Constraints on the Solar $\Delta m^2$ using Daya Bay & RENO

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Currently there is a $\sim 2\sigma$ tension between KamLAND reactor experiment and the combined Super KamiokANDE (SK) & Sudbury Neutrino Observatory (SNO) measurements of the solar mass squared difference, $\Delta m^2_{21}$. This tension needs to be resolved. Furthermore the ratio of $\Delta m^2_{21}$ to $\Delta m^2_{31}$ is needed for the precision measurement of leptonic CP violation. We demonstrate in this paper that $\Delta m^2_{31}$ can be reasonably well constrained by the currently running short baseline reactor anti-neutrino experiments, Daya Bay and RENO. Such measurements will be the only new information on this important quantity until the medium baseline reactor experiment, JUNO, gives a very precise ($< 1\%$) measurement early in the next decade.

I. INTRODUCTION

The fact that neutrinos have mass and mix is now well established by a large number of experiments. In this paper we concentrated on the mass difference squared between the two solar mass eigenstates that have the most electron neutrino, $\nu_1$ and $\nu_2$. The splitting between these two neutrinos, $\Delta m^2_{21} \equiv m_2^2 - m_1^2$, is responsible for the (anti-) neutrino oscillations observed at an L/E = 15 km/MeV and for the neutrino flavor transformations inside the sun, hence the name the solar mass squared difference.

Currently the best measurement of the solar mass squared difference, $\Delta m^2_{21}$, is from the short baseline reactor anti-neutrino experiment, KamLAND, which has determined

$$\Delta m^2_{21} = 7.50 \pm 0.20 \times 10^{-5} \text{ eV}^2, \quad (1)$$

see [1]. The only other measurement of $\Delta m^2_{21}$ comes from a combined measurement using the solar neutrino experiments principle Super KamiokANDE and SNO. This combined measurement is

$$\Delta m^2_{21} = 5.1^{+1.3}_{-1.0} \times 10^{-5} \text{ eV}^2, \quad (2)$$

from SNO [2]. Similar results can be found in SK [3] and Nu-Fit [4]. This solar neutrino determination of $\Delta m^2_{21}$ comes from the non-observation of the low energy up turn of the $^8$B neutrino survival probability by both SNO and SK and the observation of a day-night asymmetry by SK.

CPT invariance implies that the $\Delta m^2_{21}$ measured in reactor anti-neutrinos and solar neutrinos should be identical. However, at the 2$\sigma$ level there is some tension between these two determinations of this important quantity. This tension could arise from a statistical fluctuation, some error in the analysis of one or more of the experiments or new physics.

Moreover, $\Delta m^2_{21}$ is an important parameter for the determination of the CP-violating phase, $\delta$, in the long baseline neutrino$^1$ oscillation experiments (T2K [5], NOvA [6], DUNE [7], T2HK [8], T2HKK [9]) as the size of the CP violation is proportional to $\Delta m^2_{21}$ as well as other parameters. In vacuum, at the first oscillation peak for $\nu_\mu \rightarrow \nu_e$:

$$P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) - P(\nu_\mu \rightarrow \nu_e) \approx J \pi \left( \frac{\Delta m^2_{21}}{\Delta m^2_{31}} \right) \quad (3)$$

where $J = \sin 2\theta_{12} \sin 2\theta_{13} \cos \theta_{13} \sin 2\theta_{23} \sin \delta \approx 0.3 \sin \delta$ is the Jarlskog invariant.

T2K’s data point in the bi-event plane, see Fig 44 of [10],

$$N(\nu_\mu \rightarrow \nu_e) = 37 \quad \text{and} \quad N(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) = 4$$

being outside the allowed region (by about 1 $\sigma$) could be caused by $\Delta m^2_{21}$ being larger than KamLAND value. Again, it is probably a statistical fluctuation but with only one precision measurement of $\Delta m^2_{21}$, the other possibilities are not completely excluded.

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$^1$ In the rest of this paper, when referring to neutrinos, we mean neutrinos and/or anti-neutrinos.

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The future medium baseline reactor experiment JUNO will measure $\Delta m_{21}^2$ and $\sin^2 \theta_{12}$ with better than 1% precision, [11]. However, this experiment is under construction and the precision measurements of the solar neutrino oscillation parameters will not be available until approximately 5 years from now. In more than a decade from now, the DUNE & HyperK proposed experiments will make a precise measurement of $\Delta m_{21}^2$ using solar neutrinos, see [12] and [13] respectively.

