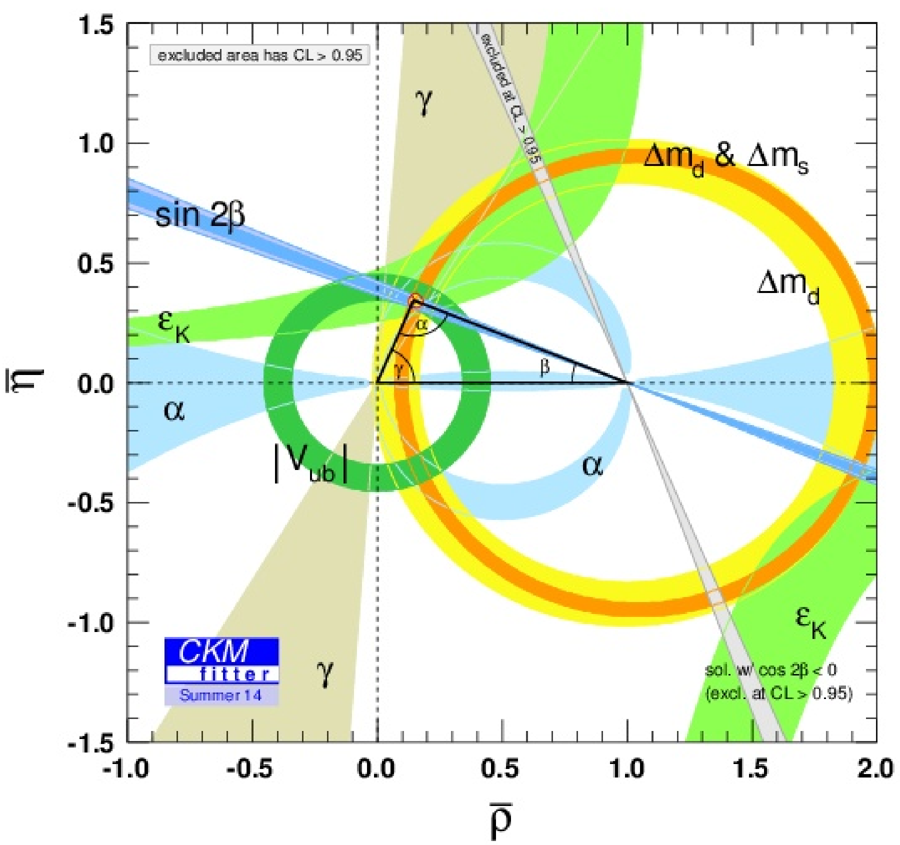
WHY?

Determine flavor fractions of neutrino mass states

Stress Test Neutrino paradigm search for new physics

Precision Neutrino Measurements:

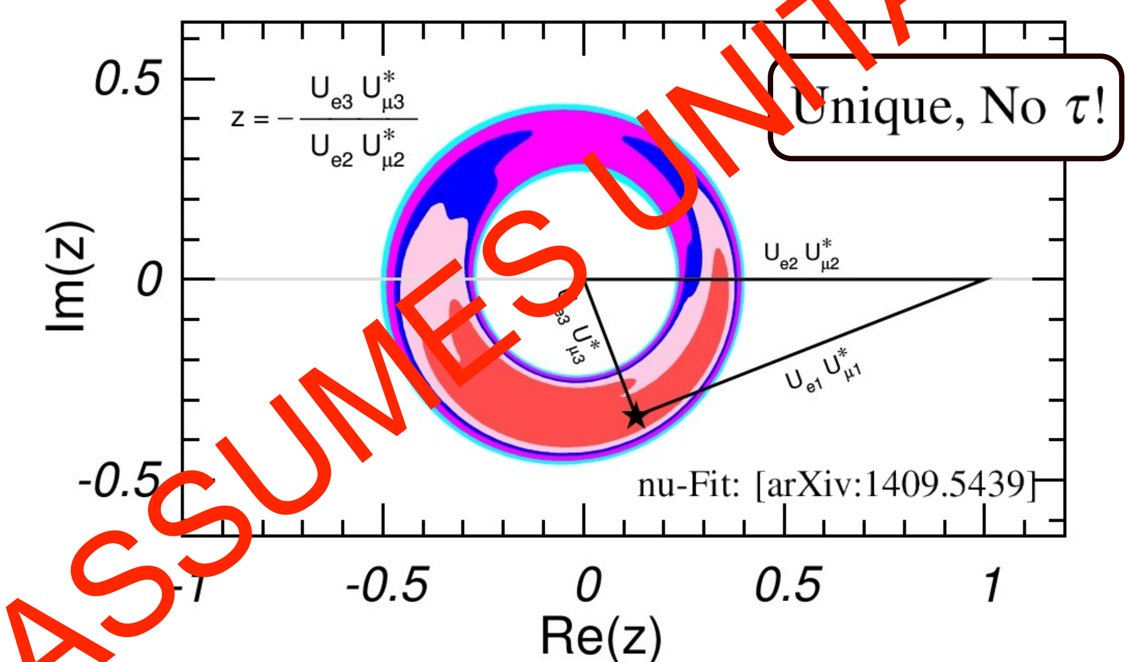
Quarks:



Unitarity *Not* assumed

Leptons:

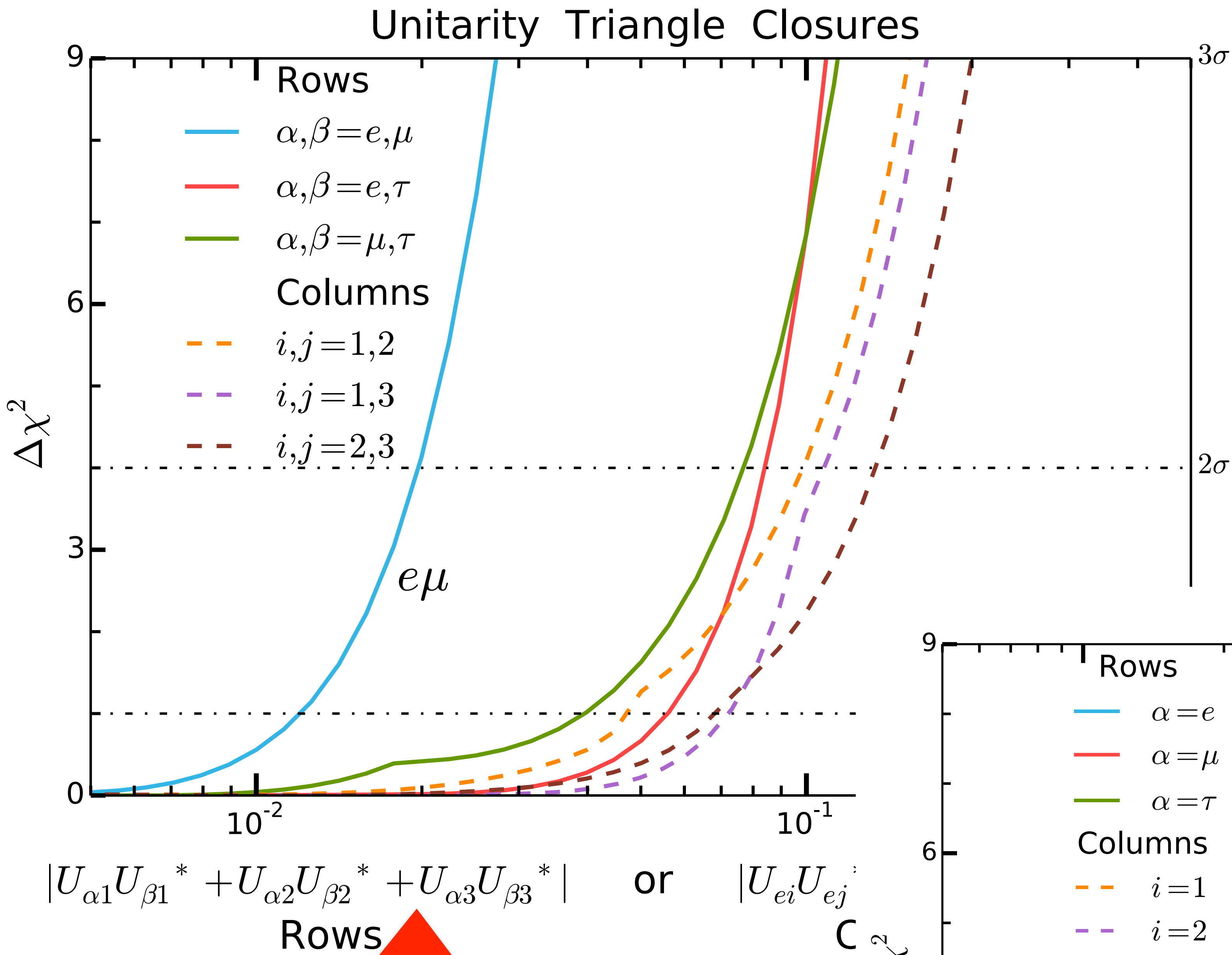
$$U_{e1}U_{\mu 2}{}^* + U_{e2}U_{\mu 2}{}^* + U_{e3}U_{\mu 2}{}^* = 0$$



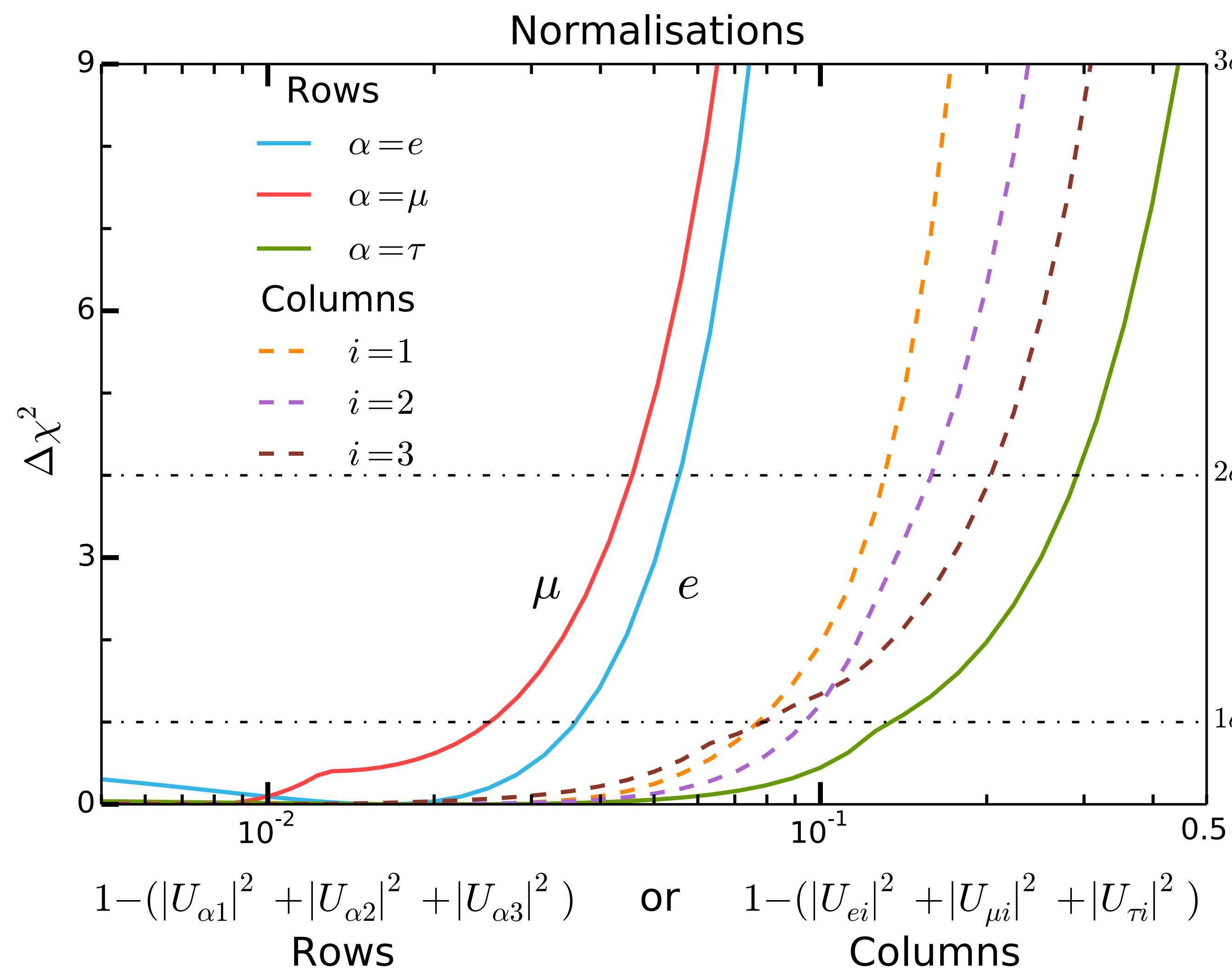
ASSUMES UNITARITY

Unitarity *Is* assumed.

$$|J| = 2 \times \text{Area}$$
$$= |s_{12}c_{12}s_{23}c_{23}s_{13}c_{13}^2 \sin\delta_{CP}|$$



LSND



M. Ross-Lonergan + SP
arXiv:1508.05095

$$U_{\rm PMNS}^{\rm Extended} = \begin{pmatrix} \overbrace{\begin{matrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{matrix}}^{U_{\rm PMNS}^{\rm 3x3}} & \cdots & U_{en} \\ & \cdots & U_{\mu n} \\ & \cdots & U_{\tau n} \\ \vdots \quad \vdots \quad \vdots & \ddots & \vdots \\ U_{s_n 1} \quad U_{s_n 2} \quad U_{s_n 3} & \cdots & U_{s_n n} \end{pmatrix}$$

Cauchy-Schwartz

$$\left| \sum_{i=1}^{3} U_{ei} U_{\mu i}^{*} \right|^2 \leq \left(1 - \sum_{i=1}^{3} |U_{ei}|^2\right) \left(1 - \sum_{i=1}^{3} |U_{\mu i}|^2\right)$$

- $\nu_\mu \to \nu_e$ Appearance
- ν_e Disappearance
- ν_μ Disappearance

- ν_μ Disappearance

MINOS+, NOvA, T2K, atmospheric neutrinos (SK and ICECUBE)

- ν_e Disappearance

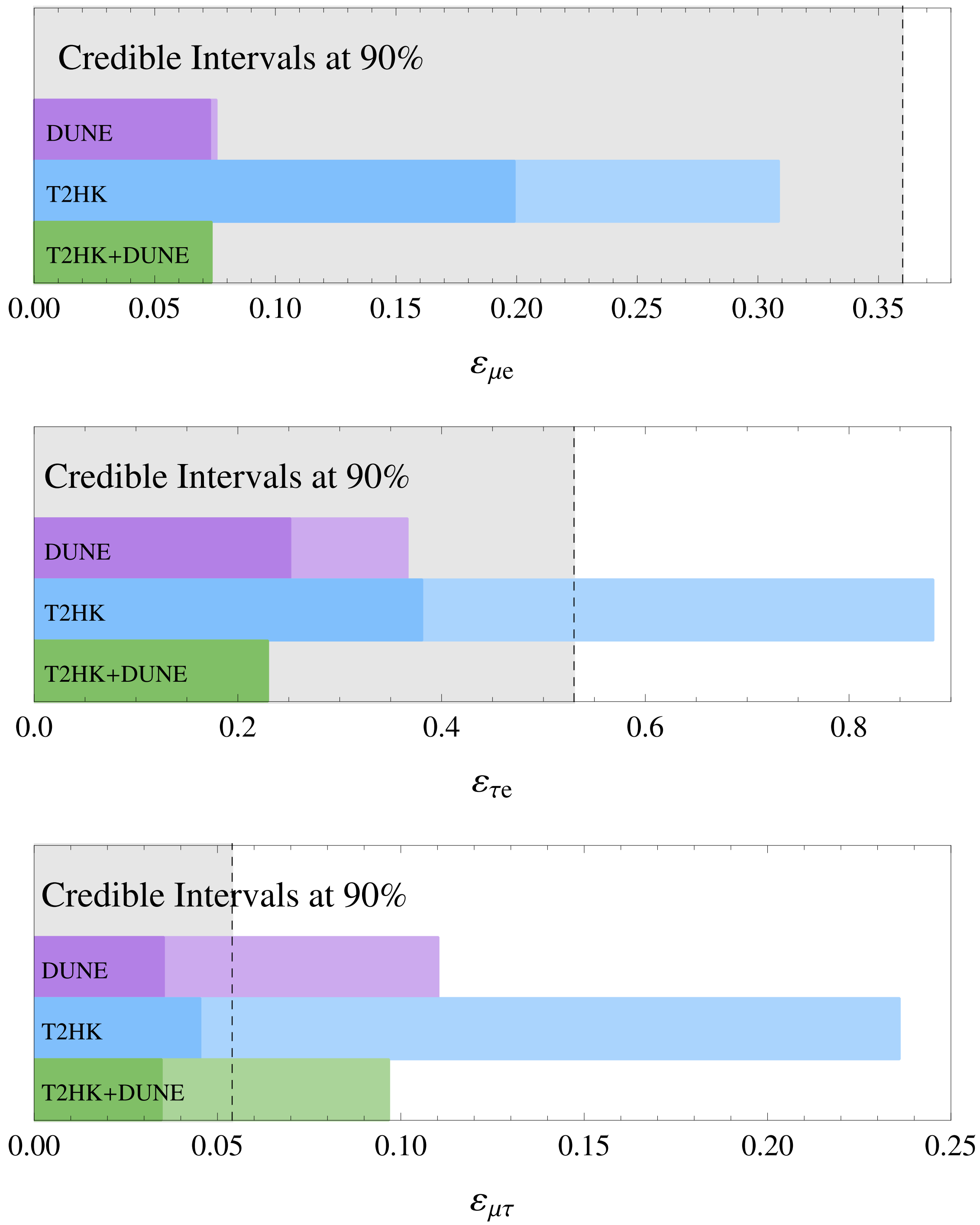
Daya Bay, RENO, many ~10m Reactor experiments & source experiments.

