Beam Breakup Mitigation by Ion Mobility in Plasma Acceleration

A. Burov, S. Nagaitsev and V. Lebedev
Fermilab, PO Box 500, Batavia, IL 60510-5011
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Moderate ion mobility provides a source of BNS damping in the plasma wakefield acceleration, which may serve as an effective remedy against the transverse instability of the trailing bunch. The ion-related BNS parameter \( \kappa \), proportional to the beam brightness, is introduced as a single parameter of the partial integro-differential equation of the bunch collective motion. This equation is further reduced to an ordinary differential equation, which solutions are shown versus its single time-space argument and the BNS parameter. It is demonstrated that conditions of the instability suppression and emittance preservation at energy efficient plasma acceleration leave for the ion BNS parameter \( \kappa \) about an order of magnitude of its possible variation along the acceleration line.

Plasma wakefield acceleration (PWA) suggests extremely high acceleration fields, so it is no surprise that this area of research attracts interest of groups working on future colliders, giving rise to many publications, targeted at resolution of multiple interrelated problems in this challenging area. A special subset of these problems is associated with stability of both driving (accelerating) and trailing (accelerated) bunches. The latter problem appears to be harder than the former, since mismatches at every change of the driving bunch between positions of the two bunches produce initial kicks for the instability development along the acceleration line for one and the same trailing bunch. From a very general point of view, the PWA trailing bunch instability belongs to the family of similar effects in linacs. Due to interactions with the surroundings, dipole perturbations at the head of the bunch leave electromagnetic wake fields behind, thus acting on the bunch tail. The kick felt by the test particle at the unit trajectory length by a unit dipole moment of the leading particle is known as the wake function, \( W_\perp(\xi) \), where \( \xi \) is the separation between the particles, see e.g. Ref. [1]. As a result, the tail dipole oscillations may grow more and more, leading to the emittance degradation. This sort of unbounded convective instability [2] is known as the beam breakup in linacs [1]. Here we are considering the acceleration of a short electron bunch in the blowout regime, a regime in which the fields of the driver (laser or an electron bunch) are so intense that they expel all plasma electrons, creating a cavity filled with pure ion plasma [3]. The longitudinal and transverse electric fields inside this cavity are used to accelerate and focus the trailing electron bunch. The excited transverse wake fields are very sensitive to the aperture radius, which is the plasma bubble radius at the bunch location, \( r_b \), for the PWA case: for the short bunches of the interest, the wake function is inversely proportional to the fourth power of this radius, \( W_\perp(\xi) \approx 8\pi P(\xi)/r_b^4 \), where \( P(\xi) \) is the Heaviside theta-function. Many details on that can be found in the recent Refs. [1][2]. To get the desired high acceleration, the plasma bubble has to be small, typically \( r_1 \approx 50 - 100 \text{nm} \), compared with \( 1 - 2 \text{cm} \) for conventional colliders; thus, with the fourth power of the aperture in the transverse wake, the transverse instability is by necessity one of the main obstacles for the PWA colliders. From this, one may correctly conclude that there must be a relation between energy efficiency and beam stability for PWA: while the former requires smaller bubbles, the latter is lost with them. Such efficiency-instability relation has been recently formulated and proved in Ref. [6]; here we reproduce this statement for the reader’s convenience.

Let \( \eta_p < 1 \) be the PWA energy efficiency, i.e. the ratio of the power of the trailing bunch acceleration to the power of the driving bunch deceleration. Further, let the wake parameter \( \eta_w \), the ratio of the bunch-averaged defocusing by the transverse wake fields to the main focusing of the bunch electrons by the ions inside the bubble. The efficiency-instability relation of Ref. [3] states that

\[
\eta_w \approx \frac{\eta_p}{4(1 - \eta_p)}.
\]