Are there any other experiments, that will constrain $\Delta m_{21}^2$ before the JUNO, DUNE, HyperK results are available? The answer is yes, the currently running short baseline (\~1.5 km) reactor anti-neutrino experiments, Daya Bay [14] and RENO [15] both have enough data already collected to constrain $\Delta m_{21}^2$ to be less than 3 times the KamLAND central value. By the end of the running time of these experiments, they will be able to constrain this parameter to less than twice the KamLAND central value. Setting a lower limit maybe possible for the Daya Bay experiment with significant improvements on their systematic uncertainties. Upper, and maybe lower, limits from Daya Bay and RENO, while not competitive with the 3% measurement by KamLAND, will add independent information to our knowledge of $\Delta m_{21}^2$.

In section II, we discuss in detail the effects of changing $\Delta m_{21}^2$ on the oscillation probability. Then in section III we explain and give the results of a stimulation of both Daya Bay and RENO using 3000 live days of data with and without systematic uncertainties followed by a conclusion.

## II. OSCILLATION PROBABILITY

The electron antineutrino disappearance probability, in vacuum, can be written as

\[ P(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 1 - P_{13} - P_{12} \quad \text{with} \]

\[ P_{13} = \sin^2 2\theta_{13} \left( \cos^2 \theta_{12} \sin^2 \Delta_{31} + \sin^2 \theta_{12} \sin^2 \Delta_{32} \right), \]

\[ P_{12} = \sin^2 2\theta_{12} \cos^2 \theta_{13} \sin^2 \Delta_{21}, \]

where $\theta_{12} \approx 33^\circ$ and $\theta_{13} \approx 8^\circ$ are the solar and reactor mixing angles respectively and the kinematic phases are given by $\Delta_{jk} \equiv \Delta m_{jk}^2 L/(4E)$. The $P_{13}$ term is associated with the atmospheric oscillation scale of 0.5 km/MeV, and the $P_{12}$ term is associated with the solar oscillation scale of 15 km/MeV.

Using the best fit values from the pdg and considering a $L/E$ range around the first oscillation minimum ($L/E = 0.5 \text{km/MeV}$), we can approximate $P_{13}$ and $P_{12}$ as follows:

\[ P_{13} \approx 0.08 \sin^2 \left( \frac{\pi}{2} \left( \frac{L/E}{0.5 \text{km/MeV}} \right) \right) \]

\[ P_{12} \approx 0.002 \left( \frac{L/E}{0.5 \text{km/MeV}} \right)^2 \left( \frac{\Delta m_{21}^2}{7.5 \times 10^{-5} \text{eV}^2} \right)^2. \]

For $\Delta m_{21}^2 = 7.5 \times 10^{-5} \text{eV}^2$, the $P_{12}$ term is essentially negligible for all $L/E < 1 \text{km/MeV}$. This encompasses the $L/E$ range of all current short baseline experiments.

However, consider the case that $\Delta m_{21}^2$ is 3 times larger than this value, i.e. $22.5 \times 10^{-5} \text{eV}^2$, then

\[ P_{12} \approx 0.02 \left( \frac{L/E}{0.5 \text{km/MeV}} \right)^2 \left( \frac{\Delta m_{21}^2}{22.5 \times 10^{-5} \text{eV}^2} \right)^2. \]

$P_{12}$ is now no longer negligible compared to $P_{13}$ at oscillation minimum ($L/E = 0.5 \text{km/MeV}$) and $P_{12}$ gets larger for $L/E > 0.5 \text{km/MeV}$ whereas $P_{13}$ is getting smaller. In fact, at $L/E = 1 \text{km/MeV}$, $P_{12}$ would be as large as $\sin^2 2\theta_{13}$ (0.08) for this value of $\Delta m_{21}^2$.

Therefore the short baseline reactor experiments can constrain $\Delta m_{21}^2$ to be less than 2 to 3 times the current best fit value depending on the experiment, Daya Bay or RENO, run time and the confidence level. Setting a lower bound on $\Delta m_{21}^2$ will be challenging for these experiments due to systematic uncertainties. As data above $L/E > 0.5 \text{km/MeV}$ is important for this constrain the experiment Double Chooz, which has no data above $L/E > 0.5 \text{km/MeV}$, is not considered.