- $\nu_\mu \to \nu_e$ Appearance

Fermilab SBN Program, T2K and NOvA: DUNE & HyperK

NSI

Stress Test
Neutrino paradigm
search for new physics



P.Coloma
arXiv:1511.06357

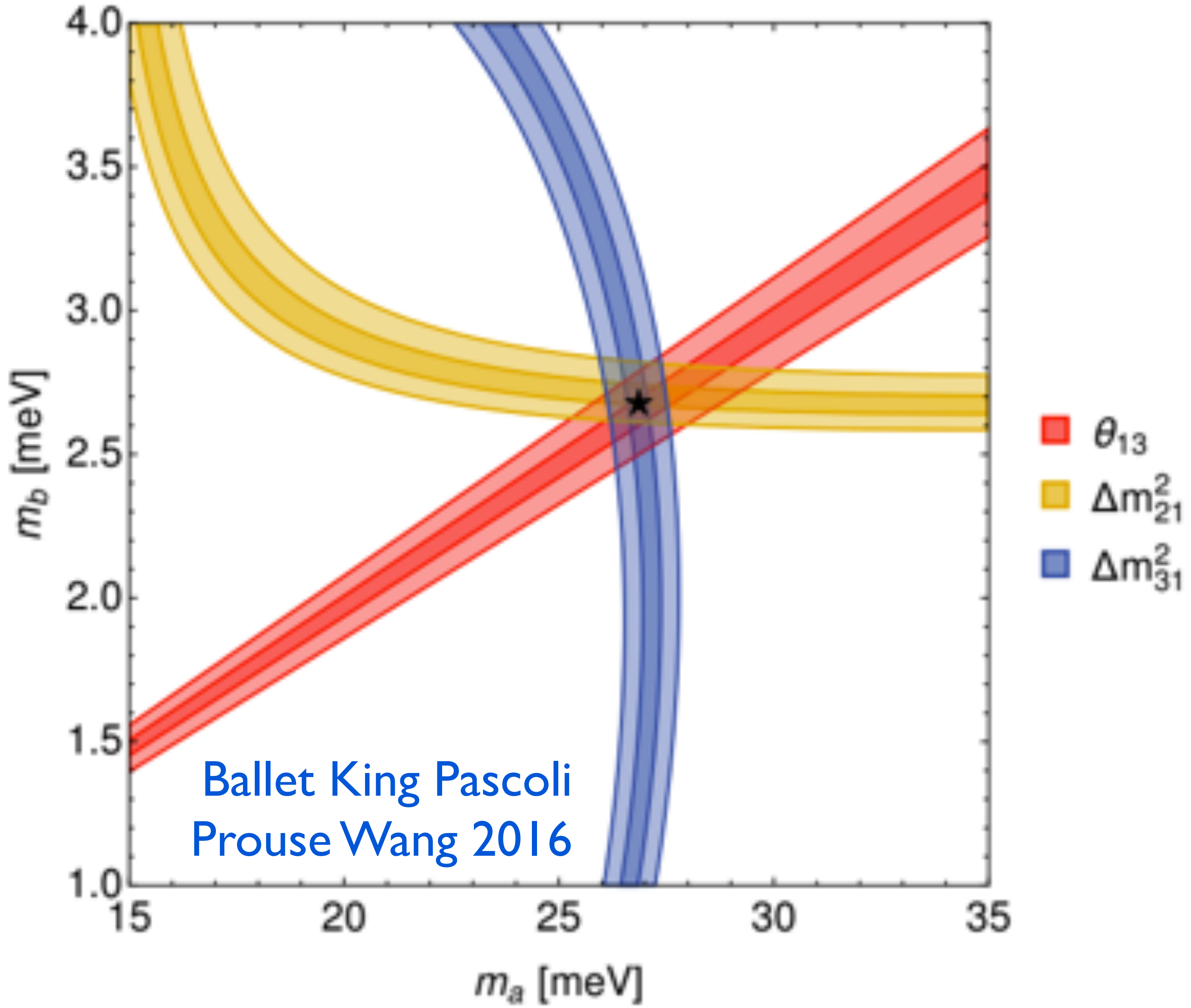
WHY?

Determine flavor fractions of neutrino mass states

Stress Test Neutrino paradigm search for new physics

Precision Neutrino Measurements:

Connection to Leptogenesis Understanding Universe



Connection to
Leptogenesis
Understanding Universe

WHY?

Determine flavor fractions of neutrino mass states

Stress Test Neutrino paradigm search for new physics

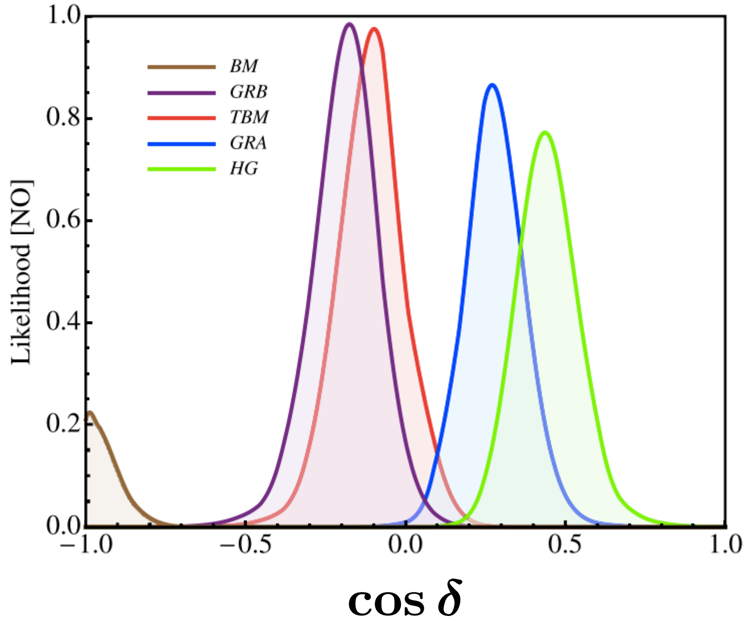
Precision Neutrino Measurements:

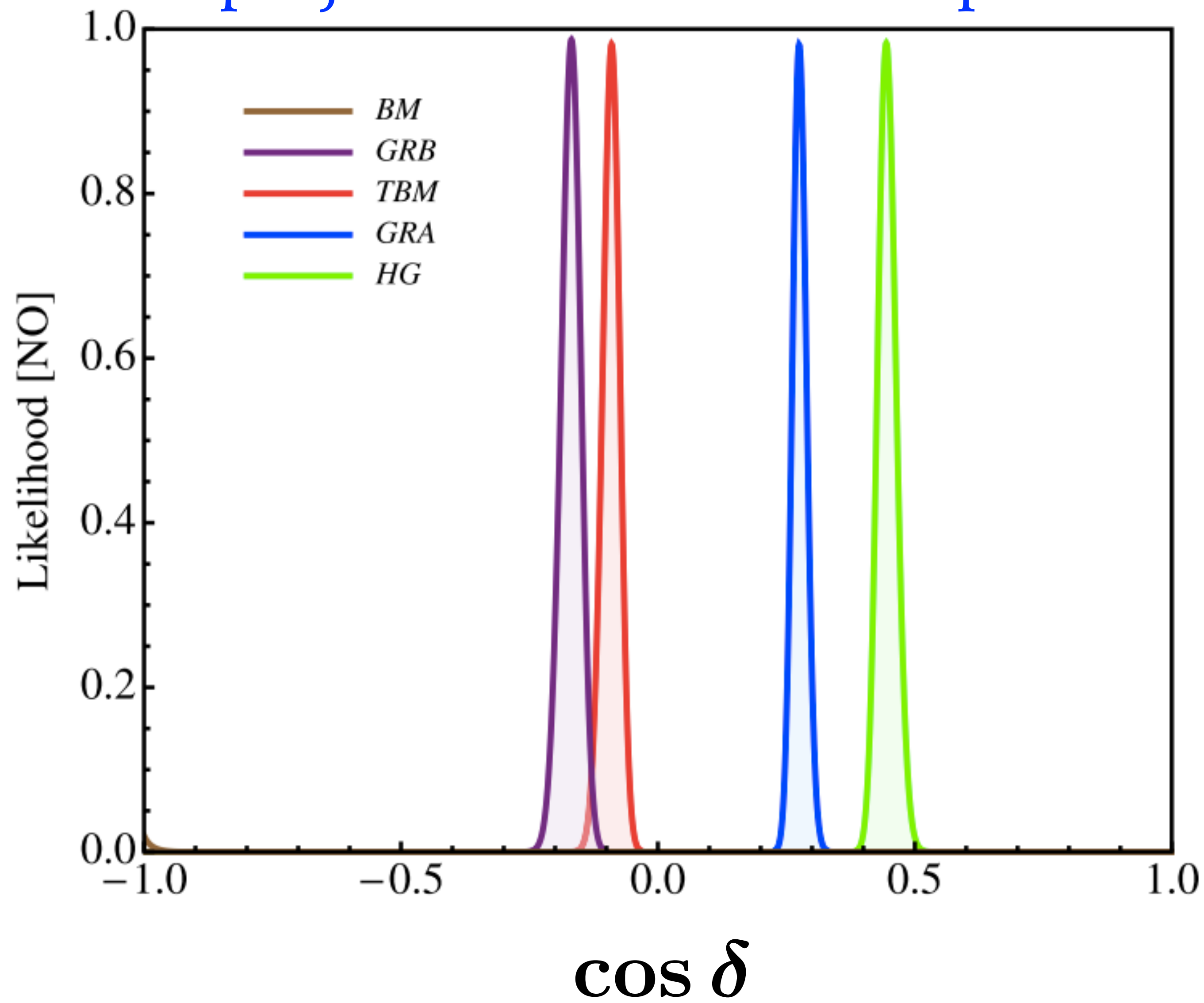
Test Theoretical Neutrino Models

Connection to Leptogenesis Understanding Universe

Predictions of flavor symmetry forms with projected measurement precision

Predictions from flavor symmetry forms with current measurement precision

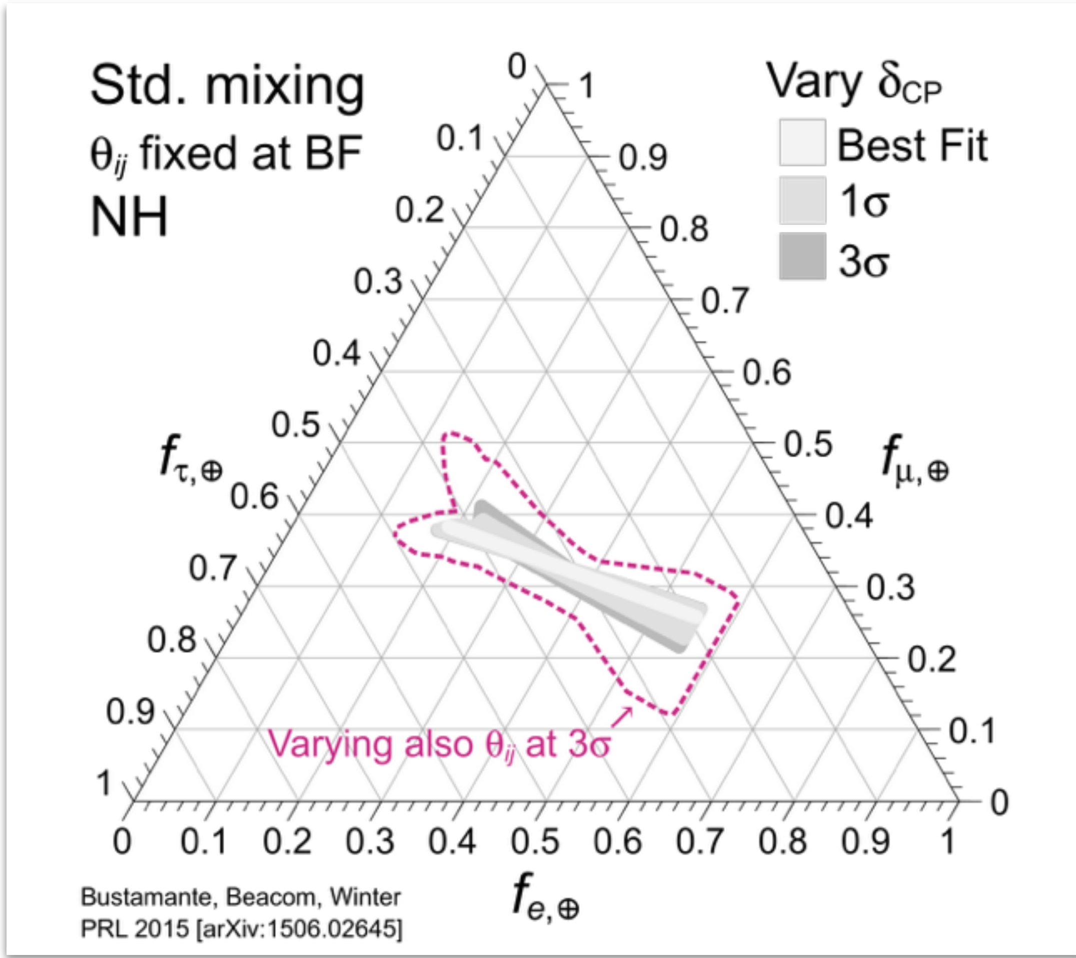


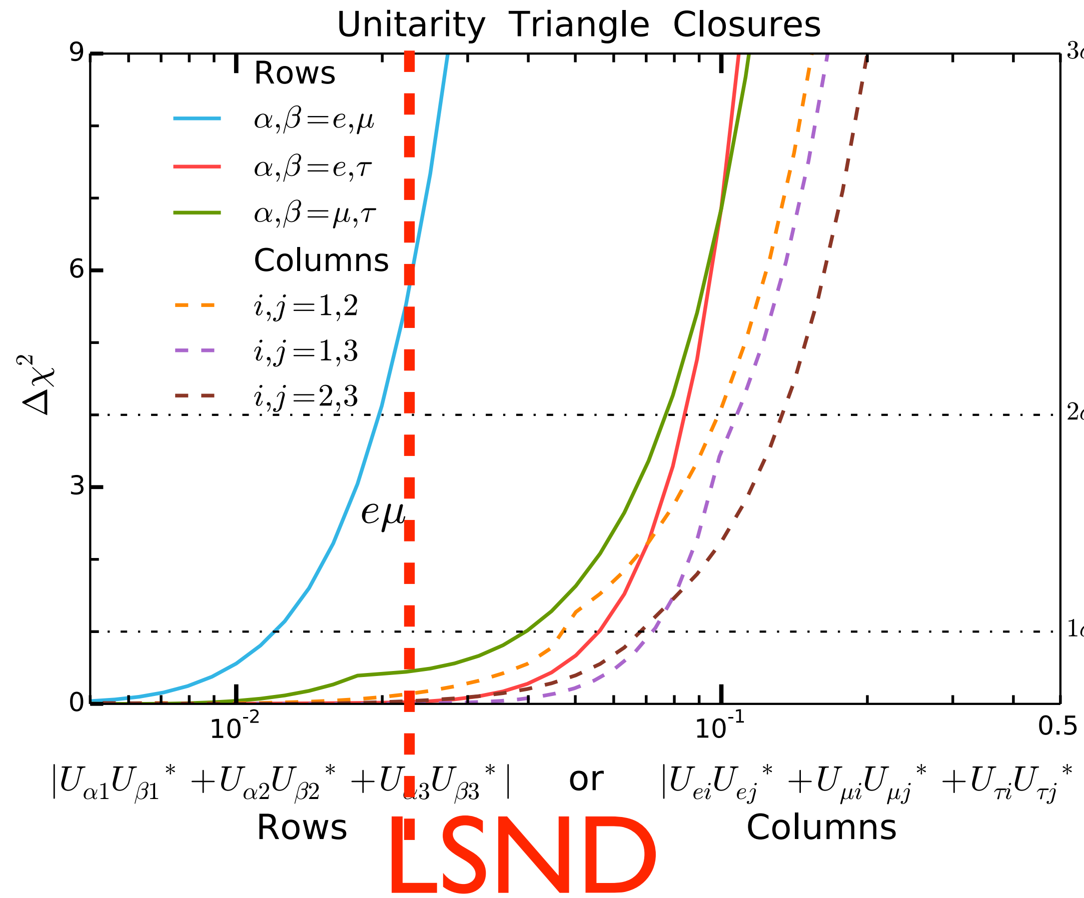


Test Theoretical Neutrino Models

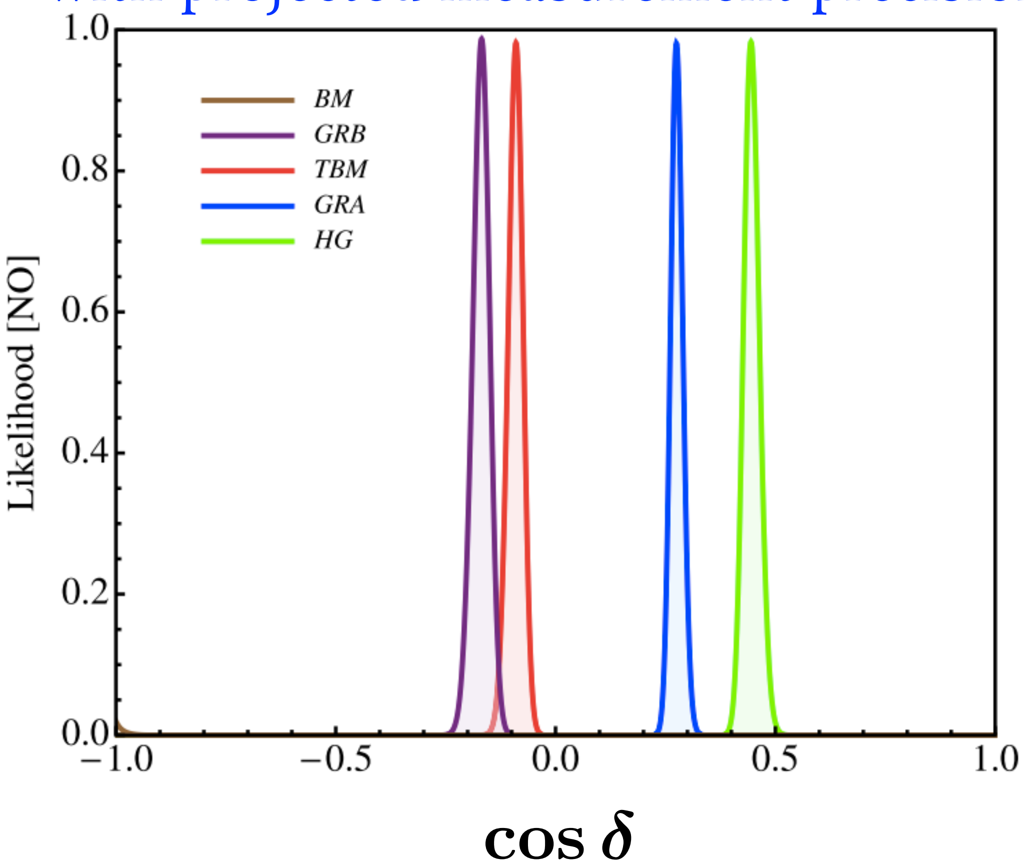
Girardi, Petcov, Titov, arXiv:1410.8056

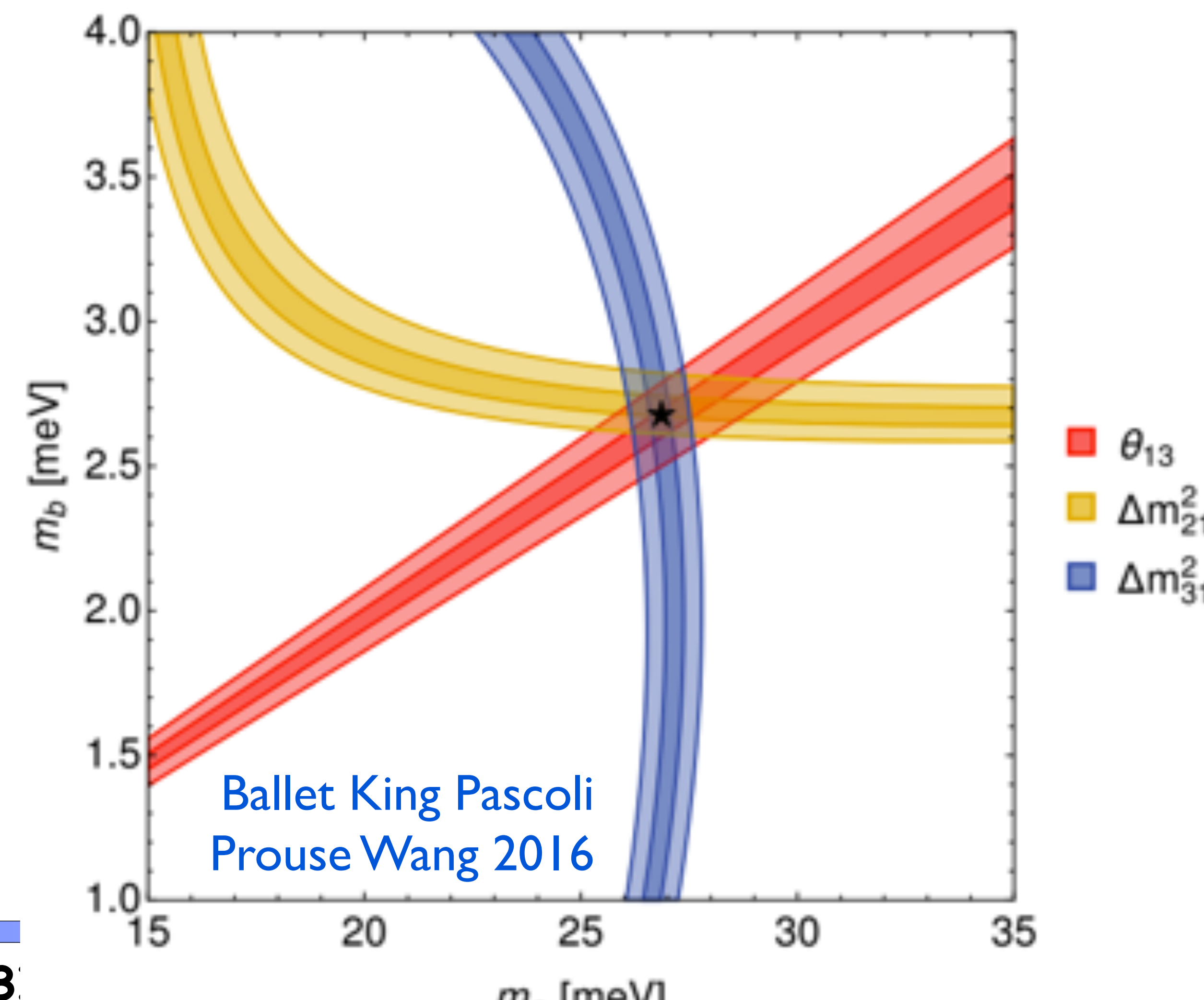
Nucl. Phys. B, Vol. 894, 733-768 (2015)





Predictions of flavor symmetry forms with projected measurement precision





Towards a better understanding of Osc. Prob.

Globes,
while a very useful tool,
is not enough !

Reactor θ_{13} Experiments

Daya Bay

RENO

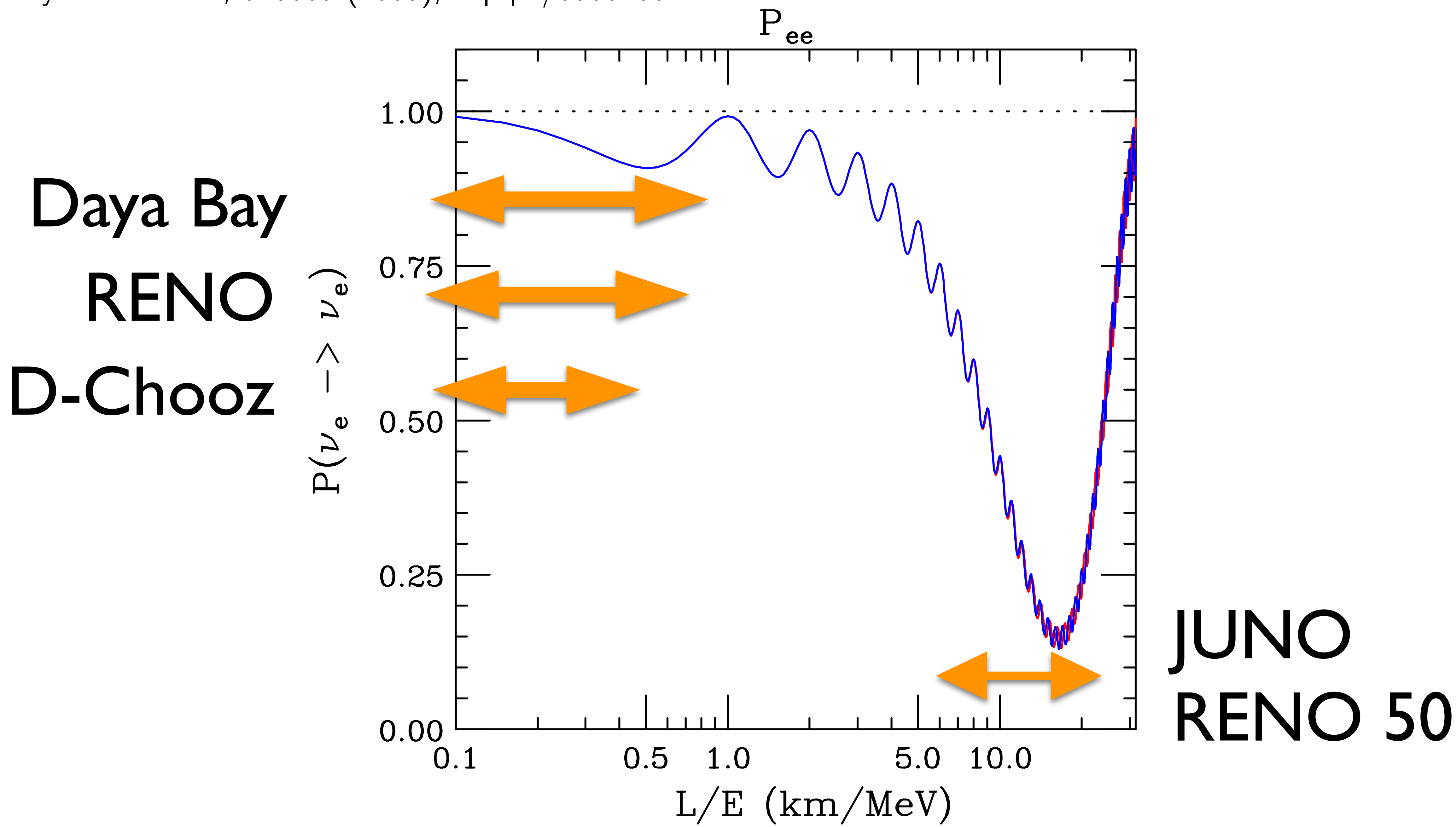
Double Chooz

What is Δm^2_{ee} ?

Stephen Parke
Fermilab

H. Nunokawa, S. J. Parke and R. Zukanovich Funchal,
"Another possible way to determine the neutrino mass hierarchy,"
Phys. Rev. D **72**, 013009 (2005), hep-ph/0503283

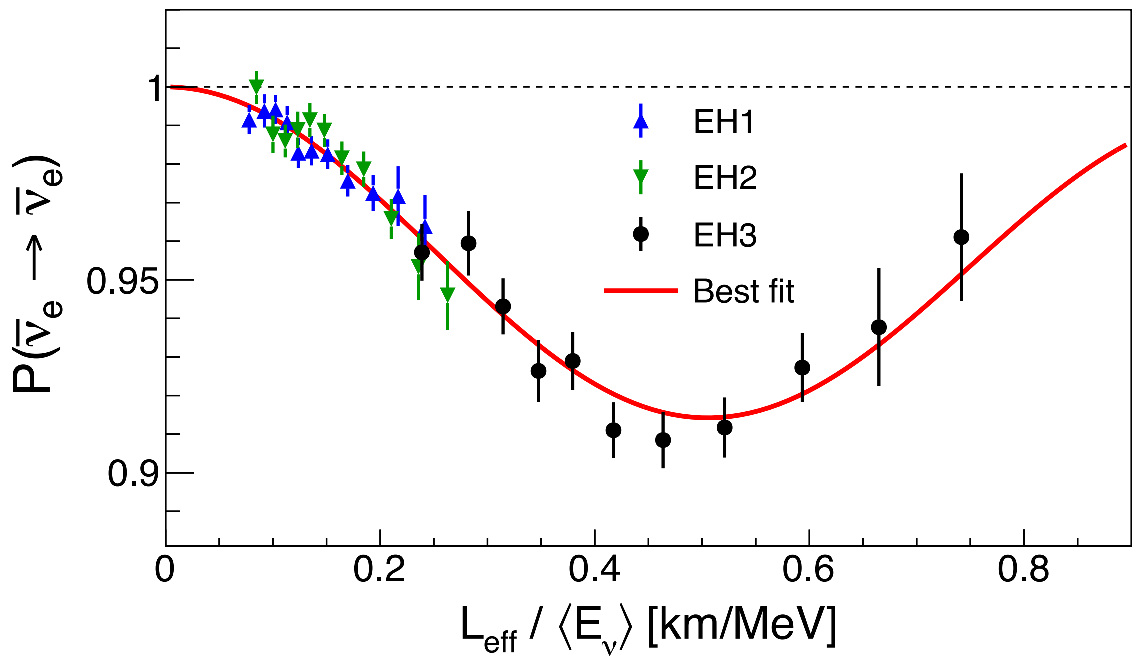
SP arXiv:1601.07464



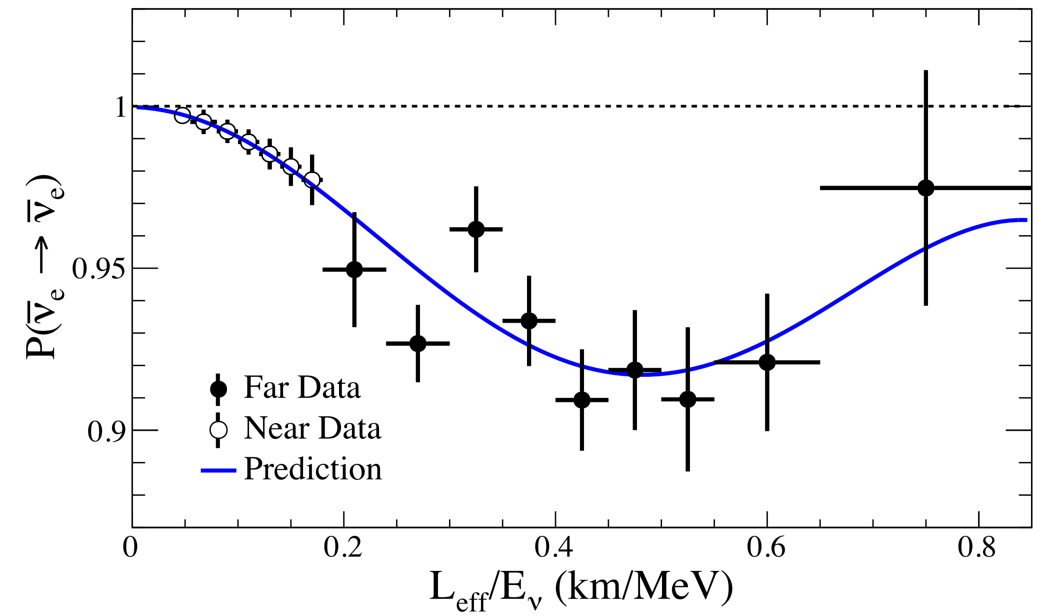
Amplitude Modulation & Phase Advancement (NO) / Retardation (IO)

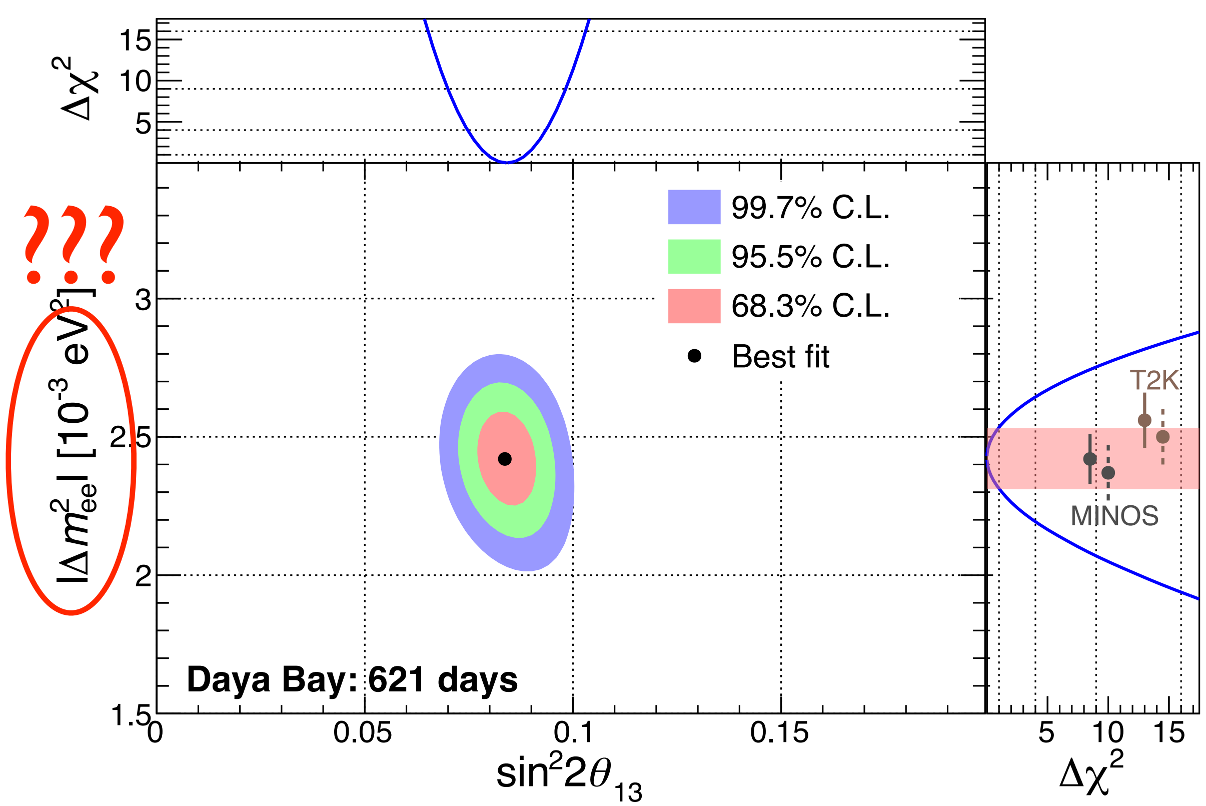
What is Δm^2_{ee} ?