Thus, an increase in the efficiency indeed brings the bunch closer to the instability threshold. To push back this limitation for a given wake parameter \( \eta_w \), BNS damping [7] can be used. The main idea of this method is based on a compensation of the wake deflecting force in every position of the bunch by additional focusing. If the two terms cancel each other for the bunch deflection as a whole, the instability would be effectively suppressed. Let us assume that the relative focusing strength \( \delta \omega / \omega \) varies along the bunch by one or another reason; then, neglecting, for simplicity sake, the variation of the bunch line density, the equation of motion for the bunch normalized local offsets \( X(\xi, \mu) \) can be presented as

\[
\frac{\partial^2 X}{\partial \mu^2} + \left( 1 + 2 \frac{\delta \omega}{\omega} \right) \frac{\partial X}{\partial \xi} = 2 \eta_w \frac{\partial}{t^2} \int_0^\xi X(\xi')(\xi - \xi')d\xi'.
\]

Here \( L_t \) is the full bunch length, \( \mu \) is the normalized time, \( d\mu = k_p d\gamma / \sqrt{2\gamma} \) with \( d\gamma \) as a differential length along the bunch motion, \( \gamma \) as the relativistic factor, \( k_p = \sqrt{4\pi n_0 e^2} \) as the relativistic Debye length; \( n_0 \) is the plasma density and \( r_e = e^2/(mc^2) \) is the electron classical radius. It follows from Eq. [2] that the bunch deflection as a whole would evolve as stable oscillations if the BNS compensation condition is fulfilled:

\[
2 \frac{\delta \omega}{\omega} = \eta \frac{\xi^2}{L_t^2},
\]

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where the longitudinal position $\xi$ is measured from the bunch head. In conventional linear accelerators, this condition can be fulfilled by momentum modulation $\delta p/p$ along the bunch, using that $2\delta \omega_\perp / \omega_\perp = \delta p/p$. For PWA though, there is an additional mechanism of the focusing variation, associated with the ion mobility in the Coulomb field of the trailing bunch. Indeed, plasma ions move in this field, causing variation of the ion 3D density $\delta n_i/n_i$. Assuming this variation to be small, it is estimated as

$$\frac{\delta n_i}{n_i} = 2\pi n_i r_i \xi^2,$$  \hfill (4)

where $r_i$ is the ion classical radius and $n_i = N_i/(\pi L_i b^2)$ is 3D density of the trailing bunch. Here we assume a simple model of a homogeneous 3D density of the bunch particles, with the full transverse radius $b$ and the full length $L_i$.

The ion density perturbations entail similar variations of the electron focusing:

$$2\frac{\delta \omega_\perp}{\omega_\perp} = \frac{\delta n_i}{n_i} = 2\pi n_i r_i \xi^2.$$  \hfill (5)

Note that both the sign and position dependence of this focusing variation are the same as BNS compensation requires. Thus, the latter can be presented as position-independent:

$$\kappa \equiv 2N_i r_i L_i / b^2 \eta_t = \frac{\mu^2}{\eta_t} = 1,$$  \hfill (6)

where $\mu_i = \sqrt{2N_i r_i L_i / b^2} \ll 1$ is the phase advance of ion's oscillations in the field of the trailing bunch.

The equation of motion (2) can be further simplified with the slow amplitudes $x = X \exp(i\mu t)$, measuring the positions inside the bunch as fractions of its full length, $\zeta = x/L_i$ and using slow time $\tau = \mu \eta_t$. Then, it is transformed to the following equation on the slow amplitudes $x(\zeta, \tau)$, with the constant initial condition,

$$\frac{\partial x}{\partial \tau} = -i\kappa \zeta^2 x + 2i \int_0^\zeta x(\zeta - \zeta') \zeta' d\zeta'; \quad x(\zeta, 0) = 1; \quad 0 \leq \zeta \leq 1.$$  \hfill (7)