Since the position of the first oscillation minimum for $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ is given by

\[ \frac{L}{E} \approx \frac{2\pi}{\Delta m_{ee}^2}, \]

where $\Delta m_{ee}^2 = c^2_{12}\Delta m_{31}^2 + s^2_{12}\Delta m_{32}^2$ (at least for small $\Delta m_{21}^2$), it is natural to write the disappearance probability in terms of $\Delta m_{ee}^2$ and $\Delta m_{21}^2$ as follows, [16] & [17]:

\[ 1 - P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \approx \cos^4 \theta_{13} \sin^2 2\theta_{12} \sin^2 \Delta_{21} \]

\[ + \sin^2 2\theta_{13} \left[ \sin^2 |\Delta_{ee}| + \sin^2 2\theta_{12} \cos^2 \Delta_{21} \sin(2\Delta_{ee}) \right. \]

\[ \left. - \frac{1}{6} \cos 2\theta_{12} \sin^2 2\theta_{13} \Delta_{21}^2 \sin(2\Delta_{ee}) + O(\Delta_{21}^4) \right]. \]

For $\Delta_{21} < 0.5$, only the first two of the terms of RHS of eq. (9) are larger than 0.005 and therefore relevant for the analysis. Since the experiments of interest, Daya Bay and RENO, have an $L/E < 1 \text{ km/MeV}$, the $\Delta_{21} < 0.5$ constraint corresponds to a $\Delta m_{21}^2 < 4 \times 10^{-4} \text{eV}^2$ or 5 times the KamLAND value of $7.5 \times 10^{-5} \text{eV}^2$. Using additional terms of eq. (9) will extend the range of applicability.

For small values of $L/E (< 0.2 \text{ km/MeV})$, where there is large statistics from the near detectors,

\[ 1 - P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \approx 4 \left[ s^2_{13}c^2_{13} + s^2_{13}c^2_{13}c^2_{13} \frac{m_{21}^2}{m_{ee}^2} \right]^2 \times \left( \frac{\Delta m_{ee}^2}{L/E} \right)^2. \]

To keep the disappearance probability the same as we vary $\Delta m_{21}^2$, at these small $L/E$, we must keep the quantity in $[\ldots]$ in the above equation unchanged. If we also keep the position of the first minima fixed by holding $\Delta m_{ee}^2$ fixed (see eq. (8)), then

\[ s^2_{13}c^2_{13} + s^2_{13}c^2_{13} = \text{constant} \approx 0.021 \]

or

\[ s^2_{13} \approx 0.021 - 2 \times 10^{-4} \left( \frac{\Delta m_{21}^2}{7.5 \times 10^{-5} \text{eV}^2} \right)^2. \]
of the setups for Daya Bay and RENO experiments, to estimate the constraints these experiments can place on $\Delta m_{21}^2$.

III. STIMULATIONS FOR DAYA BAY AND RENO USING GLOBES

Our sensitivity study on $\Delta m_{21}^2$ for the short baseline reactor experiments, Daya Bay and RENO, is performed using GLoBES [18]. In this study 3000 live days of data are assumed for both experiments and systematic uncertainties are taken into account as described in [19] for Daya Bay and [20] for RENO. Table I lists the effective baselines, $L_{\text{eff}}$, and the number of observed IBD $\nu_e$ events per day used.

To find the best fit values of $\Delta m_{21}^2$ and $\sin^2(2\theta_{13})$, a $\chi^2$ formalism with pull parameters is constructed using the far-to-near ratio method to cancel out correlated systematic uncertainties. The $\chi^2$ is given by

$$
\chi^2 = \sum_{i=1}^{N_{\text{bins}}} \frac{(O_i^{F/N} - X_i^{F/N})^2}{U_i^{F/N}} + \sum_{r=1}^{6} \left( \frac{f}{\sigma_{\text{flux}}} \right)^2 + \left( \frac{\epsilon}{\sigma_{\text{eff}}} \right)^2 + \left( \frac{s}{\sigma_{\text{scale}}} \right)^2 + \sum_{d=N,F} \left( \frac{b_d}{\sigma_{\text{bkg}}} \right)^2,
$$

where,

- $O_i^{F/N}$ is the observed far-to-near ratio of IBD $\nu_e$ events in the $i$-th $E_\nu$ bin,
- $X_i^{F/N} = X_i^{F/N}(f, \epsilon, s, b^d; \theta_{13}, \Delta m_{21}^2)$ is the expected far-to-near ratio of IBD $\nu_e$ events for a given $\Delta m_{21}^2$ and $\theta_{13}$ pair,
- $U_i^{F/N}$ is the statistical uncertainty of $O_i^{F/N}$,
- $f, \epsilon, s$, and $b^d$ are pull parameters for systematic uncertainties of neutrino flux ($\sigma_{\text{flux}}$), detection efficiency ($\sigma_{\text{eff}}$), energy scale ($\sigma_{\text{scale}}$), and background ($\sigma_{\text{bkg}}$), respectively.