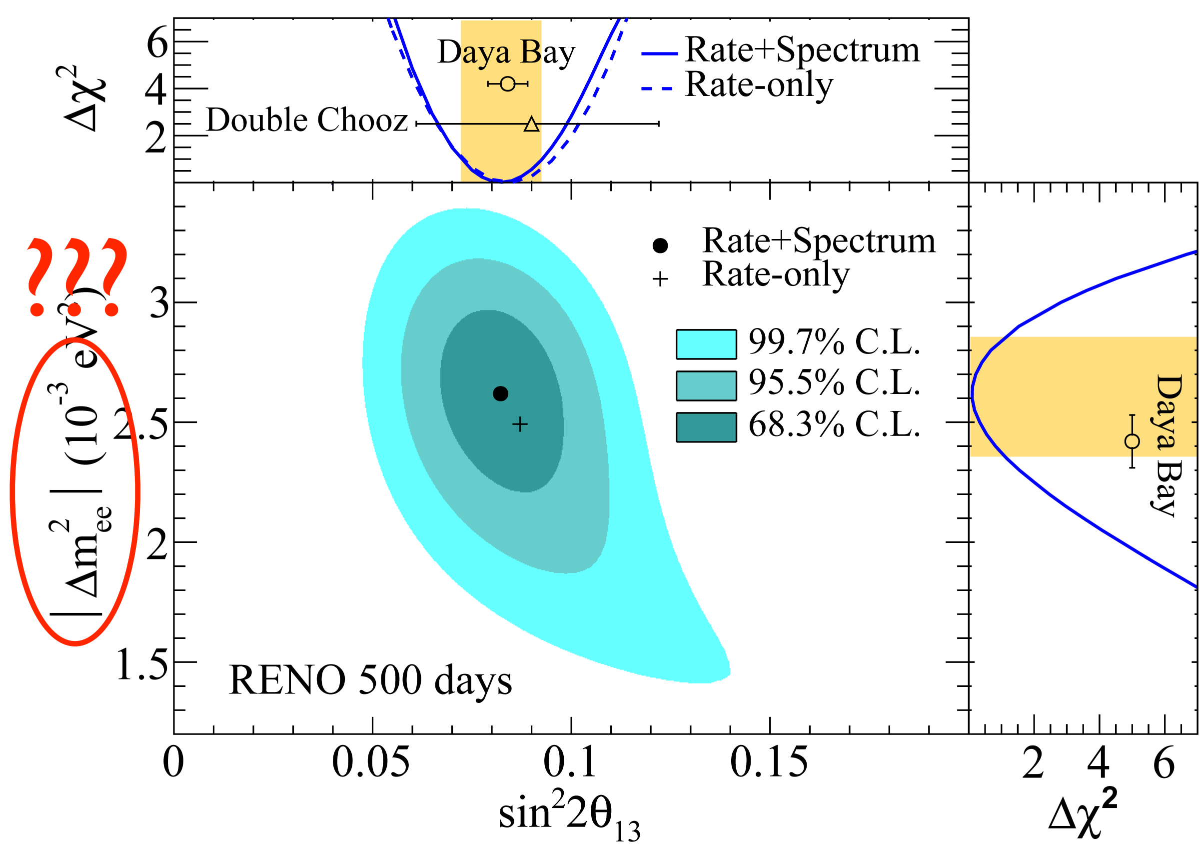
from Daya Bay: arXiv:1505.03456



from RENO arXiv:1511.05849







$\bar{\nu}_e \to \bar{\nu}_e$

- $P_{ee} \approx 1 - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21} - \sin^2 2\theta_{13}\sin^2\Delta_{ee}$ $\quad \Delta \equiv \Delta m^2 L/4E$

$+\mathcal{O}(0)$ $\quad \Delta m^2_{YY} \equiv \left(\frac{4E}{L}\right)\arcsin\left[\sqrt{(\cos^2\theta_{12}\sin^2\Delta_{31} + \sin^2\theta_{12}\sin^2\Delta_{32})}\right]$

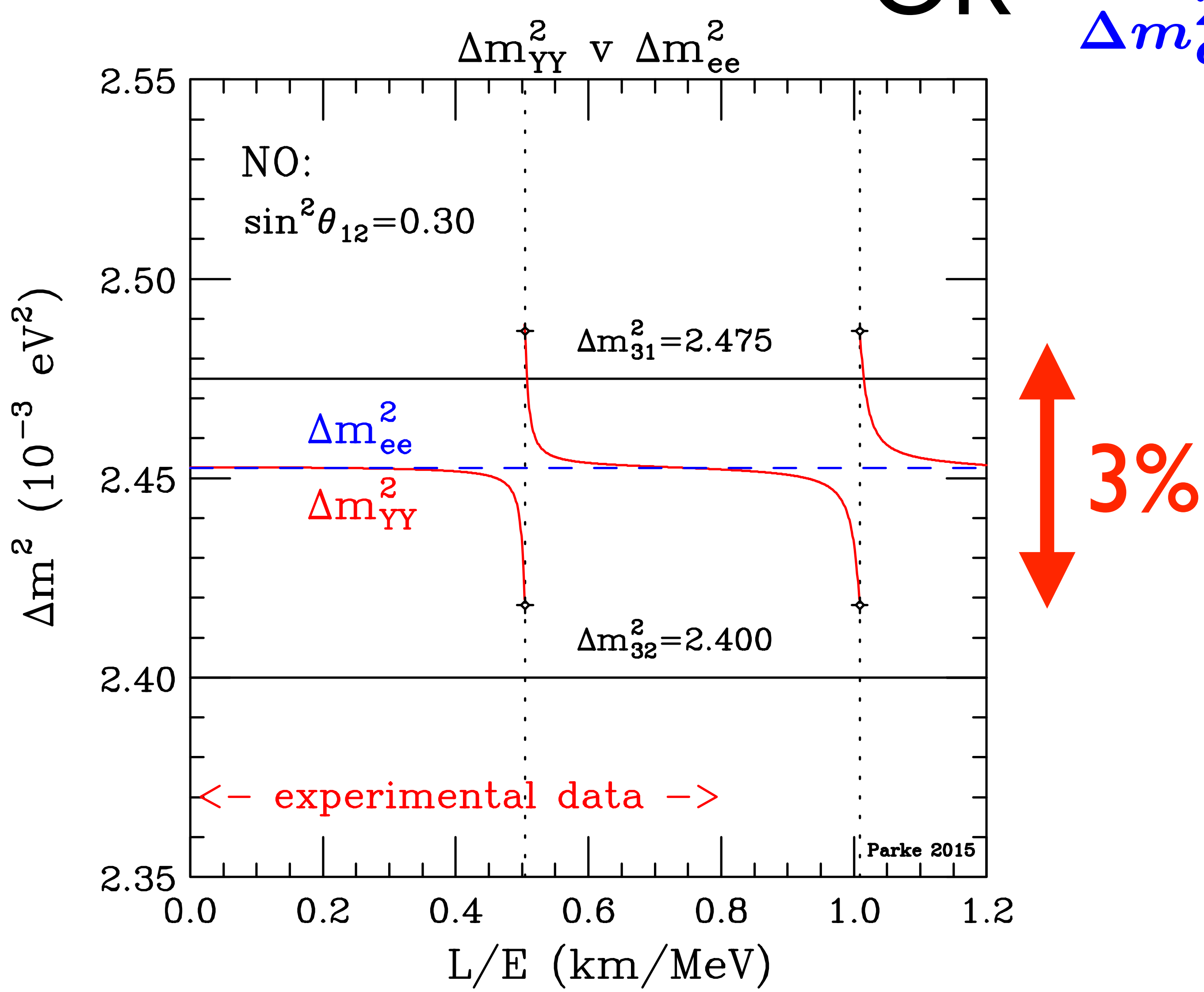
DAYA BAY

OR

$$\Delta m^2_{ee} \equiv \cos^2\theta_{12}\Delta m^2_{31} + \sin^2\theta_{12}\Delta m^2_{32}$$

$+\mathcal{O}(10^{-4})$

ν_e average !



H. Nunokawa, S. J. Parke and R. Zukanovich Funchal, "Another possible way to determine the neutrino mass hierarchy," Phys. Rev. D **72**, 013009 (2005), hep-ph/0503283

SP arXiv:1601.07464

$\nu_\mu \rightarrow \nu_e$ $\quad\quad$ $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$

T $\Updownarrow$ 

$\Updownarrow$ T

$\nu_e \rightarrow \nu_\mu$ $\quad\quad$ $\bar{\nu}_e \rightarrow \bar{\nu}_\mu$

- Running experiments:

  T2K (295km) and NOvA (810km)

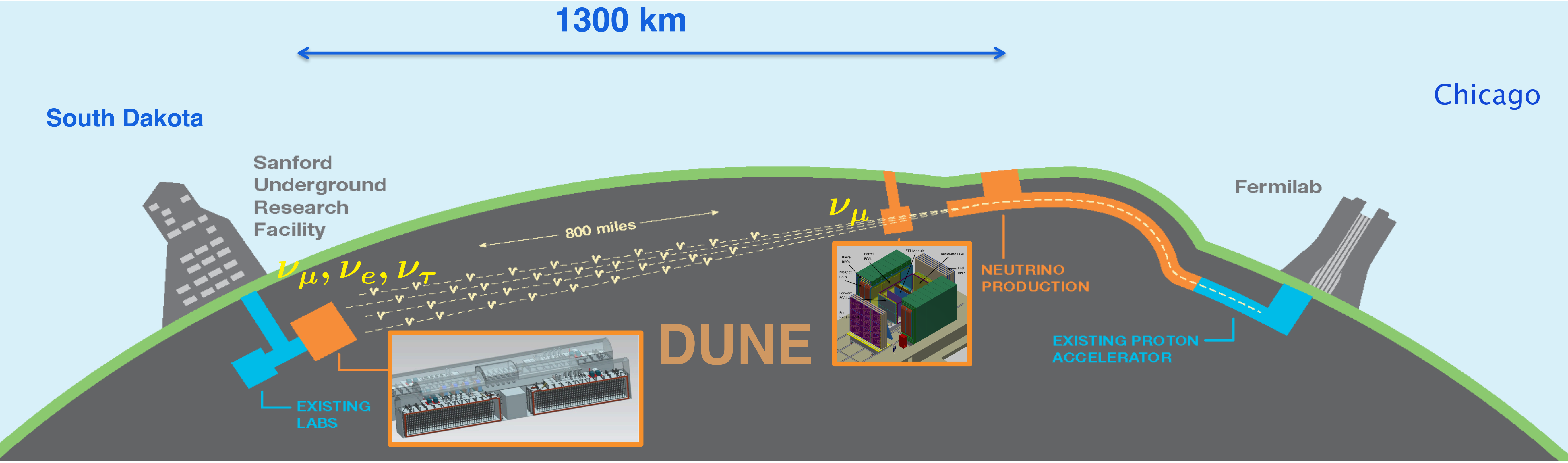
- Future experiments:

  DUNE (40 ktons LAr, 1300km)

  HyperKamiokaNDE (0.5kMtons H_2O, 295km) ← 0.2Mt + T2HKK

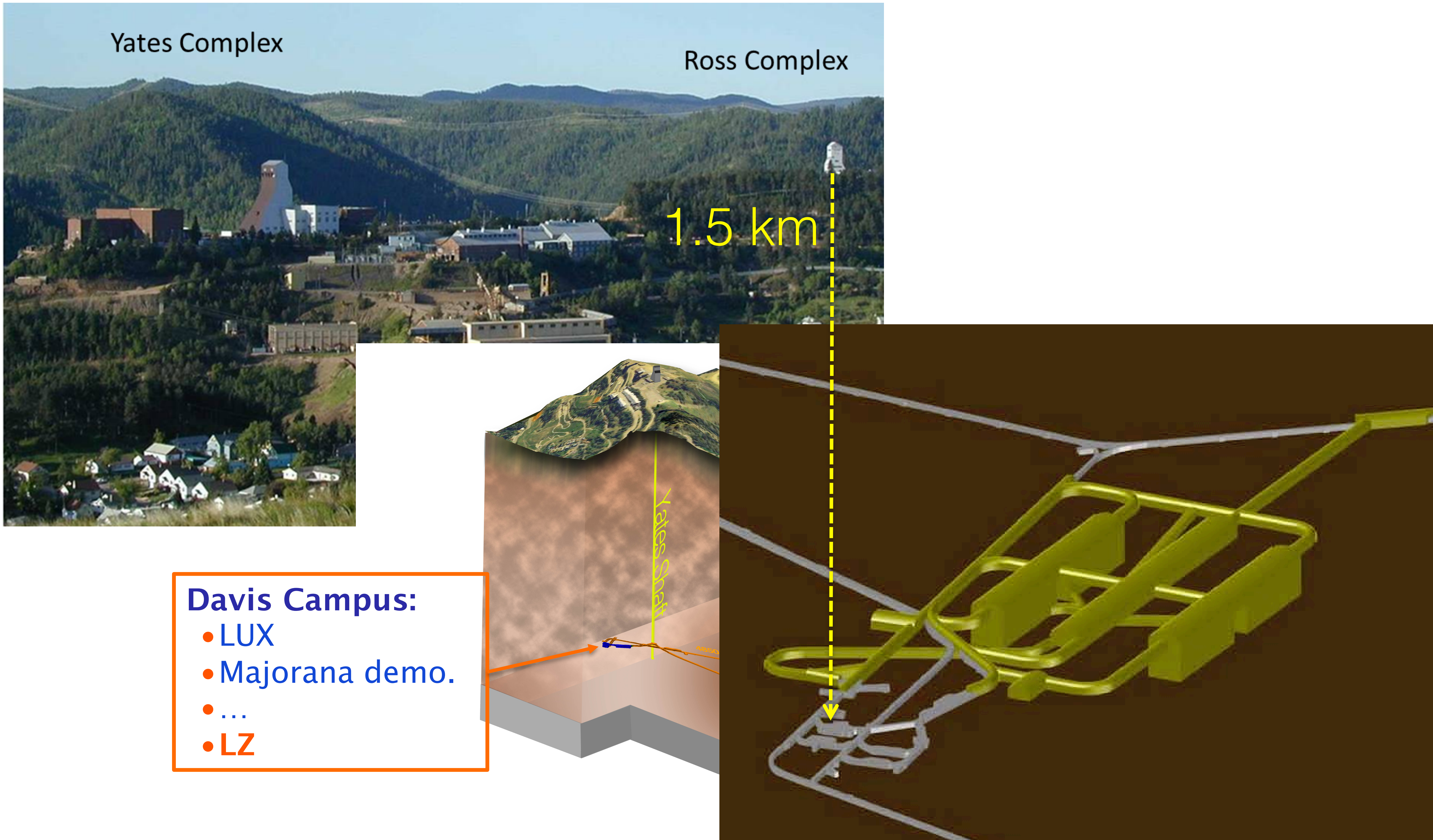
What is DUNE/LBNF ?

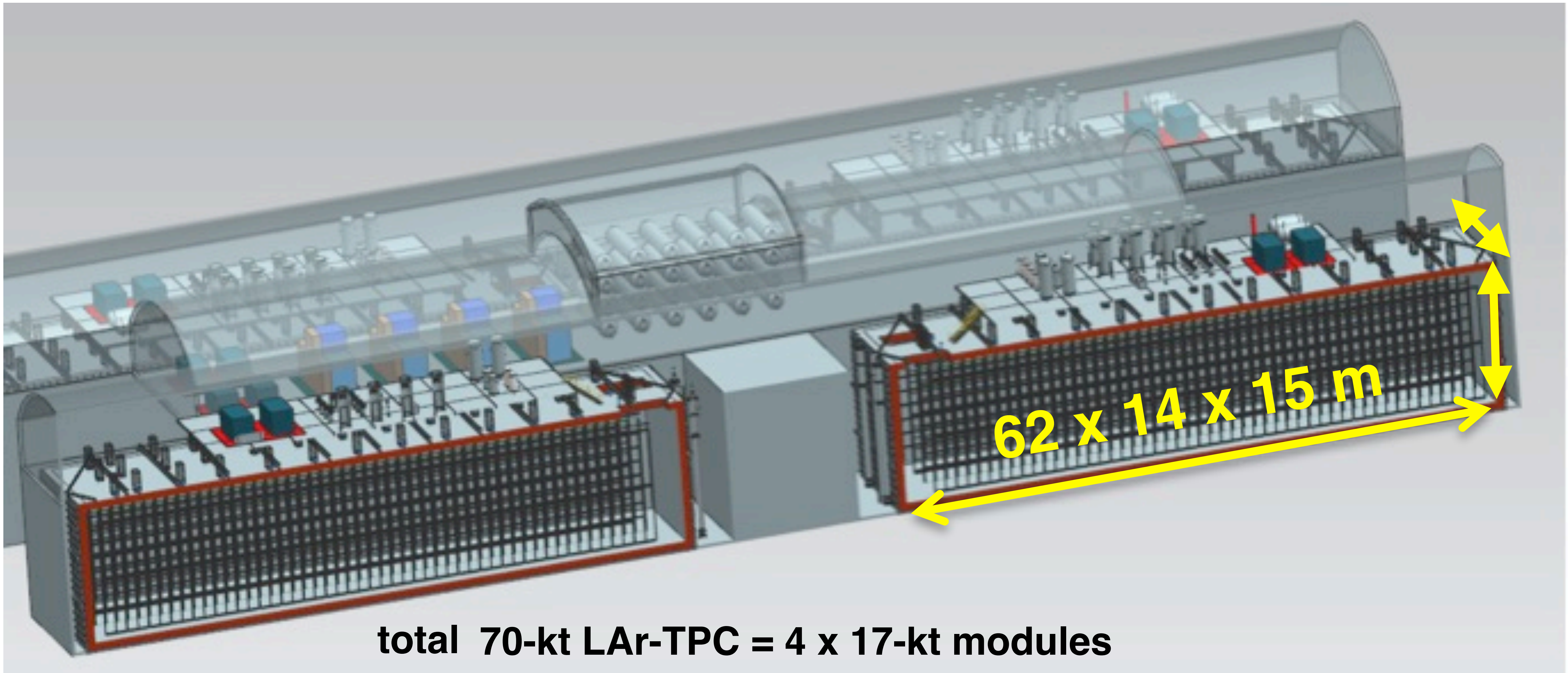
- **DUNE/LBNF will consist of**
  - **An intense (1-2 MW) neutrino beam from Fermilab**
  - **A massive (70 kton) deep underground LAr Detector South Dakota**
  - **A large Near Detector at Fermilab**
  - **A large International Collaboration (~1000 scientist)**



DUNE Far Detector site

- Sanford Underground Research Facility (SURF), South Dakota
- Four caverns on 4850ft level (~1.5km underground)