It is straightforward to show that solution of this equation $x(\zeta, \tau)$ with the specified initial condition has a scaling invariance: it depends on its space and time arguments $\zeta$ and $\tau$ as $x(\zeta, \tau) = x(1, \zeta^2 \tau) \equiv f(\zeta^2 \tau)$. In other words, the complex amplitude of the oscillations at position $\zeta$ and time $\tau$ is the same as at the bunch tail and earlier time $\zeta^2 \tau$. This means that the partial integro-differential equation (7) with the specified initial condition is equivalent to an ordinary one. Apparently, this ordinary integro-differential equation has the simplest form with the space-time argument $u = \zeta \sqrt{2} \tau$. With $x(\zeta, \tau) = g(u)$, Eq. (7) reduces then to the following form

$$\frac{dg}{du} = -i \kappa g + 2i \int_0^u g(u - u') u' du'; \quad g(0) = 1.$$  \hfill (8)

At $\kappa = 0$, i.e. without any damping, the solution at large argument, $u \gg 1$, asymptotically tends to $g(u) \simeq \exp(3i^{1/3}(u/2)^{2/3})$, omitting the pre-exponential factor.

Since the problem is reduced now to finding a function of just two parameters, $g_0(u)$, from a linear ordinary integro-differential equation (8), it can be easily solved numerically for all interesting cases; Fig. 1 presents the amplitude modulus $x(1, \tau)$ as a 3D plot for $0 \leq \kappa \leq 1.2$ and $0 \leq \tau \leq 100$. Patterns of oscillations $x(\zeta)$ along the bunch for a case of slight BNS overshooting, $\kappa = 1.2$, and more overshooting, $\kappa = 2.0$, for $\tau = 100$ are shown in Figs. 2 and 3. A couple of things are worth noting in relation to Figs. 1-3. First, the instability is dramatically weakened even with a moderate BNS parameter $\kappa \simeq 0.2 - 0.5$. Second, for overshooting $\kappa > 1$, the bunch is stable: its initial perturbation decoheres, certainly contributing to the emittance growth.

During acceleration, the ion-driven BNS parameter in-
creases, $\kappa \propto \sqrt{\zeta}$, so it cannot be the same at the beginning and the end of the acceleration. Assuming the initial and final energies differ by the factor of 100, it means that the ion-BNS parameter $\kappa = \mu_i^2/\eta_i$ increases by a factor of 10. With acceleration efficiency $\eta_p = 50\%$, the wake parameter is given by Eq. (1), resulting in $\eta_i = 0.1$. Thus, full BNS damping would occur if the ion’s phase advance is not too small, $\mu_i^2 \geq 0.1$. On the other hand, if this phase advance is not at all small, $\mu_i^2 \geq 1$, the ions collapse inside the bunch, and that leads to a dramatic emittance growth. Thus, for the specified parameters, there is an order of magnitude of possible variation of the ion-BNS parameter $\kappa$ compatible with both stabilization and emittance preservation. This, in turn, supports the possibility for the acceleration by 100 times, say, from 10 GeV to 1 TeV. Keeping in mind the significant help of an incomplete BNS compensation, demonstrated by Fig. [1] as well as approximations of this model, one may hope that this acceleration range may be significantly larger.

Transverse stabilization with the ion mobility taken into account was observed in simulations of Ref. [8] for $N_i = 4 \cdot 10^9$ cm$^{-3}$, rms bunch length $\sigma_z = 6.4 \mu$m, rms transverse size $\sigma_\perp = 0.5 \mu$m, and the energy efficiency was $\eta_p = 0.5$. According to the efficiency-instability relation (1), it corresponds to the wake parameter $\eta_i = 0.13$. For these parameters, the proton phase advance is computed as $\mu_i = 0.56$ rad, and the ion BNS parameter $\kappa = 2.6$, assuming $L_i \approx 4\sigma_z$ and $b \approx 2\sigma_\perp$. Thus, the observed stabilization is in agreement with the suggested ion-driven BNS theory.

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