The index $r$ and $d$ represent $r$-th reactor and $d$-th detector, respectively. Both Daya Bay and RENO have six reactors. For Daya Bay, two near detector sets ($N_1$ and $N_2$) are used in the last pull term of the $\chi^2$ due to their differences in the baselines, backgrounds, and systematic uncertainties [19]. As a cross check of our stimulations we have reasonably well reproduced the $\Delta m_{21}^2$ vs. $\sin^2 2\theta_{13}$ sensitivity curves for both experiments.

True values used in the signal simulation are

$$
\sin^2 \theta_{12} = 0.304, \quad \Delta m_{21}^2 = 7.65 \times 10^{-5} \text{ eV}^2, \\
\sin^2(2\theta_{13}) = 0.085, \quad \Delta m_{21}^2 = 2.50 \times 10^{-3} \text{ eV}^2.
$$

To minimize the $\chi^2$, expected values for different pairs of $\Delta m_{21}^2$ and $\sin^2(2\theta_{13})$ are compared to the simulated signal $\nu_e$ data from 1.8 to 8 MeV with 31 energy bins.
TABLE I. $L_{\text{eff}}$ and observed IBD $\nu_e$ rates for Daya Bay and RENO derived from the GLoBES settings used in this study.

<table>
<thead>
<tr>
<th>$L_{\text{eff}}$ (m)</th>
<th>Daya Bay</th>
<th>RENO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Near</td>
<td>(400.4, 512.6)</td>
<td>367.0</td>
</tr>
<tr>
<td>Far</td>
<td>1610</td>
<td>1440</td>
</tr>
<tr>
<td>IBD $\nu_e$ rate</td>
<td>Near</td>
<td>(1320, 1195)</td>
</tr>
<tr>
<td></td>
<td>Far</td>
<td>297.8</td>
</tr>
</tbody>
</table>

FIG. 2. (Color online) Contour plot of $\Delta m^2_{21}$ vs. $\sin^2 2\theta_{13}$ for the RENO experiment without (left) and with (right) systematic uncertainties. 3000 live days of data with 61 IBD $\nu_e$ events/day for the far detector were used. Magenta, blue, and green lines represent $1\sigma$, $2\sigma$, and $3\sigma$ (2 dof) allowed regions, respectively. The “+” point is the input for the stimulation.

FIG. 3. (Color online) Contour plot of $\Delta m^2_{21}$ vs. $\sin^2 2\theta_{13}$ for the Daya Bay experiment without (left) and with (right) systematic uncertainties. 3000 live days of data with 298 IBD $\nu_e$ events/day for the far detectors were used. Magenta, blue, and green lines represent $1\sigma$, $2\sigma$, and $3\sigma$ (2 dof) allowed regions, respectively. The “+” point is the input for the stimulation.
Figures 2 & 3 show the results of our stimulation for contour plots of $\Delta m_{21}^2$ vs. $\sin^2(2\theta_{13})$ sensitivities using 3000 live days of data for RENO and Daya Bay, respectively, without (left) and with (right) systematic uncertainties. Adding systematic uncertainties effects RENO less than Daya Bay, because after 3,000 days of data taking, Daya Bay has $\approx 5$ times more events in the far detector(s) than RENO, see Table I. Clearly, both of these experiments can constrain $\Delta m_{21}^2$ to be less than two to three times the KamLAND central value, i.e. $\Delta m_{21}^2 < 15 - 22 \times 10^{-5}$ eV$^2$. Setting a lower limit on $\Delta m_{21}^2$ maybe possible with Daya Bay, if an improvement in their systematic uncertainties can be achieved over those used for this stimulation. We encourage both Daya Bay and RENO to perform a measurement of $\Delta m_{21}^2$ using their more precise information on their experiments.

IV. CONCLUSION

We have argued that Daya Bay and RENO can add to the information of the solar mass squared difference, $\Delta m_{21}^2$, now. A simulation study for these experiments was performed with and without systematic uncertainties using GLoBES. We have found that $\Delta m_{21}^2$ can be reasonably well constrained by Daya Bay 3000 live days of data to be less than twice the KamLAND central value.

Without systematic uncertainties Daya Bay can exclude $\Delta m_{21}^2 = 0$ with 1σ confidence level but when current systematic uncertainties are included only an upper bound can be set. Until JUNO measures $\Delta m_{21}^2$ with great precision early next decade, we expect the $\Delta m_{21}^2$ measurement by Daya Bay can play an important role for the leptonic CP violation measurement by T2K and NOvA. A truly realistic stimulation and a true measurement of $\Delta m_{21}^2$ can only be performed by the short baseline reactor experiments, Daya Bay and RENO.

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