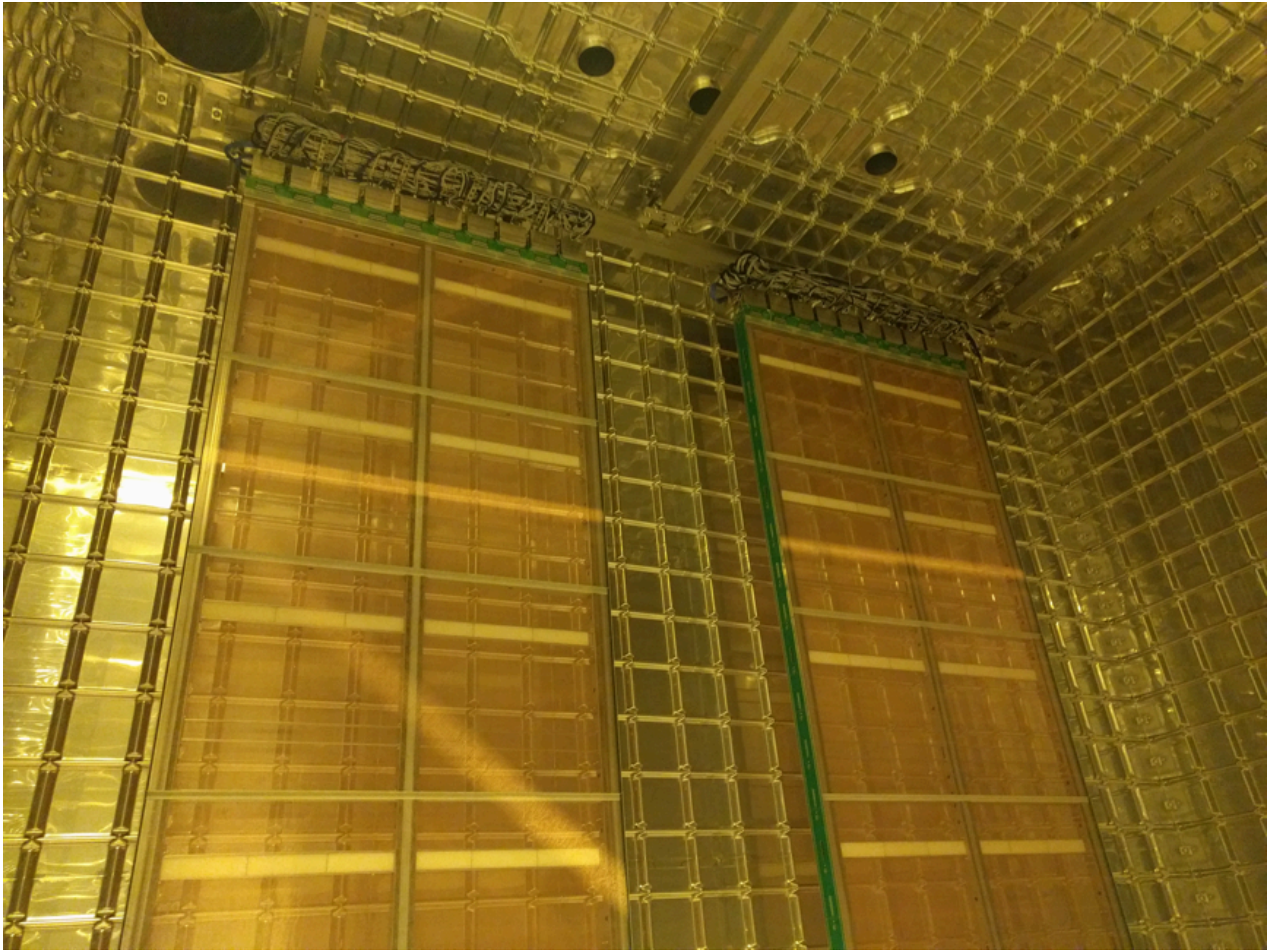


Fiducial = 4 x 10 kt

Ar from $\sim \mathbf{10}\ \mathrm{km}^{\mathbf{3}}$ of air

$=$ 300m $\times$ Area of Fermilab site (30 $\mathrm{km}^{\mathbf{2}}$)

Proto-DUNE 2/2/2018



Neutrino Oscillation Amplitudes

$$P(\nu_\alpha \to \nu_\beta) \quad = \quad |\mathcal{A}_{\alpha\beta}|^2$$

Two Flavors:

$$\mathcal{A}_{\alpha\alpha} = 1 + (2i)\; s^2_\theta \; e^{+i\Delta} \;\; \sin\Delta$$

and $\mathcal{A}_{\alpha\beta} = (2i)\; s_\theta c_\theta \; e^{-i\Delta} \;\; \sin\Delta$

$$\Delta \equiv \Delta m^2 L/4E$$

Neutrino Oscillation Amplitudes in vacuum:

"the billion $ process"

$$P(\nu_\mu \to \nu_e) \quad = \quad |\mathcal{A}_{\mu e}|^2$$

$$\mathcal{A}_{\mu e} \quad = \quad (2i)\;[\;(s_{23}s_{13}c_{13})\;[\;c_{12}^2 e^{-i\Delta_{32}} \sin\Delta_{31} + s_{12}^2 e^{-i\Delta_{31}} \sin\Delta_{32}\;]$$
$$+\;(c_{23}c_{13}s_{12}c_{12})\;e^{i\delta}\;\sin\Delta_{21}\;]$$

maintain the symmetry: $m_1^2 \leftrightarrow m_2^2$ with $\theta_{12} \to \theta_{12} \pm \pi/2$

Denton, Minakata, SP arXiv:1604.08167

$$\Delta P_{CP} = 8\;\underbrace{(s_{23}s_{13}c_{13})\;(c_{23}c_{13}s_{12}c_{12})\;\sin\delta}_{\mathbf{J}}\;\sin\Delta_{21}\;\sin\Delta_{31}\;\sin\Delta_{32}$$

$$\Delta_{32} \approx \Delta_{31}$$

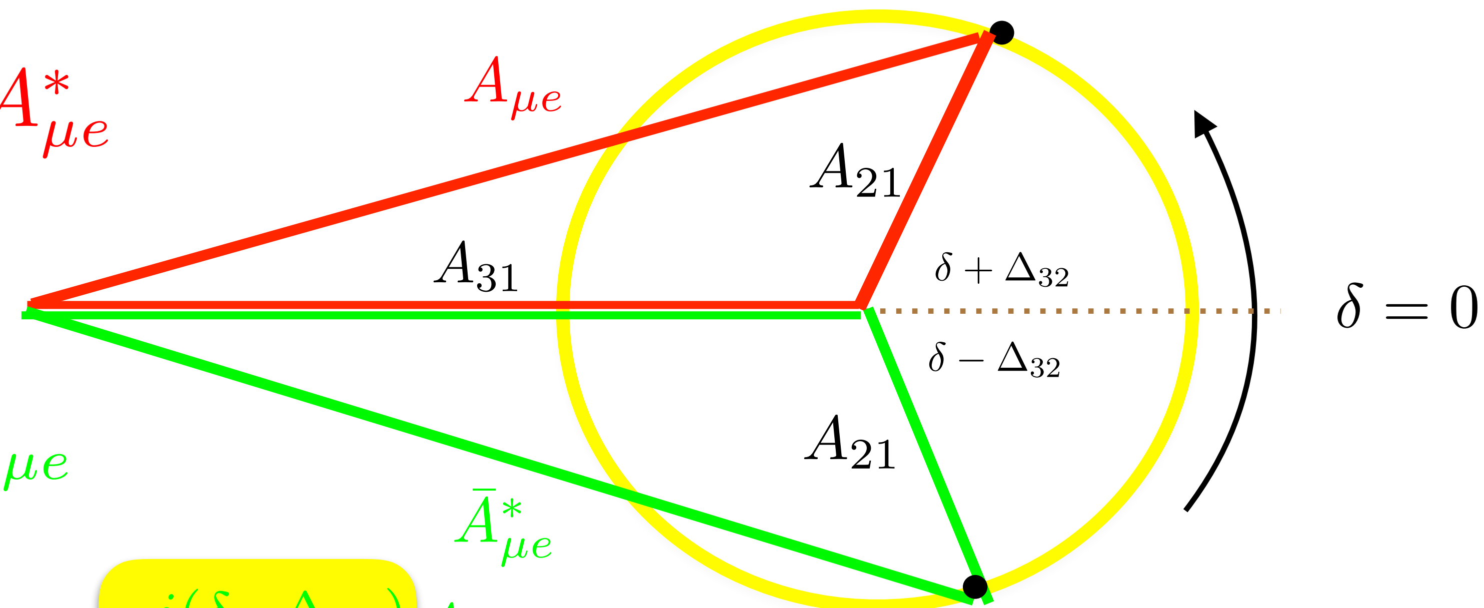
$$\mathcal{A}_{\mu e} \quad \approx \quad (2i)\;[\;(s_{23}s_{13}c_{13})\;\sin\Delta_{31} + (c_{23}c_{13}s_{12}c_{12})\;e^{i(\delta+\Delta_{31})}\;\sin\Delta_{21}\;]$$

$\nu_\mu \to \nu_e$

$A_{31} = 2 s_{23} s_{13} c_{13} \sin\Delta_{31}$ $\qquad$ $A_{21} = 2 c_{13} c_{23} s_{12} c_{12} \sin\Delta_{21}$

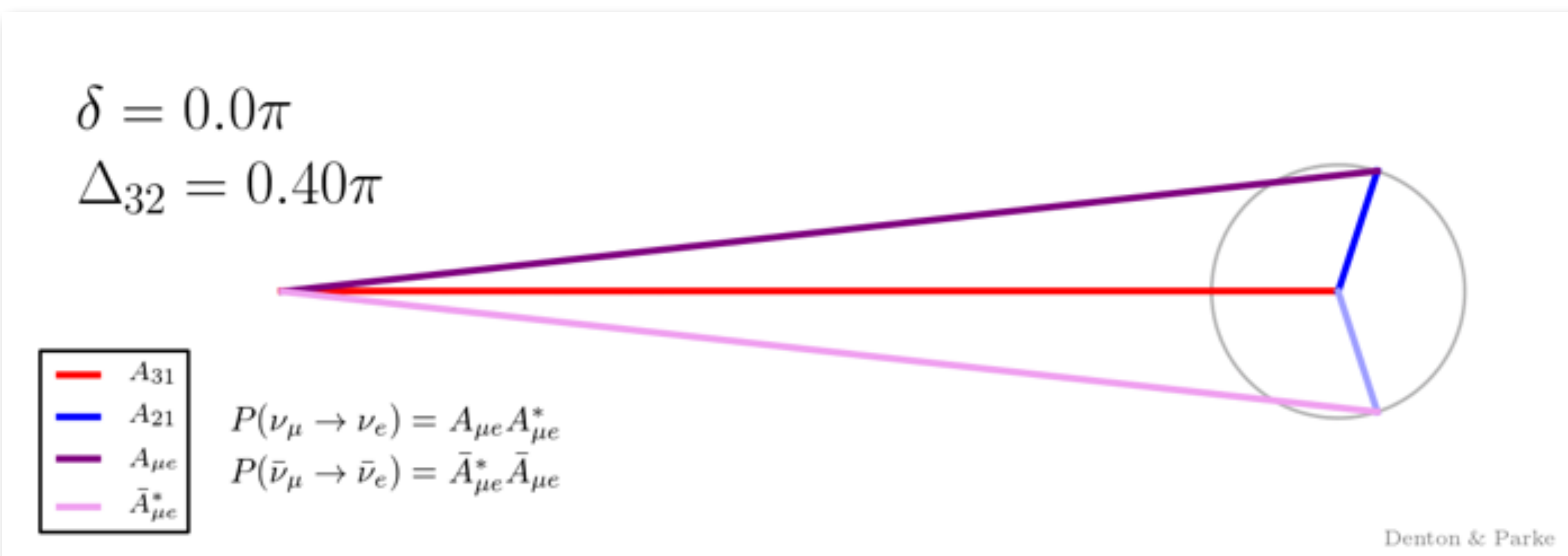
$A_{\mu e} = A_{31} + e^{i(\delta+\Delta_{32})} A_{21}$ $\qquad$ $\Delta_{ij} = \Delta m^2_{ij} L/4E$

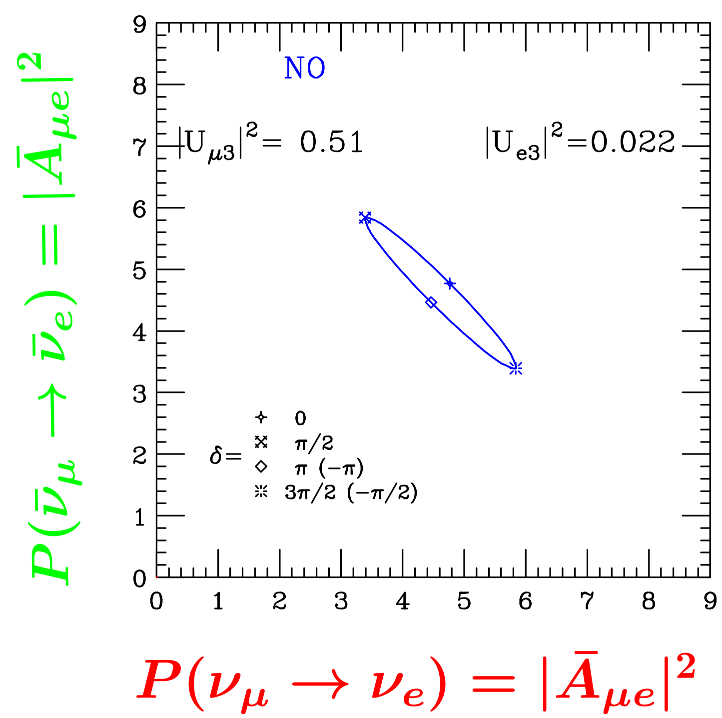
$P(\nu_\mu \to \nu_e) = A_{\mu e} A^*_{\mu e}$



$P(\bar{\nu}_\mu \to \bar{\nu}_e) = \bar{A}^*_{\mu e} \bar{A}_{\mu e}$

$\bar{A}^*_{\mu e} = A_{31} + e^{i(\delta-\Delta_{32})} A_{21}$





Matter Effects:

Neutrino Evolution in Matter:

$$i\frac{d}{dx}\nu = H\nu \quad \text{with} \quad \nu = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

$$(2E)\,H = U_{PMNS} \begin{bmatrix} 0 & 0 & 0 \\ 0 & \Delta m^2_{21} & 0 \\ 0 & 0 & \Delta m^2_{31} \end{bmatrix} U^\dagger_{PMNS} + \begin{bmatrix} a & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$a = 2\sqrt{2} G_F N_e E$$

uniform matter

- Solve Cubic Characteristic Eqn.

$$\lambda^3 - \left(a + \Delta m^2_{21} + \Delta m^2_{31}\right)\lambda^2$$
$$+[\Delta m^2_{21}\Delta m^2_{31} + a\left\{(c^2_{12} + s^2_{12}s^2_{13})\Delta m^2_{21} + c^2_{13}\Delta m^2_{31}\right\}]\,\lambda$$
$$-\ c^2_{12}c^2_{13}a\Delta m^2_{21}\Delta m^2_{31} \qquad = 0$$

IF

- $a = 0$
- or $\Delta m^2_{21} = 0$
- or $\sin\theta_{12} = 0$
- or $\sin\theta_{13} = 0$

THEN characteristic Eqn

FACTORIZES !

BUT

See Zaglauer & Schwarzer, Z. Phys. C 1988

$$\lambda_1 = \frac{1}{3}s - \frac{1}{3}\sqrt{s^2-3t}\,[u + \sqrt{3(1-u^2)}],$$

$$\lambda_2 = \frac{1}{3}s - \frac{1}{3}\sqrt{s^2-3t}\,[u - \sqrt{3(1-u^2)}],$$

$$\lambda_3 = \frac{1}{3}s + \frac{2}{3}u\sqrt{s^2-3t},$$

$$s = \Delta_{21} + \Delta_{31} + a,$$

$$t = \Delta_{21}\Delta_{31} + a[\Delta_{21}(1 - s^2_{12}c^2_{13}) + \Delta_{31}(1 - s^2_{13})],$$

$$u = \cos\left[\frac{1}{3}\cos^{-1}\left(\frac{2s^3 - 9st + 27a\Delta_{21}\Delta_{31}c^2_{12}c^2_{13}}{2(s^2-3t)^{3/2}}\right)\right],$$

here $\Delta_{ij} \equiv \Delta m^2_{ij}$

DOES NOT TRIVIALLY SIMPLIFY !

2 flavor mixing in matter

$$ax^2 + bx + c = 0$$

simple, intuitive, useful

3 flavor mixing in matter

$$ax^3 + bx^2 + cx + d = 0$$

complicated, counter intuitive, …

Stephen Parke, Fermilab 1/31/2018

arXiv:1604.08167v1 [hep-ph] 27 Apr 2016

Compact Perturbative Expressions For Neutrino Oscillations in Matter

Peter B. Denton[a,b] Hisakazu Minakata[c,d] Stephen J. Parke[a]

Addendum to "Compact Perturbative Expressions for Neutrino Oscillations in Matter"

1801.06514

Peter B. Denton,[a] Hisakazu Minakata,[b] Stephen J. Parke[c]

doi: 10.5281/zenodo.1163591

Neutrino Evolution in Matter:

$$i\frac{d}{dx}\nu = H\nu \quad \text{with} \quad \nu = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

$$(2E)\,H = U_{PMNS} \begin{bmatrix} 0 & 0 & 0 \\ 0 & \Delta m^2_{21} & 0 \\ 0 & 0 & \Delta m^2_{31} \end{bmatrix} U^\dagger_{PMNS} + \begin{bmatrix} a & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$a = 2\sqrt{2} G_F N_e E$$

$$U_{PMNS} \equiv U_{23}(\theta_{23}, 0)\, U_{13}(\theta_{13}, -\delta)\, U_{12}(\theta_{12}, 0) :=: U_{23}(\theta_{23}, \delta)\, U_{13}(\theta_{13}, 0)\, U_{12}(\theta_{12}, 0)$$

$:=:$ means equal after multiplying by a diagonal phase matrix on the left and/or right hand side.

$$i\frac{d}{dx}\,\nu' = \boxed{U^\dagger_{23}(\theta_{23}, \delta)\, H\, U_{23}(\theta_{23}, \delta)}\,\nu' \qquad \text{with } \nu' = U^\dagger_{23}(\theta_{23}, \delta) \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

Neutrino Evolution in Matter (conti):

$$U_{23}^{\dagger}(\theta_{23},\delta)\, H\, U_{23}(\theta_{23},\delta) =$$

$$\frac{1}{2E}\begin{pmatrix} a + s_{13}^2\Delta m_{31}^2 + s_{12}^2 c_{13}^2 \Delta m_{21}^2 & c_{13}s_{12}c_{12}\Delta m_{21}^2 & s_{13}c_{13}\Delta m_{31}^2 - s_{12}^2 s_{13}c_{13}\Delta m_{21}^2 \\ c_{13}s_{12}c_{12}\Delta m_{21}^2 & c_{12}^2\Delta m_{21}^2 & -s_{13}s_{12}c_{12}\Delta m_{21}^2 \\ s_{13}c_{13}\Delta m_{31}^2 - s_{12}^2 s_{13}c_{13}\Delta m_{21}^2 & -s_{13}s_{12}c_{12}\Delta m_{21}^2 & c_{13}^2\Delta m_{31}^2 + s_{12}^2 s_{13}^2 \Delta m_{21}^2 \end{pmatrix}$$

Expansions in
$$\begin{cases} s_{13} & \sim 0.15 \\ (\Delta m_{21}^2/\Delta m_{31}^2) & \sim 0.03 \\ (a/\Delta m_{31}^2) & \sim\ (E_\nu/10GeV) \end{cases}$$

Key observations:

- Don't use Δm_{31}^2
- Use $\Delta m_{ee}^2 = \Delta m_{31}^2 - s_{12}^2 \Delta m_{21}^2$
- Subtract $s_{12}^2\Delta m_{21}^2$ from all diagonal elements

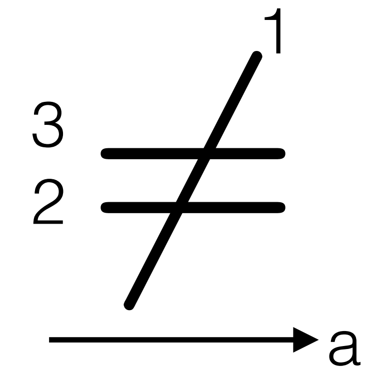
Simple but major improvements in accuracy !

Neutrino Evolution in Matter (conti):

$$U_{23}^\dagger(\theta_{23},\delta)\, H\, U_{23}(\theta_{23},\delta) = H_D + H_{OD}$$

D=diagonal OD= off-diagonal

$$(2E)\, H_D = \begin{bmatrix} a + s_{13}^2 \Delta m_{ee}^2 & & \\ & (c_{12}^2 - s_{12}^2)\Delta m_{21}^2 & \\ & & c_{13}^2 \Delta m_{ee}^2 \end{bmatrix}$$



$$\Delta m_{ee}^2 \equiv \cos^2\theta_{12}\Delta m_{31}^2 + \sin^2\theta_{12}\Delta m_{32}^2$$

$$(2E)\, H_{OD}/\Delta m_{ee}^2 = s_{13}c_{13} \begin{bmatrix} & & 1 \\ & 0 & \\ 1 & & \end{bmatrix}$$

$$+\, c_{13}\; s_{12}c_{12} \left(\frac{\Delta m_{21}^2}{\Delta m_{ee}^2}\right) \begin{bmatrix} & 1 & \\ 1 & & 0 \\ & 0 & \end{bmatrix}$$

$$-\, s_{13}\; s_{12}c_{12} \left(\frac{\Delta m_{21}^2}{\Delta m_{ee}^2}\right) \begin{bmatrix} & 0 & \\ 0 & & 1 \\ & 1 & \end{bmatrix}$$

0.15

0.015

0.002

Rotation by $U_{13}(\widetilde{\theta}_{13})$

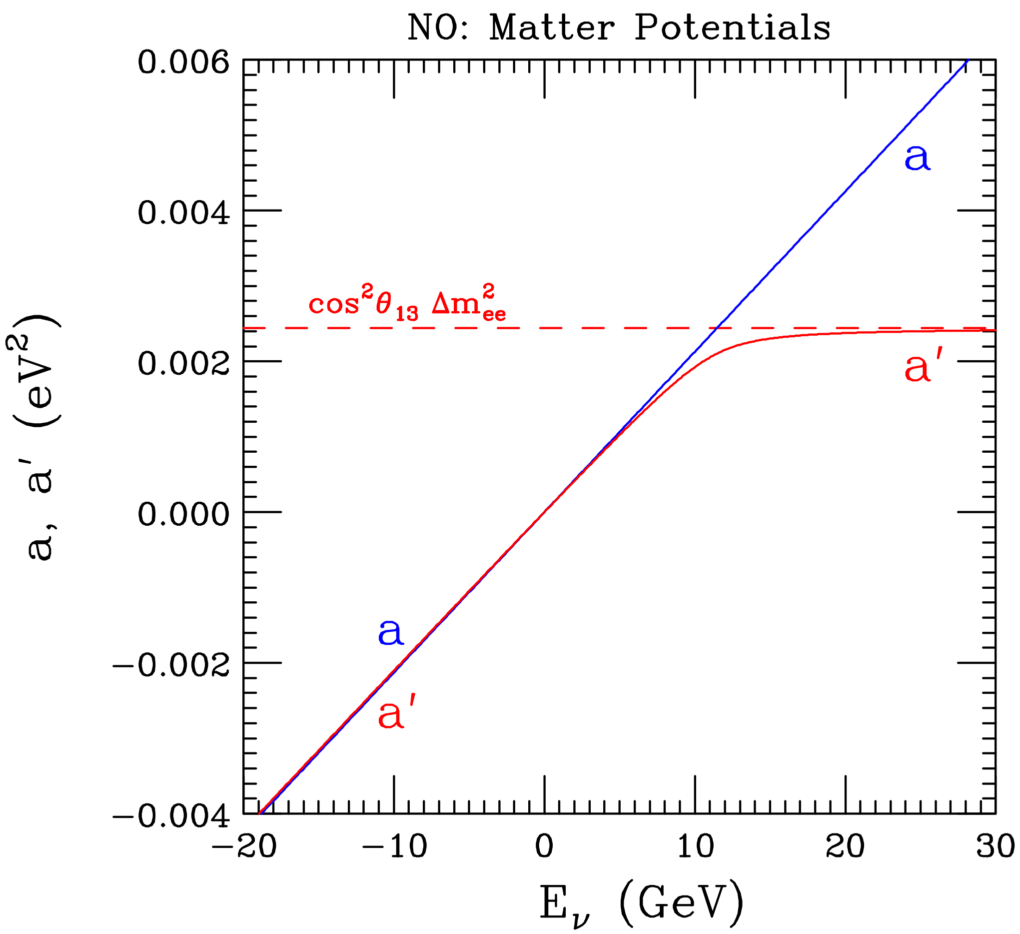
then $U_{12}(\widetilde{\theta}_{12})$

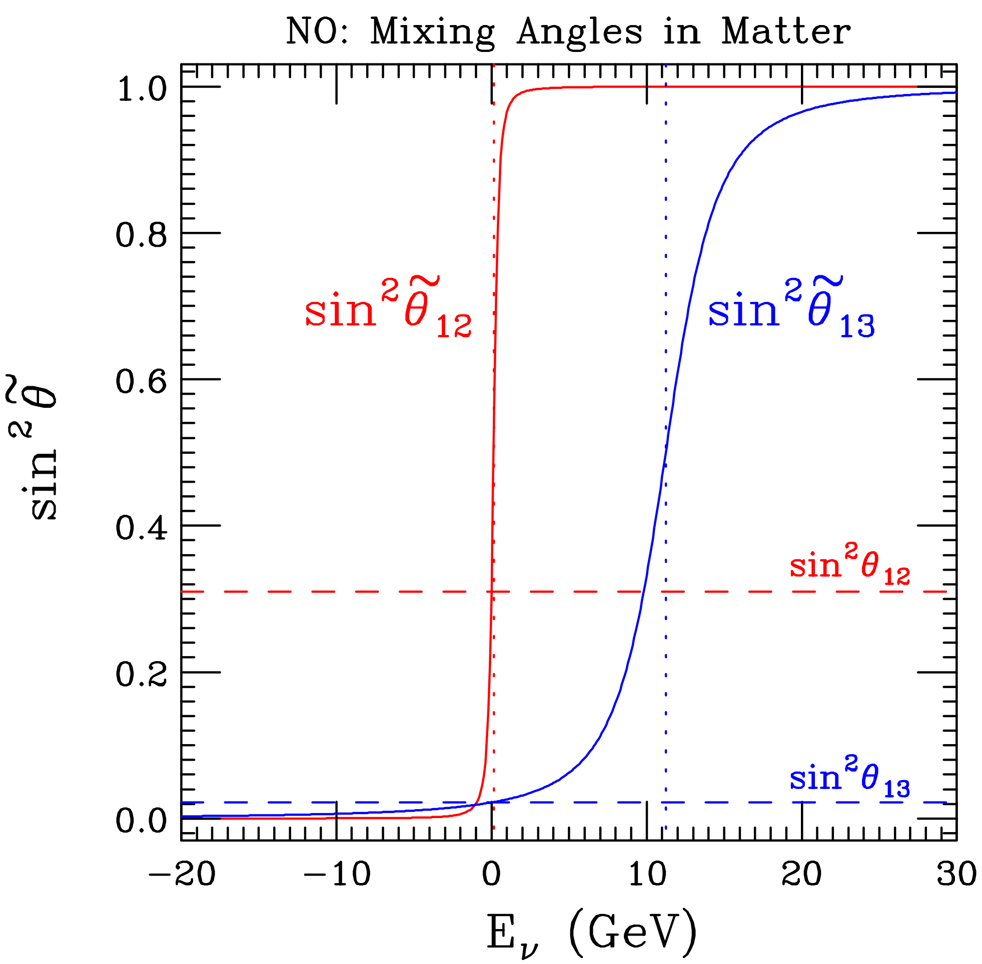
$$a = 2\sqrt{2}G_F N_e E$$

$$\cos 2\widetilde{\theta}_{13} = \frac{(\cos 2\theta_{13} - a/\Delta m^2_{ee})}{\sqrt{(\cos 2\theta_{13} - a/\Delta m^2_{ee})^2 + \sin^2 2\theta_{13}}}$$

$$a' \equiv a \cos^2 \widetilde{\theta}_{13} + \Delta m^2_{ee} \sin^2(\widetilde{\theta}_{13} - \theta_{13})$$

$$\cos 2\widetilde{\theta}_{12} = \frac{(\cos 2\theta_{12} - a'/\Delta m^2_{21})}{\sqrt{(\cos 2\theta_{12} - a'/\Delta m^2_{21})^2 + \sin^2 2\theta_{12} \cos^2(\widetilde{\theta}_{13} - \theta_{13})}}$$





Masses Squared:

$$(2E)\, H_D = \text{diag}(\, \widetilde{m^2}_1,\ \widetilde{m^2}_2,\ \widetilde{m^2}_3\ )$$

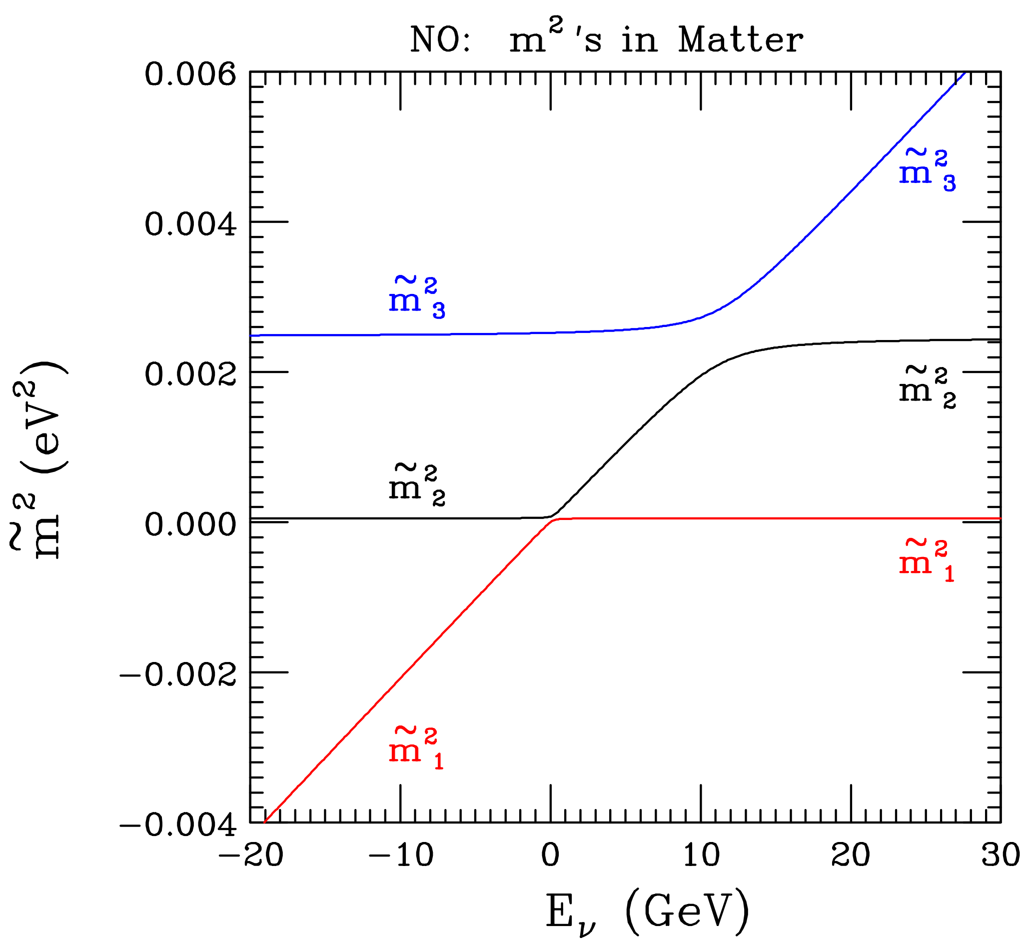
$$\widetilde{m_1^2} = \frac{1}{2}(\Delta m^2_{21} - \Delta\widetilde{m^2}_{21} + a'\,)$$

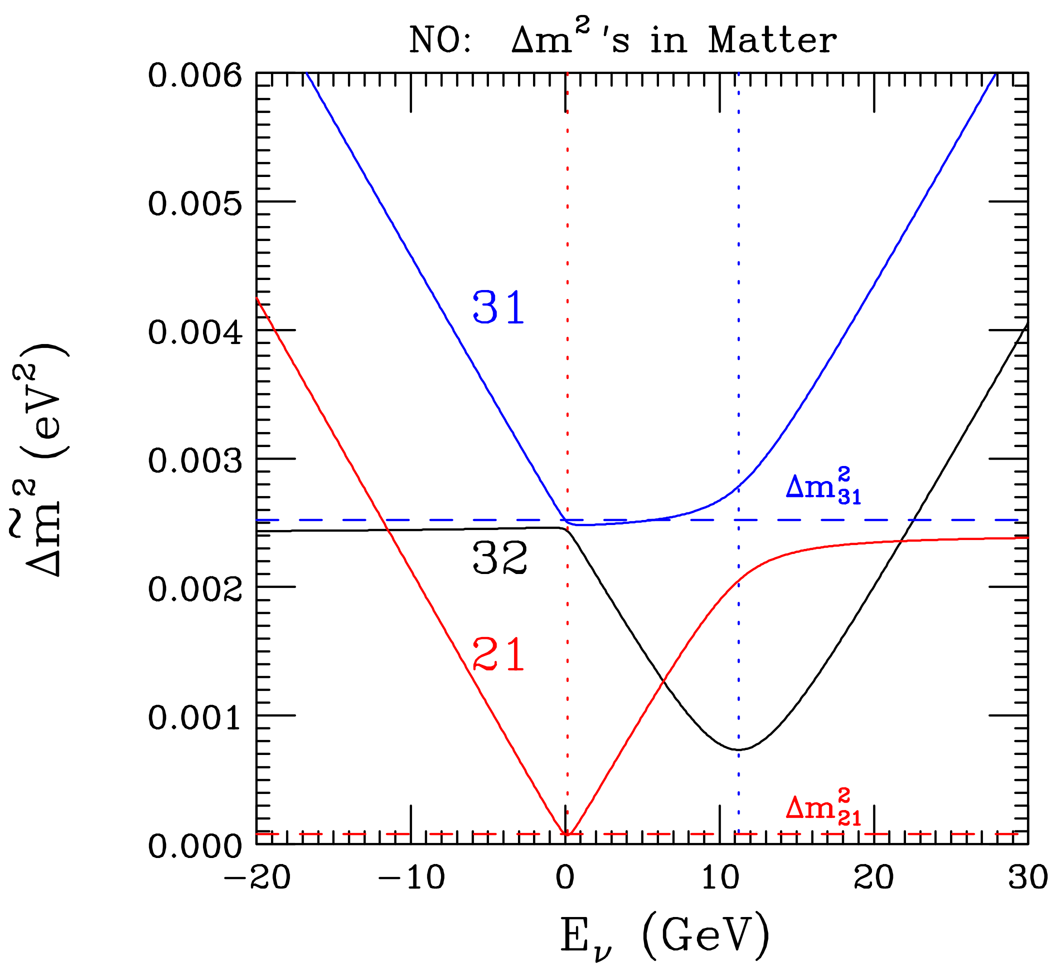
$$\widetilde{m_2^2} = \frac{1}{2}(\Delta m^2_{21} + \Delta\widetilde{m^2}_{21} + a'\,)$$

$$\widetilde{m_3^2} = \Delta m^2_{31} + (\,a - a'\,)$$

$$\Delta\,\widetilde{m^2}_{21} = \Delta m^2_{21}\sqrt{(\cos 2\theta_{12} - a'/\Delta m^2_{21})^2 \;+\; \sin^2 2\theta_{12}\cos^2(\tilde{\theta}_{13} - \theta_{13})} \;\approx |\,\Delta m^2_{21}\,\cos 2\theta_{12} - a'\,|$$

when $|a'| \gg \Delta m^2_{21}$





vacuum $\Rightarrow$ matter

$$\begin{aligned} \Delta m^2_{jk} &\rightarrow \Delta \widetilde{m^2}_{jk} \\ \theta_{13} &\rightarrow \widetilde{\theta}_{13} \\ \theta_{12} &\rightarrow \widetilde{\theta}_{12} \\ \theta_{23} &\rightarrow \theta_{23} \\ \delta &\rightarrow \delta \end{aligned}$$

0th order !

$$P^{vac}_{\nu_\alpha \rightarrow \nu_\beta}(\Delta m^2_{31}, \Delta m^2_{21}, \theta_{13}, \theta_{12}, \theta_{23}, \delta)$$

$$\Rightarrow P^{mat}_{\nu_\alpha \rightarrow \nu_\beta}(\Delta \widetilde{m^2}_{31}, \Delta \widetilde{m^2}_{21}, \widetilde{\theta}_{13}, \widetilde{\theta}_{12}, \theta_{23}, \delta)$$

Intuitive and Analytically simple !

$$U_{23}^\dagger(\theta_{23},\delta)\, H\, U_{23}(\theta_{23},\delta) = H_D + H_{OD}$$

What about H_{OD} ?

$$(2E)\, H_{OD}/\Delta m^2_{ee} \quad = \quad \sin(\widetilde{\theta}_{13} - \theta_{13})\; s_{12}c_{12} \left(\frac{\Delta m^2_{21}}{\Delta m^2_{ee}}\right) \begin{bmatrix} & & -\widetilde{s}_{12} \\ & & \widetilde{c}_{12} \\ -\widetilde{s}_{12} & \widetilde{c}_{12} & \end{bmatrix}$$

$\widetilde{s}_{12} \equiv \sin\widetilde{\theta}_{12}$, etc

$$\sin(\widetilde{\theta}_{13} - \theta_{13}) \approx s_{13}c_{13}\left(\frac{a}{\Delta m^2_{ee}}\right) \sim 0.03\,\left(\frac{E}{2\ \mathrm{GeV}}\right)\left(\frac{\rho}{3\ \mathrm{g.cm^{-3}}}\right)$$

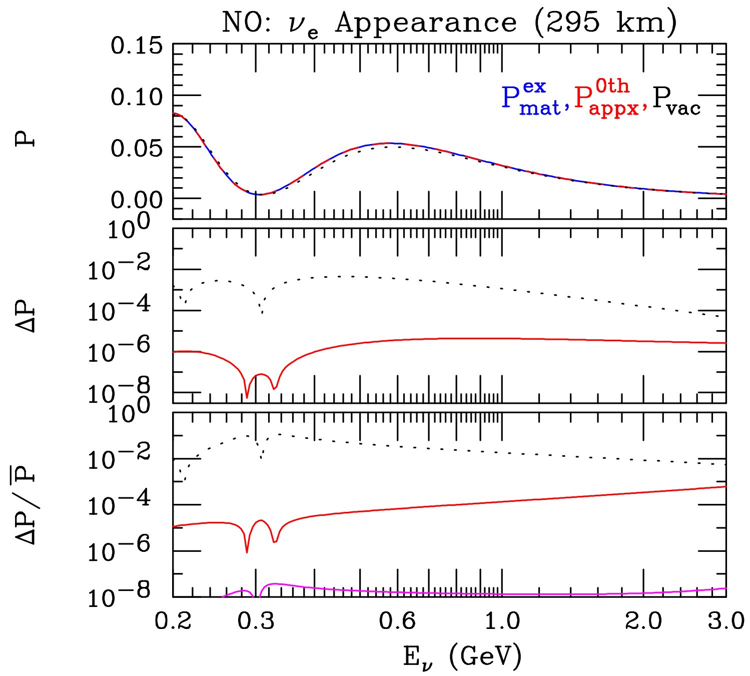
0.015

Vanishes in Vacuum

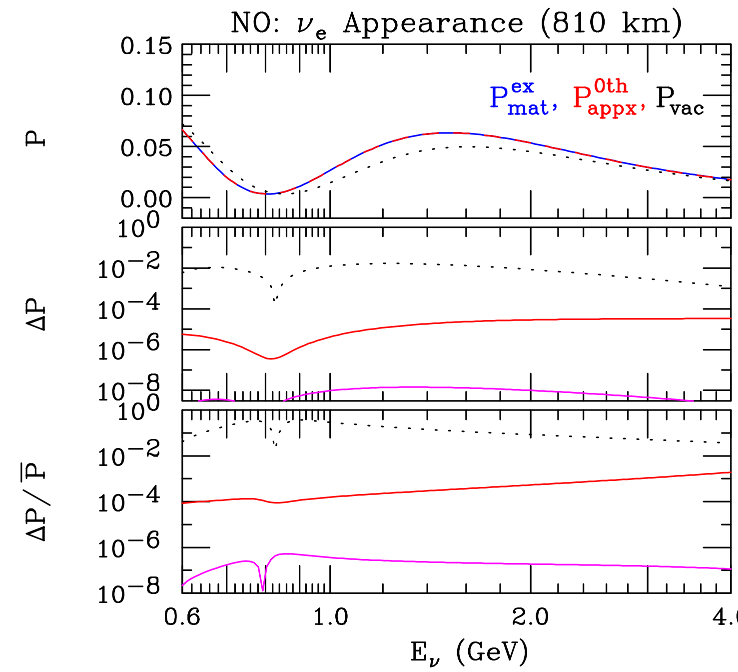
4×10^{-4} for $E = 2$ GeV and $\rho = 3$ g.cm^{-3}

Perturbation Theory !!!

T2K/HK



NOvA

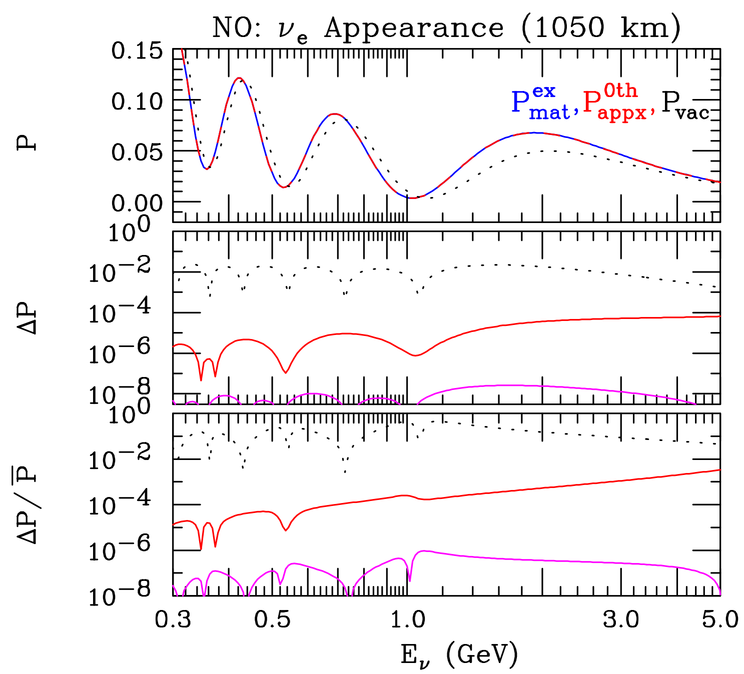


0th order

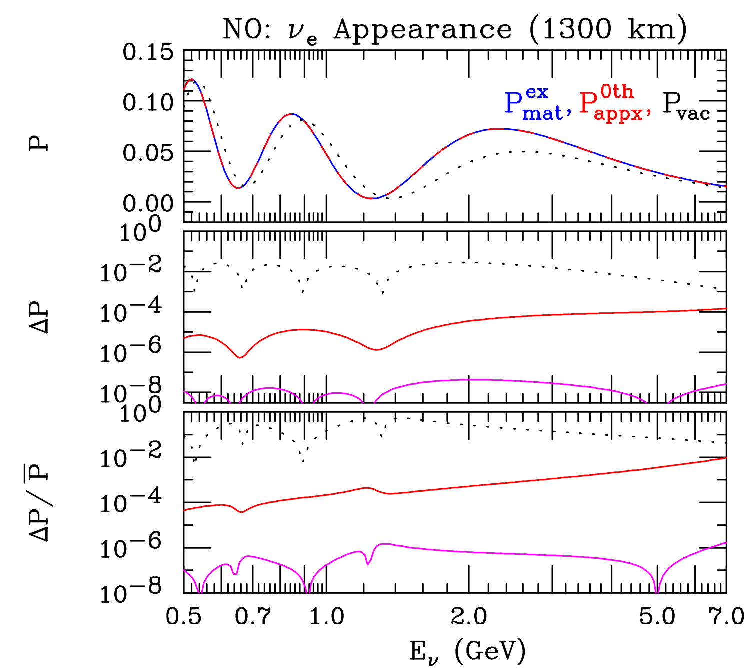
1st order

2nd order

T2HKK



DUNE



Top panel: P^{ex}_{mat}, P^{0th}_{appx} and P_{vac}

Middle and bottom panels:

black dotted lines	red solid lines	magenta solid lines
$\Delta P = \lvert P^{ex}_{mat} - P_{vac}\rvert$	$\Delta P = \lvert P^{ex}_{mat} - P^{0th}_{appx}\rvert$	$\Delta P = \lvert P^{ex}_{mat} - P^{1st}_{appx}\rvert$
$\overline{P} = \frac{1}{2}(P^{ex}_{mat} + P_{vac})$	$\overline{P} = \frac{1}{2}(P^{ex}_{mat} + P^{0th}_{appx})$	$\overline{P} = \frac{1}{2}(P^{ex}_{mat} + P^{1st}_{appx})$

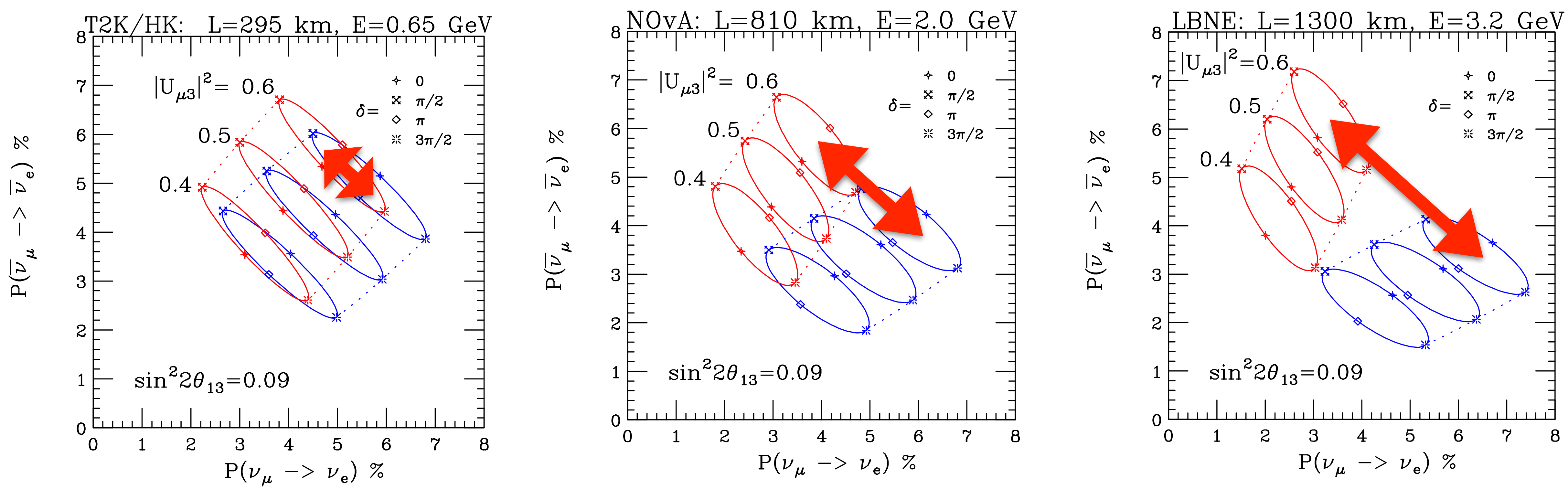
Correlations between $\nu_\mu \to \nu_e \quad \bar{\nu}_\mu \to \bar{\nu}_e$

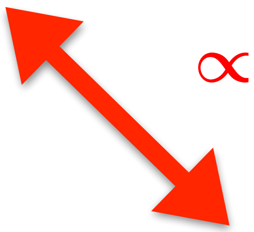
Normal Ordering — Inverted Ordering

$\nu_\mu \to \nu_\mu$ gives: $\sin^2 2\theta_{\mu\mu} \equiv 4|U_{\mu 3}|^2(1-|U_{\mu 3}|^2) = 0.96 \ - \ 1.00$

$|U_{\mu 3}|^2 \leftrightarrow (1-|U_{\mu 3}|^2)$ degeneracy !

T2K/HK — NOvA — DUNE (Same L/E as NOνA)



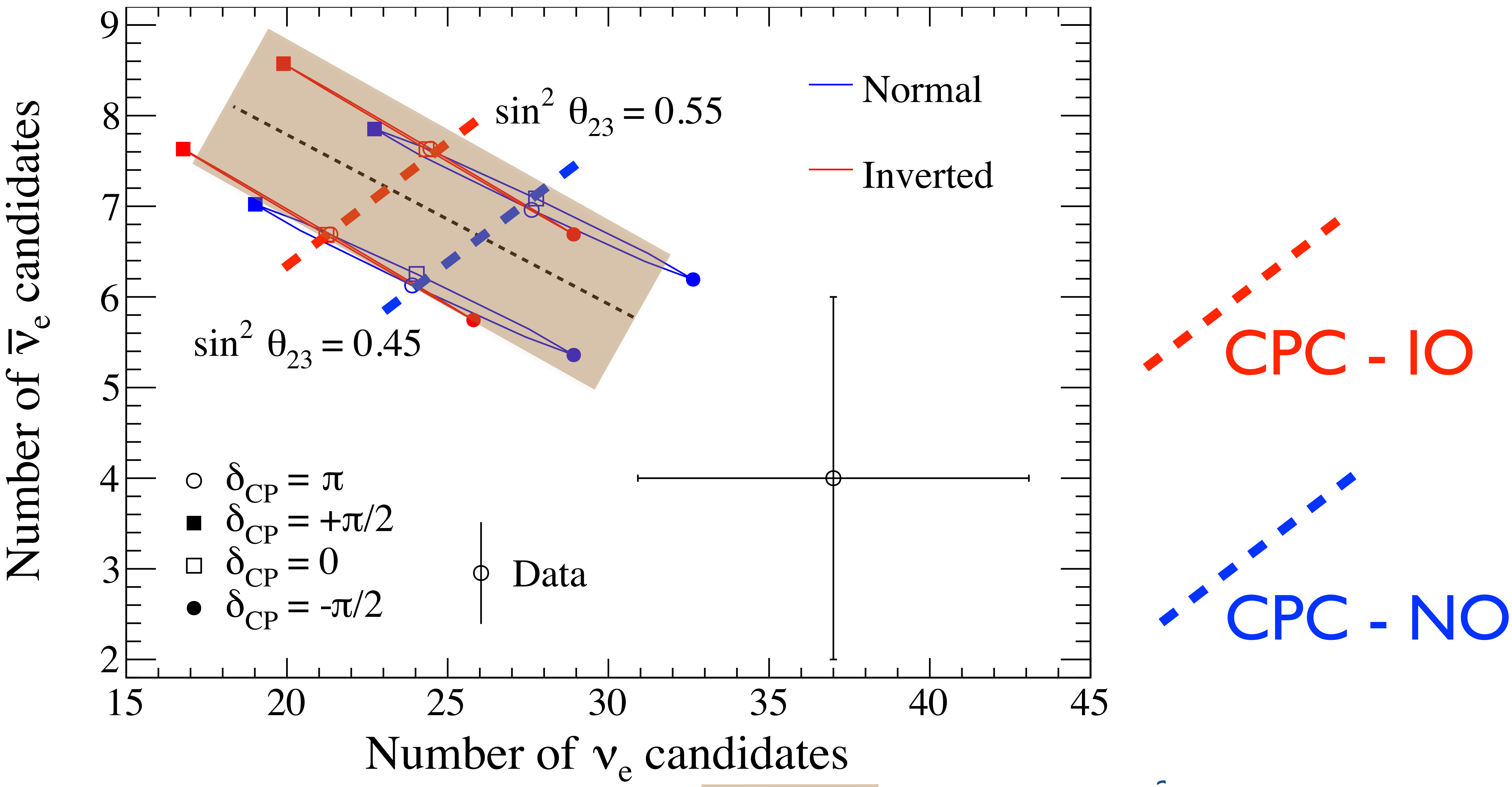


$\propto \rho L \ \sin^2\theta_{23}$

$$\sin\delta_{NO} - \sin\delta_{IO} = \tan\theta_{23} \times \begin{cases} 0.48 & \text{T2K} \\ 1.62 & \text{NO}\nu\text{A} \\ 2.60 & \text{DUNE} \end{cases}$$

O. Mena & SP hep-ph/0408070

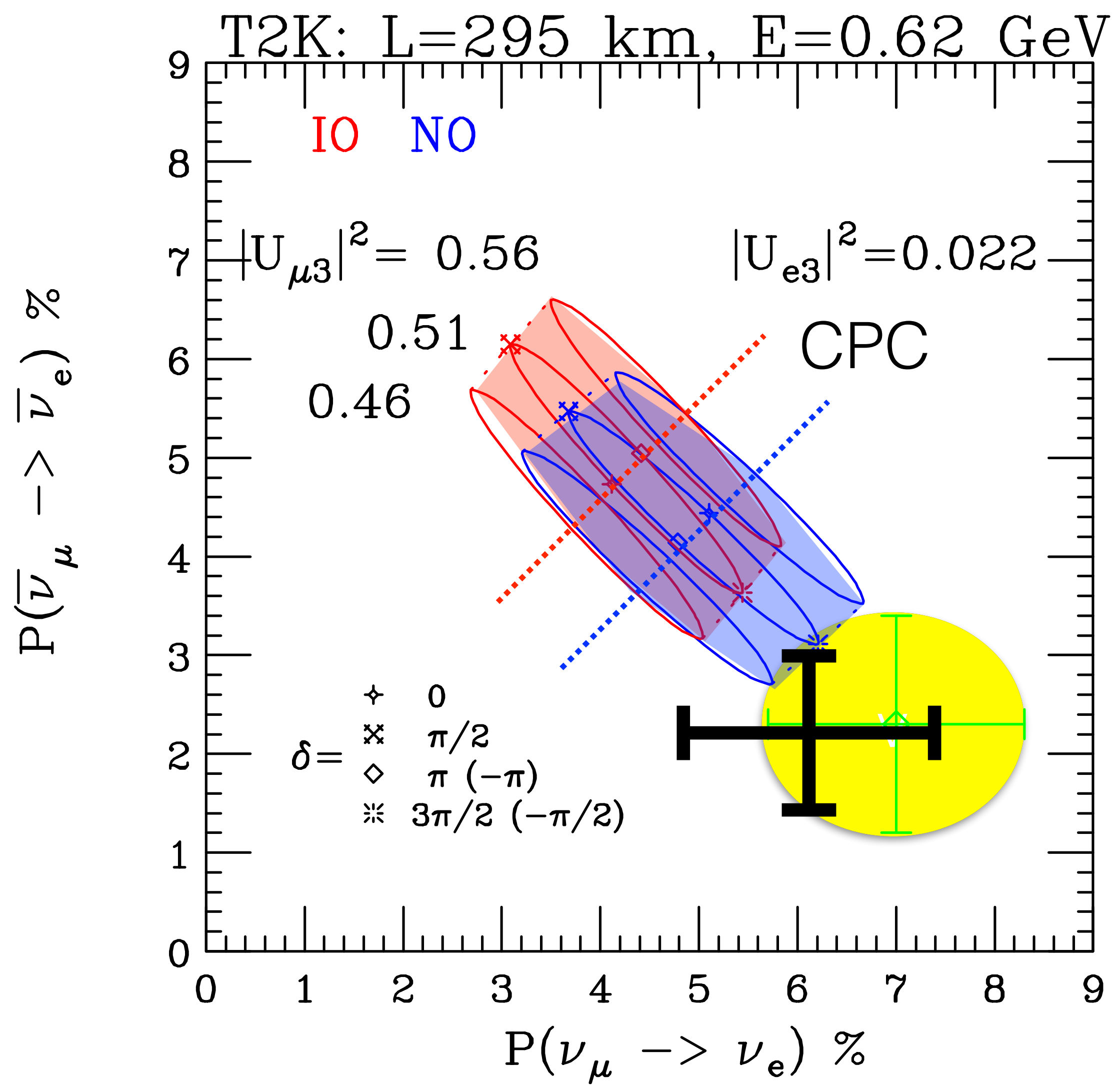
T2K

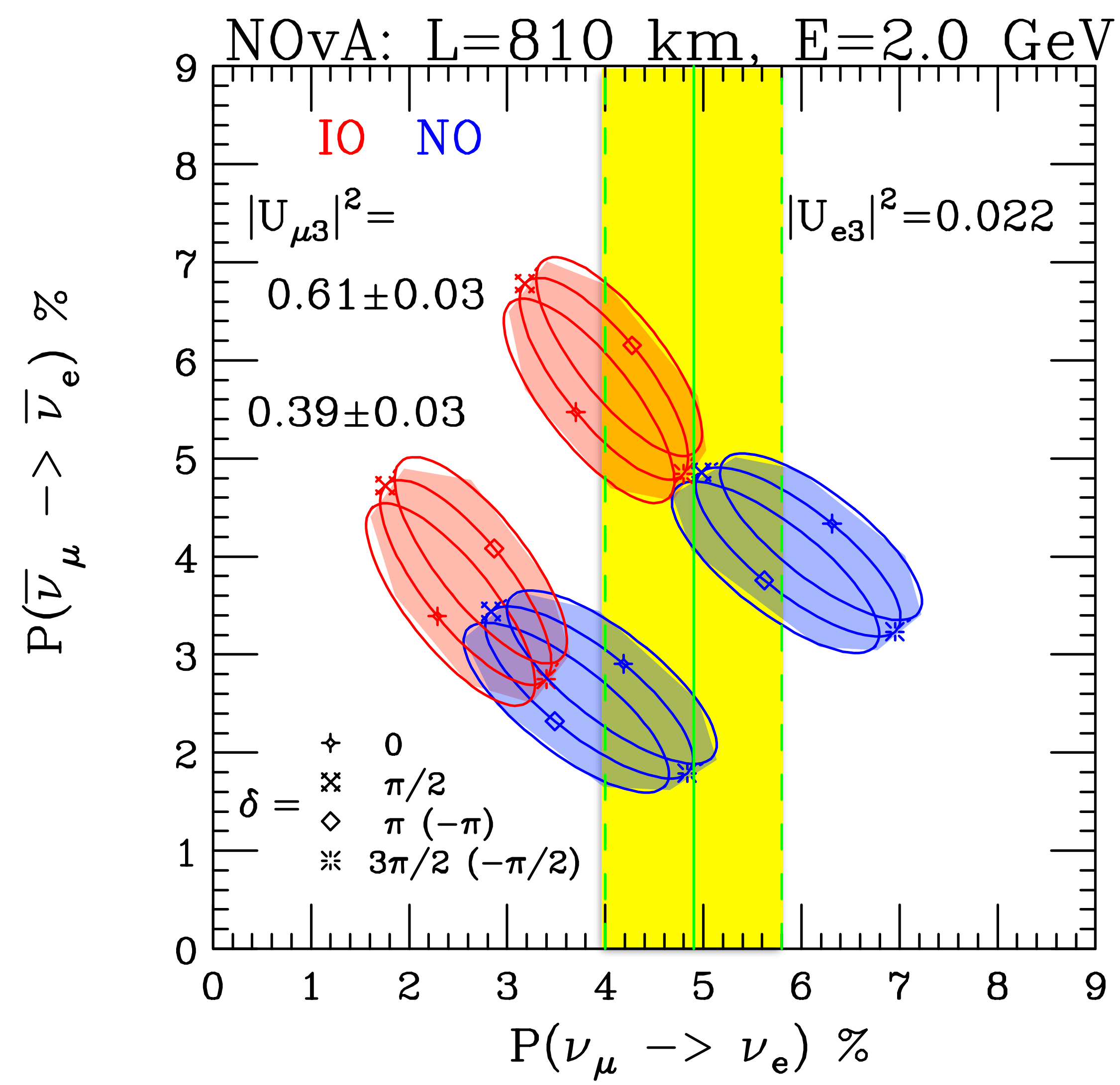


All except appearance !

T2K & NOvA

Number of Events proportional to Oscillation Probability





Needs an update !

1 sigma: NO IO Appearance data

Summary:

- from Nu1998 to now, tremendous exp. progress on Neutrino SM: more at Nu2018

- LSND Sterile Nu's neither confirmed or ruled out at acceptable CL: - ultra short baseline reactor exp.

- Great Theoretical progress on understand many aspects of Quantum Neutrino Physics: - Oscillations, Decoherence, Osc. Probabilities in Matter, Leptogenesis, ...

- Still searching for convincing model of Neutrino masses and mixings: with testable and confirmed predictions !

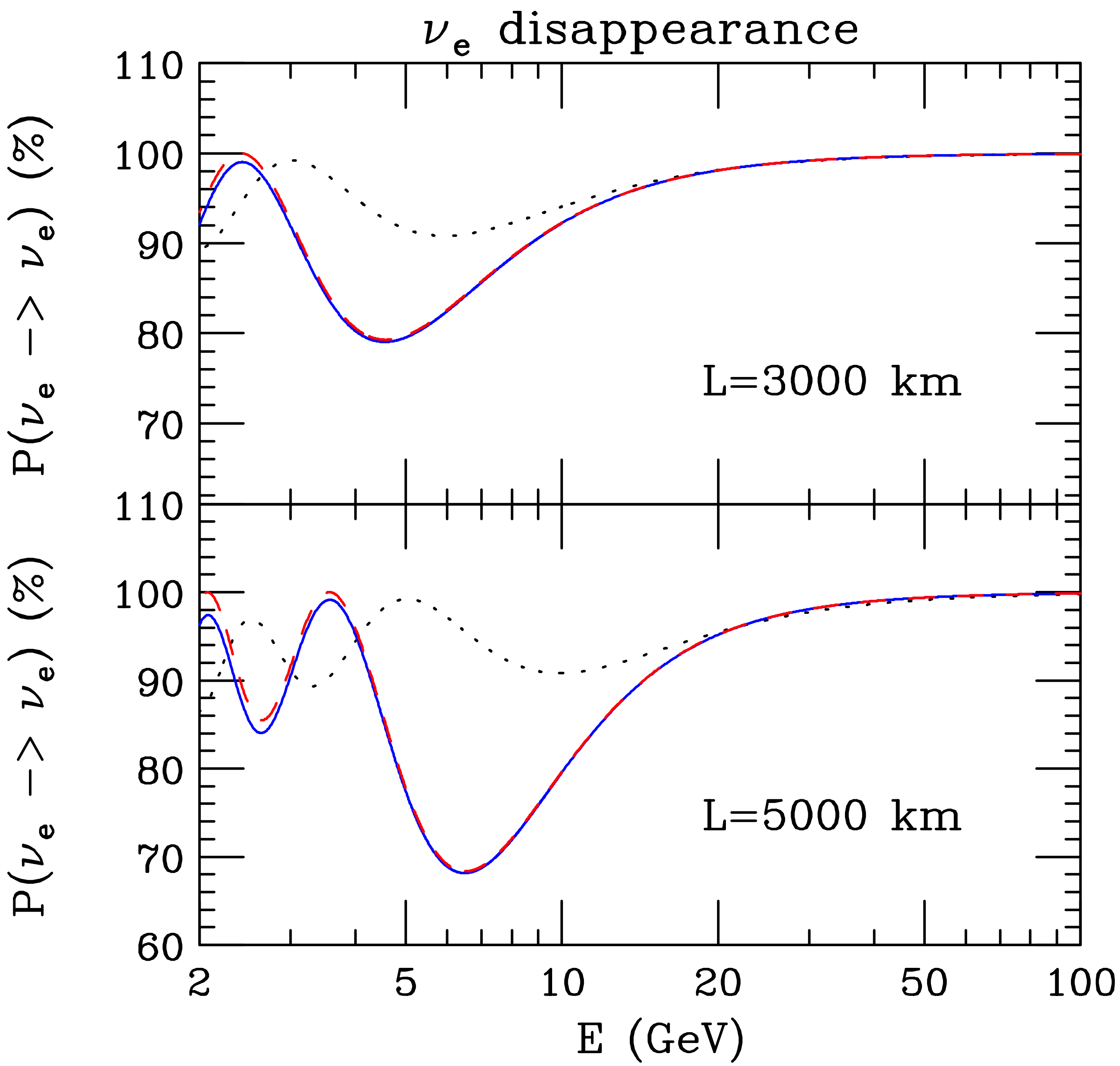
extras

arXiv:1505.01826

ν_e Survival Probability:

$\widetilde{\theta}_{13}$

$$P(\nu_e \to \nu_e) \;=\; 1 - \sin^2 2\widetilde{\theta}_{13}\; \sin^2 \frac{\Delta \widetilde{m^2}_{ee} L}{4E} - \ldots$$



$$\sin 2\widetilde{\theta}_{13} = \frac{\sin^2 2\theta_{13}}{[(\cos 2\theta_{13} - a/\Delta m^2_{ee})^2 + \sin^2 2\theta_{13}]}$$

$$\Delta \widetilde{m^2}_{ee} = \Delta m^2_{ee}\sqrt{(\cos 2\theta_{13} - a/\Delta m^2_{ee})^2 + \sin^2 2\theta_{13}}$$

depth of first minimum

$$\sin 2\theta_{13} \quad \to \quad \sin 2\widetilde{\theta}_{13}$$

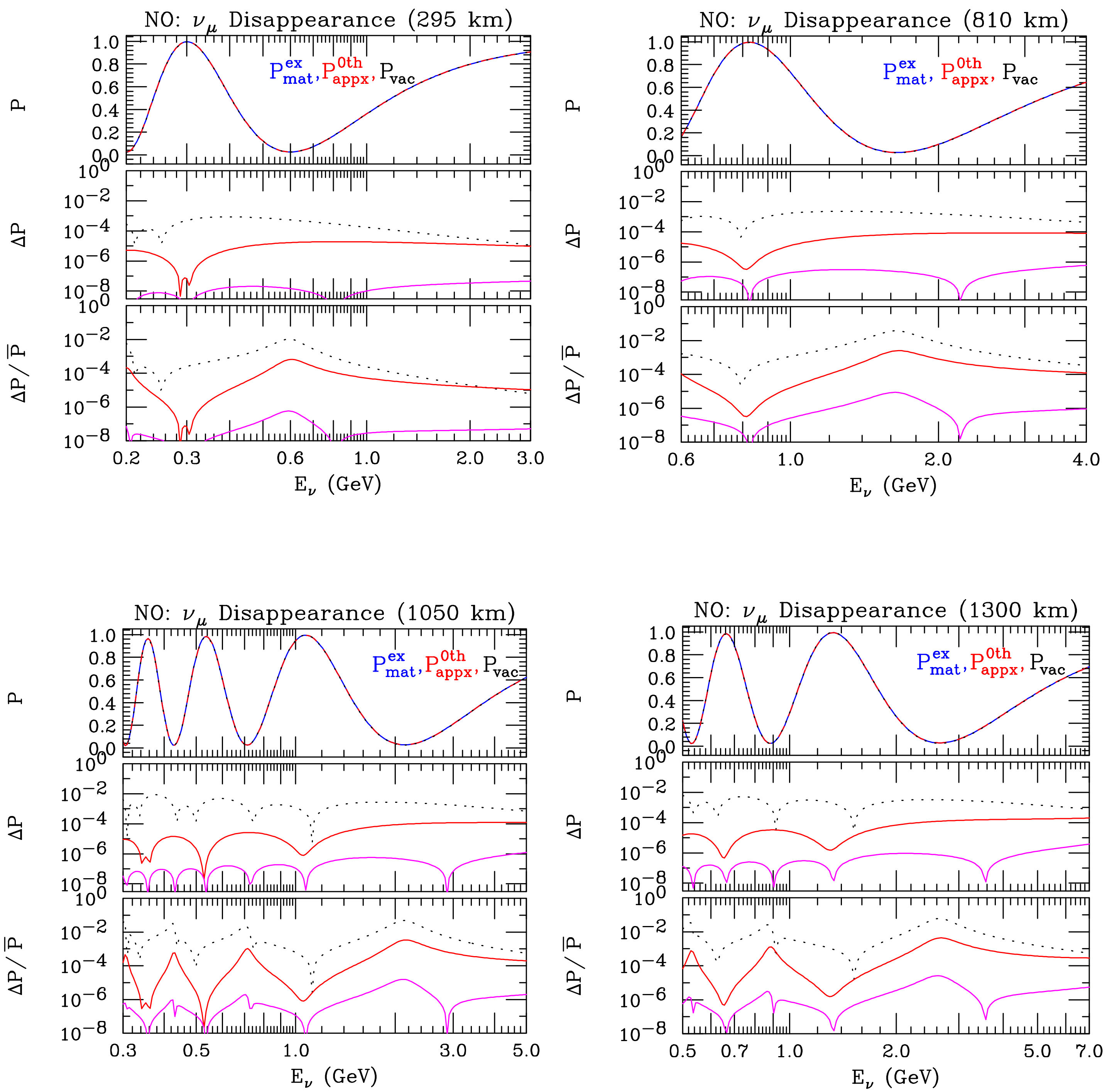
energy at first minimum

$$\frac{\Delta m^2_{ee} L}{2\pi} \quad \to \quad \frac{\Delta \widetilde{m^2}_{ee} L}{2\pi}$$

exact - approx - vaccum

T2K/HK

NOvA



0th order

1st order

2nd order

T2HKK

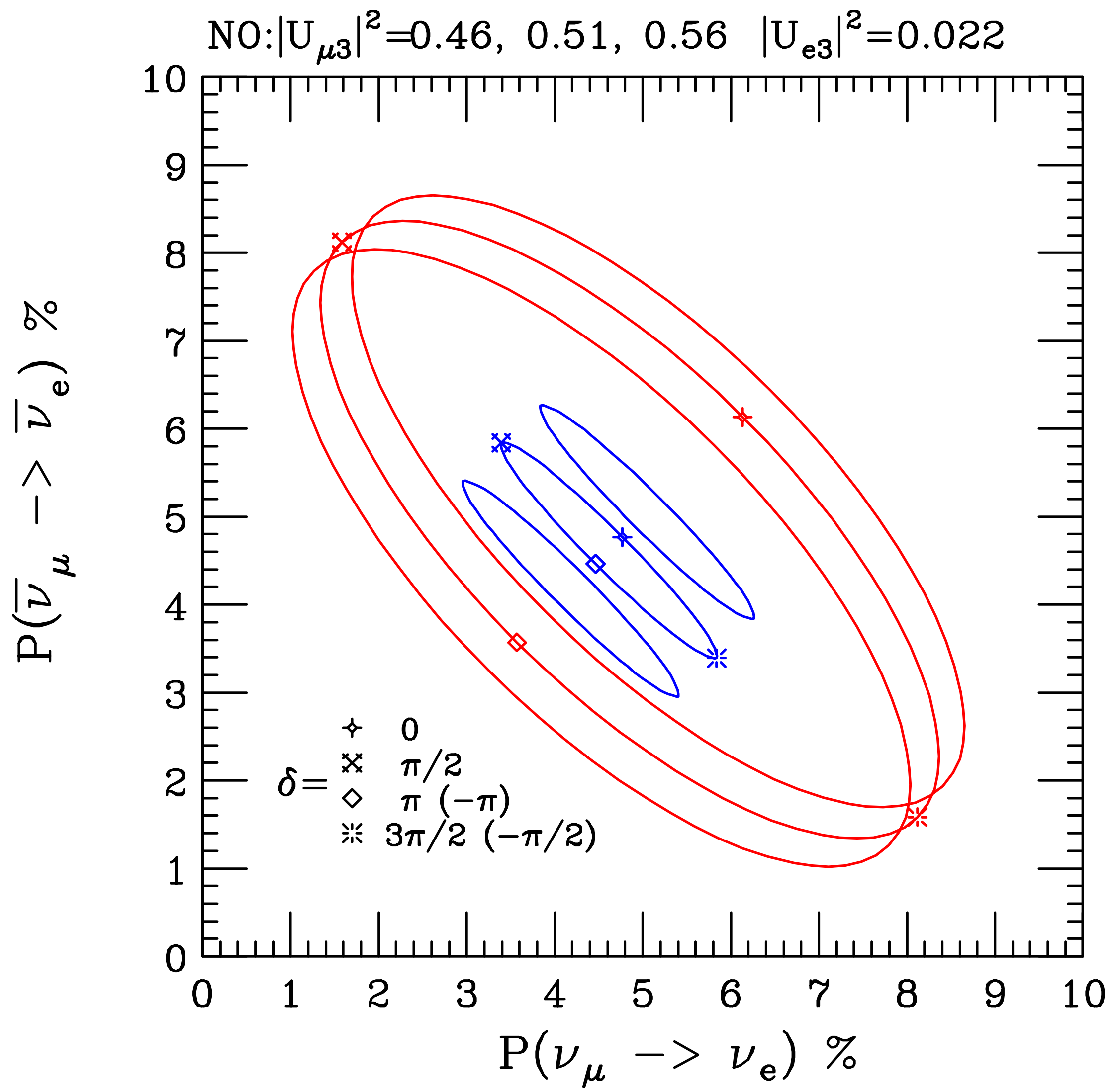
DUNE

Top panel: P^{ex}_{mat}, P^{0th}_{appx} and P_{vac}

Middle and bottom panels:

black dotted lines	red solid lines	magenta solid lines
$\Delta P = \lvert P^{ex}_{mat} - P_{vac}\rvert$	$\Delta P = \lvert P^{ex}_{mat} - P^{0th}_{appx}\rvert$	$\Delta P = \lvert P^{ex}_{mat} - P^{1st}_{appx}\rvert$
$\overline{P} = \frac{1}{2}(P^{ex}_{mat} + P_{vac})$	$\overline{P} = \frac{1}{2}(P^{ex}_{mat} + P^{0th}_{appx})$	$\overline{P} = \frac{1}{2}(P^{ex}_{mat} + P^{1st}_{appx})$

2nd Osc Max: (vacuum)



$$A \equiv \frac{\overline{P} - P}{\overline{P} + P}$$

$$A_1 \approx 0.30 \sin\delta$$

$$A_2 \approx 0.75 \sin\delta$$

$$\frac{A_2}{A_1} \approx 2.5 \qquad \frac{\sqrt{N_2}}{\sqrt{N_1}} \approx \frac{1}{3}$$

Approximately same uncertainty on δ
until systematic uncertainities dominate at 1st OM !

ESSnuSB, T2HKK